N=2 BPS black holes from the double copy

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based on work done in collaboration with:
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Goals

- The goal of this program is to ultimately generate new (super)gravitational solutions.
- Work at linear level (for now).
- Work in $\mathcal{N} = 2$ (can be easily truncated/extended)
- Dictionary requirements:
  - general $\rightarrow$ not restricted to any specific solution.
  - Lorenz-covariant $\rightarrow$ work in most convenient coordinates
  - incorporates scalar geometry

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Convolution

- Give convolution-based dictionary [Anastasiou, Borsten, Duff, Hughes, SN 2014]

$$[f \star g](x) = \int d^4 y f(y) g(x-y).$$

and is a consequence of the momentum-space origin of squaring: product in momentum space is convolution in position space!

- Importantly, is **doesn't** obey the Leibnitz rule:

$$\partial_\mu (f \star g) = (\partial_\mu f) \star g = f \star (\partial_\mu g)$$

Spectator

- Non-abelian indices are contracted with the "spectator"

$$z_{\mu\nu} = A_\mu{}^a \star \Phi^{-1}_{a\tilde{a}} \star \tilde{A}_\nu{}^{\tilde{a}}$$

where $\Phi_{ii'}$ is a bi-adjoint scalar field [Hodges 2013,Cachazo 2014], same as one in [Luna, Monteiro, Nicholson, Ochirov, O'Connell, Westerberg, White 2016]

- Have to put it on the RHS to ensure correct d.o.f. counting:

$$\Phi^{a\tilde{a}} \star z_{\mu\nu}(h, B, \phi) = A_\mu{}^a \star \tilde{A}_\nu{}^{\tilde{a}}$$

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- Have to put it on the RHS to ensure correct d.o.f. counting:

$$\text{mom. space:} \quad \Phi^{a\tilde{a}} \otimes z_{\mu\nu}(h, B, \phi) = A_\mu{}^a \otimes \tilde{A}_\nu{}^{\tilde{a}}$$

  othewise $z_{\mu\nu}$ cannot reproduce the most general expressions for $h_{\mu\nu}, B_{\mu\nu}, \phi$. OK for solutions with a lot of symmetry though.

Spectator

- Non-abelian indices are contracted with the "spectator"

$$z_{\mu\nu} = A_\mu{}^a \star \Phi^{-1}_{a\tilde{a}} \star \tilde{A}_\nu{}^{\tilde{a}}$$

where $\Phi_{ii'}$ is a bi-adjoint scalar field [Hodges 2013,Cachazo 2014], same as one in [Luna, Monteiro, Nicholson, Ochirov, O'Connell, Westerberg, White 2016]

- For notational symplicity:

$$z_{\mu\nu} = A_\mu{}^a \star \Phi^{-1}_{a\tilde{a}} \star \tilde{A}_\nu{}^{\tilde{a}} \equiv A_\mu \star \tilde{A}_\nu$$

set-up

- Work in $\mathcal{N}=2$ supergravity coupled to n_V vector multiplets.
- Scalars live in special Kahler manifold

$$\frac{G}{H}=\frac{SU(1,1)}{U(1)}\otimes\frac{SO(n_v-1,2)}{SO(n_v-1)\otimes SO(2)}\otimes\frac{SU(2)}{SU(2)}$$

- Convenient to work with projective coordinates X^I, $I=0,...n_V$

set-up

- Work in the superconformal approach to matter-coupled $\mathcal{N}=2$ supergravity.
- Kinetic terms determined from prepotential $F(X)$.

$$\mathcal{L}(X) = -N_{IJ} D_\mu X^I D^\mu \bar{X}^J$$

where $N_{IJ} = -i\left(F_{IJ} - \bar{F}_{IJ}\right) \quad , \quad F_{IJ} = \frac{\partial^2 F(X)}{\partial X^I \partial X^J}$.

