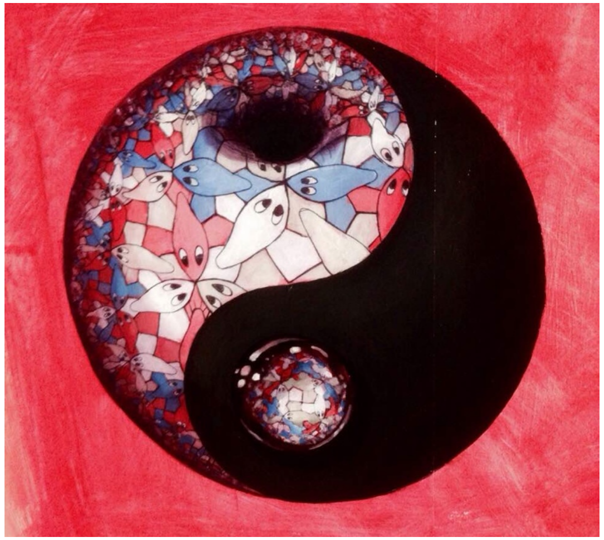
Applied string theory: understanding strange metals with virtual black holes

Koenraad Schalm

Institute Lorentz for Theoretical Physics, Leiden University



NWO
Netherlands Organisation for Scientific Research

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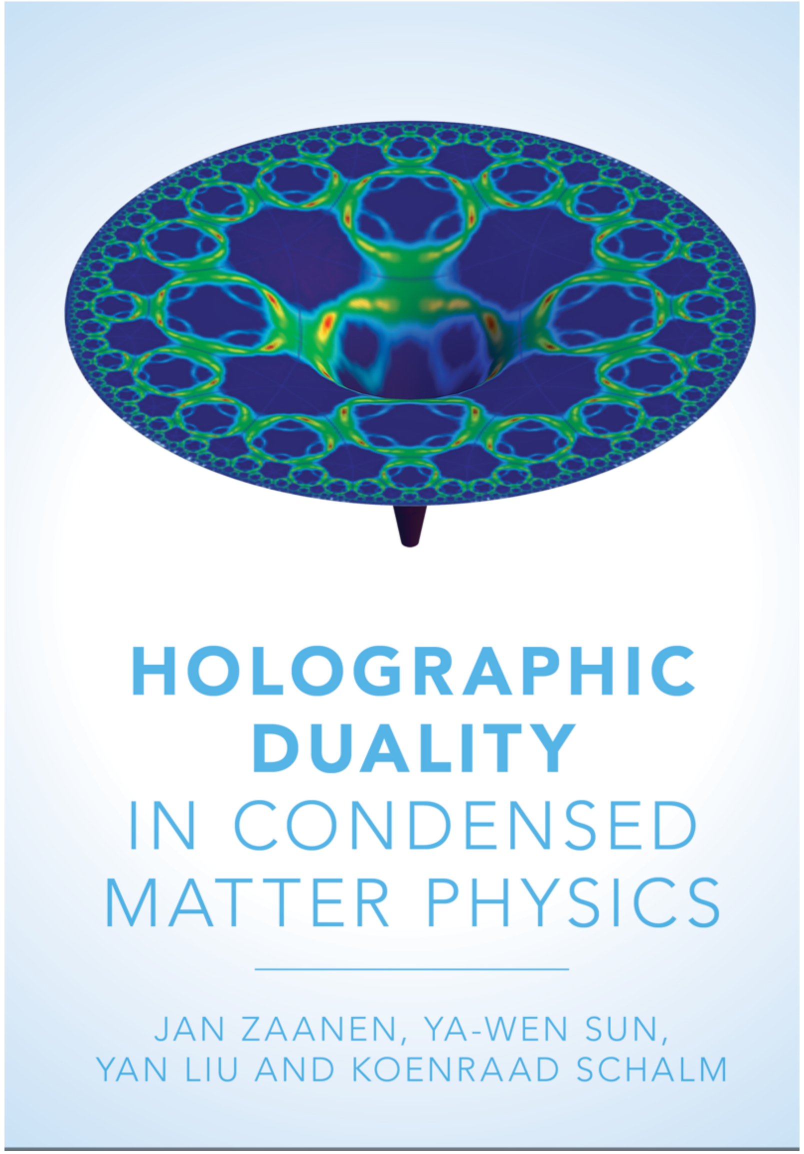
Fall 2015



HOLOGRAPHIC
DUALITY
IN CONDENSED
MATTER PHYSICS
JAN ZAANEN, YA-WEN SUN,
YAN LIU AND KOENRAAD SCHALM

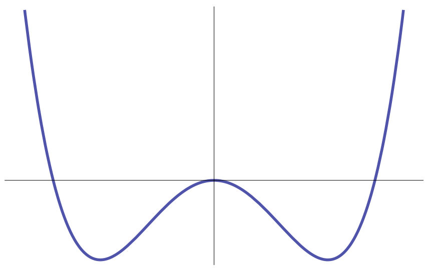
- AdS/CFT applied to condensed matter:

1. Generating functional for new non-trivial *unknown* IR fixed points

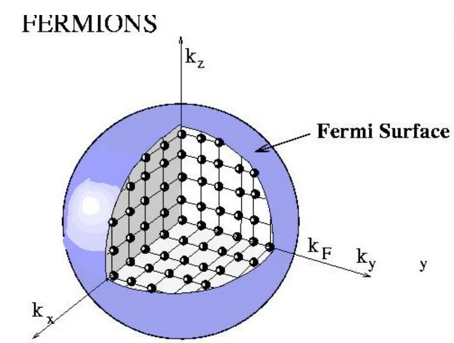
2. Far superior method to compute *real time* finite temperature/density correlation functions

Condensed Matter in a Nutshell

- Systems at finite density
  - The generic bosonic ground state
  - Macroscopic properties: SSB
  - $m = 0$ Goldstone boson

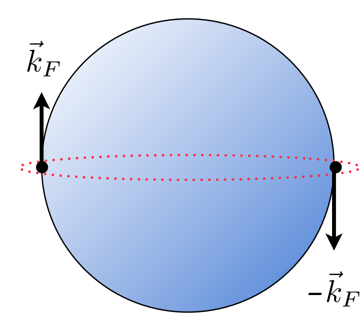


- The generic fermionic ground state
  - Macroscopic properties: Fermi gas/liquid
  - $m = 0$ Quasiparticle



$$G(\omega, k) = \langle \Psi^\dagger \Psi \rangle = \frac{1}{\omega + \mu - \frac{k^2}{2m}} = \frac{1}{\omega - v_F(k - k_F) + \ldots}$$

