Soft Heisenberg Hair

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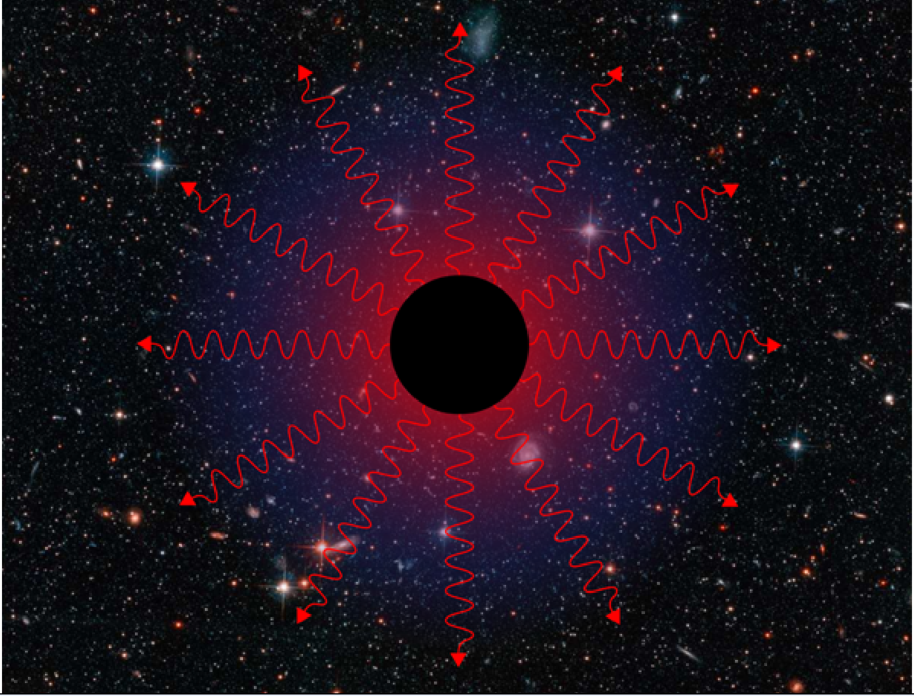
FWF

1603.04824, 1607.00009, 1607.05360, 1611.09783



Two simple punchlines

1. Heisenberg algebra

$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

fundamental not only in quantum mechanics
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but also in near horizon physics of (higher spin) gravity theories

2. Black hole microstates identified as specific "soft hair" descendants

based on work with

- Hamid Afshar [IPM Teheran]
- Stephane Detournay [ULB]
- Wout Merbis [TU Wien]
- Blagoje Oblak [ULB / ETH]
- Alfredo Perez [CECS Valdivia]
- Stefan Prohazka [TU Wien]
- Shahin Sheikh-Jabbari [IPM Teheran]
- David Tempo [CECS Valdivia]
- Ricardo Troncoso [CECS Valdivia]

Outline

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Motivation

Near horizon boundary conditions

Explicit construction of BTZ microstates

Discussion

Black hole microstates

Bekenstein–Hawking

$$S_{\text{BH}} = \frac{A}{4G_N}$$

- Motivation: microscopic understanding of generic black hole entropy

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- Microstate counting from CFT_2 symmetries (Strominger, Carlip, ...) using Cardy formula
- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

warped CFT: Detournay, Hartman, Hofman '12
Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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- Main idea: consider near horizon symmetries for non-extremal horizons

  Related ideas pursued e.g. by
  - Donnay, Giribet, Gonzalez, Pino '15
  - Hawking, Perry, Strominger '16

  Postpone comparison with related approaches after discussing our approach

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- Main idea: consider near horizon symmetries for non-extremal horizons
- Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Meaning of coordinates:

- ρ: radial direction ($\rho = 0$ is horizon)
- $\varphi \sim \varphi + 2\pi$: angular direction
- v: (advanced) time

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Recall scale invariance

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of Rindler metric

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  suggestion in `1511.08687`

  We make this choice in this talk!

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- Work in 3d Einstein gravity in Chern–Simons formulation

$$I_{\text{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^\pm \wedge \mathrm{d}A^\pm + \tfrac{2}{3} A^\pm \wedge A^\pm \wedge A^\pm \rangle$$

with $sl(2)$ connections $A^\pm$ and $k = \ell/(4G_N)$ with AdS radius $\ell = 1$

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Review of general aspects of boundary conditions (bc's)

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Example:

$$\Phi(x \to \infty) = 0$$

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Example: Brown-Henneaux type of bc's (aAdS_3):

$$\mathrm{d}s^2_{\mathrm{aAdS}} = \mathrm{d}\rho^2 + \big(e^{2\rho}\eta_{\mu\nu} + \gamma_{\mu\nu} + \mathcal{O}(e^{-2\rho})\big)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

with $\delta\gamma =$ arbitrary

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- Local diffeos and gauge trafos fall into three classes:
  1. Trafos that violate bc's (forbidden)
  2. Trafos that preserve bc's and remain pure gauge (trivial)
  3. Trafos that preserve bc's but are not pure gauge at the asymptotic boundary (asymptotic symmetries)

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- Consistency means they are finite, integrable, non-trivial and conserved (in time)

AdS_3 bc's

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- Compere–Song–Strominger (2013): Virasoro plus $u(1)$ current algebra
- Troessaert (2013): 2 Virasoros plus 2 $u(1)$ current algebras
- Avery–Poojary–Suryanarayana (2013): Virasoro plus $sl(2)$ current algebra
- Donnay–Giribet–Gonzalez–Pino (2015): centerless warped conformal
- Afshar–Detournay–DG–Oblak (2015): twisted warped conformal

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Our near horizon bc's simpler than any of the above!