- The scalar fields X^I satisfy the Einstein frame constraint

$$N_{IJ} X^I \bar{X}^J = -1$$

- The gaugini Ω_i^I are constrained by the S-supersymmetry gauge fixing condition

$$\bar{X}^I N_{IJ} \Omega_i^J = 0 \; . \tag{1}$$

On-shell Lorenz-covariant Dictionary

Start with a simple example: tensor on-shell $\mathcal{N}=2$ SYM with a gauge field. At the level of helicity states, we get the content of the $\mathcal{N}=2$ supergravity multiplet coupled to a vector multiplet:

	$\tilde{A}^-$	$\tilde{A}^+$
A^-	g^-	φ_0
λ_i^-	ψ_i^-	χ_i^+
σ^+, σ^-	$A_{0,1}^-$	$A_{0,1}^+$
λ_i^+	χ_i^-	ψ_i^+
A^+	φ_1	g^+

What theory are we building?

Work with quadratic prepotential $F = -iX^0 X^1$. We linearise the spacetime metric and the scalar fields as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$X^I = \langle X^I \rangle + \delta X^I$$

In the following, in order to avoid cluttering of notation, we will denote the fluctuations δX^I simply by X^I.

SYM transformations

$\mathcal{N} = 2$ SYM transformations:

$$\delta A_\mu^a = \left(\frac{1}{2}\varepsilon^{ij}\bar{\epsilon}_i\gamma_\mu\lambda_j^a + h.c.\right) + \partial_\mu\alpha^a + f^a{}_{bc}A_\mu^b\theta^c \,,$$

$$\delta\lambda_i^a = \gamma^\mu\partial_\mu\sigma^a\epsilon_i + \frac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}^{a-}\varepsilon_{ij}\epsilon^j + f^a{}_{bc}\lambda_i^b\theta^c \,,$$

$$\delta\sigma^a = \frac{1}{2}\bar{\epsilon}^i\lambda_i^a + f^a{}_{bc}\phi^b\theta^c \,,$$

where $f^a{}_{bc}$ denote the structure constants of the global non-Abelian group G.

SUSY transformations sugra

$$\delta_Q h_{\mu\nu} = \bar{\epsilon}^i \gamma_{(\mu}\psi_{\nu)i} + h.c.\ ,$$

$$\delta_Q \psi^i_\mu = -\frac{1}{4}\gamma^{ab}\partial_{[a}h^-_{b]\mu}\epsilon^i - \frac{1}{16}T^-_{\alpha\beta}\gamma^{\alpha\beta}\gamma_\mu \varepsilon^{ij}\epsilon_j\ ,$$

$$\delta_Q W^0_\mu = \frac{1}{2}\varepsilon^{ij}\bar{\epsilon}_i\gamma_\mu\Omega^0_j + \varepsilon^{ij}\bar{\epsilon}_i\psi_{\mu j}\langle X^0\rangle + h.c.\ ,$$

$$\delta_Q W^1_\mu = -\frac{\langle \bar{X}^1\rangle}{2\langle \bar{X}^0\rangle}\varepsilon^{ij}\bar{\epsilon}_i\gamma_\mu\Omega^0_j + \varepsilon^{ij}\bar{\epsilon}_i\psi_{\mu j}\langle X^1\rangle + h.c.\ ,$$

$$\delta_Q \Omega^{0i} = \gamma^\mu\partial_\mu \bar{X}^0\epsilon^i + \frac{1}{4}\gamma^{\mu\nu}\mathcal{F}^{0+}_{\mu\nu}\varepsilon^{ij}\epsilon_j\ ,$$

$$\delta_Q X^0 = \frac{1}{2}\bar{\epsilon}^i\Omega^0_i\ ,$$

Equations of Motion

Fields decouple at linear level - v simple!

