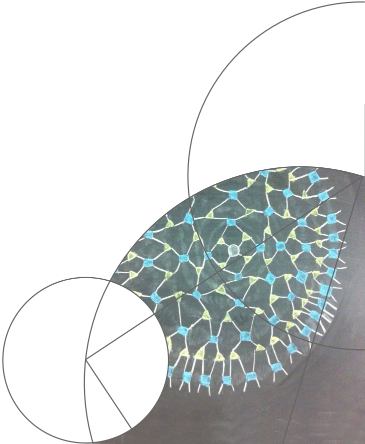
Holographic complexity

Javier Martín García

Based on:
J. L. F. Barbón & J. M-G. 1510.00349
& Ongoing work...



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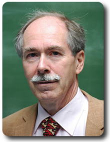
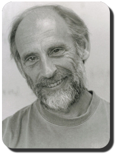
January 19, 2017

Holography



- Quantum gravity in d+1 dimensions must be described by a non gravitational theory in d dimensions.
  - ⋆ Translation between both theories should be possible: DICTIONARY
  - ⋆ Not so easy to find precise examples of holographic theories.
- Succesfull example: AdS/CFT
  - ⋆ Finding the entries of the dictionary is manageable.



- ⋆ 1997: Today:

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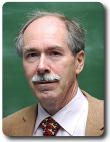

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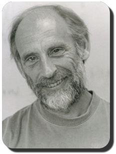

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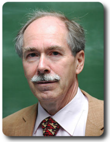

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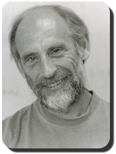

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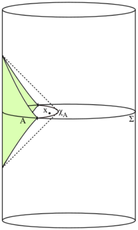
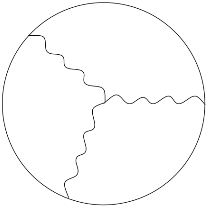
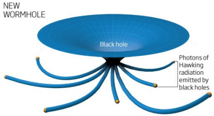
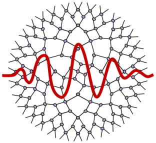
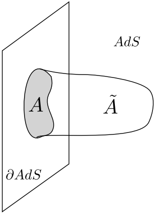


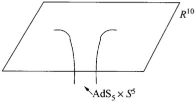


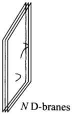


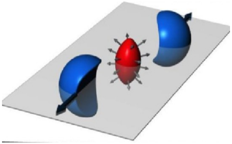


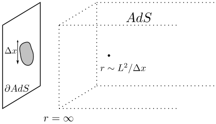


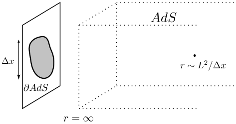
$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 L^2}$$











DICTIONARY:
CHAPTER QI

AdS	CFT
	Entanglement?
	Complexity?

Quantum entanglement

- Entanglement in QM
  - ⋆ Failure of making the decomposition $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$
  - ⋆ Quantified by entanglement entropy $S_A = -\mathrm{Tr}_A[\rho_A \log \rho_A]$
  - ⋆ Characterization of operator correlations

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \quad \Leftrightarrow \quad \langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle = 0.$$

- Tensor Network
  - ⋆ 2-Qubit entangled state $|\Psi\rangle = \sum_{i,j} \alpha_{ij} \, |\psi_i\rangle_A \, |\psi_j\rangle_B$.
  - ⋆ General entangled state $|\Psi\rangle = \sum_{\mu_1,\dots\mu_n} c_{\mu_1,\dots\mu_n} \, |\psi_{\mu_1}\rangle_1 \dots |\psi_{\mu_n}\rangle_n$

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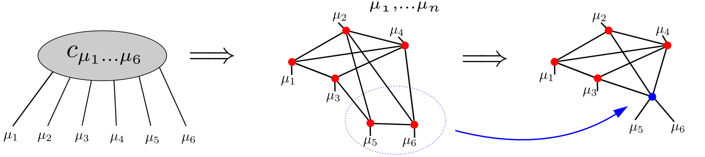
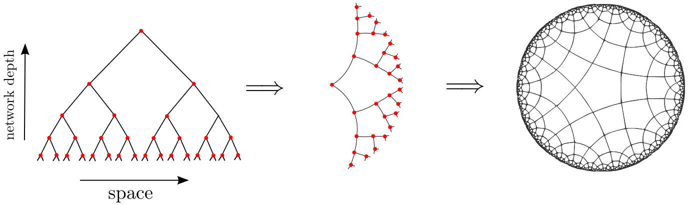


Entanglement in the spacetime picture

- Tensor network for a CFT
  - ⋆ Scale invariance $\Longrightarrow$ Tree-like structure TN
  - ⋆ New (non-physical) dimension: Network depth
  - ⋆ Mimics a discretized hyperbolic space
- Holographic entanglement entropy
  - ⋆ Entanglement between two regions $\Longleftrightarrow$ number of links cut to separate the TN in two.
  - ⋆ Ryu-Takayanagi formula $S_A = \dfrac{\text{Area}(\tilde{A})}{4G}$.



