Breaking the sound barrier from holography

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based on

C. Hoyos, N. Jokela, D. Rodríguez Fernández, A. Vuorinen
arXiv: 1511.01002, Phys. Rev. D.94.106008

and

C. Ecker, C. Hoyos, N. Jokela, D. Rodríguez Fernández, A. Vuorinen
Work in progress

Iberian Strings 2017.
Instituto Superior Técnico, Lisbon/Portugal.

Summary

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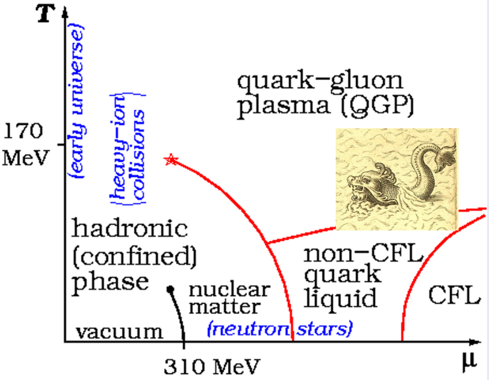
Phenomenological Motivation



Full phase Diagram of QCD yet unclear

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Intermediate densities? Neutron Stars

- Not available with CET, pQCD
- Lattice QCD does not work in the $T \ll \mu$ regime
- Phenomenological models must be interpolated

To find a model that can describe **strongly coupled (non perturbative) deconfined matter at those densities**

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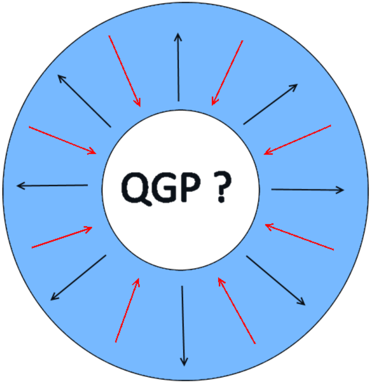
HOLOGRAPHY

Caveats

- Baryons are difficult $\rightarrow$ Restrict ourselves to the deconfined phase
- Holography not exact QCD: sistematic uncertainty $\rightarrow$ Universality of the Quark Gluon Plasma

Thermodynamic notions...

- Compress the fluid $\implies$ Increase ε
- P also increases $\implies$ Opposes compression $\rightarrow$ large $\frac{\partial P}{\partial \varepsilon}$ $\rightarrow$ "Stiff" fluid



To support **gravitational collapse**, Neutron Stars (NS) must have a **stiff EoS**

1. No significant amount of quark matter can exist inside a NS
2. $(v_s)_{\text{QGP}} > 1/\sqrt{3}$ at those densities

Theoretical Motivation

- No holographic model UV complete found so far that exhibits $v_s > 1/\sqrt{3}$. **Universal Bound?**
- Lifshitz theories, $v_s^2 = z/(d-1) \implies$ Not-relativistic

$3+1$-dimensional D-brane intersections

- $D4 - D6 \rightarrow v_s^2 = 1/2$
- $D5 - D5 \rightarrow v_s = 1$
- $D4 - D8$ (Sakai-Sugimoto model) $\rightarrow v_s^2 = 2/5$

However $\rightarrow$ After compactification to $3+1$ dimensions, 4-Dim. dynamics entangled with additional d.o.f. from color branes

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Not a bona fide 4-dim. QFT?

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Set-up

We start from the action

$$\mathcal{S} = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\,\left[R - L^2F^2 - |D\phi|^2 + V(\phi)\right],$$

$$D_\mu = \partial_\mu - iqA_\mu, \quad V(\phi) = -\frac{12}{L^2} + m^2|\phi|^2, \quad m^2 = \Delta(\Delta - 4),$$

and

$$dS^2 = \frac{L^2}{r^2 f(r)}dr^2 + \frac{r^2}{L^2}e^{2A}\left(-f(r)dt^2 + dx^i dx^j \delta_{ij}\right), \quad A_\mu dx^\mu = A_0(r)dt$$

$$\left.\begin{array}{l}\text{Absence of scalar field} \rightarrow \text{AdS-RN solution (CFT)} \\ \text{Scalar field alone cannot yield } v_s^2 > 1/3\end{array}\right\}\textit{Coupled}$$

[A.Cherman, T. Cohen, A.Nellore]

We combine both!

