On SO(3)-gauged maximal d=8 supergravities

Óscar Lasso Andino

Instituto de Física Teórica UAM-CSIC
Universidad Autónoma de Madrid

19/01/2017

Iberian Strings 2017
Lisbon - Portugal

A work in collaboration with Tomás Ortín.
Based on arXiv:1605.05882 arXiv:1605.09629.

Overview

1. Introduction.
2. Abelian deformations.
   - First abelian deformation.
3. Summary of relations for the generic ungauged, massles Abelian theory
4. Non-Abelian and massive deformations: the tensor hierarchy.
   - The embedding tensor formalism.
   - Gauging the global symmetries using the embedding tensor.
5. Summary of relations for the gauged theory
6. Consistency.
   - EOMs. and Bianchi identities
7. The 8-dimensional supergravities with SO(3) gaugings
   - Supergravity in 11-dimensions.
   - Supergravity in 8-dimensions

Goals

- The construction of a generic (up to second order in derivatives) 8-dimensional theories with Abelian gauge symmetry and non-trivial Chern-Simons terms compatible with the existence of a group of electric-magnetic duality rotations of the equations of motion(in 8 dimensions it must be a subgroup of the symplectic group). There are previous works in different dimensions,d=3,d=4,d=5,d=6,d=9. Bergshoeff, Hartong, Hohm, Hubscher, Ortín, 2009. Hartong, Hohm, Hubscher, Ortín, 2009. Hubscher, Ortín, Shahbazi, 2014. Hubscher, Ortín, Shahbazi, 2014. Fernandez-Melgarejo,Ortín, Torrente-Lujan, 2012.

Goals

- The construction of a generic (up to second order in derivatives) 8-dimensional theories with Abelian gauge symmetry and non-trivial Chern-Simons terms compatible with the existence of a group of electric-magnetic duality rotations of the equations of motion(in 8 dimensions it must be a subgroup of the symplectic group). There are previous works in different dimensions,d=3,d=4,d=5,d=6,d=9. Bergshoeff, Hartong, Hohm, Hubscher, Ortín, 2009. Hartong, Hohm, Hubscher, Ortín, 2009. Hubscher, Ortín, Shahbazi, 2014. Hubscher, Ortín, Shahbazi, 2014. Fernandez-Melgarejo,Ortín, Torrente-Lujan, 2012.

- The general gauging of the global symmetry group using the embedding-tensor fomalism including the possibility of adding Stückelberg couplings consistent with the above-mentioned electric-magnetic duality.

Goals

- The construction of a generic (up to second order in derivatives) 8-dimensional theories with Abelian gauge symmetry and non-trivial Chern-Simons terms compatible with the existence of a group of electric-magnetic duality rotations of the equations of motion(in 8 dimensions it must be a subgroup of the symplectic group). There are previous works in different dimensions,d=3,d=4,d=5,d=6,d=9. Bergshoeff, Hartong, Hohm, Hubscher, Ortín, 2009. Hartong, Hohm, Hubscher, Ortín, 2009. Hubscher, Ortín, Shahbazi, 2014. Hubscher, Ortín, Shahbazi, 2014. Fernandez-Melgarejo,Ortín, Torrente-Lujan, 2012.

- The general gauging of the global symmetry group using the embedding-tensor fomalism including the possibility of adding Stückelberg couplings consistent with the above-mentioned electric-magnetic duality.

- A simplification/sitematization of the construction of maximal 8-dimensional supergravities with SO(3) gaugings. Salam and Sezgin (1985), Alonso-Alberca,Messen, Ortín (2000), Alonso-Alberca,Bergshoeff, Gran, Linares, Ortín (2003)

Ungauged $d = 8$ theories

The field content of a generic $d = 8$ theory:

Ungauged $d = 8$ theories

The field content of a generic $d = 8$ theory:

- the metric $g_{\mu\nu}$,
- scalar fields ϕ^x,
- 1-form fields $A^I = A^I{}_\mu dx^\mu$,
- 2-form fields $B_m = \frac{1}{2} B_{m\,\mu\nu} dx^\mu \wedge dx^\nu$ and
- 3-form fields $C^a = \frac{1}{3!} C^a{}_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho$.

The way

What is the simplest theory one can construct with these fields?.

The simplest field strengths are the exterior derivatives:

$$F^I \equiv dA^I\,, \qquad H_m \equiv dB_m\,, \qquad G^a \equiv dC^a\,.$$

The field content of a generic $d = 8$ theory:

- the metric $g_{\mu\nu}$,
- scalar fields ϕ^x,
- 1-form fields $A^I = A^I{}_\mu dx^\mu$,
- 2-form fields $B_m = \frac{1}{2} B_{m\,\mu\nu} dx^\mu \wedge dx^\nu$ and
- 3-form fields $C^a = \frac{1}{3!} C^a{}_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho$.

The way

What is the simplest theory one can construct with these fields?.

The simplest field strengths are the exterior derivatives:

$$F^I \equiv dA^I\,, \qquad H_m \equiv dB_m\,, \qquad G^a \equiv dC^a\,.$$

The most general gauge-invariant action is:

$$S = \int \left\{ \star 1 R + \tfrac{1}{2} \mathscr{G}_{xy} d\phi^x \wedge \star d\phi^y - \tfrac{1}{2} \mathscr{M}_{IJ} F^I \wedge \star F^J + \tfrac{1}{2} \mathscr{M}^{mn} H_m \wedge \star H_n \right.$$

$$\left. - \tfrac{1}{2} \mathfrak{Im}\mathscr{N}_{ab} G^a \wedge \star G^b - \tfrac{1}{2} \mathfrak{Re}\mathscr{N}_{ab} G^a \wedge G^b \right\}\,,$$

where the kinetic matrices $\mathscr{G}_{xy}, \mathscr{M}_{IJ}, \mathscr{M}^{mn}, \mathfrak{Im}\mathscr{N}_{ab}$ as well as the matrix $\mathfrak{Re}\mathscr{N}_{ab}$ are scalar-dependent.