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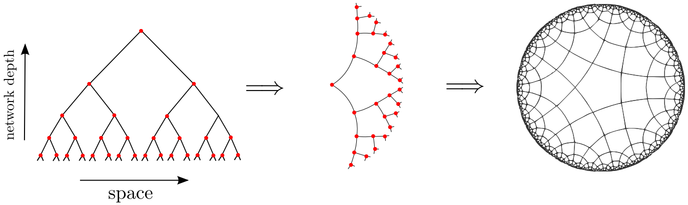

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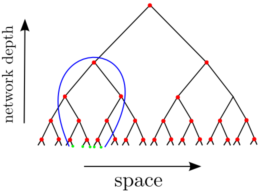
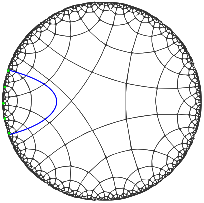


$\Longrightarrow$



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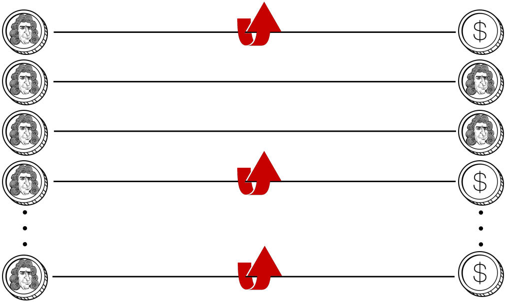
- Entanglement and spacetime connectivity
  - ⋆ Eternal AdS black hole $|\Psi\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle \otimes |E_i\rangle \Longrightarrow$Two entangled BH connected by a wormhole
  - ⋆ Classicaly connected spacetime $\Longleftrightarrow$ Superposition of disconnected ones
  - ⋆ ER=EPR: Any entangled system is connected by a wormhole

DICTIONARY: CHAPTER QI

AdS	CFT
Connectivity	**Entanglement**
Minimal surfaces	S_A
ER	EPR
	Complexity?

Computational complexity

- Computer science: Minimum number of 'simple' operations needed to get a state from a 'simple' reference one. Example: Given a string of K bits, get some state 011010.. from 000000...



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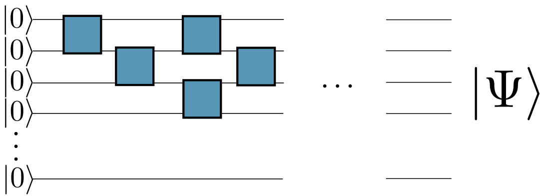

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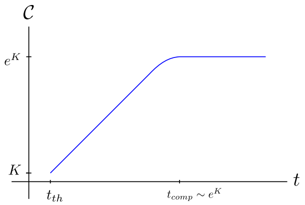
- Growth of complexity
  - ⋆ Quantum complexity keeps growing after thermalization



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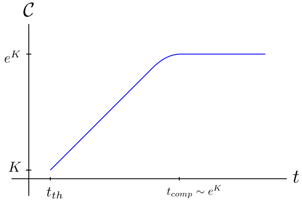
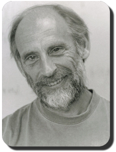
  - ⋆ $\frac{d\mathcal{C}}{dt} \propto TS$

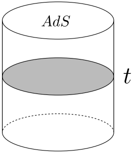


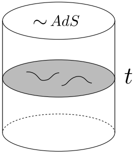
Complexity in the spacetime picture

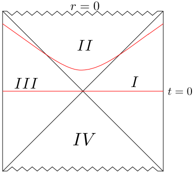


L. Susskind

- Tensor networks
  - ⋆ Complex states have bigger tensor networks
  - ⋆ Complexity ⇔ size of the tensor network
- Volume/complexity relation: The complexity of $|\psi(t)\rangle$ is proportional to the volume of a maximal slice in the dual spacetime that passes through t.
- Black holes
  - ⋆ Growth of quantum complexity is encoded as the growth of the Einstein-Rosen Brigde







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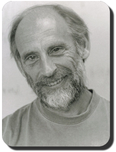

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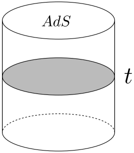