- Instabilities: BCS superconductivity

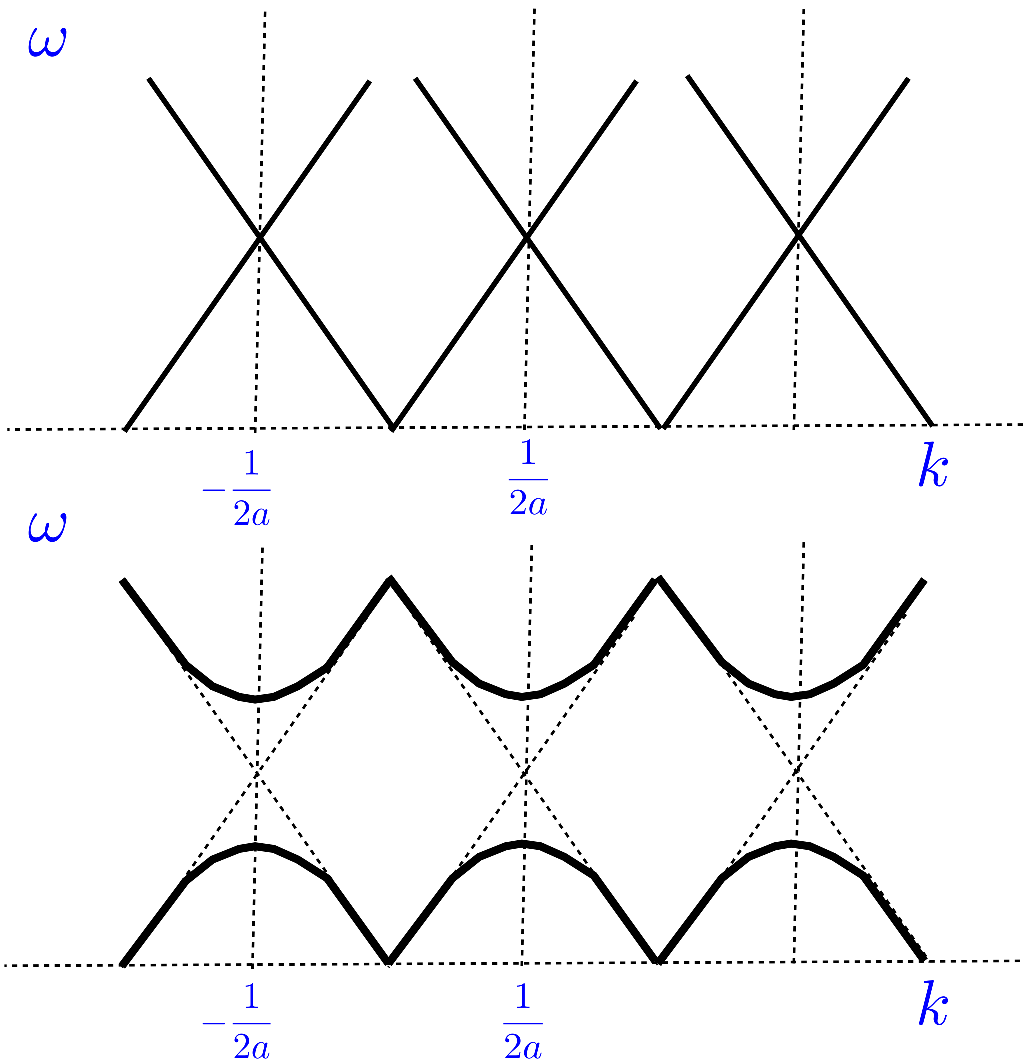


Quantum Condensed Matter Physics - Lecture Notes

Chetan Nayak

Lattice effects at low energy

- Bands from eigenvalue repulsion



$$\Psi(x) = \int_{-1/2a}^{1/2a} \frac{dk}{2\pi} \sum_{\ell \in Z} \Phi^{(\ell)}(k) e^{i(k+\ell K)x}$$

“Umklapp”

"Lattice" effects at low energy

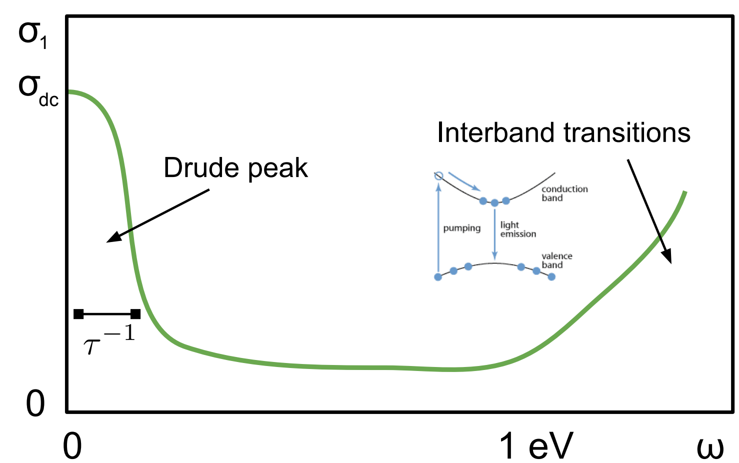
- Broken translations affect transport

cf. Davison

  - Drude model

$$\frac{d\vec{p}}{dt} = q\vec{E} - \frac{\vec{p}}{\tau}$$

$$\sigma(\omega) \sim \frac{1}{1/\tau - i\omega}$$



- Origin translational symmetry breaking
  - Lattice — distinct lattice momentum — k_L
  - Impurities ("disorder") — "ensemble average" — $\langle\langle \cdot \rangle\rangle = \int d^d k_L \, \langle \cdot \rangle_{k_L}$
  - Translational symmetry breaking can be *weak* or *strong*

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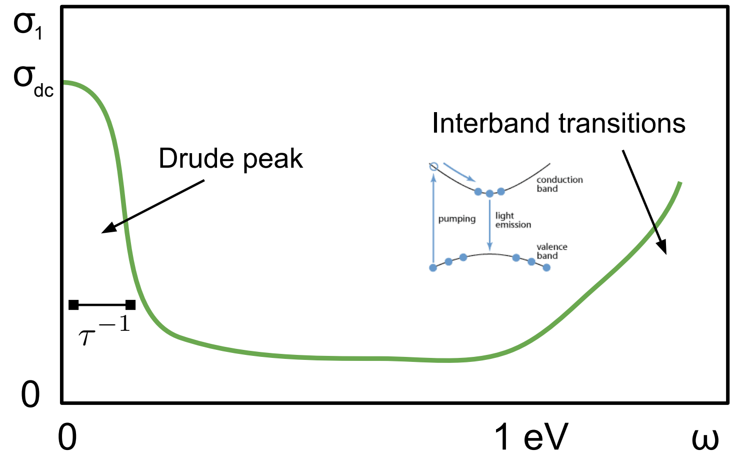

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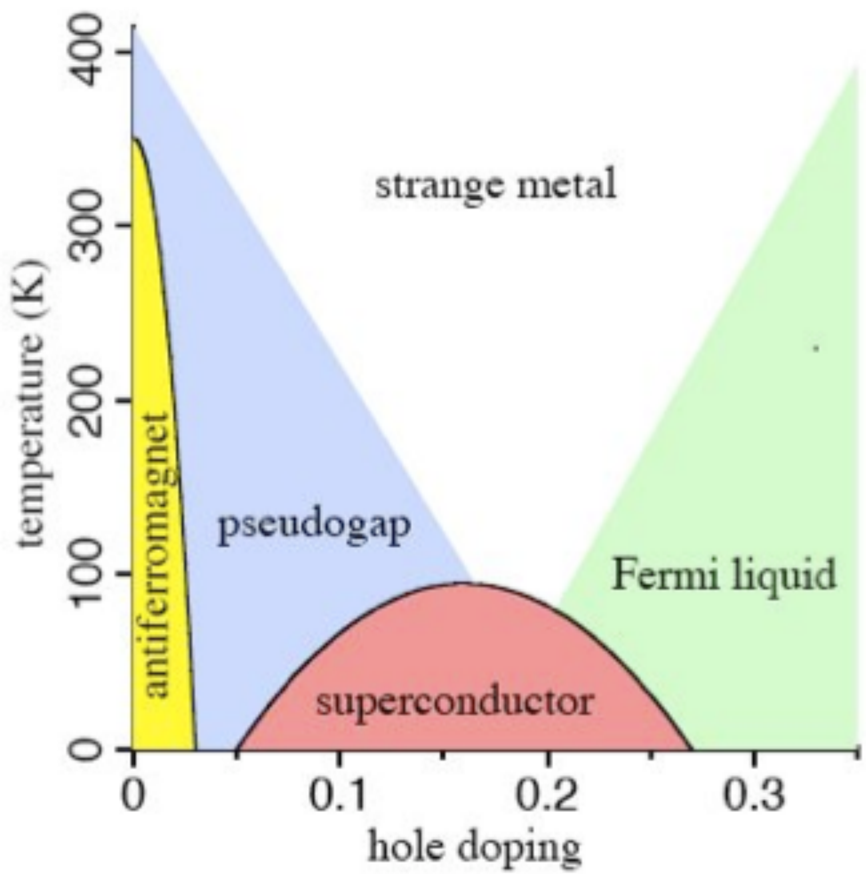
What is the strange metal?

The strange metal in high Tc cuprates



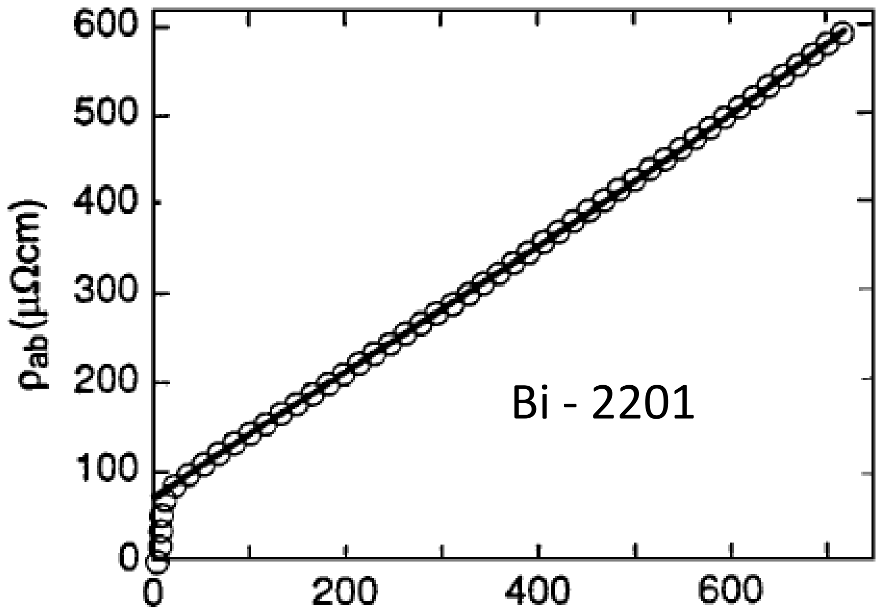
What is the theory of the strange metal?

