

Bootstrapping $\mathcal{N} = 2$ SCFTs

Madalena Lemos



Iberian Strings
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Based on:

1312.5344 w/ C. Beem, P. Liendo, W. Peelaers, L. Rastelli and B. van Rees

1511.07449 w/ P. Liendo

1612.01536 w/ P. Liendo, C. Meneghelli and V. Mitev

1702.xxxxx w/ M. Cornagliotto and V. Schomerus

Outline

- 1 The (Super)conformal Bootstrap Program
- 2 A solvable subsector
- 3 Constraining the space of $\mathcal{N} = 2$ SCFTs
- 4 $4d$ $\mathcal{N} = 3$ SCFTs
- 5 Summary and Outlook

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The (Super)conformal Bootstrap Program

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Can we bootstrap specific theories?

- Particularly helpful if theory is uniquely fixed by a set of discrete data
- Only tool available for finite N non-Lagrangian theories

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- ▶ Step 1: Solve this subsector
- ▶ Step 2: Full blown numerics for the rest

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Chiral algebra

Organize operators in representations of superconformal algebra

$$\{ \mathcal{O}_{\Delta, (j_1, j_2)}, \quad \}$$

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→ Meromorphic!

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Example: free hypermultiplet

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$$q(z, \bar{z}) = u_I Q' = Q(z, \bar{z}) + \bar{z} \tilde{Q}^*(z, \bar{z}),$$

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$$\rightarrow q(z, \bar{z}) \tilde{q}(0) \sim \bar{z} \tilde{Q}^*(z, \bar{z}) \tilde{Q}(0) \sim \frac{\bar{z}}{z\bar{z}} = \frac{1}{z}$$

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Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu}$

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↪ $c_{2d} = -12c_{4d}$

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$4d \mathcal{N} = 2$ SCFTs with a flavor symmetry

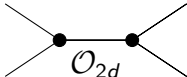
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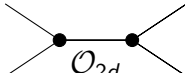
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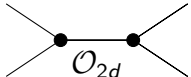
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assumptions: interacting theory, unique stress tensor

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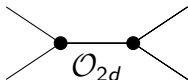
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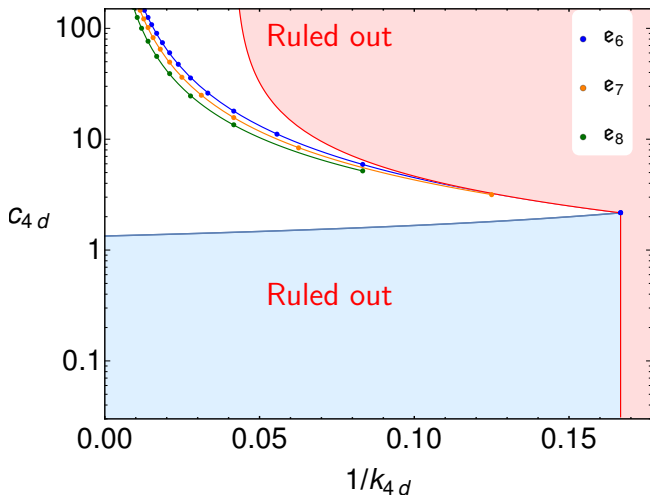
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$4d \mathcal{N} = 2$ SCFTs with E_6 flavor symmetry

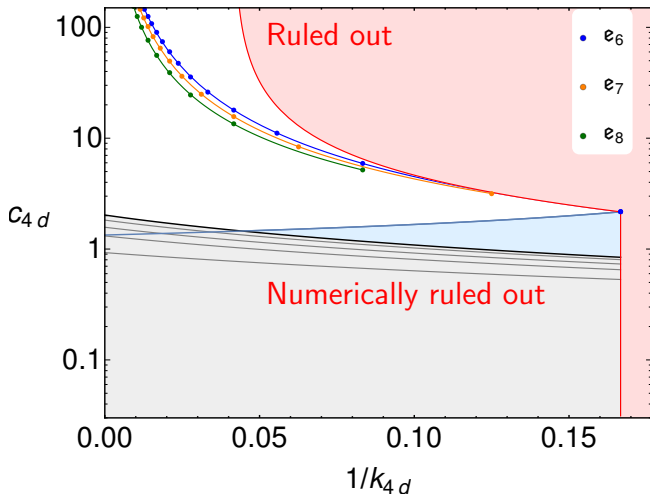
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[Beem, ML, Liendo, Peelaers, Rastelli, van Rees] [ML, Liendo]