- SYM

$$\partial_\mu F^{\mu\nu a} = \partial_\mu ({}^*F)^{\mu\nu a} = 0 \,,$$
$$\not{\partial} \lambda_i^a = 0 \,,$$
$$\Box \sigma^a = 0 \,,$$

- Supergravity

$$R_{\mu\nu} = 0$$
$$\gamma^{\mu\nu\rho} \partial_\nu \psi_\rho^i = 0 \Rightarrow \gamma^\mu \psi_{\mu\nu} = 0$$

Ansatz

Work with field strengths - eliminates gauge freedom issues by working with gauge invariant quantities.

	$\tilde{A}^-$	$\tilde{A}^+$
A^-	g^-	φ_0
λ_i^-	ψ_i^-	χ_i^+
σ^+, σ^-	$A_{0,1}^-$	$A_{0,1}^+$
λ_i^+	χ_i^-	ψ_i^+
A^+	φ_1	g^+

Ansatz

Work with field strengths - eliminates gauge freedom issues by working with gauge invariant quantities. Ansatz:

$$\boxed{a\psi^i_{\mu\nu} + 2b\gamma_{[\nu}\partial_{\mu]}\Omega^{0i} \equiv \varepsilon^{ij}\lambda_j \star \tilde{F}_{\mu\nu}}$$

This is the most general ansatz for the fermions.

Ansatz

Work with field strengths - eliminates gauge freedom issues by working with gauge invariant quantities. Ansatz:

$$a\psi^{i}_{\mu\nu}+2b\gamma_{[\nu}\partial_{\mu]}\Omega^{0i}\equiv\varepsilon^{ij}\lambda_{j}\star\tilde{F}_{\mu\nu}+c\varepsilon^{ij}\lambda_{j}\star^{*}\tilde{F}_{\mu\nu}+d\gamma_{\mu\nu}\varepsilon^{ij}\lambda_{j}\star\partial^{\rho}\tilde{A}_{\rho}$$

Dictionary derivation

Work with field strengths - eliminate gauge freedom issues by working with gauge invariant quantities.

- Ansatz:

$$a\psi^i_{\mu\nu} + 2b\gamma_{[\nu}\partial_{\mu]}\Omega^{0i} \equiv \varepsilon^{ij}\lambda_j \star \tilde{F}_{\mu\nu}$$

- Contract with γ^μ to get the dictionary entry for the gaugini.
- Apply SUSY to get dictionary entries for all other fields!
- At every step, we check equations of motions.
- We find a constraint:

$$\partial_\mu(\varphi \star \tilde{A}^\mu) = 0 \quad \Leftrightarrow \varphi \star \partial_\mu \tilde{A}^\mu = 0$$

Full Dictionary

$$
\begin{aligned}
aR_{\mu\nu\alpha\beta} &= -\frac{1}{2}\left[F_{\mu\nu}\star\tilde{F}_{\alpha\beta}+F_{\alpha\beta}\star\tilde{F}_{\mu\nu}-4\eta_{[\alpha[\mu}\partial_{\nu]}\partial_{\beta]}A^{\rho}\star\tilde{A}_{\rho}\right]\\
a\psi^{i}_{\mu\nu} &= \varepsilon^{ij}\lambda_j\star\tilde{F}^{-}_{\mu\nu}\\
aT^{-}_{\mu\nu} &= -4\sigma\star\tilde{F}^{-}_{\mu\nu}\\
b\mathcal{F}^{0+}_{\mu\nu} &= -\sigma\star\tilde{F}^{+}_{\mu\nu}\\
b\partial_\mu\Omega^{0i} &= \frac{1}{2}\varepsilon^{ij}\gamma^{\rho}\lambda_j\star\tilde{F}^{+}_{\mu\rho}\\
b\partial_\mu\bar{X}^{0} &= \frac{1}{2}F^{-}_{\mu\rho}\star\tilde{A}^{\rho}
\end{aligned}
$$