Why is a strange metal "strange"? e.g. Anderson, Physics Today 2013

- Linear-in-T resistivity

$$\rho \equiv \frac{1}{\sigma} \sim T$$

$$\rho_{metal} \sim T^2$$



Martin et al, PRB41 (1990) 846

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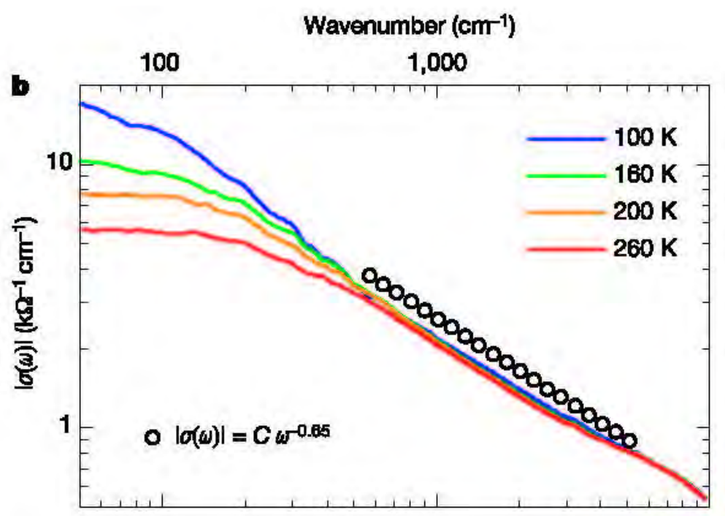
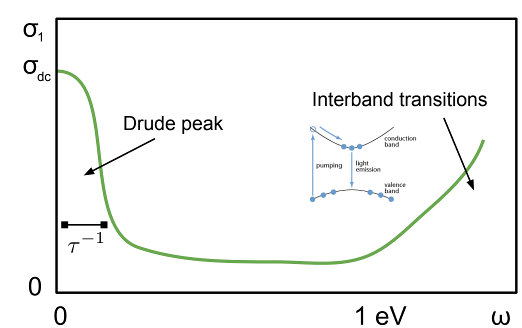
$$\sigma(\omega) \sim \omega^{-2/3}$$

$$\sigma(\omega)_{metal} \sim C$$

Van der Marel et al, Nature 425, 271 (2003)





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- Hall angle vs DC conductivity scaling

$$\theta = \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2} \qquad\qquad \theta_{metal} \sim \sigma_{DC,metal} \sim \frac{1}{T}$$

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$$\sigma \sim \sigma_I + \sigma_{II}$$

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- The Fermion Green's function

$$G(\omega, k) = \frac{1}{\omega - v_F k + \Sigma(\omega, k)}$$

The marginal Fermi liquid

- The Fermion Green's function

$$G(\omega,k) = \frac{1}{\omega - v_F k + \Sigma(\omega,k)}$$

- Strange metal: postulate phenomenologically

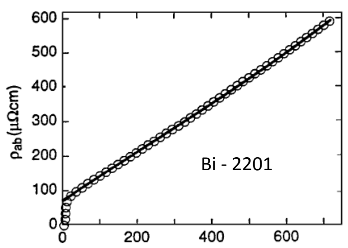
Varma, Littlewood, Schmitt-Rink, Abrahams, Ruckenstein, PRL63 (1989) 1996

$$\Sigma(\omega) = \lambda\,\omega\ln\omega$$

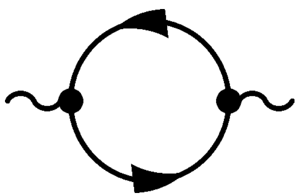
- The marginal Fermi liquid Green's function

$$G(\omega, k) = \frac{1}{\omega - v_F k + \lambda\,\omega \ln \omega}$$

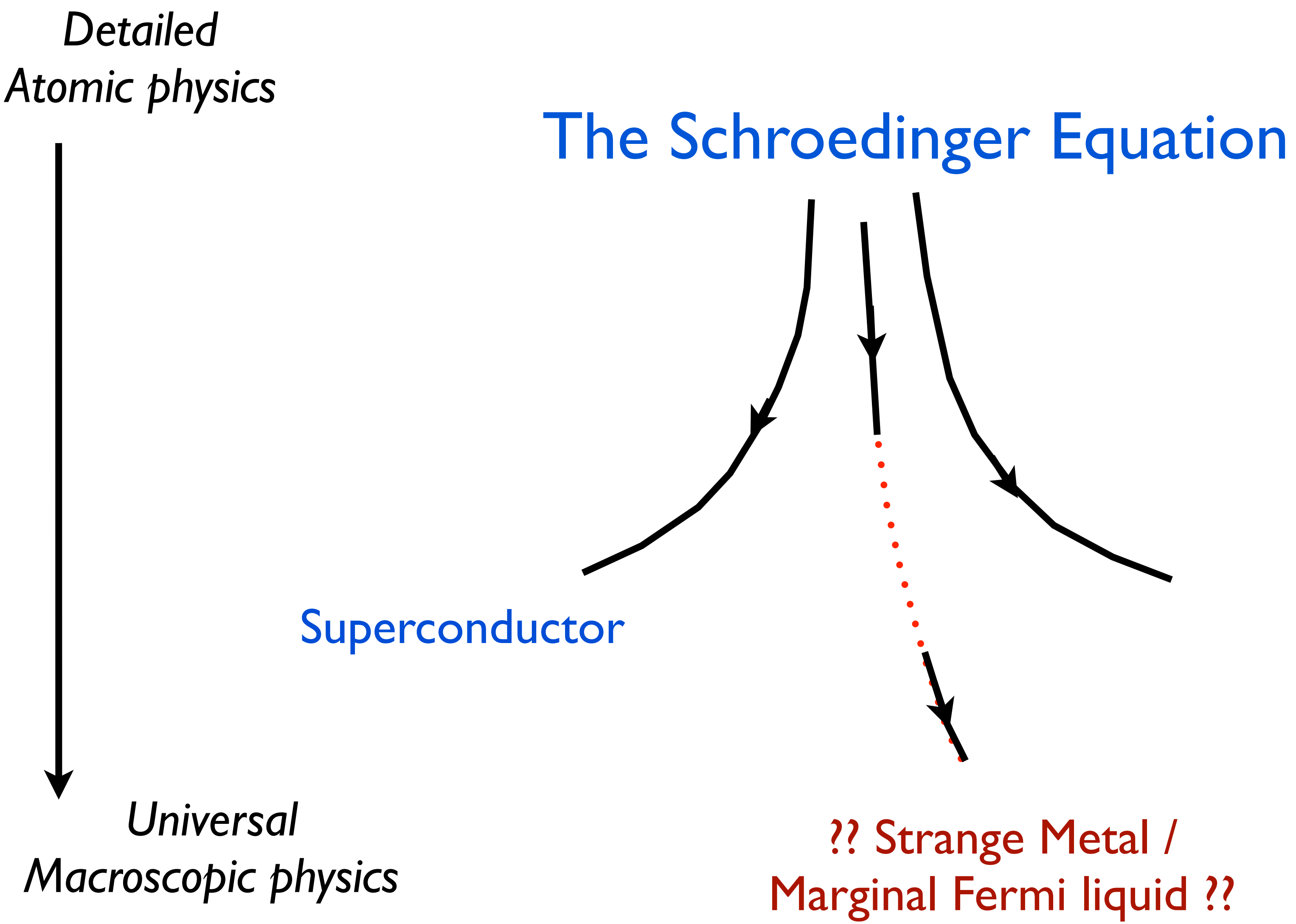
- Explains numerous features: notably T-linear resistivity