Equations of motion

The equations of motion of the 3-forms C^a are

$$\frac{\delta S}{\delta C^a} = -d\frac{\delta S}{\delta G^a} = 0\,, \qquad \frac{\delta S}{\delta G^a} = R_a \equiv -\mathfrak{Re}\mathcal{N}_{ab}G^b - \mathfrak{Im}\mathcal{N}_{ab}\star G^b\,.$$

The equations of motion of the 3-forms C^a are

$$\frac{\delta S}{\delta C^a} = -d\frac{\delta S}{\delta G^a} = 0\,, \qquad \frac{\delta S}{\delta G^a} = R_a \equiv -\Re\mathfrak{e}\mathcal{N}_{ab}G^b - \Im\mathfrak{m}\mathcal{N}_{ab}\star G^b\,.$$

These equations can be solved locally by introducing a set of dual 3-forms C_a.

$$dC_a \equiv R_a\,.$$

Moreover, we can built a vector containing the fundamental and dual 3-forms:

$$(C^i) \equiv \begin{pmatrix} C^a \\ C_a \end{pmatrix}, \qquad G^i \equiv dC^i\,,$$

so that the equations of motion and the Bianchi identities for the fundamental field strengths take the simple form

$$dG^i = 0\,.$$

First abelian deformation.

$$G^a = dC^a + d^a{}_l{}^m F^l B_m\,,$$

$$\delta_\sigma A^l = d\sigma^l\,, \qquad \delta_\sigma B_m = d\sigma_m\,, \qquad \delta_\sigma C^a = d\sigma^a - d^a{}_l{}^m F^l \sigma_m\,.$$

First abelian deformation.

$$G^a = dC^a + d^a{}_l{}^m F^l B_m\,,$$

$$\delta_\sigma A^l = d\sigma^l\,, \qquad \delta_\sigma B_m = d\sigma_m\,, \qquad \delta_\sigma C^a = d\sigma^a - d^a{}_l{}^m F^l \sigma_m\,.$$

Problems!

- The action remains gauge-invariant but the formal symplectic invariance is broken: if we do not modify the action, the dual 4-form field strengths are just $G_a = dC_a$ and $\mathrm{Sp}(2n_3, \mathbb{R})$ cannot rotate these into G^a
- Furthermore, the 1-form and 2-form equations of motion do not have a symplectic-invariant form.

First abelian deformation.

$$G^a = dC^a + d^a{}_l{}^m F^l B_m \,,$$

$$\delta_\sigma A^l = d\sigma^l \,, \qquad \delta_\sigma B_m = d\sigma_m \,, \qquad \delta_\sigma C^a = d\sigma^a - d^a{}_l{}^m F^l \sigma_m \,.$$

Problems!

- The action remains gauge-invariant but the formal symplectic invariance is broken: if we do not modify the action, the dual 4-form field strengths are just $G_a = dC_a$ and $\mathrm{Sp}(2n_3, \mathbb{R})$ cannot rotate these into G^a
- Furthermore, the 1-form and 2-form equations of motion do not have a symplectic-invariant form.

Solution:Add a CS term to the action

$$S_{CS} = \int \{-d_{al}{}^m dC^a F^l B_m\} \,,$$

Many things change

New equation of motion:

$$-d\frac{\delta S}{\delta dC^a} = 0\,, \qquad \frac{\delta S}{\delta dC^a} = R_a - d_{al}{}^m F^l B_m\,.$$

Many things change

New equation of motion:

$$-d\frac{\delta S}{\delta dC^a} = 0\,, \qquad \frac{\delta S}{\delta dC^a} = R_a - d_{al}{}^m F^l B_m\,.$$

The local solution is now

$$dC_a \equiv R_a - d_{al}{}^m F^l B_m\,,$$

Many things change

New equation of motion:

$$-d\frac{\delta S}{\delta dC^a}=0\,,\qquad\qquad \frac{\delta S}{\delta dC^a}=R_a-d_{al}{}^mF^lB_m\,.$$

The local solution is now

$$dC_a\equiv R_a-d_{al}{}^mF^lB_m\,,$$

The dual, gauge-invariant, field strength now is:

$$R_a=dC_a+d_{al}{}^mF^lB_m\equiv G_a\,.$$

$(C^i)=\begin{pmatrix}C^a\\C_a\end{pmatrix}$ transforms linearly as a symplectic vector if $(d^i{}_l{}^m)\equiv\begin{pmatrix}d^a{}_l{}^m\\d_{al}{}^m\end{pmatrix}$ also does.

We can define the symplectic vector of 4-form field strengths

$$G^i = dC^i + d^i{}_l{}^m F^l B_m \,,$$

We can define the symplectic vector of 4-form field strengths

$$G^i = dC^i + d^i{}_l{}^m F^l B_m \,,$$

invariant under the deformed gauge transformations

$$\delta_\sigma A^l = d\sigma^l \,, \qquad \delta_\sigma B_m = d\sigma_m \,, \qquad \delta_\sigma C^i = d\sigma^i - d^i{}_l{}^m F^l \sigma_m \,.$$

We can define the symplectic vector of 4-form field strengths

$$G^i = dC^i + d^i{}_l{}^m F^l B_m \,,$$

invariant under the deformed gauge transformations

$$\delta_\sigma A^l = d\sigma^l \,, \qquad \delta_\sigma B_m = d\sigma_m \,, \qquad \delta_\sigma C^i = d\sigma^i - d^i{}_l{}^m F^l \sigma_m \,.$$

Problem!

The deformed gauge transformations do not leave invariant the CS term.

We can define the symplectic vector of 4-form field strengths

$$G^i = dC^i + d^i{}_I{}^m F^I B_m \,,$$

invariant under the deformed gauge transformations

$$\delta_\sigma A^I = d\sigma^I \,, \qquad \delta_\sigma B_m = d\sigma_m \,, \qquad \delta_\sigma C^i = d\sigma^i - d^i{}_I{}^m F^I \sigma_m \,.$$

Problem!

The deformed gauge transformations do not leave invariant the CS term.

Solution

Add another term of the form

$$S_{CS} = \int \{ -d_{aI}{}^m dC^a F^I B_m - \tfrac{1}{2} d_{aI}{}^m d^a{}_J{}^m F^{IJ} B_{mn} \} \,,$$

We can define the symplectic vector of 4-form field strengths

$$G^i = dC^i + d^i{}_I{}^m F^I B_m \,,$$

invariant under the deformed gauge transformations

$$\delta_\sigma A^I = d\sigma^I \,, \qquad \delta_\sigma B_m = d\sigma_m \,, \qquad \delta_\sigma C^i = d\sigma^i - d^i{}_I{}^m F^I \sigma_m \,.$$

Problem!