Full Dictionary

$$R_{\mu\nu\alpha\beta} = F_{\mu\nu} \star \tilde{F}_{\alpha\beta} + F_{\alpha\beta} \star \tilde{F}_{\mu\nu} - 4\eta_{[\alpha[\mu}\partial_{\nu]}\partial_{\beta]} A^{\rho} \star \tilde{A}_{\rho}$$

$$\psi^{i}_{\mu\nu} = -2\varepsilon^{ij}\lambda_j \star \tilde{F}^{-}_{\mu\nu}$$

$$T^{-}_{\mu\nu} = 8\sigma \star \tilde{F}^{-}_{\mu\nu}$$

$$b\mathcal{F}^{0+}_{\mu\nu} = -\sigma \star \tilde{F}^{+}_{\mu\nu}$$

$$b\partial_{\mu}\Omega^{0i} = \frac{1}{2}\varepsilon^{ij}\gamma^{\rho}\lambda_j \star \tilde{F}^{+}_{\mu\rho}$$

$$b\partial_{\mu}\bar{X}^{0} = \frac{1}{2}F^{-}_{\mu\rho} \star \tilde{A}^{\rho}$$

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SYM BPS solutions

BPS condition:

$$k\,\epsilon_i = \varepsilon_{ij}\,\gamma_0\,\epsilon^j,$$

where k denotes a phase factor with an appropriate chiral weight. We set the spinors to zero, and hence their variation must vanish:

$$\delta_Q\lambda_i = \gamma^\mu\partial_\mu\sigma\epsilon_i + \frac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}\varepsilon_{ij}\epsilon^j = 0$$

SYM BPS Solutions

In Cartesian coordinates, the BPS configurations we consider are described by

$$\begin{aligned}
\partial_t(\sigma\bar{k}) &= 0\ , \\
\partial_i Im(\sigma\bar{k}) &= 0\ , \\
F_{ti} &= -2\partial_i Re(\sigma\bar{k})\ . \\
F_{ij} &= 0 \quad , \quad i,j = x,y,z\ .
\end{aligned}$$

We find it convenient to work with Eddington-Finkelstein type coordinates (u, r, θ, ϕ), with u given by $u = t + r$. The gauge potential A_u reads

$$A_u = 2Re(\sigma\bar{k})\ .$$

The above is a purely electric solution.

Dyonic BPS black hole solutions

At non-linear level, the line element is of the form

$$ds^2 = -e^{2g}dt^2 + e^{-2g}(dr^2 + r^2 d\Omega_2^2) \ ,$$

with $e^{-2g} = i(\bar{Y}^I F_I(Y) - Y^I \bar{F}_I(\bar{Y}))$, where $Y^I = e^{-g} X^I \bar{k}$ are determined by the attractor equations:

$$Y^I - \bar{Y}^I = iH^I \ ,$$
$$F_I(Y) - \bar{F}_{\bar{I}}(\bar{Y}) = iH_I \ ,$$

where the (H_I, H^I) denote harmonic functions

$$H_I = h_I + \frac{q_I}{r}, \quad H^I = h^I + \frac{p^I}{r} \quad , \quad I = 0, 1 \ ,$$

For the prepotential $F(X) = -iX^0 X^1$, the solutions are supported by electric charges (q_0, q_1) and by magnetic charges (p^0, p^1).

Dyonic BPS black hole solutions

We work in Eddington-Finkelstein type coordinates and perform a weak field approximation: only keep terms up to $\mathcal{O}(r^{-1})$ in the fields. The solution is then

$$h_{\mu\nu} = diag\,(\frac{Q}{r}, 0, Qr, Qrsin^2\theta)\ ,$$

$$F^0_{ur} = \frac{Qh_1 - q_1}{r^2},\ \ F^1_{ur} = \frac{Qh_0 - q_0}{r^2},\ \ F^I_{\theta\phi} = p^I \sin\theta, \quad I = 0, 1$$

$$\bar{k}\,\partial_\mu X^0 = -\frac{\left(q_1 - ip^0 - \frac{1}{2}Q(h_1 - ih^0)\right)}{2}\,\partial_\mu \frac{1}{r}\ .$$

where

$$Q = h_0 q_1 + h_1 q_0 + h^0 p^1 + h^1 p^0\ .$$

and the background scalar fields are

$$\langle X^0 \bar{k}\rangle = -\frac{1}{2}\left(h_1 - ih^0\right), \langle X^1 \bar{k}\rangle = -\frac{1}{2}\left(h_0 - ih^1\right)$$