Explicit specification of our bc's in diagonal gauge

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho)\left(\mathrm{d}+\mathfrak{a}_{\pm}(x^0,\, x^1)\right)b_{\pm}(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate ρ and boundary coordinates $(x^0, x^1) \sim (v, \varphi)$

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- Standard AdS_3 approach: highest weight gauge

$$\mathfrak{a} \sim L_+ + \mathcal{L}(x^0,\, x^1) L_- \qquad b(\rho) = \exp\left(\rho L_0\right)$$

$sl(2)$: $[L_n,\, L_m] = (n-m) L_{n+m}, \quad n, m = -1, 0, 1$

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  $sl(2)$: $[L_n,\,L_m] = (n-m)L_{n+m}, \quad n,m = -1,0,1$
- For near horizon purposes diagonal gauge useful:

  $$\mathfrak{a} \sim \mathcal{J}(x^0,\,x^1)\,L_0$$

- Precise boundary conditions (ζ: chemical potential):

  $$\mathfrak{a} = (\mathcal{J}\;\mathrm{d}\varphi + \zeta\;\mathrm{d}v)\;L_0 \qquad \delta\mathfrak{a} = \delta\mathcal{J}\;\mathrm{d}\varphi\,L_0$$

  and $b = \exp\left(\frac{1}{\zeta}\,L_+\right)\cdot\exp\left(\frac{\rho}{2}\,L_-\right)$. (assume constant ζ for simplicity)

Near horizon metric

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$$g_{\mu\nu} = \frac{1}{2} \left\langle \left(A_\mu^+ - A_\mu^- \right) \left(A_\nu^+ - A_\nu^- \right) \right\rangle$$

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yields ($f := 1 + \rho/(2a)$)

$$\mathrm{d}s^2 = -2a\rho f \,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho \\ + 4\omega\rho f\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \frac{2\rho}{a} f(\gamma^2 - \omega^2)\right] \mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$

For simplicity set $\Omega = 0$ and $a = \mathrm{const.}$ in metric above

EOM imply $\partial_v \mathcal{J}^\pm = \pm\partial_\varphi \zeta^\pm$; in this case $\partial_v \mathcal{J}^\pm = 0$

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Comments:

- Recover desired near horizon metric

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- Two state-dependent functions (γ, ω) as usual in 3d gravity

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- $\gamma = \gamma(\varphi)$: "black flower"

Canonical boundary charges

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- Zero mode charges: mass and angular momentum

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- Zero mode charges: mass and angular momentum

Background independent result for Chern–Simons yields

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Meaningful near horizon boundary conditions and non-trivial theory!

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos

Most general trafo

$$\delta_\epsilon \mathfrak{a} = \mathrm{d}\epsilon + [\mathfrak{a},\, \epsilon] = \mathcal{O}(\delta\mathfrak{a})$$

that preserves our boundary conditions for constant ζ given by

$$\epsilon = \epsilon^+ L_+ + \eta L_0 + \epsilon^- L_-$$

with

$$\partial_v \eta = 0$$

implying

$$\delta_\epsilon \mathcal{J} = \partial_\varphi \eta$$

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- Expand charges in Fourier modes

$$J_n^{\pm} = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^{\pm}(\varphi)$$

What should we expect?

- Virasoro? (spacetime is locally AdS_3)
- BMS_3? (Rindler boundary similar to scri)
- warped conformal algebra? (this is what we found for Rindleresque holography and what Donnay, Giribet, Gonzalez, Pino found in their near horizon analysis)

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- Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \tfrac{i}{kn}\,(J_{-n}^+ + J_{-n}^-) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

  yields Heisenberg algebra (with Casimirs X_0, P_0)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$
$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Brief list of generalizations

Heisenberg algebras as near horizon symmetries arise not only in AdS_3 Einstein gravity, but also in ...

- ... flat space Einstein gravity in three dimensions
  Afshar, DG, Merbis, Perez, Tempo, Troncoso '16
- ... higher spin gravity in three dimensions
  DG, Perez, Prohazka, Tempo, Troncoso '16
- ... higher derivative gravity in three dimensions
  Setare, Adami '16
- ... general relativity (in four dimensions)
  Afshar, DG, Sheikh-Jabbari '16

Conclusions about near horizon symmetry algebra fairly general!