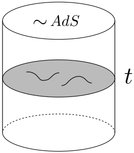

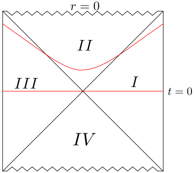

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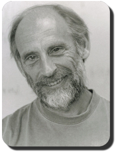

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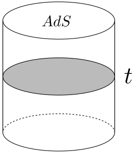
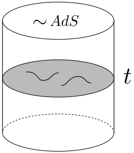
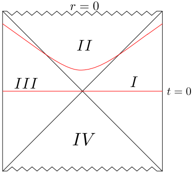






Dictionary:
Chapter QI

AdS	CFT
Connectivity	**Entanglement**
Minimal surfaces	S_A
ER	EPR
Maximal volumes	**Complexity**

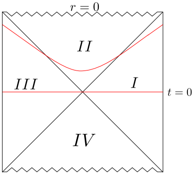
General features

- Topological black holes
  - ⋆ Metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + \frac{r^2}{l^2}dH^2_{d-1}$$

$$f(r) = -1 + \frac{r^2}{l^2} - \frac{\mu}{r^{d-2}}$$

  - ⋆ Dual to two CFT's on a hyperboloid [Emparan]
  - ⋆ Degenerate system: $\lim_{T\to 0} S \neq 0$



- Slicing conditions
  - ⋆ Spacelike Cauchy sufaces
  - ⋆ 'Nice' slices: Stay away from singularities
  - ⋆ Asymptotically match constant t surfaces far away
  - ⋆ Foliation of the entire exterior region
  - ⋆ Asymptote to constant r_m surface in the interior for long times.

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- Topological black holes
  - ⋆ Metric

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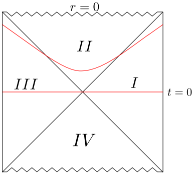

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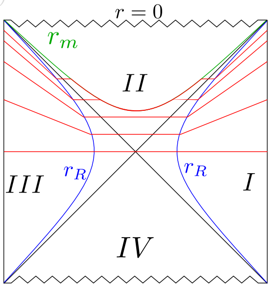
  - ⋆ Spacelike Cauchy sufaces
  - ⋆ 'Nice' slices: Stay away from singularities
  - ⋆ Asymptotically match constant t surfaces far away
  - ⋆ Foliation of the entire exterior region
  - ⋆ Asymptote to constant r_m surface in the interior for long times.

Non-extremal regime

- Features
  - ⋆ $T_H \gg 1$
  - ⋆ One horizon. Schwarzschild-Ads-like topology.
- Metric Patching
  - ⋆ Exterior $(r \gg 1) \Longrightarrow f_E(r) = -1 + r^2 \Longrightarrow$ Constant t surface
  - ⋆ Rindler region $(r \sim r_h) \Longrightarrow f_R(r) = 4\pi T_H(r - r_h) \Longrightarrow$ Horizontal planes (in X,T coordinates)
  - ⋆ Interior $(r \ll r_h)$
- Results
  - ⋆ Exterior: Constant contribution
  - ⋆ Wormhole: $V \sim \sqrt{\frac{d}{1-2^{-1/d}}} G_N S T_H t$



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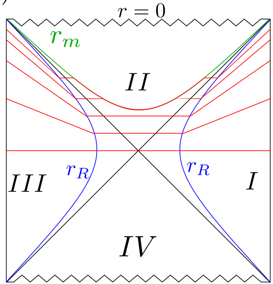

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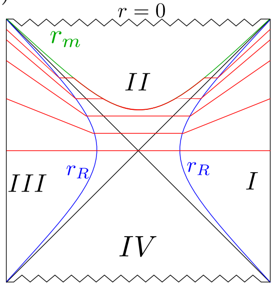