The thermodynamical quantities (μ, T) read

$$\lim_{r\to\infty} A_0(r) = \mu, \quad \frac{r_H^2}{4\pi L^2} f'(r_H) e^{A(r_H)} = T,$$

Near the boundary ($\Delta \notin \mathbb{N}$),

$$\phi \sim \left(\frac{L^2}{r}\right)^{4-\Delta} \widetilde{\phi}_{(0,0)} + \frac{L^{2\Delta}}{r^\Delta}\phi_{(0,0)} + \cdots, \quad f \sim 1 + \frac{L^8}{r^4} f_{(4,0)} + \cdots$$

from the AdS/CFT dictionary, $\frac{\delta S_{\text{ren}}}{\delta \gamma_{\mu\nu}}$ yields $\langle T^{\mu\nu}\rangle$,

Renormalized operators

$$\varepsilon = \langle T^{00}\rangle = -\frac{L^3}{16\pi G_5}\left[3f_{(4,0)} - \Big(\Delta - 2\Big)(\Delta - 4)\widetilde{\phi}_{(0,0)}\phi_{(0,0)}\right]$$

$$P = \langle T^{ii}\rangle = -\frac{L^3}{16\pi G_5}\left[f_{(4,0)} + \Big(\Delta - 2\Big)(\Delta - 4)\widetilde{\phi}_{(0,0)}\phi_{(0,0)}\right]$$

1. Probe approximation, $\phi \ll$, $(A, f, A_0) \to (0, f_{\mathrm{AdSRN}}, A_{0\,\mathrm{AdSRN}})$.
   $v_s = \frac{1}{\sqrt{3}} + \delta v_s$ $\boxed{\text{Analytical } (\mu/T \to \infty)/\text{Numerical}}$
2. Backreacted solution. $\boxed{\text{Numerical}}$. Maximum v_s?

Probe limit, near extremal

- Capture the behavior of the speed of sound when $\mu \gg T$
- Black Hole $\sim AdS_2 \times \mathbb{R}^3$
- Analytical solution through the Matching procedure

Probe limit, non-near extremal

- Chance of examining almost the full (μ, T) spectrum
- Check whether instabilities arise at finite μ/T
- Resort to numerics

Non probe limit

Shooting technique + Newton method

Probe limit, near extremal

In the probe approximation, the scalar field decouples from the Einstein equations and,

$$f_{\mathrm{AdSRN}} = 1 + \frac{Q^2}{r^6} - \frac{M}{r^4}, \; A_{0\,\mathrm{AdSRN}} = \mu\left(1 - \frac{r_H^2}{r^2}\right), \; A_{\mathrm{AdSRN}} = 0$$

with only a single D.O.E. to solve ($\nu = \Delta - 2$),

$$\phi'' + \left(\frac{5}{r} + \frac{f'}{f}\right)\phi' + \left(\frac{4-\nu^2}{r^2 f} + \frac{q^2 L^4}{r^4 f^2} A_0^2\right)\phi = 0\ .$$

In the near extremal limit, we can apply the Matching procedure and obtain an analytical solution, regular at the horizon, provided

$$\boxed{\Delta > 2 + \sqrt{1 + \frac{q^2}{2}} \geq 3}$$

which agrees with the BF condition in AdS_2

$$m_{\mathrm{eff}}^2 R_{\mathrm{AdS}_2}^2 \geq -\frac{1}{4}$$

by taking $R_{\mathrm{AdS}_2}^2 = \frac{R^2}{12}, m_{\mathrm{eff}}^2 = m^2 - \frac{q^2}{12R^2}$

Isothermal speed of sound

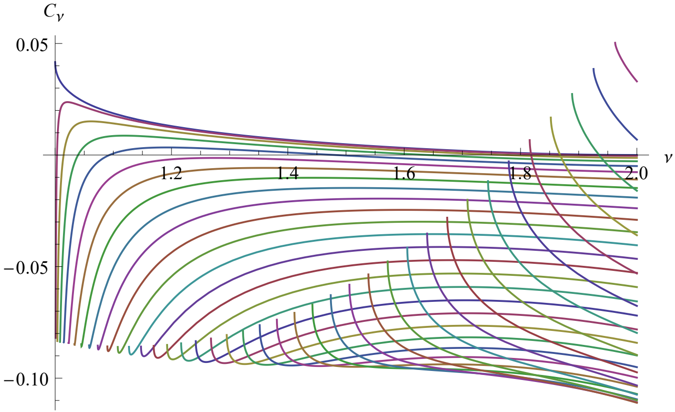
$$v_s^2 = \frac{\partial P_r}{\partial \epsilon_r} = \frac{\partial_{\mu_r} P_r}{\partial_{\mu_r} \varepsilon_r}$$

with

$$(\varepsilon_r, P_r) = \frac{(\varepsilon, P)}{\frac{L^3}{16\pi G_5} \widetilde{\phi}_{(0,0)}^{\frac{4}{4-\Delta}}}, \qquad (\mu_r, t_r) = \frac{(\mu, T)}{\frac{L^3}{16\pi G_5} \widetilde{\phi}_{(0,0)}^{\frac{1}{4-\Delta}}}$$

