AN ABSTRACT THEORY OF PHYSICAL MEASUREMENTS

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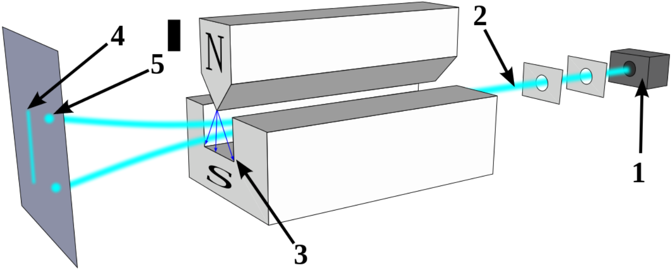
TQFT Seminar, IST, on March 19, 2021

Measurement z

One run yields *one bit* of *classical information*:

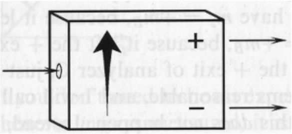
$$\begin{aligned} 0 &= \text{down} \\ 1 &= \text{up} \end{aligned}$$



Stern–Gerlach analyzer

Information-induced order

z



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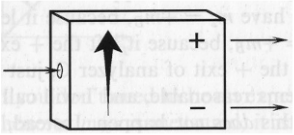
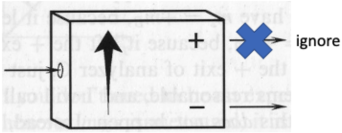


z^- (can only yield 0)



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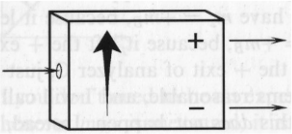

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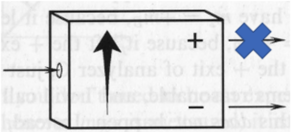
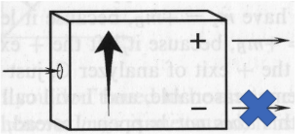


ignore

z^+ (can only yield 1)



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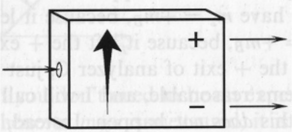
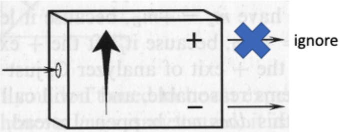


z^- (can only yield 0)



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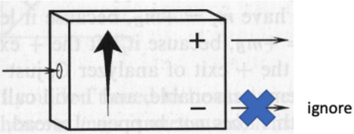


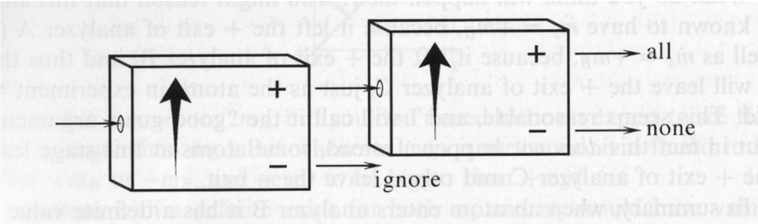
Disjunction

$$z = z^- \vee z^+$$

Temporal/causal order

Can have *sequentially composed* measurements:

zz^{+}



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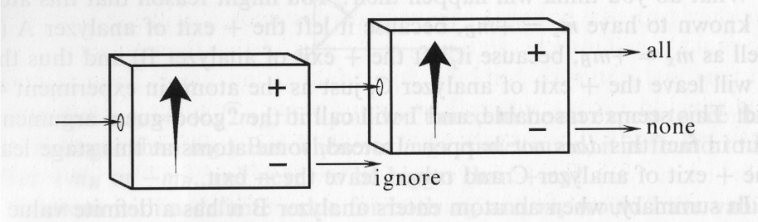

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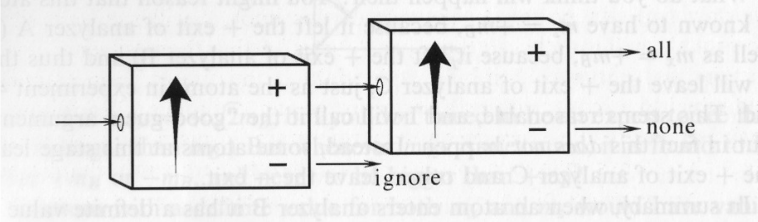
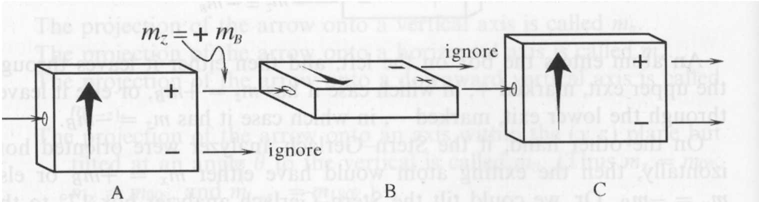