The deformed gauge transformations do not leave invariant the CS term.

Solution

Add another term of the form

$$S_{CS} = \int \{-d_{aI}{}^m dC^a F^I B_m - \tfrac{1}{2} d_{aI}{}^m d^a{}_J{}^m F^{IJ} B_{mn}\} \,,$$

Constraints

$$d_{a(I}{}^{[m} d^a{}_{J)}{}^{m]} = 0 \,, \text{ so } d_{i(I}{}^{(m} d^i{}_{J)}{}^{m)} = 0 \,.$$

EOM for the 1-forms

Using the duality relation $R_a = G_a$ the equations of motion of the 1-forms can be written in the form

$$\frac{\delta S}{\delta A^I} = d\left\{\mathscr{M}_{IJ} \star F^J + d_{iI}{}^m G^i B_m + \tfrac{1}{2} d_{iI}{}^m d^i{}_J{}^m F^J B_{mn}\right\} = 0\,,$$

EOM for the 1-forms

Using the duality relation $R_a = G_a$ the equations of motion of the 1-forms can be written in the form

$$\frac{\delta S}{\delta A^I} = d\left\{\mathcal{M}_{IJ} \star F^J + d_{iI}{}^m G^i B_m + \tfrac{1}{2} d_{iI}{}^m d^i{}_J{}^m F^J B_{mn}\right\} = 0\,,$$

The solution

$$\tilde{F}_I \equiv d\tilde{A}_I + d_{iI}{}^m G^i B_m + \tfrac{1}{2} d_{iI}{}^m d^i{}_J{}^m F^J B_{mn}\,,$$

$$\tilde{F}_I = -\mathcal{M}_{IJ} \star F^J\,,$$

$$d\tilde{F}_I = d_{iI}{}^m G^i H_m\,,$$

where $\tilde{A}_I$ is a set of 5-forms.

EOM for the 1-forms

Using the duality relation $R_a = G_a$ the equations of motion of the 1-forms can be written in the form

$$\frac{\delta S}{\delta A^I} = d\left\{\mathscr{M}_{IJ}\star F^J + d_{iI}{}^m G^i B_m + \tfrac{1}{2} d_{iI}{}^m d^i{}_J{}^m F^J B_{mn}\right\} = 0\,,$$

The solution

$$\begin{aligned}
\tilde{F}_I &\equiv d\tilde{A}_I + d_{iI}{}^m G^i B_m + \tfrac{1}{2} d_{iI}{}^m d^i{}_J{}^m F^J B_{mn}\,,\\
\tilde{F}_I &= -\mathscr{M}_{IJ}\star F^J\,,\\
d\tilde{F}_I &= d_{iI}{}^m G^i H_m\,,
\end{aligned}$$

where $\tilde{A}_I$ is a set of 5-forms.

Then:

$$\frac{\delta S}{\delta A^I} = -\left\{d\tilde{F}_I - d_{iI}{}^m G^i H_m\right\}.$$

EOM for the 2-forms

$$\frac{\delta S}{\delta B_m} = -\left\{ d\tilde{H}^m + d_{il}{}^m G^i F^l \right\}.$$

The Solution

Using the duality relation $R_a = G_a$ and following the same steps for the 2-forms , we find

$$\begin{aligned}
\tilde{H}^m &= d\tilde{B}^m + d^i{}_l{}^m F^l C_i \,, \\
\tilde{H}^m &= \mathscr{M}^{mn} \star H_n \,, \\
d\tilde{H}^m &= -d_{il}{}^m G^i F^l \,,
\end{aligned}$$

EOM for the 2-forms

$$\frac{\delta S}{\delta B_m} = -\left\{ d\tilde{H}^m + d_{il}{}^m G^i F^l \right\}.$$

The Solution

Using the duality relation $R_a = G_a$ and following the same steps for the 2-forms , we find

$$\begin{aligned} \tilde{H}^m &= d\tilde{B}^m + d^i{}_l{}^m F^l C_i \,, \\ \tilde{H}^m &= \mathscr{M}^{mn} \star H_n \,, \\ d\tilde{H}^m &= -d_{il}{}^m G^i F^l \,, \end{aligned}$$

This completes the first abelian deformation!

Field strengths

$$
\begin{aligned}
F^I &= dA^I \,. \\
H_m &= dB_m - d_{mIJ} F^I A^J \,, \\
G^i &= dC^i + d^i{}_I{}^m F^I B_m - \tfrac{1}{3} d^i{}_I{}^m d_{mJK} A^I F^J A^K \,, \\
\tilde{H}^m &= d\tilde{B}^m + d^i{}_I{}^m C_i F^I + d^{mnp} B_n (H_p + \Delta H_p) + \tfrac{1}{12} d^i{}_I{}^m d_{iJ}{}^n A^{IJ} \Delta H_n \,, \\
\tilde{F}_I &= d\tilde{A}_I + 2 d_{mIJ} A^J (\tilde{H}_m - \tfrac{1}{2}\Delta \tilde{H}_m) - \left(d^i{}_I{}^m B_m - \tfrac{1}{3} d^i{}_J{}^m d_{mIK} A^{JK} \right) (G_i - \tfrac{1}{2} \Delta G_i) \\
& - \tfrac{1}{3} \left(d^i{}_I{}^m d_{mJK} - d^i{}_K{}^m d_{mIJ} \right) F^J A^K C_i - d^{mnp} d_{mIJ} A^J B_n H_p \\
& + \tfrac{1}{24} \left(d^i{}_K{}^m d_{iL}{}^n d_{mIJ} + 2 d^i{}_{[I|}{}^m d_{i|K]}{}^n d_{mJL} \right) F^J A^{KL} B_n + \tfrac{1}{24} d^i{}_J{}^m d_{iK}{}^n d_{mIL} A^{JKL} dB_n \\
& - \tfrac{1}{180} d^i{}_L{}^n d_{iQ}{}^m d_{mIJ} d_{nPK} A^{JKLQ} F^P \,,
\end{aligned}
$$

Gauging the global symmetries of the theory

The most general possibilities can be explored using the embedding tensor formalism