Martin et al, PRB41 (1990) 846

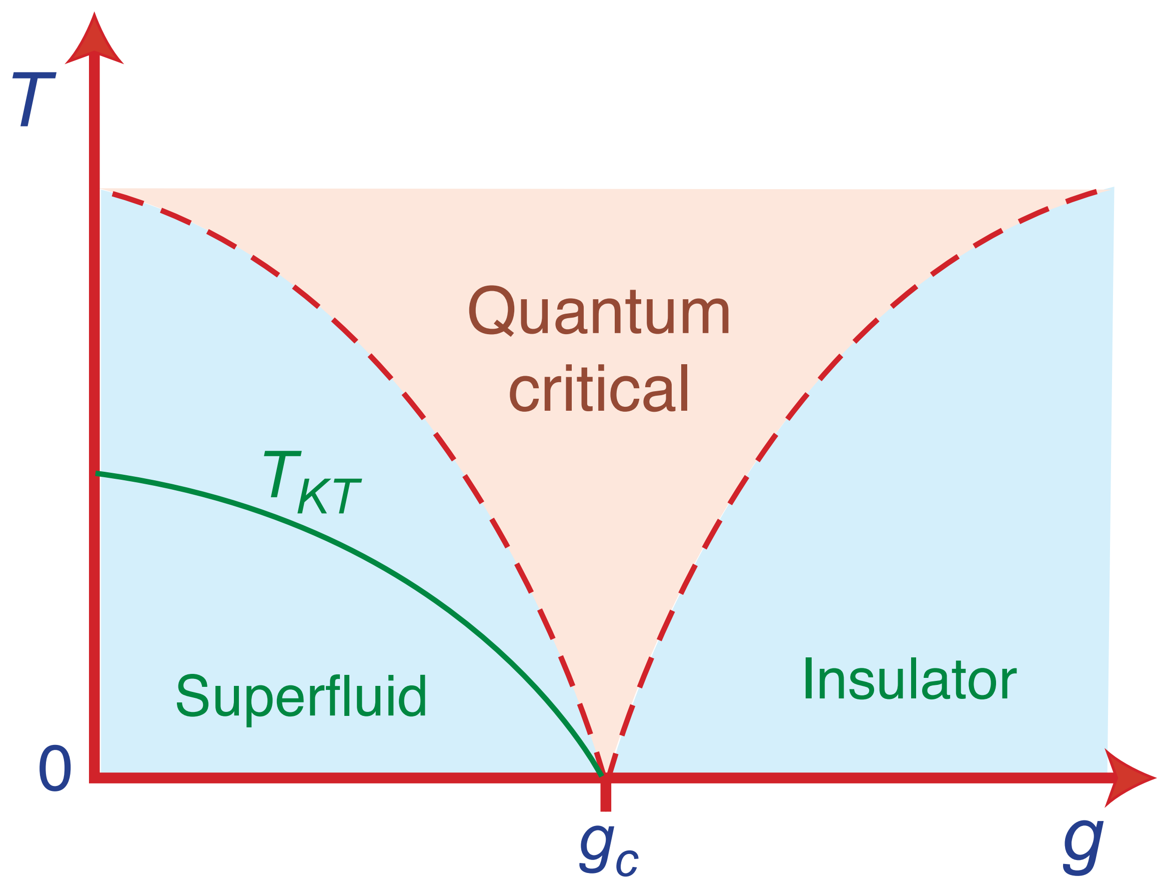
$\sigma(\omega) =$ 

- Assumes Fermi-surface-excitations are responsible for transport


Detailed
Atomic physics
The Schroedinger Equation
Superconductor
Fermi liquid
Universal
Macroscopic physics
?? Strange Metal /
Marginal Fermi liquid ??

Why are non-Fermi liquids hard?

- Non-Fermi Liquids
- Physics controlled by a Quantum Critical Point [conjecture]
- Theory without quasiparticles: Qualitatively Different Macroscopics



Sachdev

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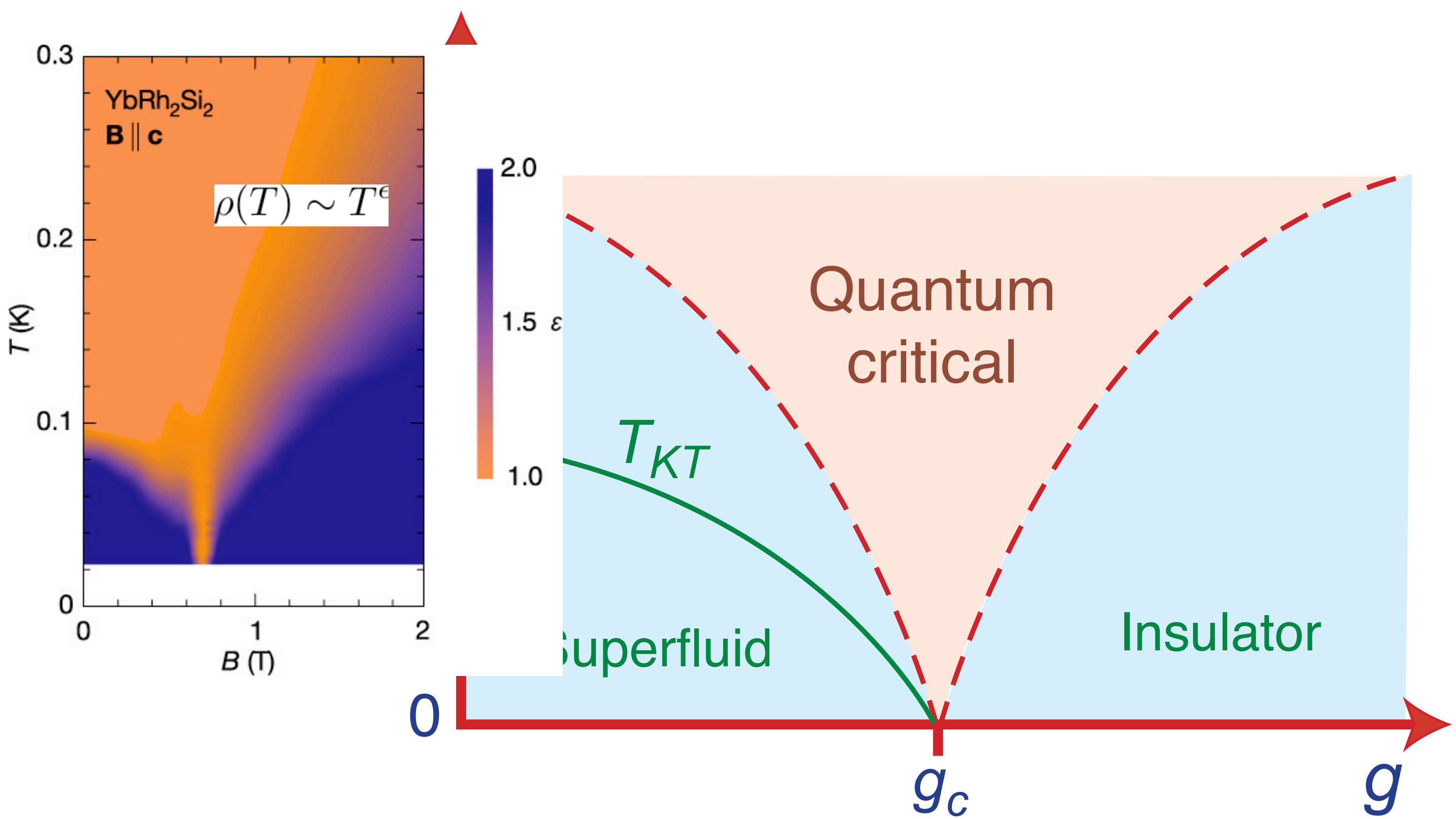
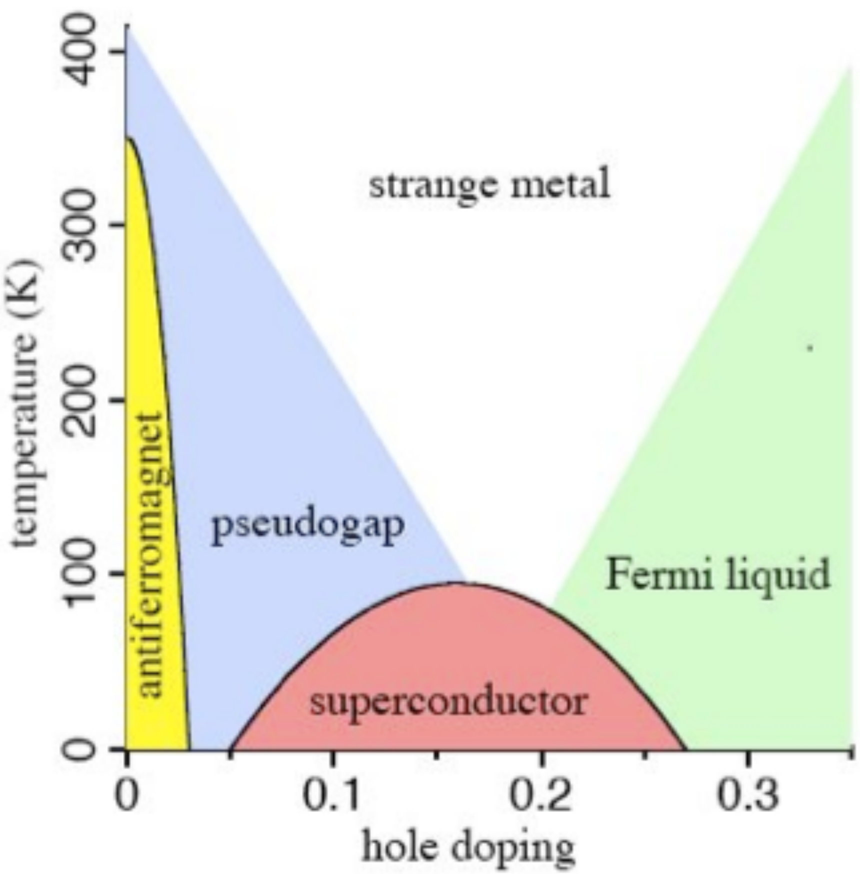