$zx^{+}z^{+}$



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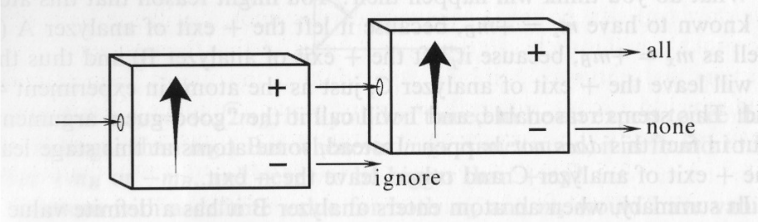

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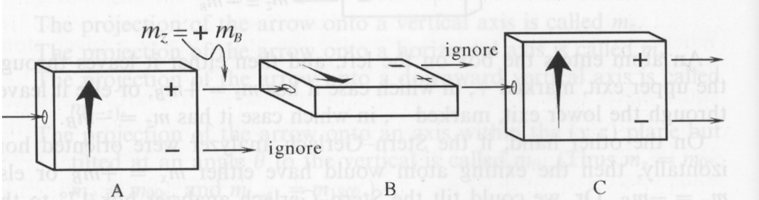

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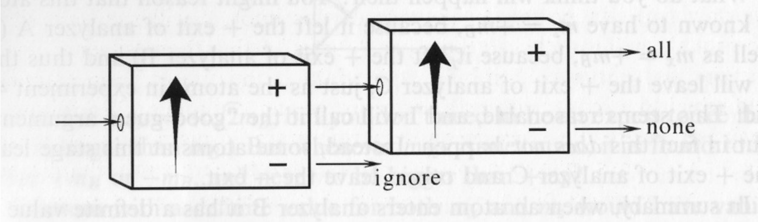
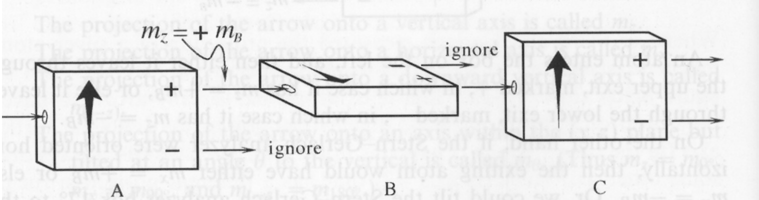


$$zx^{+}z^{+} \quad = \quad z^{-}x^{+}z^{+} \vee z^{+}x^{+}z^{+} \quad \neq \quad z$$



- Topological and algebraic structure of *spaces* of measurements.

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- *Multiple runs* of z also yield *statistical information*:

$$0 \text{ occurs in } M \text{ runs}$$

$$1 \text{ occurs in } N \text{ runs}$$

Will not address this in this talk (no measure-theoretic structure).

Outline

1. The measurement problem

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2. Rationale

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4. Classical measurement spaces

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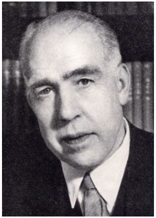
4. Classical measurement spaces
5. Quantizations and observers

Part 1 — The measurement problem

"... the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation."

– *Niels Bohr, The quantum postulate and the recent development of quantum theory, Supplement to "Nature," April 14, 1928.*



"...the description of the experimental arrangement and the recording of observations must be given in plain language, suitably refined by the usual physical terminology. This is a simple logical demand, since by the word "experiment" we can only mean a procedure regarding which we are able to communicate to others what we have done and what we have learnt."