Cordaro, Fré,Gualtieri, Termonia and Trigiante (1998). Nicolai and Samtleben (2001).De Wit and Samtleben (2001). De Wit, Samteblen and Trigiante (2003)

Bonus

The tensor hierarchy

De Wit and Samtleben (2005). De Wit, Nicolai and Samteblen (2008). Bergshoeff, hartong, Hohm,Huubscher and Ortín (2009). De Wit and Zalk (2009)

- It turns out that all couplings that deform an ungauged supergravity into a gauged one, can be given in terms of the embedding tensor.
- Gauged supergravities are classified by the embedding tensor, subject to a number of algebraic or group-theoretical constraints.
- The embedding tensor ${\Theta_M}^{\alpha}$ pairs the generators t_{α} of the group G with the vector fields ${A_{\mu}}^M$ used for the gauging.

$$ {A_{\mu}}^M {\Theta_M}^{\alpha} , $$

- The d-tensors $d_{mIJ}, {d^i}_I{}^m, d^{mnp}$ are invariant under the global symmetry group.

We promote the global parameters α^A to local ones $\alpha^A(x)$ and we make the identifications:

The embedding tensor and the global parameters

$$\alpha^A \equiv \sigma^I \vartheta_I{}^A .$$

The embedding tensor and the $1-forms$

The gauge fields for these symmetries are given by

$$A^A \equiv A^I \vartheta_I{}^A .$$

The derivatives transform covariantly under gauge transformations $\delta_\sigma = \sigma^I \vartheta_I{}^A \delta_A$ provided that the embedding tensor is gauge-invariant

$$\delta_\sigma \vartheta_I{}^A = 0\,,$$

and provided that the 1-forms transform as

$$\delta_\sigma A^I = \mathscr{D}\sigma^I + \Delta A^I\,, \quad \text{where} \quad \left\{ \begin{array}{rcl} \Delta A^I \vartheta_I{}^A & = & 0\,, \\ \mathscr{D}\sigma^I & = & d\sigma^I - A^J X_J{}^I{}_K \sigma^K\,, \end{array} \right.$$

The gauge invariance of the embedding tensor leads to the so-called *quadratic constraint*

$$\vartheta_J{}^B \left[T_B{}^K{}_I \vartheta_K{}^A - f_{BC}{}^A \vartheta_I{}^C \right] = 0\,.$$

To determine ΔA^I we have to construct the gauge-covariant 2-form field strengths F^I.

Field Strengths

$$F^I = dA^I - \tfrac{1}{2} X_J{}^I{}_K A^{JK} + Z^{I\,m} B_m\,,$$

$$H_m = \mathscr{D} B_m - d_{mIJ} dA^I A^J + \tfrac{1}{3} X_J{}^M{}_K A^{IJK} + Z_{im} C^i\,,$$

$$G^i = \mathscr{D} C^i + d^i{}_I{}^n \left[F^I B_n - \tfrac{1}{2} Z^{I\,p} B_n B_p + \tfrac{1}{3} d_{nJK} dA^J A^{KI} + \tfrac{1}{12} d_{mMJ} X_K{}^M{}_L A^{IJKL} \right] - Z_{im} \tilde{H}^m$$

$$\tilde{H}^m = \mathscr{D} \tilde{B}^m - d_{iI}{}^m F^I C^i + d^{mnp} B_n \left(H_p + \Delta H_p - 2 Z_{ip} C^i \right)$$

$$+ d^m{}_{IJK} dA^I dA^J A^K$$

$$+ \left(\tfrac{1}{12} d_{iJ}{}^m d^j{}_K{}^n d_{nIL} - \tfrac{3}{4} d^m{}_{IJM} X_K{}^M{}_L \right) dA^I A^{JKL}$$

$$+ \left(\tfrac{3}{20} d^m{}_{NPM} X_I{}^N{}_J - \tfrac{1}{60} d_{iM}{}^m d^i_I{}^n d_{nPJ} \right) X_K{}^P{}_L A^{IJKLM}$$

$$+ Z^{Im} \tilde{A}_I\,,$$

Constraints

- First of all we have the gauge-invariance constraints

$$\mathcal{Q}_{IJ}{}^{A}\,,\ \mathcal{Q}_{I}{}^{Jm}\,,\ \mathcal{Q}_{Iim}\,,$$

- Secondly, we have the global-invariance constraints

$$\mathcal{Q}_{AmIJ}\,,\ \mathcal{Q}_{A}{}^{i}{}_{I}{}^{m}\,,$$

- Thridly we have the orthogonality constraints between the three deformation tensors

$$\begin{aligned}
\mathcal{Q}^{mA} &\equiv -Z^{Im}\vartheta_{I}{}^{A}\,,\\
\mathcal{Q}_{i}{}^{I} &\equiv Z_{im}Z^{Im}\,,\\
\mathcal{Q}_{mn} &\equiv Z_{im}Z^{i}{}_{n}\,.
\end{aligned}$$

- Next, we have the constraints relating the gauge transformations to the d-tensors

$$\mathcal{Q}_{I}{}^{J}{}_{K} \equiv X_{(I}{}^{J}{}_{K)} - Z^{Km} d_{mIJ}\,,$$

$$\mathcal{Q}_{I}{}^{m}{}_{n} \equiv X_{I}{}^{m}{}_{n} + 2 d_{mIJ} Z^{Jn} + Z_{im} d^{i}{}_{I}{}^{m}\,,$$

$$\mathcal{Q}_{Iij} \equiv -X_{Iij} - 2 Z_{(i|m} d_{|j)I}{}^{m}\,,$$

- Finally, we have the constraints that related the d-tensors amongst them

$$\mathcal{Q}^{imn} \equiv d^{i}{}_{I}{}^{[m|} Z^{I|n]} + Z^{i}{}_{p} d^{pmn}\,,$$

$$\mathcal{Q}_{IJ}{}^{mn} \equiv \tfrac{1}{2} d^{i}{}_{(I|}{}^{m} d_{i|J)}{}^{n} + d^{mnp} d_{pIJ} + 3 d^{[m|}{}_{IJK} Z^{K|n]}\,,$$