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smallest interacting $c_{4d} = \frac{15}{12}$

Chiral algebra

- ▶ $4d \mathcal{N} \geq 3$: some of the extra supercharges commute with \mathbb{Q}
 - ↪ $4d \mathcal{N} = 4 \Rightarrow 2d$ “small” $\mathcal{N} = 4$ chiral algebra
 - ↪ $4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]

$2d \mathcal{N} = 2$ **Stress tensor** $\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$

→ Local, interacting $\mathcal{N} = 3$ SCFT

$$c_{4d} \geq \frac{13}{24} \quad [\text{Cornagliotto, ML, Schomerus}]$$

↪ Not saturated by any known SCFT

smallest interacting $c_{4d} = \frac{15}{12}$

↪ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs [Beem, Rastelli, van Rees] [Liendo, Ramirez, Seo]

Outline

- 1 The (Super)conformal Bootstrap Program
- 2 A solvable subsector
- 3 Constraining the space of $\mathcal{N} = 2$ SCFTs
- 4 $4d$ $\mathcal{N} = 3$ SCFTs
- 5 Summary and Outlook**

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→ New constraints on the space of allowed $\mathcal{N} \geq 2$ SCFTs

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Other analytic approaches?

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Other analytic approaches?

- Is c bounded by k for large k ?

Thank you!

Backup slides

Outline

Conformal bootstrap

Constraining the space of $\mathcal{N} = 2$ SCFTs

Bootstrapping $\mathcal{N} = 3$ SCFTs

Conformal Bootstrap

Conformal field theory defined by

Set of local operators and their correlation functions

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CFT data

$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\}$ and

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Conformal field theory defined by

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$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\}$ and

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} \mathcal{O}_k(0)$$

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$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k\text{prim.}} \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} c(x, \partial_x)\mathcal{O}_k(0)$$

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→ n -point function by recursive use of the OPE until

$$\langle \mathbb{1} \rangle = 1$$

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- Finite radius of convergence
- n -point function by recursive use of the OPE until $\langle \mathbb{1} \rangle = 1$

CFT data strongly constrained

- ▶ Unitarity
- ▶ Associativity of the operator product algebra

Conformal Bootstrap

Crossing Symmetry

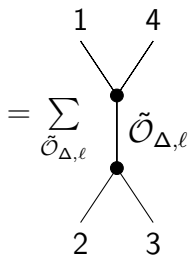
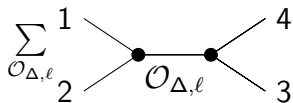
$$\langle (\mathcal{O}_1(x_1) \mathcal{O}_2(x_2)) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle =$$

$$\sum_{\mathcal{O}_{\Delta,\ell}} \begin{array}{c} 1 \\ \diagdown \\ \bullet \\ \diagup \\ 2 \end{array} \begin{array}{c} \text{---} \\ \bullet \\ \text{---} \\ \mathcal{O}_{\Delta,\ell} \end{array} \begin{array}{c} \diagup \\ \bullet \\ \diagdown \\ 3 \end{array} \begin{array}{c} 4 \\ \diagdown \\ \bullet \\ \diagup \\ 3 \end{array}$$

Conformal Bootstrap

Crossing Symmetry

$$\langle \mathcal{O}_1(x_1) (\mathcal{O}_2(x_2) \mathcal{O}_3(x_3)) \mathcal{O}_4(x_4) \rangle =$$