– *Niels Bohr (1958), Quantum physics and philosophy—causality and complementarity (pp. 1–7) Woodbridge: Ox Bow Press (Reprinted in The Philosophical writings of Niels Bohr, Essays 1958–1962 on atomic physics and human knowledge originally, Wiley 1963).*

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- Hilbert space $H_S = \mathbb{C}^2$
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Time evolution in $H_S \otimes H_A$

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$$\Big\downarrow e^{-i\hat{H}\Delta t}$$

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"But in any case, no matter how far we calculate — to the mercury vessel, to the scale of the thermometer, to the retina, or into the brain, at some time we must say: and this is perceived by the observer. That is, we must always divide the world into two parts, the one being the observed system, the other the observer.

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"What exactly qualifies some physical systems to play the role of 'measurer'? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system... with a PhD?"

– *John S. Bell, Against 'measurement', Phys. World 3 (1990).*

Interpretations and variants

- *Realist*: decoherence, many-worlds, stochastic collapse, gravity-induced collapse, de Broglie–Bohm mechanics, contextual topos-based models...
- *Epistemic/subjective*: "Copenhagen" (partially), QBism...
- "New interpretations appear every year. None ever disappear."
  – David Mermin

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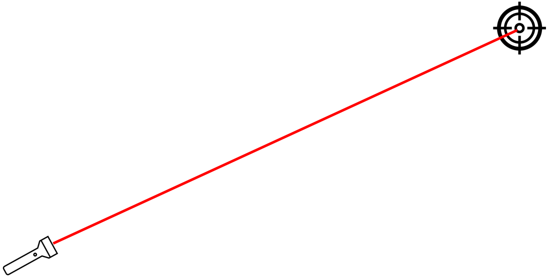
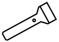
Idea: define *spaces* of *abstract* measurements.