$$\mathcal{Q}_{iIJK} \equiv Z_{im} d^{m}{}_{IJK} - d_{i(I|}{}^{m} d_{m|JK)}\,.$$

EOMs and Bianchi identities

$$\frac{\delta S}{\delta B_m} = \mathscr{B}(B_m)$$
$$\frac{\delta S}{\delta C^i} = \mathscr{B}(C_i)$$

$$\frac{\delta S}{\delta A^I} = B(\tilde{A}_I) + \mathscr{B}(B_m) A^K d_{mKI} - \mathscr{B}(C_i)(-d_I^i{}^n B_n + \frac{1}{3} d^i{}_J{}^n d_{nIK} A^{JK})$$

$$\upsilon_I^A k_A{}^x(\phi) \frac{\delta S}{\delta \phi^x} = \upsilon_I^A (\mathscr{B}(\phi^x) + \mathscr{B}(C_i) T_A{}^j{}_i C_j + \mathscr{B}(B_m) T_I{}^n{}_m B_n)$$

where

$$\upsilon_I^A \mathscr{B}(\phi^x) = \upsilon_I^A (\mathscr{D} K_A + T_A{}^I{}_J F^J \tilde{F}_I + T_A{}^m{}_n \tilde{H}^n H_m - \frac{1}{2} T_{A_{ij}} G^{ij})$$

$$B(\tilde{A}_I) = -\mathscr{D}\tilde{F}_I + 2 d_{mIJ} F^J \tilde{H}^m + d_i{}^m{}_I G^i H_m - 3 d^m{}_{IJK} F^{JK} H_m + \upsilon_I^A K_A$$
$$\mathscr{B}(B_m) = -\mathscr{D}\tilde{H}^m - d_j{}^m{}_I G^j F^I + d^{mnp} H_{np} + d^m{}_{IJK} F^{IJK} + Z^{\mathrm{Im}} \tilde{F}_I$$
$$\mathscr{B}(C_i) = -\mathscr{D} G_i + d_i{}^m{}_I F^I H_m - Z_{im} \tilde{H}^m$$

The 8-dimensional supergravities with SO(3) gaugings

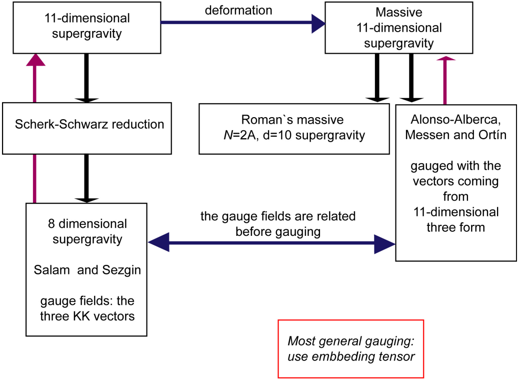
- All these theories will be equivalent from an 8-dimensional point of view: they are all related by $SL(2,\mathbb{R})$ duality transformations that can be understood as a different changes of variables. Dibietto, Fernández-Melgarejo, Marquéz, Roest (2012).

- All these theories will be equivalent from an 8-dimensional point of view: they are all related by $SL(2,\mathbb{R})$ duality transformations that can be understood as a different changes of variables. Dibietto, Fernández-Melgarejo, Marquéz, Roest (2012).
- The simplest mechanical procedure to obtain them from 11-dimensional supergravity would be to perform the standard Scherk-Schwarz reduction that gives an 8-dimensional SO(3)-gauged maximal supergravity in which the 3 Kaluza-Klein vectors play the role of gauge fields and then perform the $SL(2,\mathbb{R})$ duality transformations mentioned above. Salam and Sezgin (1985).

- All these theories will be equivalent from an 8-dimensional point of view: they are all related by $SL(2,\mathbb{R})$ duality transformations that can be understood as a different changes of variables. Dibietto, Fernández-Melgarejo, Marquéz, Roest (2012).
- The simplest mechanical procedure to obtain them from 11-dimensional supergravity would be to perform the standard Scherk-Schwarz reduction that gives an 8-dimensional SO(3)-gauged maximal supergravity in which the 3 Kaluza-Klein vectors play the role of gauge fields and then perform the $SL(2,\mathbb{R})$ duality transformations mentioned above. Salam and Sezgin (1985).

Problem!

Technically complicated (because of electric-magnetic duality). The Kaluza-Klein triplet of vector fields are the first component of a $SL(2,\mathbb{R})$ doublet and, after the duality transformations, the gauge fields are no longer the first component of that doublet, but a general linear combination of the first and the second.



Salam and Sezgin, (1985),
Alonso-Alberca,Messen, Ortín, (2001),
Alonso-Alberca, Bergshoeff, Gran, Linares, Ortín, Roest, (2003),
Puigdomènech, de Roo, (2008).

Cremmer and Julia (1978)
The bosonic fields of $N=1, d=11$ supergravity are:

$$\left\{ \hat{\hat{e}}_{\hat{\hat{\mu}}}{}^{\hat{\hat{a}}}, \hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}} \right\}.$$

The field strength of the 3-form is

$$\hat{\hat{G}} = 4\partial\,\hat{\hat{C}}\,,$$

and is obviously invariant under the gauge transformations

$$\delta\,\hat{\hat{C}} = 3\partial\,\hat{\hat{\chi}}\,,$$

where $\hat{\hat{\chi}}$ is a 2-form.
The action for these bosonic fields is

$$\hat{\hat{S}} = \int d^{11}\hat{\hat{x}}\sqrt{|\hat{\hat{g}}|}\left[\hat{\hat{R}} - \tfrac{1}{2\cdot 4!}\,\hat{\hat{G}}^2 - \tfrac{1}{6^4}\,\frac{1}{\sqrt{|\hat{\hat{g}}|}}\,\hat{\hat{\varepsilon}}\partial\,\hat{\hat{C}}\partial\,\hat{\hat{C}}\,\hat{\hat{C}}\right].$$

.

$\mathscr{N} = 2,\ d = 8$ supergravity

- $\mathscr{N} = 2$, $d = 8$ supergravity can be obtained by direct dimensional reduction of 11-dimensional supergravity on T^3.

$\mathcal{N} = 2,\ d = 8$ supergravity

- $\mathcal{N} = 2$, $d = 8$ supergravity can be obtained by direct dimensional reduction of 11-dimensional supergravity on T^3.