Sachdev

Custers et al, *Nature* **424**, 524 (2003)

Experimental Evidence

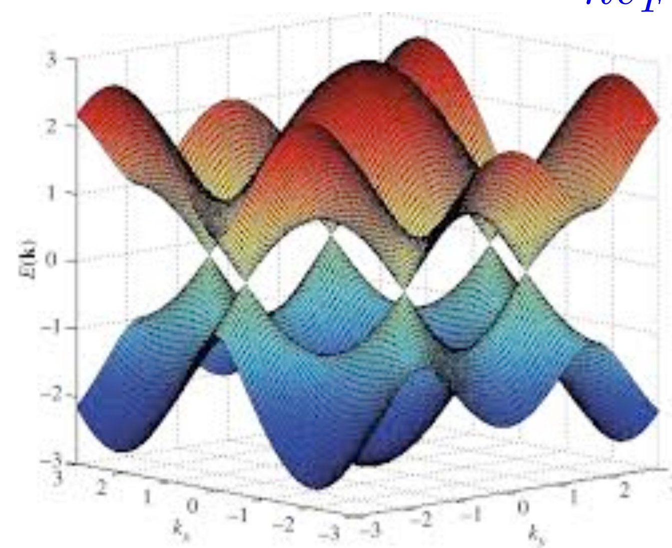
- Non-Fermi Liquids
- a.k.a. “strange metals”; a.k.a “quantum critical fermions”

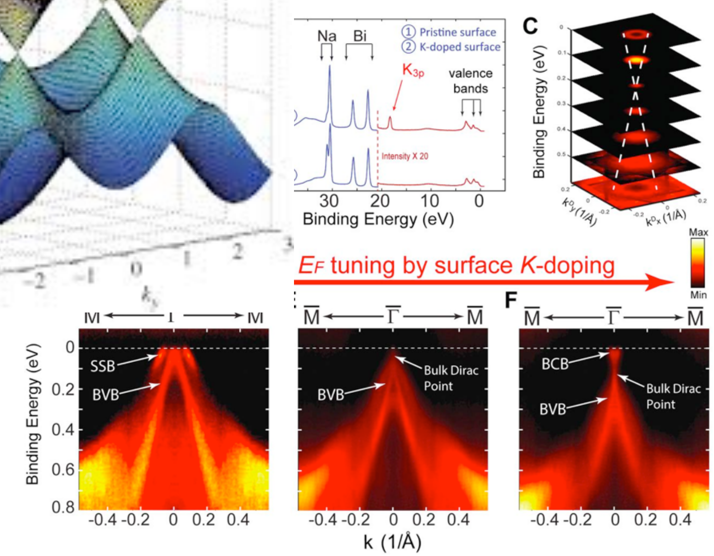


Phase diagram of a high Tc cuprate

$$\alpha = \frac{e^2}{\hbar v_F} \simeq 0.5 - 2$$

e.g. Mueller, Sachdev



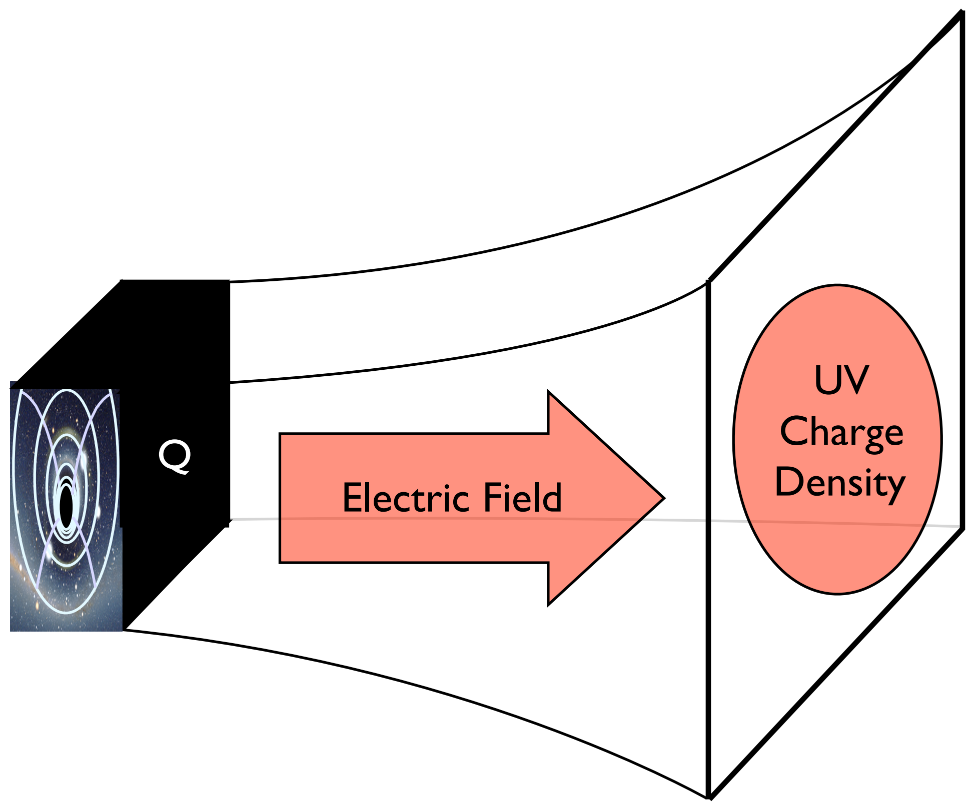


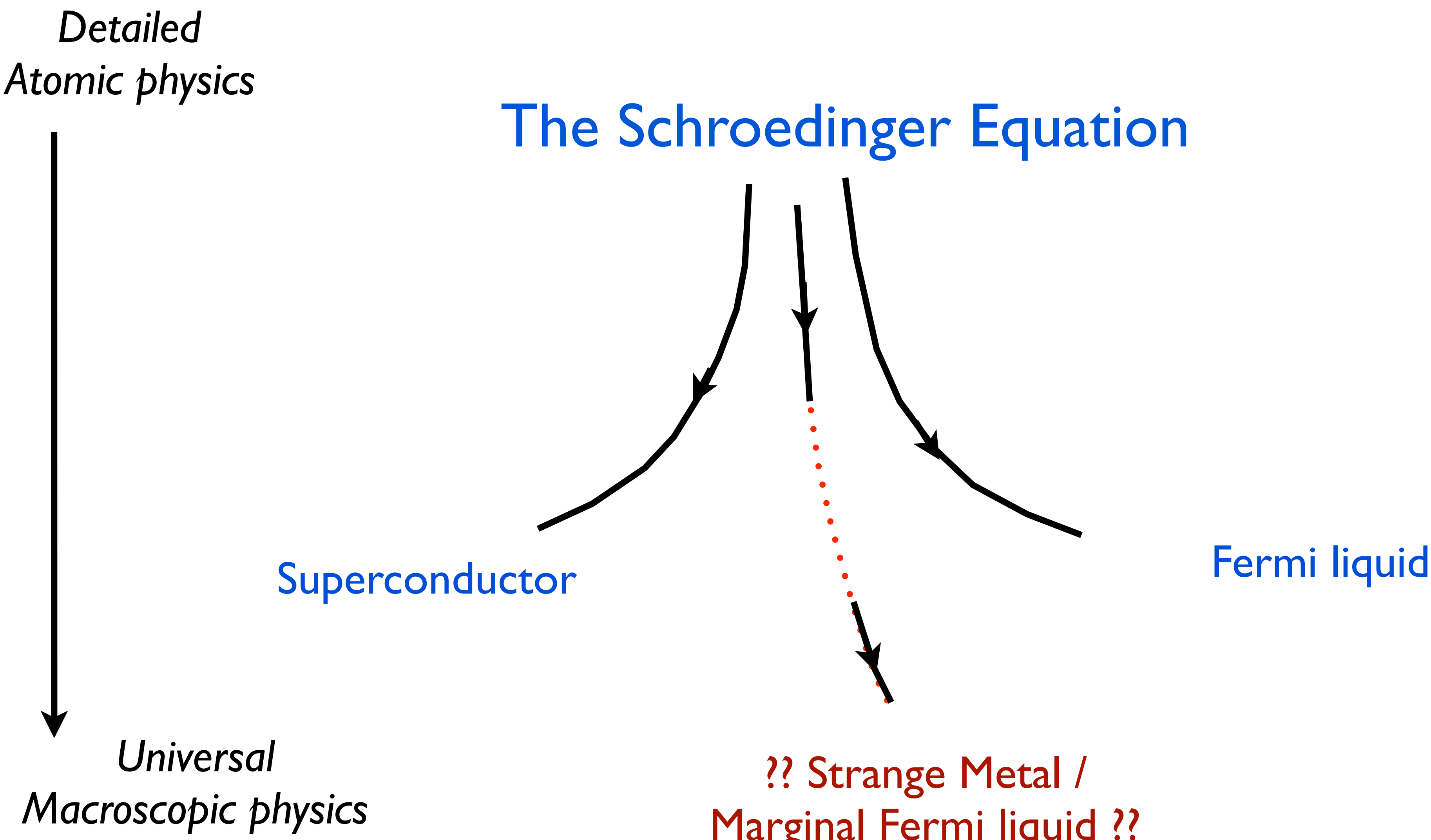
Graphene and other Dirac/Weyl semimetals

Can holography give a theory of a strange metal?