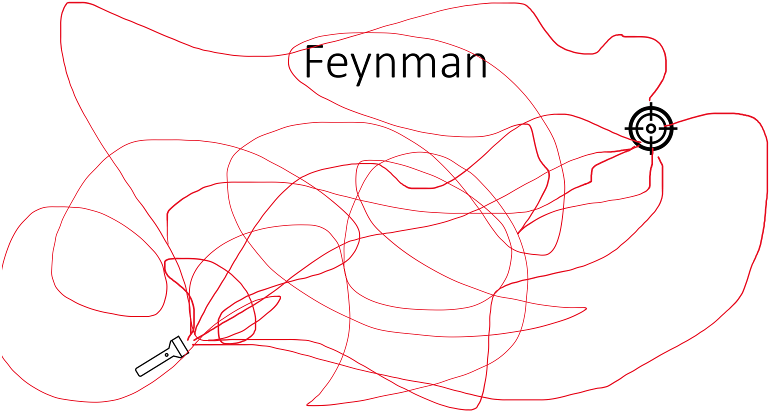
Classical physics



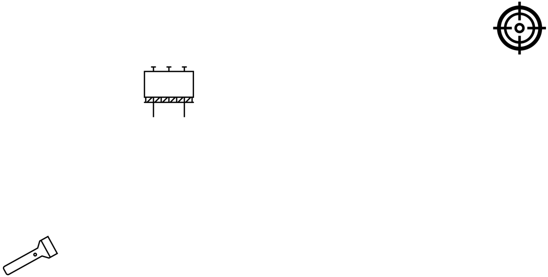
Copenhagen



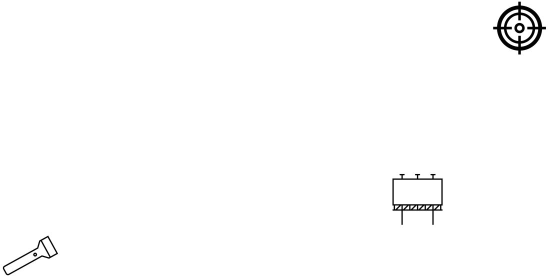
Feynman



Measurement 1

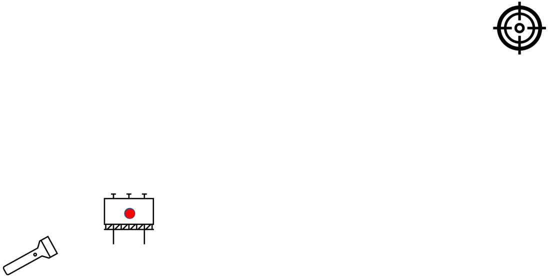


Measurement 2

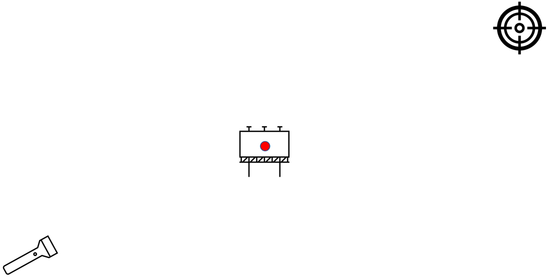


Measurement 3

Measurement 4

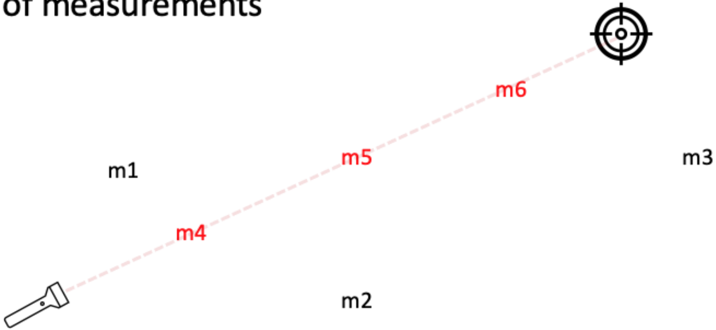


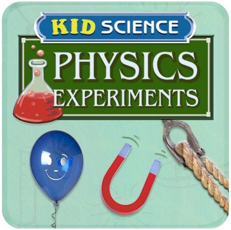
Measurement 5



Measurement 6

Geometry in “space” of measurements

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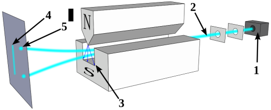
EXPERIMENTS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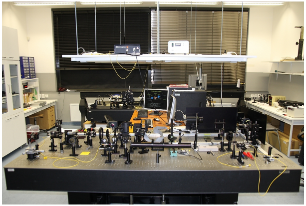

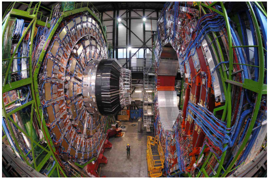

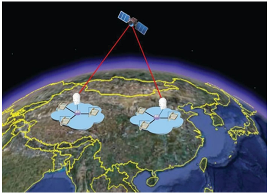

“Definition”

A *measurement* is a finite physical procedure, performed with an experimental apparatus, in the course of which a finite amount of *communicable classical information* is recorded.

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(Based on arXiv:2102.01712)

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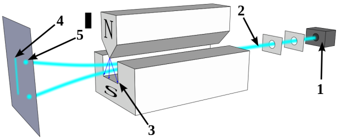

$$
\begin{aligned}
z &\in U^+ \cap U^- \\
z^+ &\in U^+ \setminus U^- \\
z^- &\in U^- \setminus U^+
\end{aligned}
$$

Topology

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- Quotient space of abstract measurements is T_0.

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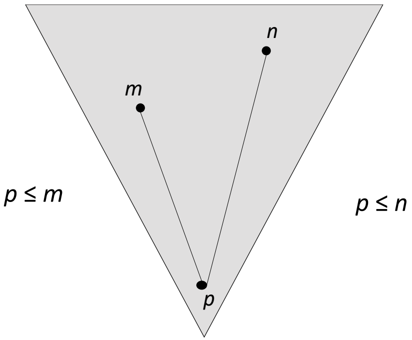
- The open sets are *upper-closed* in the specialization order (and also inaccessible by directed joins):



Disjunctions

Disjunctions are *joins* (suprema) in the specialization order:

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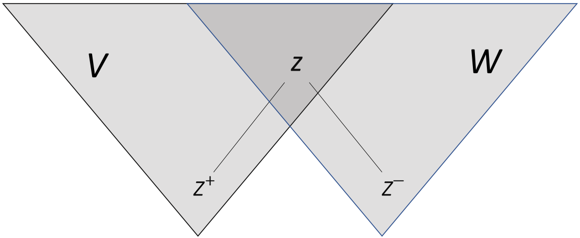
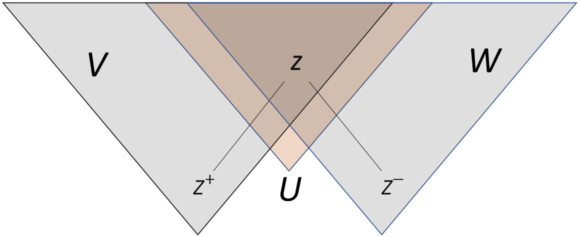


Disjunctions

Disjunctions are *continuous* operations because $V \vee W = V \cap W$:



Definition

A *sober lattice* is a sober space L whose specialization order has a least element 0 and a continuous binary join operation $\vee : L \times L \to L$.