Near-extremal regime

- Features
  - ⋆ $T_H < \frac{1}{2\pi}$
  - ⋆ Two horizons $r_\pm$. Timelike singularity. Degenerate system.
- Metric Patching
  - ⋆ Exterior ($r \gg 1$) $\Longrightarrow$ Constant t surface
  - ⋆ $AdS_{1+1} \times H^{d-1}$ region ($r_h \ll r \ll 1$) $\Longrightarrow$ Constant t surface
  - ⋆ Rindler region ($r \sim r_h$) $\Longrightarrow$ Horizontal planes (in X,T coordinates)
  - ⋆ Interior ($r \ll r_h$)
- Results
  - ⋆ Exterior: Constant contribution
  - ⋆ $AdS_{1+1} \times H^{d-1}$ region: Constant divergent contribution $\sim logT^{-1}$
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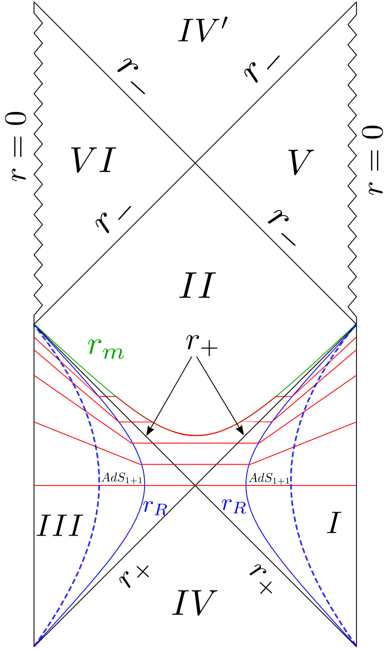
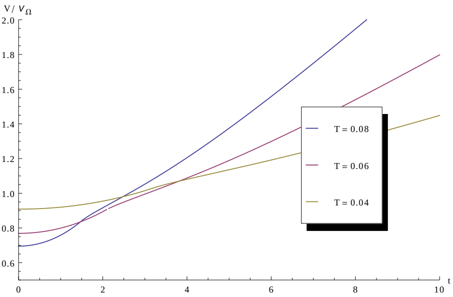
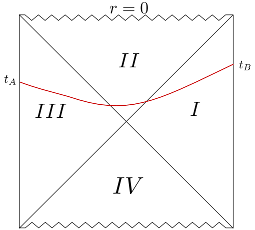

IV'
r_-
r_-
$r=0$
VI
V
$r=0$
r_-
r_-
II
r_m
r_+
AdS_{1+1}
AdS_{1+1}
r_R
r_R
III
I
$r_\times$
$r_\times$
IV


V/ V_Ω
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0
2
4
6
8
10
t
T = 0.08
T = 0.06
T = 0.04

From volume to action

- Unpleasant features of volume/complexity duality
  - ⋆ Arbitrary scale $\mathcal{C} \sim \frac{V}{G\ell}$, $\ell \sim \ell_{AdS}, r_h, \ldots$
  - ⋆ Why should the maximal slice play a preferred role?
- A higher dimensional object might solve the problem
  - ⋆ $\mathcal{C} \sim \frac{V\ell_{AdS}}{G\ell_{AdS}^2} \sim \frac{\mathcal{W}}{G\ell_{AdS}^2} \sim \frac{\Lambda}{G}\int\sqrt{g}dV \sim \mathcal{A}$
  - ⋆ Consider all possible foliations



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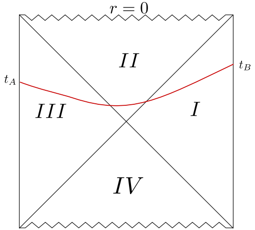

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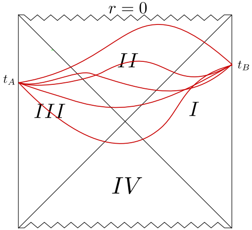

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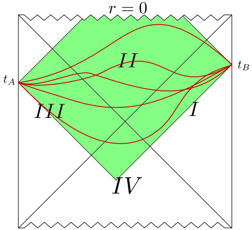

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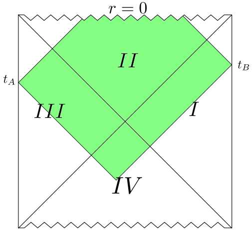
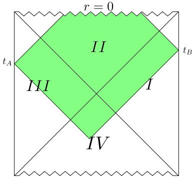


The Action/complexity proposal

- The complexity of $|\psi(t)\rangle$ is given by the on-shell action of its gravitational dual evaluated on the WdW patch corresponding to t.

$$\mathcal{C} = \frac{\mathcal{A}}{\pi\hbar}$$

[Brown, Roberts, Susskind, Swingle, Zhao]



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  - ⋆ Absence of arbitrary scales
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  - ⋆ Recovers the nice features of V/C
    - BH complexity growth: $\frac{d\mathcal{C}}{dt}=2M \sim ST$
    - Shockwave tests [Susskind, Stanford]
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  - ⋆ The YGH term is ill-defined for null surfaces and joints. New prescriptions for these quantities still ambigous and *ad hoc.* [Lehner, Myers, Poisson, Sorkin...]