- The scalars of the theory parametrize the coset spaces $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ and $\mathrm{SL}(3,\mathbb{R})/\mathrm{SO}(3)$ and the U-duality group of the theory is $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(3,\mathbb{R})$ and its fields are either invariant or transform in the fundamental representations of both groups.

$\mathcal{N} = 2,\ d = 8$ supergravity

- $\mathcal{N} = 2,\ d = 8$ supergravity can be obtained by direct dimensional reduction of 11-dimensional supergravity on T^3.

- The scalars of the theory parametrize the coset spaces $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ and $\mathrm{SL}(3,\mathbb{R})/\mathrm{SO}(3)$ and the U-duality group of the theory is $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(3,\mathbb{R})$ and its fields are either invariant or transform in the fundamental representations of both groups.

- We use the indices $i, j, k = 1, 2$ for $\mathrm{SL}(2,\mathbb{R})$ doublets and $m, n, p = 1, 2, 3$ for $\mathrm{SL}(3,\mathbb{R})$ triplets.

$\mathcal{N} = 2,\ d = 8$ supergravity

- $\mathcal{N} = 2,\ d = 8$ supergravity can be obtained by direct dimensional reduction of 11-dimensional supergravity on T^3.

- The scalars of the theory parametrize the coset spaces $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ and $\mathrm{SL}(3,\mathbb{R})/\mathrm{SO}(3)$ and the U-duality group of the theory is $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(3,\mathbb{R})$ and its fields are either invariant or transform in the fundamental representations of both groups.

- We use the indices $i, j, k = 1, 2$ for $\mathrm{SL}(2,\mathbb{R})$ doublets and $m, n, p = 1, 2, 3$ for $\mathrm{SL}(3,\mathbb{R})$ triplets.

- The bosonic fields are

$$g_{\mu\nu}, C, B_m, A^{i\,m}, a, \varphi, \mathcal{M}_{mn},$$

where C is a 3-form, B_m a triplet of 2-forms, $A^{i\,m}$, a doublet of triplets of 1-forms (six in total), a and φ are the axion and dilaton fields which can be combined into the axidilaton field

$$\tau \equiv a + ie^{-\varphi},$$

The bosonic action is:

$$
\begin{aligned}
S \quad = \quad & \int \Big\{ -\star R + \tfrac{1}{4}\mathrm{Tr}\left(d\mathscr{M}\mathscr{M}^{-1}\wedge\star d\mathscr{M}\mathscr{M}^{-1}\right) + \tfrac{1}{4}\mathrm{Tr}\left(d\mathscr{W}\mathscr{W}^{-1}\wedge\star d\mathscr{W}\mathscr{W}^{-1}\right) \\
& + \tfrac{1}{2}\mathscr{W}_{ij}\mathscr{M}_{mn}F^{im}\wedge\star F^{jn} + \tfrac{1}{2}\mathscr{M}^{mn}H_m\wedge\star H_n + \tfrac{1}{2}e^{-\varphi}G^1\wedge\star G^1 - \tfrac{1}{2}aG^1G^1 \\
& + \tfrac{1}{3}G^1\left[H_mA^{2m} - B_mF^{2m} + \tfrac{1}{2}\varepsilon_{mnp}F^{2m}A^{1n}A^{2p}\right] \\
& + \tfrac{1}{3}H_mF^{2m}\left[C^1 + \tfrac{1}{6}\varepsilon_{mnp}A^{1m}A^{1n}A^{2p}\right] \\
& + \tfrac{1}{3!}\varepsilon^{mnp}H_mH_n\left(B_p - \tfrac{1}{2}\varepsilon_{pqr}A^{1q}A^{2r}\right)\Big\} \,.
\end{aligned}
$$

and the field strengths

$$
\begin{aligned}
F^{im} &= dA^{im}\,, \\
H_m &= dB_m + \tfrac{1}{2}\varepsilon_{ij}\varepsilon_{mnp}F^{in}A^{jp}\,, \\
G^1 &= dC^1 + F^{1m}B_m + \tfrac{1}{6}\varepsilon_{ij}\varepsilon_{mnp}A^{1m}F^{in}A^{jp}\,.
\end{aligned}
$$

- The only structure constants that we need to know explicitly are those of the $SO(3)$ subgroup

$$[T_m, T_n] = f_{mn}{}^p T_p = -\varepsilon_{mn}{}^p T_p \,,$$

- The indices $I, J, \ldots$ must be replaced by composite indices im, jn etc. where $i, j, \ldots = 1, 2$ and $m, n, \ldots = 1, 2, 3$ are indices in the fundamental representations of $\mathrm{SL}(2, \mathbb{R})$ and $\mathrm{SL}(3, \mathbb{R})$, respectively.

The SO(3) gaugings of $\mathcal{N}=2$, $d=8$ supergravity

- The only structure constants that we need to know explicitly are those of the $SO(3)$ subgroup

$$[T_m, T_n] = f_{mn}{}^p T_p = -\varepsilon_{mn}{}^p T_p\,,$$

- The indices $I, J, \dots$ must be replaced by composite indices im, jn etc. where $i, j, \dots = 1,2$ and $m, n, \dots = 1,2,3$ are indices in the fundamental representations of $\mathrm{SL}(2,\mathbb{R})$ and $\mathrm{SL}(3,\mathbb{R})$, respectively.
- In the electric 3-forms the lower index 1 is equivalent to an upper index 2: $C_1 = \varepsilon_{12} C^2 = C^2$ and, therefore $(C^i) = \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} = \begin{pmatrix} C^1 \\ C^2 \end{pmatrix}$. On the other hand, $C_i \equiv \varepsilon_{ij} C^j$.