Fact: any sober lattice is a complete lattice (all joins exist).

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M is a *stably Gelfand quantale*.

Definition

A *measurement space* M is a topological involutive semigroup which is a sober lattice and for all $m, n, p \in M$ satisfies:

1. $0m = 0$;
2. $(m \vee n)p = mp \vee np$;
3. $mm^*m = m$ whenever $mm^*m \leq m$.

Shorter definition: a *measurement space* is a *sober involutive semiring* that satisfies condition 3.

Even shorter definition: a *measurement space* is a *sober stably Gelfand quantale*.

EXAMPLES

Measurement spaces from C*-algebras

Theorem

Let A be a C*-algebra. The involutive quantale $\mathrm{Max}\, A$ [Mulvey 1989], with the lower Vietoris topology, is a measurement space.

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- $P^* = \{a^* \mid a \in P\}$
- The lower Vietoris topology has a subbasis of open sets

$$\widetilde{U} = \{P \in \operatorname{Max} A \mid P \cap U \neq \emptyset\}$$

where U is open in A.

For the spin 1/2 example: $A = M_2(\mathbb{C})$.

$$z^+ = \left\langle \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\rangle \qquad z^- = \left\langle \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\rangle$$

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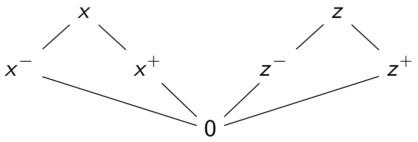
Fragment of the specialization order:



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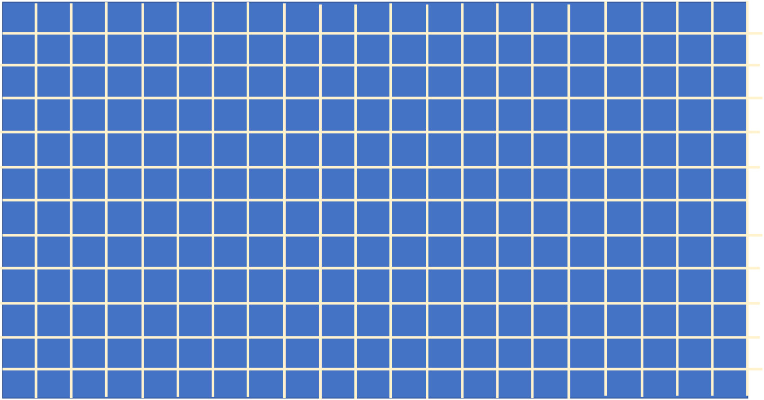
The "picture" of the wall as a *space with points* emerges from an integrated *mental image* that translates mathematically to a *geometric model*.

Example 2 — Adding partial symmetries



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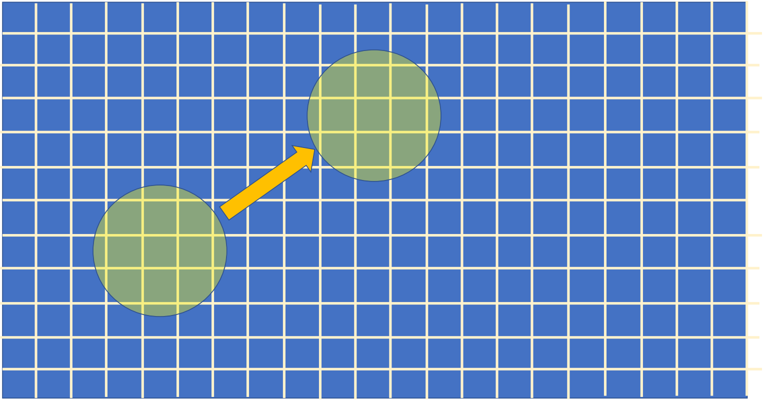