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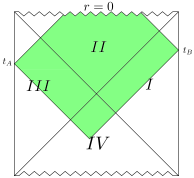

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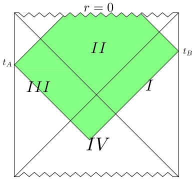

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A curiosity

- 'Extended' black hole thermodynamics

$$\text{Smarr formula: } (d-2)M = (d-1)TS - 2PV$$

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  - ⋆ $\delta S_{\text{bulk}} \sim PV\delta t$
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DICTIONARY:
CHAPTER QI

AdS	CFT
Connectivity	**Entanglement**
Minimal surfaces	S_A
ER	EPR
Maximal volumes	**Complexity**$_a$
WDW action	**Complexity**$_b$

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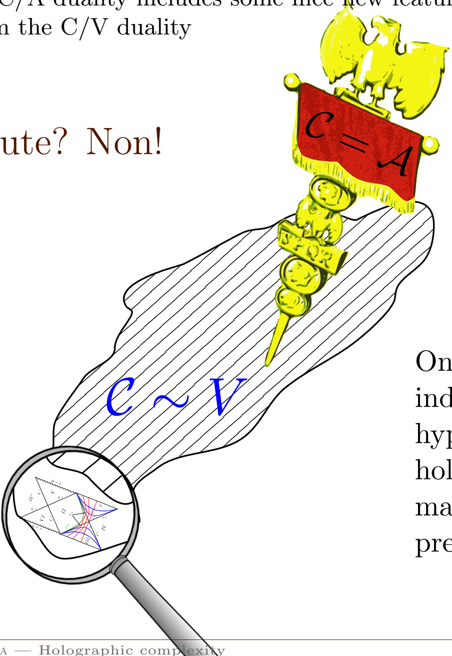
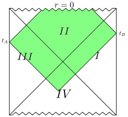
Toute? Non!



One small set of indomitable hyperbolic black holes still resist to match both prescriptions

WdW action for hyperbolic black holes



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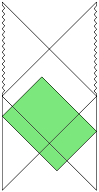

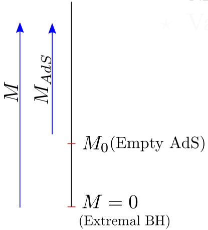

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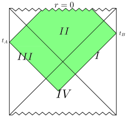

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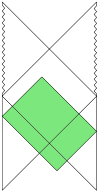

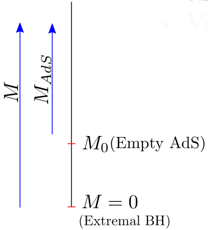

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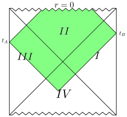

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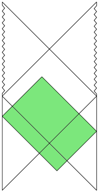

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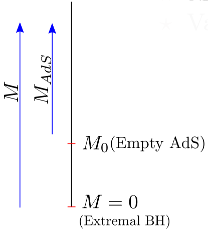

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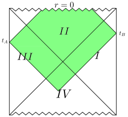

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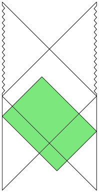

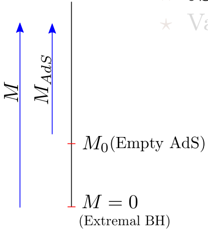

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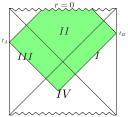

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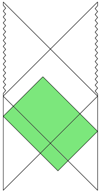
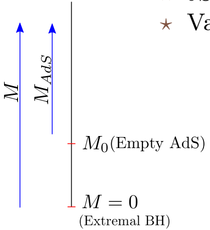
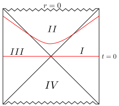
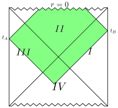




Comparison between both prescriptions





General features	Complexity/Volume	Complexity/Action
Definition	Preferred slicing	Covariant definition
Scale	Ambigous	Fixed
TN interpretation	# of tensors	?
$d\mathcal{C}/dt$ in BH	$\sim ST$	$2M$
Technicalities	Optimization	Boundary terms

Cold hyp. BH		
$\mathcal{C}$ in AdS_{1+1} throat	$\sim \log T$	Finite
$d\mathcal{C}/dt$	$\sim S_0 T$	0

Conclusions & Outlook

- Quantum complexity might be a interesting tool to explore black hole interiors, but its definition is not yet clear
  - ⋆ Need for a precise definition in the continuum CFT
  - ⋆ Decide which of the holographic proposals (if any) is the correct one
- Topological black holes might be a useful diagnostic tool to select the correct prescription
  - ⋆ If C/V duality is correct, we need to explain the exotic properties of the effective CQM vacuum
  - ⋆ If C/A duality is correct, we need to explain why cold hyperbolic black holes do not compute

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Questions

Muito obrigado!