- AdS/CFT:
- a dual gravitational description of a (strongly) interacting quantum field theory.

$$\text{Systems at finite temperature/density} \quad = \quad \text{AdS charged black hole}$$

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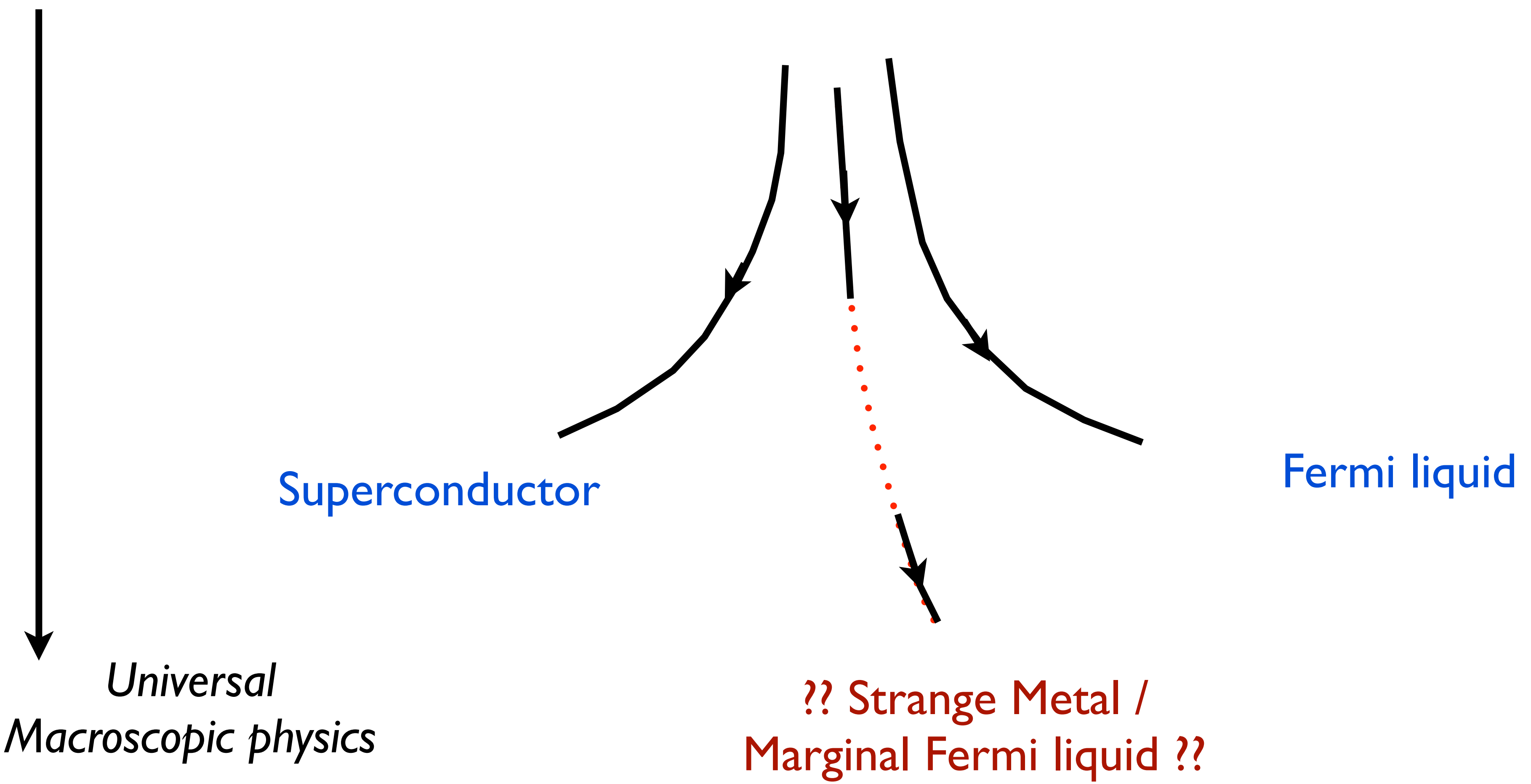

Superconductor
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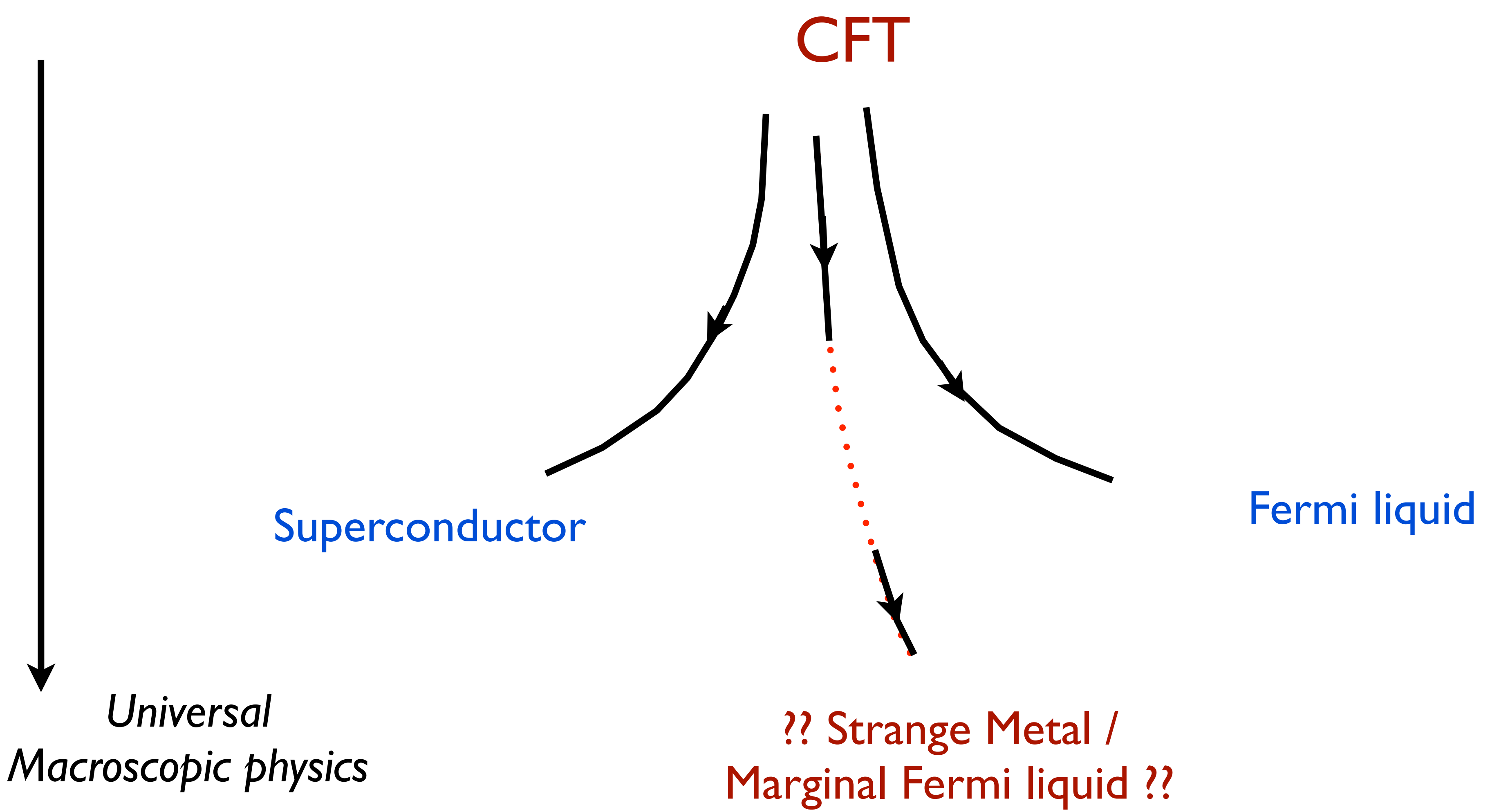