Conformal Bootstrap

Crossing Symmetry

$$\langle (\mathcal{O}_1(x_1) \mathcal{O}_2(x_2)) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle =$$

$$= \sum_{\tilde{\mathcal{O}}_{\Delta, l}} \tilde{\mathcal{O}}_{\Delta, l}$$

$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\mathcal{O}_{\Delta, l}} \lambda_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_{\Delta, l}} \lambda_{\mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_{\Delta, l}} g_{\Delta, l}(u, v) =$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

Conformal Bootstrap

Crossing Symmetry

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$$\frac{1}{x_{14}^{2\Delta_{\tilde{\mathcal{O}}}} x_{23}^{2\Delta_{\tilde{\mathcal{O}}}}} \sum_{\tilde{\mathcal{O}}_{\Delta, l}} \lambda_{\mathcal{O}_1 \mathcal{O}_4 \tilde{\mathcal{O}}_{\Delta, l}} \lambda_{\mathcal{O}_2 \mathcal{O}_3 \tilde{\mathcal{O}}_{\Delta, l}} g_{\Delta, l}(v, u)$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

Conformal Bootstrap

→ Solve crossing equations for *all* four-point functions

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[Rattazzi, Rychkov, Tonni, Vichi]

- ▶ Solving \Rightarrow constraining

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→ Guess for the spectrum

Conformal Bootstrap

→ Solve crossing equations for *all* four-point functions

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↪ there's a large gap in the spectrum

Conformal Bootstrap

→ Solve crossing equations for *all* four-point functions

[Rattazzi, Rychkov, Tonni, Vichi]

- ▶ Solving \Rightarrow constraining
 - Guess for the spectrum
 - \hookrightarrow there's a large gap in the spectrum
 - Can it ever define a consistent CFT?

Conformal Bootstrap

→ Solve crossing equations for *all* four-point functions

[Rattazzi, Rychkov, Tonni, Vichi]

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Sum rule: identical scalars ϕ

Conformal Bootstrap

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 - Guess for the spectrum
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Sum rule: identical scalars ϕ

→ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

$$1 = \sum_{\substack{\mathcal{O}_{\Delta\ell} \neq \mathbb{1} \\ \mathcal{O} \in \phi\phi}} \lambda_{\phi\phi\mathcal{O}}^2 \underbrace{\frac{u^{\Delta\phi} g_{\Delta,\ell}(v, u) - v^{\Delta\phi} g_{\Delta,\ell}(u, v)}{v^{\Delta\phi} - u^{\Delta\phi}}}_{F_{\Delta,\ell}}$$

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Sum rule

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- ▶ Find Functional Ψ such that
 - ↪ $\psi \cdot 1 < 0$ ($\mathbb{1}$)
 - ↪ $\psi \cdot F_{\Delta,\ell}(u, v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum

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- ▶ Truncate

$$\psi = \sum_{m,n}^{m,n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n |_{z=\bar{z}=\frac{1}{2}}$$

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→ Increase $\Lambda \Rightarrow$ bounds get stronger

Conformal Bootstrap

Sum rule

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- ▶ Find Functional Ψ such that

$$\hookrightarrow \psi \cdot 1 < 0 \quad (\mathbb{1})$$

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→ Spectrum is inconsistent \Rightarrow rule out CFT

- ▶ Truncate

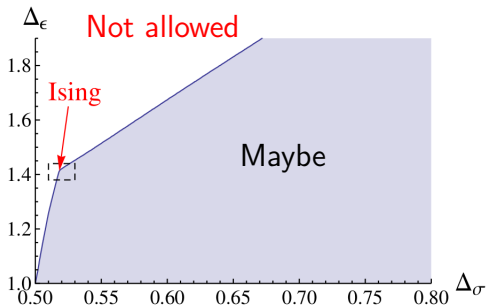
$$\psi = \sum_{\substack{m,n \leq \Lambda \\ m,n}} a_{mn} \partial_z^m \partial_{\bar{z}}^n \Big|_{z=\bar{z}=\frac{1}{2}}$$