Speed of sound in the probe limit, near extremal

$$v_s^2 \sim \frac{1}{3}\left(1 + 4C_v(\Delta, q)\mu_r^{2(\Delta-4)}\right) \implies \text{if } C_v > 0, v_s > 1/\sqrt{3}$$

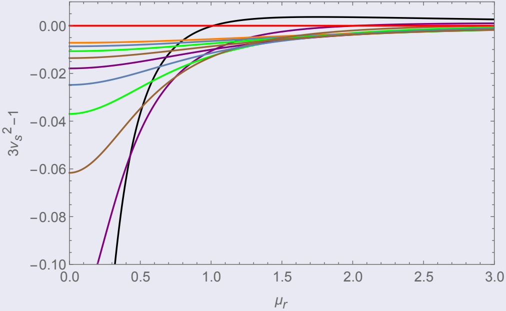


So, there is a region at which $C_\nu > 0$. If the bound is violated, it will reach $1/\sqrt{3}$ *from above*.

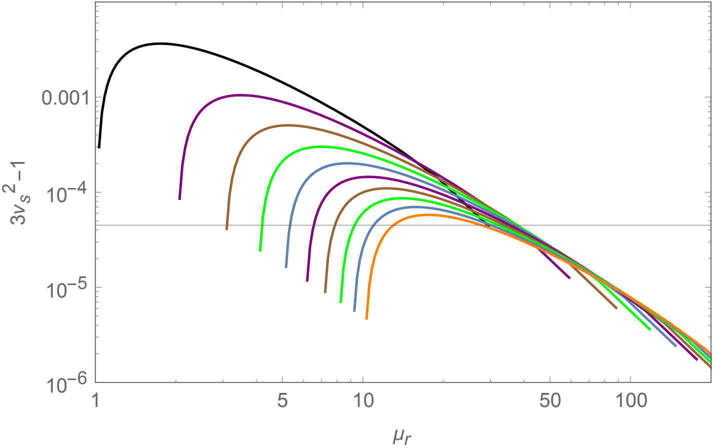
Probe limit, non-near extremal

This time, we numerically solve the scalar equation imposing regularity at the horizon and shoot toward the boundary

Speed of sound in the probe limit, Non-near extremal



$t_r = 0{,}1$, $t_r = 1$. Curves in between have intermediate values separated by $\Delta t_r = 0{,}1$ steps, $q \approx 10^{-5}$, $\Delta = 3{,}1$.



Conformal deviation as a function of μ_r in logarithmic scale

Stability of the solution

We care for physical solutions $\rightarrow$ Check stability!

Stability against formation of an homogeneous condensate

Why?

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Is usually the first kind of instability

At $\mu = 0$

- State dual to a thermal state of a CFT $\implies$ Stable
- $\nexists$ regular+normalizable solution at $\omega = 0$

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- $\exists$ regular+normalizable solutions at $\omega \in \mathbb{C}^-$

As we increase μ

- The mode will migrate to the upper complex plane $\mathbb{C}^+$, until
- A $\omega = 0$ normalizable+regular solution will $\exists$ (massless mode)
- Further increase $\mu \implies \exists$ mode at $\mathbb{C}^+$

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$$\boxed{\exists \text{ Unstable quasinormal mode}}$$

First moment when $\widetilde{\phi}_{(0,0)} = 0$ at a certain μ

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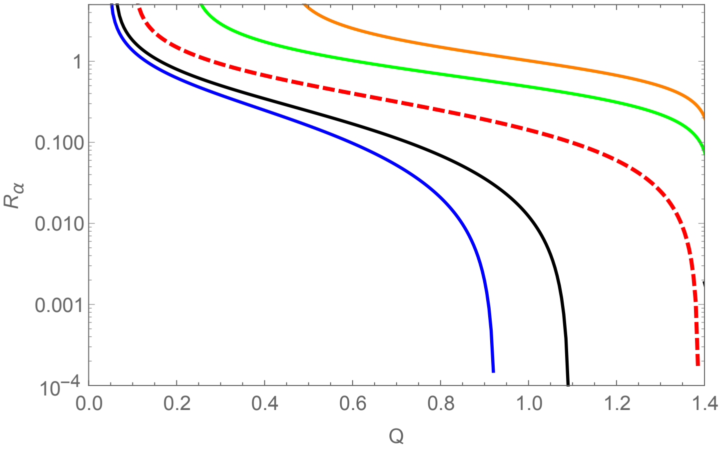
$$\boxed{\textbf{Onset of the instability!}}$$



Ratio non-normalizable/normalizable coefficients as a function of $\mathcal{Q} \propto \mu_r/t_r$, $\Delta = 3{,}1$. q=0.5, $q = 1$, $q = 4$, $q = 5$, $q = 1{,}64 \rightarrow$ critical value at which the first instability appears.