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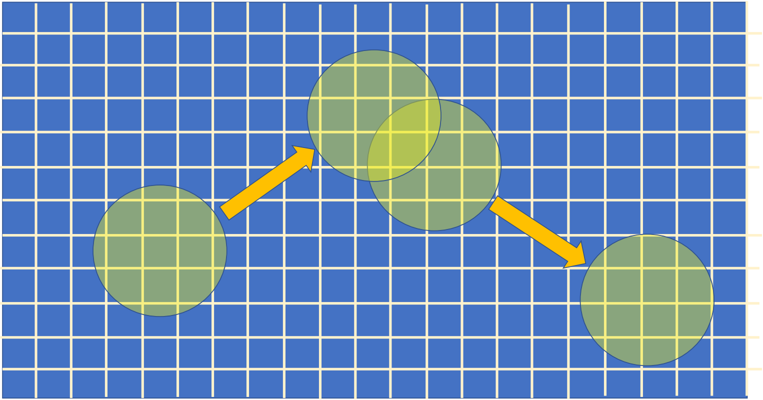

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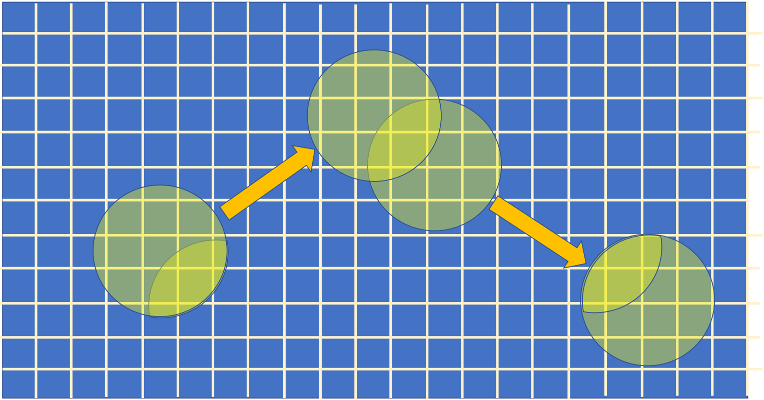

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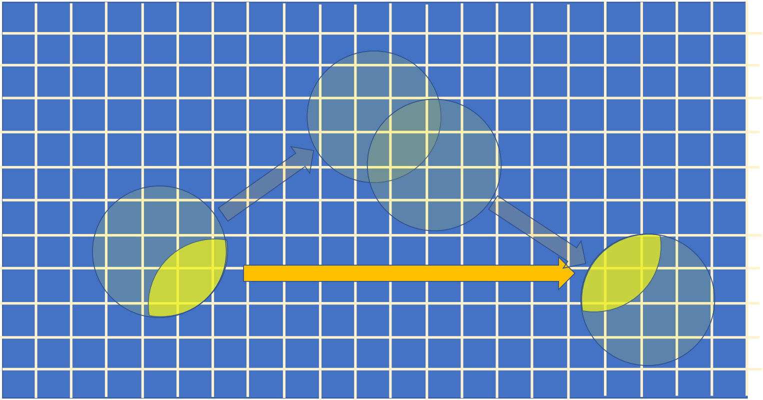

The composition is associative and defines a *pseudogroup* (a complete and infinitely distributive *inverse semigroup* [cf. Mark V. Lawson, *Inverse Semigroups: The Theory of Partial Symmetries*, WS, 2002]).

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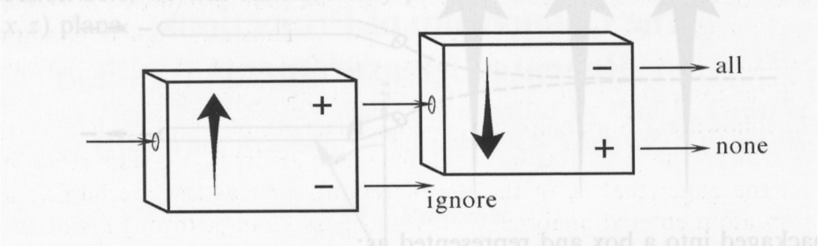
Example

$$\text{flip } z^+ \quad = \quad z^- \text{ flip } z^+$$



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- Or define G to be the (non-étale) pair groupoid $G_0 \times G_0$.