Double Copy for Solutions

LHS solution:

$$A_u = \frac{d_1}{r}, \quad A_r = 0$$
$$2Re(\sigma\bar{k}) = \frac{d_1}{r}, \quad Im(\sigma\bar{k}) = 0$$

RHS solution:

$$\tilde{A}_u = \delta^{(4)}(u, r, \theta, \phi), \quad \tilde{A}_r = d_2\, \delta^{(4)}(u, r, \theta, \phi)$$

Spectator:

$$\phi_{a\tilde{a}} = V_{a\tilde{a}}\, \delta^{(4)}(u, R, \theta, \phi)$$

This reproduces solution on previous page via the dictionary- we get dionic BPS sugra solutions from electric BPS SYM solutions!

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n vectors

	$\tilde{A}^-$	$\tilde{A}^+$	$\tilde{\sigma}^a$
A^-	g^-	φ_0	A^{a-}
λ_i^-	ψ_i^-	χ_i^+	χ_i^{a-}
σ^+, σ^-	$A_{0,1}^-$	$A_{0,1}^+$	σ^{a+}, σ^{a-}
λ_i^+	χ_i^-	ψ_i^+	χ_i^{a+}
A^+	φ_1	g^+	A^{a+}

Table: On-shell $(\mathcal{N}=2)_{SYM} \times [(\mathcal{N}=0)_{SYM} + (n_V - 1)\tilde{\sigma}] = (\mathcal{N}=2)_{sugra} + n_V(\mathcal{N}=2)_{SYM}$

n vectors - dictionary

$$
\begin{aligned}
aR_{\mu\nu\alpha\beta} &= -\frac{1}{2}\left[F_{\mu\nu}\star\tilde{F}_{\alpha\beta}+F_{\alpha\beta}\star\tilde{F}_{\mu\nu}-4\eta_{[\alpha[\mu}\partial_{\nu]}\partial_{\beta]}A^{\rho}\star\tilde{A}_{\rho}\right]\\
a\psi^{i}_{\mu\nu} &= \varepsilon^{ij}\lambda_j\star\tilde{F}^{-}_{\mu\nu}\\
F^{I-}_{\mu\nu} &= -2\frac{\langle\bar{X}^I\rangle}{a}\sigma\star\tilde{F}^{-}_{\mu\nu}+l^I\bar{\sigma}\star\tilde{F}^{-}_{\mu\nu}+r^I{}_aF^{-}_{\mu\nu}\star\tilde{\sigma}^a\\
\partial_\mu\Omega^I_i &= \frac{l^I}{2}\varepsilon_{ik}\gamma^\rho\lambda^k\star\tilde{F}^{-}_{\rho\mu}+r^I{}_a\partial_\mu\lambda_i\star\tilde{\sigma}^a\\
\partial_\mu\bar{X}^I &= -\frac{\bar{l}^I}{2}F^{-}_{\mu\rho}\star\tilde{A}^{\rho}+\bar{r}^I{}_a\partial_\mu\bar{\sigma}\star\tilde{\sigma}^a\ ,
\end{aligned}
$$

n vectors - dictionary parameters

The parameters satisfy the constraints

$$\langle N_{IJ}\bar{X}^I\rangle l^J = 0 \,,$$
$$\langle N_{IJ}\bar{X}^I\rangle r^J{}_a = 0 \,.$$

Candidates

$$l^I \propto \langle D_1 X^I\rangle \quad , \quad r^I{}_a \propto \langle D_a X^I\rangle \,.$$

where D_A is the Kähler covariant derivative

$$D_A X^I = \partial_A X^I + \frac{1}{2}\left(\partial_A K\right) X^I$$

Add magnetic charges on SYM side, we get multi-centred black hole solutions.[Cardoso,SN,Nampuri 2016]