→ Increase $\Lambda \Rightarrow$ bounds get stronger

→ Always true bounds

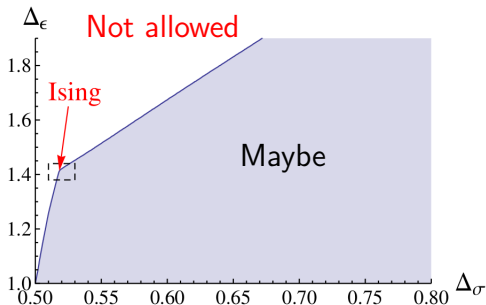
3d Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



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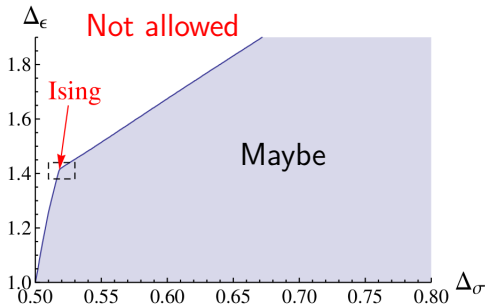
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→ Saturated by 3d Ising model

3d Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



- Saturated by 3d Ising model
- 3d Ising lives at “kink”

Outline

Conformal bootstrap

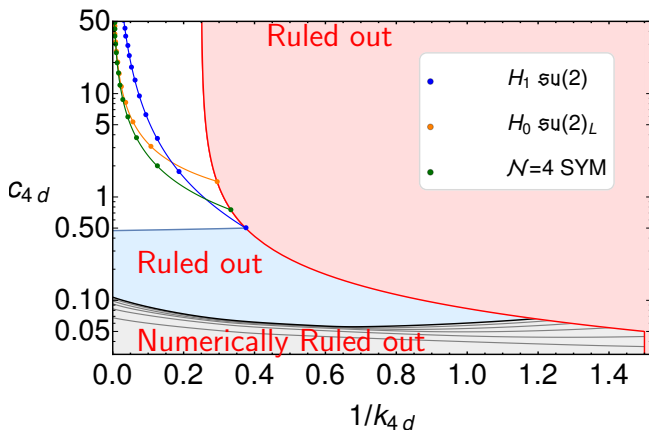
Constraining the space of $\mathcal{N} = 2$ SCFTs

Bootstrapping $\mathcal{N} = 3$ SCFTs

What is the space of consistent SCFTs?

4d $\mathcal{N} = 2$ SCFTs with $SU(2)$ flavor symmetry

$\langle TTTT \rangle$, $\langle J^a J^b J^c J^d \rangle$, $\langle TTJ^a J^b \rangle$



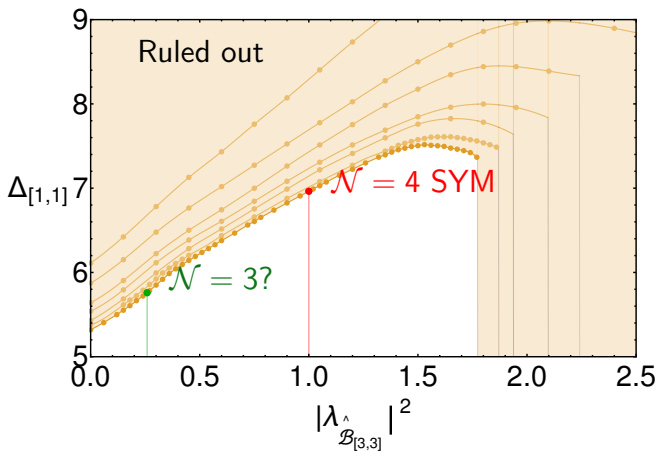
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Conformal bootstrap

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Bootstrapping $\mathcal{N} = 3$ SCFTs

$4d \mathcal{N} = 3$ SCFTs with $c = \frac{15}{12}$



[ML, Liendo, Meneghelli, Mitev]