CFT
Superconductor
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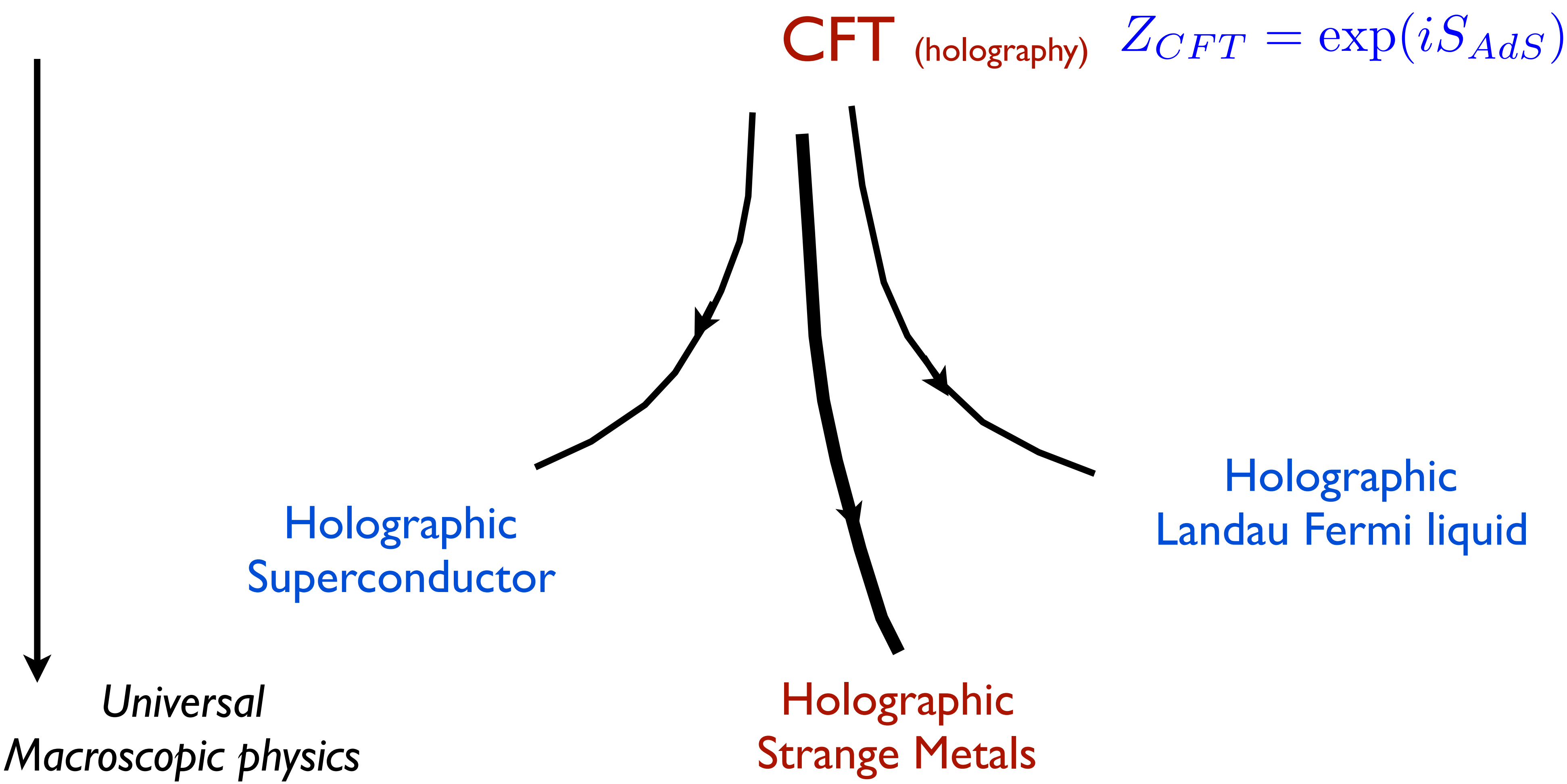
CFT (holography)
$Z_{CFT} = \exp(iS_{AdS})$
Holographic
Superconductor
Holographic
Strange Metals
Holographic
Landau Fermi liquid
Universal
Macroscopic physics

Holography describes new states of matter

- Holographic prediction:

  Emergent scale invariant hyperscaling violating theories

$$S = \frac{1}{2\kappa^2}\int d^{d+2}x\sqrt{-g}\left(R - \frac{Z(\Phi)}{4}F_{\mu\nu}F^{\mu\nu} - 2|\partial_\mu\Phi|^2 - V(\Phi)\right)$$

$$ds^2 = \frac{L^2}{r^2}\left[r^{2\theta/(d-\theta)}dr^2 - r^{-2d(z-1)/(d-\theta)}dt^2 + dx^2\right] \qquad A_t = Q\ r^{\zeta-z}$$

$$t \to \lambda^z t\ , \quad x \to x$$

Lifshitz quantum critical theory supported by an ordered state

$$s_{AdS-BH} \sim T^{(d-\theta)/z}$$

- At finite T , $z \sim \infty$, and quantum criticality is ultralocal

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Lifshitz quantum critical theory supported by an ordered state

- Experimental signature: Quantum critical sector

Lots of power law scaling

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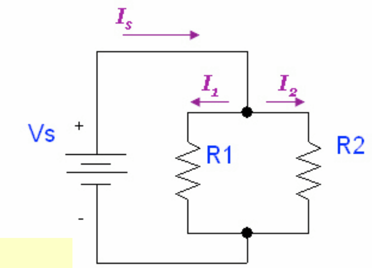
Lifshitz quantum critical theory supported by an ordered state

- Experimental signature: Thermoelectric response

$$\sigma = \sigma_{ccs} + \sigma_{relax}$$



Inverse Matthiessen law: two independent sectors

Strange metal without quasiparticles

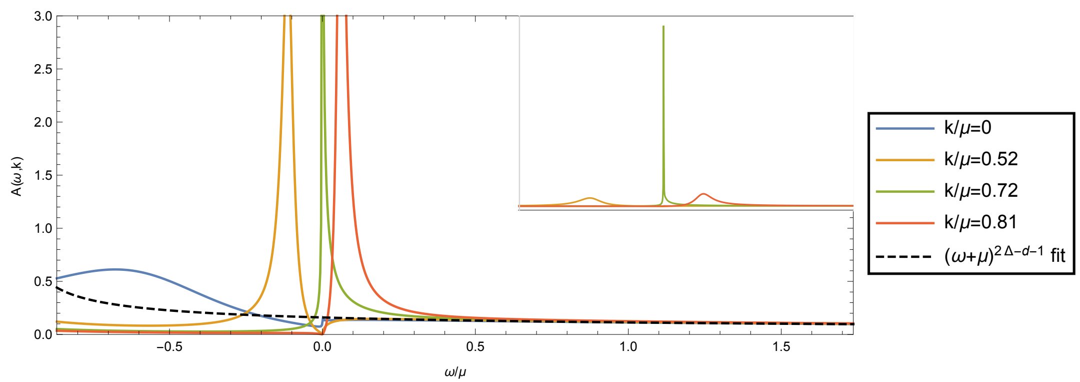
Holographic strange metals

- The single fermion function from AdS/CFT

Cubrovic, Zaanen, Schalm; Science 325 (2009) 439
Faulkner, Liu, McGreevy, Vegh PRD 83 (2011) 125002, Science 329 (2010) 1043

$$G(\omega, k) = \frac{1}{\omega - v_F k + \Sigma(\omega, k)}$$

$$A(\omega, k) = -\frac{1}{\pi}\mathrm{Im}G(\omega, k)$$



- The groundstate has a clear Fermi surface

Holographic strange metals

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$$G(\omega,k) = \frac{Z}{\omega - v_F(k-k_F) - e^{i\gamma}\omega^{2\nu_{k_F}}} + \ldots$$

- The exponent $\nu_{k_F} \sim \sqrt{\frac{1}{\xi^2} + k_F^2}$ is a free parameter

- Fermi surface excitations disperse as

$$\omega \sim (k-k_F)^z \quad \text{with} \quad z = \begin{cases} 1/2\nu_{k_F} & \nu_{k_F} < 1/2 \\ 1 & \nu_{k_F} = 1/2 \\ 1 & \nu_{k_F} > 1/2 \end{cases}$$