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N=0

Truncate dictionary to $\mathcal{N} = 0$:

$$R_{\mu\rho\nu\sigma} = F_{\mu\rho} \star \tilde{F}_{\nu\sigma} + F_{\nu\sigma} \star \tilde{F}_{\mu\rho} - 2[\eta_{[\sigma[\mu} F_{\rho]\lambda} \star \tilde{F}^{\lambda}{}_{\nu]} + \eta_{[\rho[\nu} F_{\sigma]\lambda} \star \tilde{F}^{\lambda}{}_{\mu]}]$$

$$\partial_\mu X = \frac{1}{2} F^{+}_{\mu\rho} \star \tilde{A}^{\rho}$$

How does this match with usual $A_\mu \star A_\nu \to h_{\mu\nu}, B_{\mu\nu}, \phi$? We have

$$Re(X) \propto \phi = A_\rho \star \tilde{A}^\rho$$

$$Im(\partial_\mu X) \propto \varepsilon_{\mu\nu\rho\sigma} \partial^\nu B^{\rho\sigma} = \varepsilon_{\mu\nu\rho\sigma} \partial^\nu A^\rho \star \tilde{A}^\sigma$$

Can consider limits $A_\mu = \tilde{A}_\mu$ etc...

N=0

$$R_{\mu\rho\nu\sigma} = F_{\mu\rho} \star \tilde{F}_{\nu\sigma} + F_{\nu\sigma} \star \tilde{F}_{\mu\rho} - 2[\eta_{[\sigma[\mu} F_{\rho]\lambda} \star \tilde{F}^{\lambda}{}_{\nu]} + \eta_{[\rho[\nu} F_{\sigma]\lambda} \star \tilde{F}^{\lambda}{}_{\mu]}]$$

- Both sides gauge invariant.
- *No gauge fixing* - hope for getting full diffeomorphisms in the non-linear theory.
- Crucially, when A_μ and $\tilde{A}_\mu$ satisfy YM equation, $h_{\mu\nu}$ will satisfy Einstein equation, i.e. $R^{lin}_{\mu\nu} = 0$!
- Potential tool for generating new solutions.
- Can be used as starting point for lots of dictionaries!

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Conclusions

- We have a candidate tool for generating new solutions.
- Next step: go to higher order in perturbation theory. (helps to work with gauge-invariant objects).
- Add cosmological constant to dictionary, gauged supergravities, connection with AdS/CFT...

Thank you!

Gauge Invariance

- LHS
  Linearised diffeo: $\delta h_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$.
  Linearised Riemann tensor:

$$R^{lin}_{\mu\nu\alpha\beta} = -2\partial_{[\alpha}\partial_{[\mu}h_{\nu]\beta]} \Rightarrow \delta R^{lin}_{\mu\nu\alpha\beta} = 0$$

- RHS (with $\partial_\mu \tilde{A}^\mu = 0 \Rightarrow \Box\tilde{\alpha} = 0$) First two terms obvious.
  Last term:

$$\begin{aligned}\delta\partial_\beta A^\rho \star \tilde{A}_\rho = \partial_\beta\partial^\rho\alpha \star \tilde{A}_\rho = \partial_\beta\alpha \star \partial^\rho\tilde{A}_\rho = 0 \\ = \partial_\beta A^\rho \star \partial_\rho\tilde{\alpha} = \partial_\beta\partial_\rho A^\rho \star \tilde{\alpha} = \Box A_\beta \star \tilde{\alpha} = 0\end{aligned}$$

Peel off Derivatives

$$h_{\mu\nu} = A_{(\mu} \star \tilde{A}_{\nu)} - \frac{1}{2}\left(A_\alpha \star \tilde{A}^\alpha\right)\eta_{\mu\nu}$$