Part 5 — Quantizations and observers

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Communication: each observer *approximates* measurements done by others:

$$\alpha : M \to \mathcal{O}$$

Definition

Let M be a measurement space. An *observer* of M is a pair $(\mathcal{O}, \boldsymbol{\alpha})$ in which $\mathcal{O}$ is a classical subspace of measurements (e.g., $\cong \mathcal{O}(G)$) and $\boldsymbol{\alpha} : M \to \mathcal{O}$ is a topological retraction onto $\mathcal{O}$ such that for all $m, n \in M$ and $\omega \in \mathcal{O}$

$$\begin{aligned} \boldsymbol{\alpha}(m \vee n) &= \boldsymbol{\alpha}(m) \vee \boldsymbol{\alpha}(n) \\ \boldsymbol{\alpha}(m^*) &= \boldsymbol{\alpha}(m)^* \\ \boldsymbol{\alpha}(0) &= 0 \\ \boldsymbol{\alpha}(\omega m) &= \omega \boldsymbol{\alpha}(m) \end{aligned}$$

The observer is *persistent* if it further satisfies

$$\boldsymbol{\alpha}(m \omega n) = \boldsymbol{\alpha}(m) \omega \boldsymbol{\alpha}(n),$$

and *localizable* if $m \leq \boldsymbol{\alpha}(m)$ for all $m \in M$.

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Let M be a measurement space. An *observer* of M is a pair $(\mathcal{O}, \boldsymbol{\alpha})$ in which $\mathcal{O}$ is a classical subspace of measurements (e.g., $\cong \mathcal{O}(G)$) and $\boldsymbol{\alpha} : M \to \mathcal{O}$ is a topological retraction onto $\mathcal{O}$ such that for all $m, n \in M$ and $\omega \in \mathcal{O}$

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- If $(\mathcal{O}', \boldsymbol{\alpha}')$ is another observer, the restriction $\boldsymbol{\alpha}|_{\mathcal{O}'} : \mathcal{O}' \to \mathcal{O}$ translates measurements of an observer to the other.

Example

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The embedding $\iota : \mathcal{O}(G) \to \operatorname{Max} A$ *given by* $U \mapsto \overline{C_c(U)}$ *is continuous and preserves composition, involution and joins.*

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Theorem (from [R 2018a])

The pair $(\mathcal{O}, \boldsymbol{\alpha})$ *is an observer of* $\operatorname{Max} A$.

If G *is compact: (1) the observer is localizable; (2) if* G *is principal with discrete orbits the observer is persistent; (3) if the observer is persistent then* G *is principal.*

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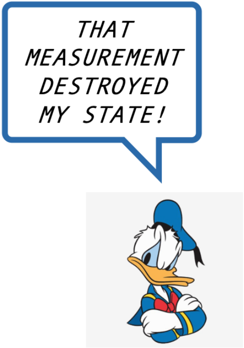
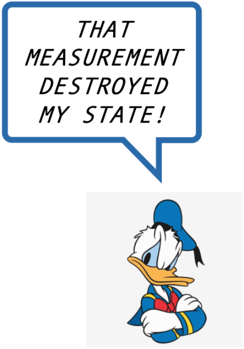
- Example: in $M_2(\mathbb{C})$

$$\underbrace{\left\langle \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\rangle}_{z^+} \left\langle \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \right\rangle \underbrace{\left\langle \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\rangle}_{x^+} = \underbrace{\left\langle \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \right\rangle}_{z^+ \leftarrow x^+}$$


THAT MEASUREMENT DESTROYED MY STATE!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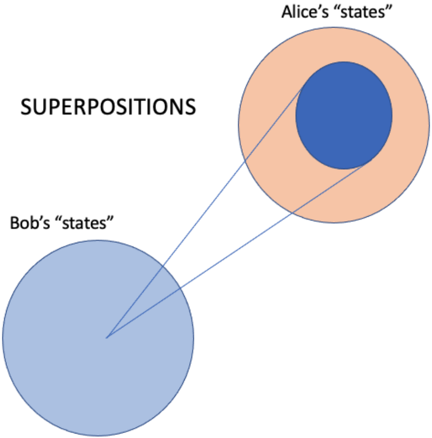
Alice’s “states”
SUPERPOSITIONS
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Part 6 — Wrapping up

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- The model is both realist and operational.
- The open sets correspond to the classical information that can be extracted from measurements.
- On the other hand, measurements are defined in terms of the classical information they yield.
- Neither information nor measurement takes precedence: measurement spaces bootstrap a definition of both.

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- Open problems related to C*-algebras and Fell bundles; statistical interpretation; dynamics; geometrization of observers...