Backreacted solution

ϕ not restricted to be small $\Longrightarrow$ $\boxed{\text{Full EOMS}}$

- Near the boundary,

$$\phi \sim \left(\frac{L^2}{r}\right)^{4-\Delta} \tilde{\phi}_{(0,0)} + \frac{L^{2\Delta}}{r^{\Delta}}\phi_{(0,0)}, \quad f \sim 1 + \frac{L^8}{r^4} f_{(4,0)},$$

$$A \sim 0, \quad A_0 \sim \mu + \frac{L^4}{r^2} A_{0(2,0)}$$

- Near the horizon,

$$\phi \sim \phi_H^{(0)}, \quad f \sim f_H^{(1)}(r - r_H), \quad A \sim A_H^{(0)} + A_H^{(1)}(r - r_H),$$

$$A_0 \sim A_{0}{}_H^{(1)}(r - r_H)$$

We can employ the shooting technique + Newton method to find the set of constants

$$\left\{\widetilde{\phi}_{(0,0)}, \phi_{(0,0)}, \cancel{f_{(4,0)}}, \cancel{A_{0(2,0)}}, \phi_H^{(0)}, f_H^{(1)}, A_H^{(0)}, \cancel{A_{0H}^{(1)}}\right\}$$

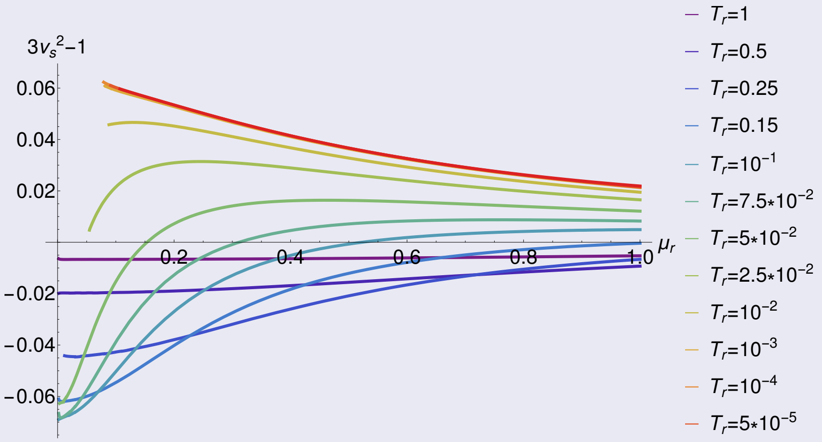
at a given (T, μ)

It is necessary to consider some initial vector

$$\boxed{\text{Solutions from probe limit at mild } (\mu_r, t_r)}$$

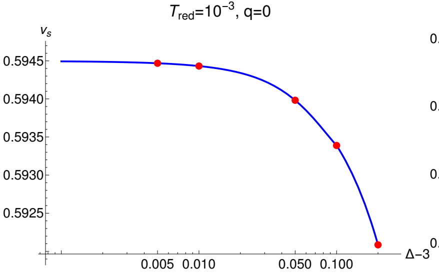
Still to examine stability $\rightarrow$ Fluctuations at $k^\mu \neq 0$
Quasinormal modes & transport coefficients? Work in progress!

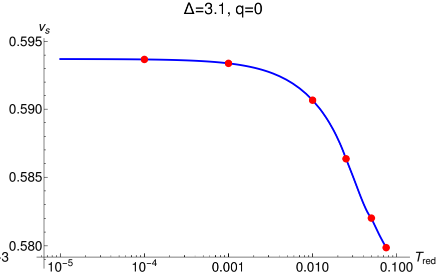
Speed of sound in the non-probe limit



$$q = 0,\ \Delta = 3{,}1$$

Maximum speed of sound





$\frac{1}{\sqrt{3}} \approx 0{,}577$, so there is not a large deviation

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Conclusions and future work

- We have found examples of consistent UV complete relativistic theories that can have a speed of sound larger than its conformal value.

- Mixed phases?

- Further models? $\rightarrow$ Find the largest possible speed of sound $\Longrightarrow$ We can qualitatively describe the "deconfined portion" of a NS

- If one can find an holographic model that fits with the deconfined portion of a NS, one can study its transport phenomena

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$$\boxed{\text{Love numbers}}$$

THANK YOU!