- The only structure constants that we need to know explicitly are those of the $SO(3)$ subgroup

$$[T_m, T_n] = f_{mn}{}^{p} T_p = -\varepsilon_{mn}{}^{p} T_p \,,$$

- The indices $I, J, \ldots$ must be replaced by composite indices im, jn etc. where $i, j, \ldots = 1, 2$ and $m, n, \ldots = 1, 2, 3$ are indices in the fundamental representations of SL$(2, \mathbb{R})$ and SL$(3, \mathbb{R})$, respectively.
- In the electric 3-forms the lower index 1 is equivalent to an upper index 2: $C_1 = \varepsilon_{12} C^2 = C^2$ and, therefore $(C^i) = \begin{pmatrix} C^1 \\ C_1 \end{pmatrix} = \begin{pmatrix} C^1 \\ C^2 \end{pmatrix}$. On the other hand, $C_i \equiv \varepsilon_{ij} C^j$.
- Comparing the field strengths of this theory with those of the generic ungauged theory we get that the d-tensors can be constructed entirely in terms of the U-duality invariant tensors $\delta^i{}_j, \varepsilon_{ij}, \delta^m{}_n, \varepsilon_{mnp}$:

$$d_{mIJ} \quad \rightarrow \quad d_{m\,in\,jp} = -\tfrac{1}{2} \varepsilon_{mnp} \varepsilon_{ij} \,,$$

$$d^i{}_I{}^m \quad \rightarrow \quad d^i{}_{jn}{}^m = \delta^i{}_j \delta^m{}_n \,.$$

Moreover

$$d^i{}_{(I|}{}^m d_{i|J)}{}^n = -2 d^{mnp} d_{pIJ} \,, \quad \Rightarrow \quad d^{mnp} = +\tfrac{1}{2} \varepsilon^{mnp} \,.$$

- In this theory, the embedding tensor has the form $\vartheta_{im}{}^{A}$. We know there are at least two possible $\mathrm{SO}(3) \subset \mathrm{SL}(3,\mathbb{R})$ gaugings of this theory:

- In this theory, the embedding tensor has the form ${\vartheta_{im}}^{A}$. We know there are at least two possible $SO(3) \subset SL(3,\mathbb{R})$ gaugings of this theory:

  1. Salam and Sezgin's (SS), in which the 3 vector fields A^{1m} coming from the metric of 11-dimensional supergravity (that is, the 3 Kaluza-Klein (KK) vector fields) are used as gauge fields.

- In this theory, the embedding tensor has the form $\vartheta_{im}{}^{A}$. We know there are at least two possible $\mathrm{SO}(3) \subset \mathrm{SL}(3,\mathbb{R})$ gaugings of this theory:
  1. Salam and Sezgin's (SS), in which the 3 vector fields A^{1m} coming from the metric of 11-dimensional supergravity (that is, the 3 Kaluza-Klein (KK) vector fields) are used as gauge fields.
  2. The AAMO gauging in which the 3 gauge fields are the A^{2m} coming from the 3-form of 11-dimensional supergravity.

- In this theory, the embedding tensor has the form $\vartheta_{im}{}^{A}$. We know there are at least two possible $SO(3) \subset \mathrm{SL}(3,\mathbb{R})$ gaugings of this theory:
  1. Salam and Sezgin's (SS), in which the 3 vector fields A^{1m} coming from the metric of 11-dimensional supergravity (that is, the 3 Kaluza-Klein (KK) vector fields) are used as gauge fields.
  2. The AAMO gauging in which the 3 gauge fields are the A^{2m} coming from the 3-form of 11-dimensional supergravity.
- The Salam-Sezgin gauging corresponds to choosing an embedding tensor whose only non-vanishing components are $\vartheta_{im}{}^{n} = g\delta_i{}^{1}\delta_m{}^{n}$ where g is the coupling constant, and the AAMO gauging corresponds to the choice $\vartheta_{im}{}^{n} = g\delta_i{}^{2}\delta_m{}^{n}$.

- In this theory, the embedding tensor has the form $\vartheta_{im}{}^{A}$. We know there are at least two possible $\mathrm{SO}(3) \subset \mathrm{SL}(3,\mathbb{R})$ gaugings of this theory:
  1. Salam and Sezgin's (SS), in which the 3 vector fields A^{1m} coming from the metric of 11-dimensional supergravity (that is, the 3 Kaluza-Klein (KK) vector fields) are used as gauge fields.
  2. The AAMO gauging in which the 3 gauge fields are the A^{2m} coming from the 3-form of 11-dimensional supergravity.
- The Salam-Sezgin gauging corresponds to choosing an embedding tensor whose only non-vanishing components are $\vartheta_{im}{}^{n} = g\delta_i{}^1\delta_m{}^n$ where g is the coupling constant, and the AAMO gauging corresponds to the choice $\vartheta_{im}{}^{n} = g\delta_i{}^2\delta_m{}^n$.
- From the 8-dimensional supergravity point of view, one could use any other $\mathrm{SL}(2,\mathbb{R})$ transformed of the A^{1m} triplet as gauge fields.The corresponding embedding tensor has the form

$$\vartheta_{im}{}^{n} = v_i\delta_m{}^n\,,$$

where v_i is a 2-component vector transforming in the fundamental of the electric-magnetic $\mathrm{SL}(2,\mathbb{R})$ duality group

- In this theory, the embedding tensor has the form $\vartheta_{im}{}^{A}$. We know there are at least two possible $SO(3) \subset SL(3,\mathbb{R})$ gaugings of this theory:
  1. Salam and Sezgin's (SS), in which the 3 vector fields A^{1m} coming from the metric of 11-dimensional supergravity (that is, the 3 Kaluza-Klein (KK) vector fields) are used as gauge fields.
  2. The AAMO gauging in which the 3 gauge fields are the A^{2m} coming from the 3-form of 11-dimensional supergravity.
- The Salam-Sezgin gauging corresponds to choosing an embedding tensor whose only non-vanishing components are $\vartheta_{im}{}^{n} = g\delta_{i}{}^{1}\delta_{m}{}^{n}$ where g is the coupling constant, and the AAMO gauging corresponds to the choice $\vartheta_{im}{}^{n} = g\delta_{i}{}^{2}\delta_{m}{}^{n}$.
- From the 8-dimensional supergravity point of view, one could use any other $SL(2,\mathbb{R})$ transformed of the A^{1m} triplet as gauge fields.The corresponding embedding tensor has the form

$$\vartheta_{im}{}^{n} = v_{i}\delta_{m}{}^{n} \, ,$$

  where v_i is a 2-component vector transforming in the fundamental of the electric-magnetic $SL(2,\mathbb{R})$ duality group
- The SO(3) gauge fields are combinations of the two triplets of vector fields

$$\vartheta_{in}{}^{m} A^{in} = v_{i} A^{im} \, ,$$

  and include, as limiting cases, the SS and the AAMO theories.