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- Construct near horizon Virasoro through standard Sugawara construction

$$\mathcal{L}_n^\pm \equiv \sum_{p\in\mathbb{Z}} :\mathcal{J}_{n-p}^\pm\, \mathcal{J}_p^\pm:$$

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Near horizon Hilbert space

- Denote "near horizon" generators with calligraphic letters
- Near horizon algebra (conveniently rescaled)

$$[\mathcal{J}_n^\pm,\, \mathcal{J}_m^\pm] = \tfrac{1}{2}\, n\, \delta_{n,-m}$$

- Near horizon Hilbert space: define vacuum by highest weight conditions

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- Call this "near horizon symmetry algebra" (note: independent from ℓ)

Soft hair

- Generic descendant of vacuum:

$$|\Psi(\{n_i^\pm\})\rangle = \prod_{\{n_i^\pm>0\}} \big(\mathcal{J}^+_{-n_i^+}\mathcal{J}^-_{-n_i^-}\big)|0\rangle$$

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- Will exploit this property to provide cut-off on soft hair spectrum!

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- Microstates = all states in near horizon Hilbert space obeying equations above

Horizon fluffs as microstates

We are now ready to identify all BTZ microstates

- Vector space $\mathcal{V}_{\mathcal{B}}$ of BTZ microstates defined by

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- Useful observation:

$$\Delta^\pm = \langle\mathcal{B}|L_0^\pm|\mathcal{B}\rangle \approx \frac{1}{c}\langle\mathcal{B}|\mathcal{L}_0^\pm|\mathcal{B}\rangle = \frac{1}{c}\sum_i n_i^\pm = \frac{1}{c}\,\mathcal{E}_{\mathcal{B}}^\pm$$

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- Agrees with Bekenstein–Hawking and Cardy formula

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Generalization to four dimensions

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Compare with near horizon construction of Donnay, Giribet, Gonzalez, Pino '15

- Near horizon algebra similar to but different from BT-BMS$_4$:

$$[\mathcal{Y}_n^\pm,\, \mathcal{Y}_m^\pm] = (n-m)\, \mathcal{Y}_{n+m}^\pm$$
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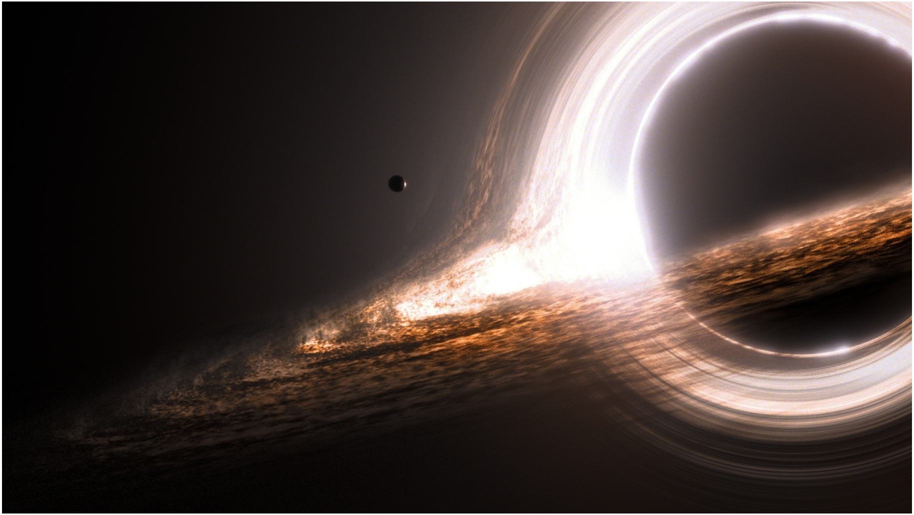
- Proves existence of soft Heisenberg hair in 4d

Microstates of non-extremal Kerr?



Main challenge: how to provide (controlled) cut-off on soft hair spectrum in four dimensions?

Thanks for your attention!

- H. Afshar, D. Grumiller and M.M. Sheikh-Jabbari "Near Horizon Soft Hairs as Microstates of Three Dimensional Black Holes," 1607.00009.
- H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso "Soft Heisenberg hair on black holes in three dimensions," Phys.Rev. **D93** (2016) 101503(R); 1603.04824.

Thanks to Bob McNees for providing the LaTeX beamerclass!

Map to asymptotic variables

- Usual asymptotic AdS_3 connection with chemical potential μ:

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- Get Virasoro with non-zero central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

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Near horizon boundary conditions natural for near horizon observer

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Punchline: our proposal is Bohr-type quantization of spectrum

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- Spectral flow and discrete conic spaces generated by $\mathcal{J}_r^\pm$ ($r = 1, 2, \ldots c-1$), the "horizon fluffs"

On log corrections

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$$S = S_0 + \# \cdot \ln S_0 + \mathcal{O}(1)$$

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- Mismatch in coefficients; not sure yet if bug or feature