We have found a set of deformation parameters which are a solution for all contraints

$$\vartheta_{im}{}^{n} = v_i \delta_m{}^{n}\,, \quad Z^{imn} = v^i \delta^{mn}\,, \quad Z_{im} = 0$$

Thank you!

Bandos,Ortín 2016,....

- The 6-form potentials are expected to be the duals of the scalars: requires the introduction of as many 6-forms D_A as generators of global transformations δ_A leaving the equations of motion (not just the action) invariant.

- The 7-form field strengths K_A are the Hodge duals of the piece $j_A^{(\sigma)}(\phi)$ of the Noether–Gaillard–Zumino (NGZ) conserved 1-form currents $j_A = j_A^{(\sigma)}(\phi) + \Delta j_A$ associated to those symmetries.

$$K_A \equiv \star j_A^{(\sigma)} ,$$

The Bianchi identity is

$$dK_A = -d \star j_A^{(\sigma)} = T_A{}^I{}_J F^J \tilde{F}_I + T_A{}^m{}_n \tilde{H}^n H_m - \tfrac{1}{2} T_{Aij} G^{ij} .$$

The action

- The kinetic terms in the action

$$S^{(0)} = \int \{ -\star R + \tfrac{1}{4}\mathrm{Tr}\left(\mathscr{D}\mathscr{M}\mathscr{M}^{-1} \wedge \star \mathscr{D}\mathscr{M}\mathscr{M}^{-1}\right) + \tfrac{1}{4}\mathrm{Tr}\left(d\mathscr{W}\mathscr{W}^{-1} \wedge \star d\mathscr{W}\mathscr{W}^{-1}\right)$$

$$+\tfrac{1}{2}\mathscr{W}_{ij}\mathscr{M}_{mn}F^{im} \wedge \star F^{jn} + \tfrac{1}{2}\mathscr{M}^{mn}H_m \wedge \star H_n + \tfrac{1}{2}e^{-\varphi}G \wedge \star G - \tfrac{1}{2}aG \wedge G - V \}$$

- We add

$$S^{(1)} = \int \{ -dC^1 \Delta G^2 - \tfrac{1}{2}\Delta G^1 \Delta G^2 - \tfrac{1}{12}\varepsilon^{mnp}B_m \mathscr{D}B_n \mathscr{D}B_p + \tfrac{1}{4}\varepsilon^{mnp}B_m H_n H_p$$

$$-\tfrac{1}{24}\varepsilon_{ij}A^{im}A^{jn}\Delta H_m \mathscr{D}B_n \} ,$$

- Another correction

$$S^{(2)} = \int \{ -\tfrac{1}{12}v_i(F^{im} - v^i B_m)B_m B_n B_n + \tfrac{1}{4}\varepsilon^{mnp}B_m \Delta H_n \Delta H_p - \tfrac{1}{2}\varepsilon_{ij}\Box G^i \Box F^{jm} B_m$$

$$+\tfrac{1}{24}\varepsilon_{ij}A^{im}A^{in}\mathscr{D}B_m \Delta H_n \} .$$

- The scalar potential must satisfy:

$$k_A{}^x \frac{\partial V}{\partial \phi^x} = Y_A{}^\sharp \frac{\partial V}{\partial c^\sharp},$$

where the index $\sharp$ labels the deformations $c^\sharp$, which, in this case, are just $\vartheta_{im}{}^A, Z^{imn}$ and Z_{im}.

- The scalar potential must satisfy:

$$k_A{}^x \frac{\partial V}{\partial \phi^x} = Y_A{}^\sharp \frac{\partial V}{\partial c^\sharp}\,,$$

where the index $\sharp$ labels the deformations $c^\sharp$, which, in this case, are just $\vartheta_{im}{}^A, Z^{imn}$ and Z_{im}.

- No general rules available in the literature to construct the fermion shifts of any gauged supergravity. Castellani, Ceresole, Ferrara, DAuria, Fré and Maina,(1986), (1985). Bandos and Ortn (2016).

The scalar potential

- The scalar potential must satisfy:

$$k_A{}^x \frac{\partial V}{\partial \phi^x} = Y_A{}^\sharp \frac{\partial V}{\partial c^\sharp}\,,$$

where the index $\sharp$ labels the deformations $c^\sharp$, which, in this case, are just $\vartheta_{im}{}^A, Z^{imn}$ and Z_{im}.

- No general rules available in the literature to construct the fermion shifts of any gauged supergravity. Castellani, Ceresole, Ferrara, DAuria, Fré and Maina,(1986), (1985). Bandos and Ortn (2016).
- They can be written in terms of the *dressed structure constants* of the gauge group.

- The scalar potential must satisfy:

$$k_A{}^x \frac{\partial V}{\partial \phi^x} = Y_A{}^\sharp \frac{\partial V}{\partial c^\sharp},$$

where the index $\sharp$ labels the deformations $c^\sharp$, which, in this case, are just $\vartheta_{im}{}^A, Z^{imn}$ and Z_{im}.

- No general rules available in the literature to construct the fermion shifts of any gauged supergravity. Castellani, Ceresole, Ferrara, DAuria, Fré and Maina,(1986), (1985). Bandos and Ortn (2016).
- They can be written in terms of the *dressed structure constants* of the gauge group.

The scalar potential

$$V = -\tfrac{1}{4} S_{IJ} S^{*\,IJ} + \tfrac{1}{8} \delta^{\mathbf{mn}} N_{\mathbf{m}}{}^{I}{}_{J} N^{*}_{\mathbf{n}\,I}{}^{J} = -\tfrac{1}{2} \mathscr{W}^{ij} v_i v_j \left[\mathrm{Tr}(\mathscr{M})^2 - 2\mathrm{Tr}(\mathscr{M}^2) \right],$$

where $\mathscr{W}^{ij}$ is the $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ symmetric matrix, and where we have used

$$\mathscr{M}_{mn} \equiv L_m{}^{\mathbf{p}} L_n{}^{\mathbf{p}}, \quad \text{so that} \quad T = \mathrm{Tr}(\mathscr{M}), \quad \text{and} \quad T^{\mathbf{mn}} T^{\mathbf{mn}} = \mathrm{Tr}(\mathscr{M}^2).$$