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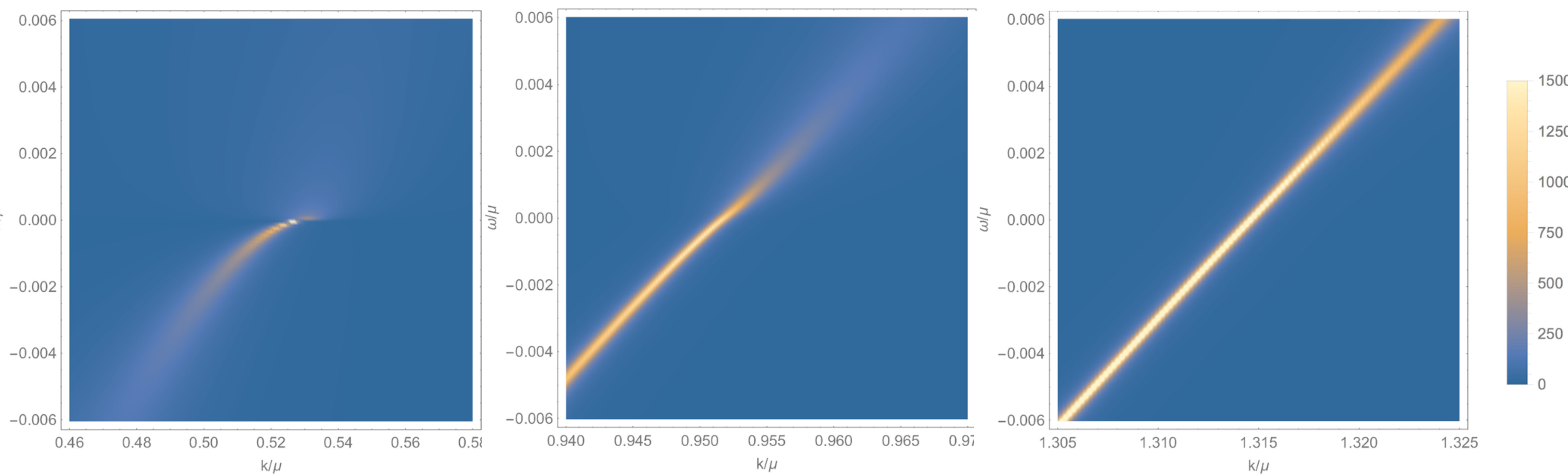
artificially broadenend

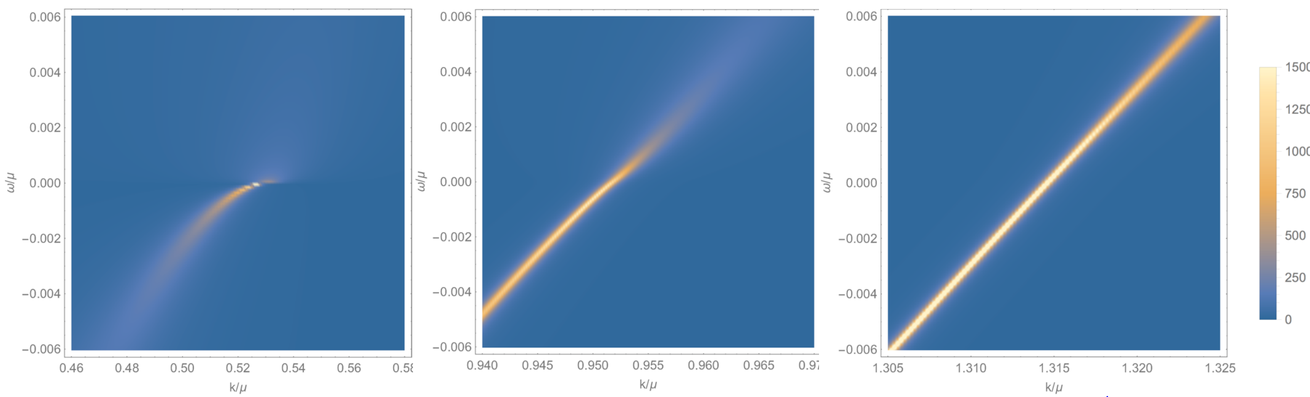


$\nu_{k_F} < 1/2$ Non-Fermi Liquid

$\nu_{k_F} = 1/2$ Marginal Fermi Liquid

$\nu_{k_F} > 1/2$ Fermi Liquid

Holographic strange metals

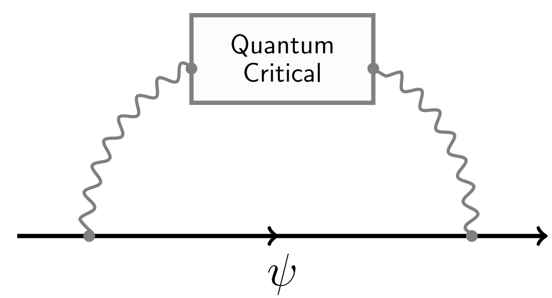


$\nu_{k_F} < 1/2$ $\nu_{k_F} = 1/2$ $\nu_{k_F} > 1/2$

$$\Sigma \sim \omega^{2\nu_{k_F}}$$

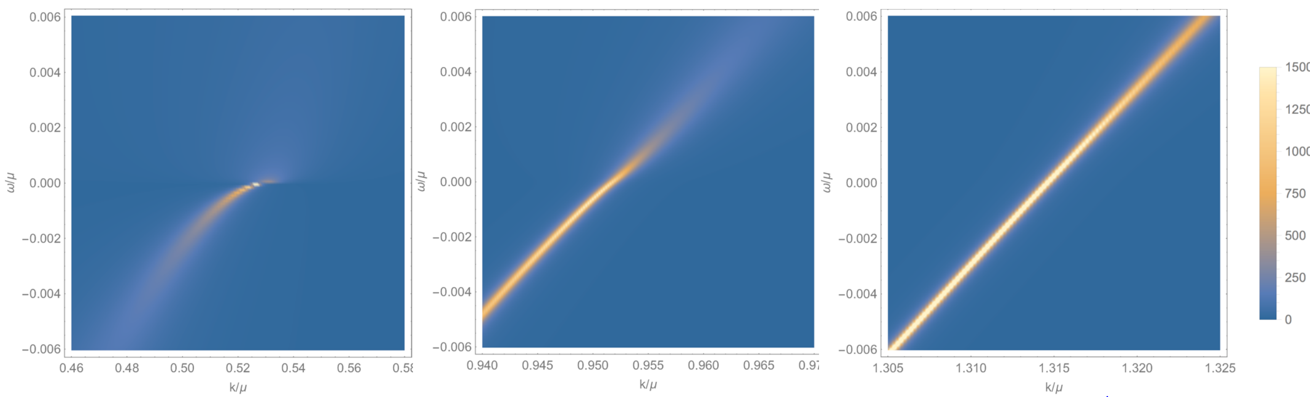
- The $\nu_{k_F} < 1/2$ NFL is a system *without* quasiparticles

- Physics: the probe fermion interacts with a quantum critical sector



- Transport does not follow from FS excitations (alone). The quantum critical sector contributes significantly

Holographic strange metals



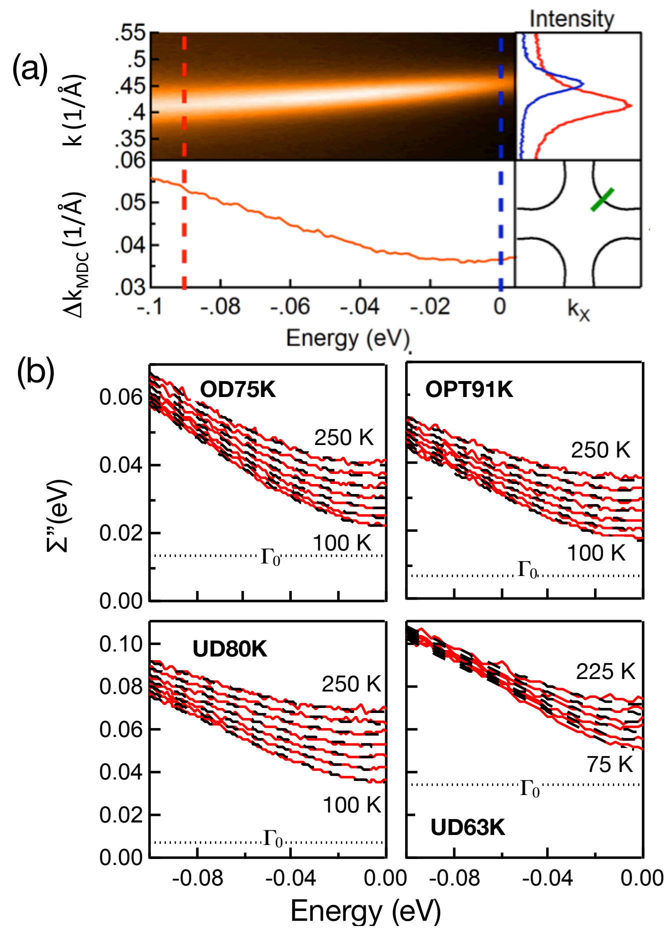
$\nu_{k_F} < 1/2$ $\quad$ $\nu_{k_F} = 1/2$ $\quad$ $\nu_{k_F} > 1/2$

$$\Sigma \sim \omega^{2\nu_{k_F}}$$

- Are such holographic self-energies and dispersions measured in experiment?

Reber, et al ,..Desseau et
arXiv:1508.06252, 1509.01556

This situation changed recently with the introduction of ultra-resolution laser-ARPES [18] ..., which bypasses the unknowns of the ARPES lineshape and removes much of the effects of the heterogeneous "dirt" effects that are for example observed in STM experiments [22] (see supplementary materials). Combined with new methods for removing nonlinearities in the electron detection [23], a quantitative analysis of the small ... or scattering rates (a self-energy effect) ... in an ARPES measurement.



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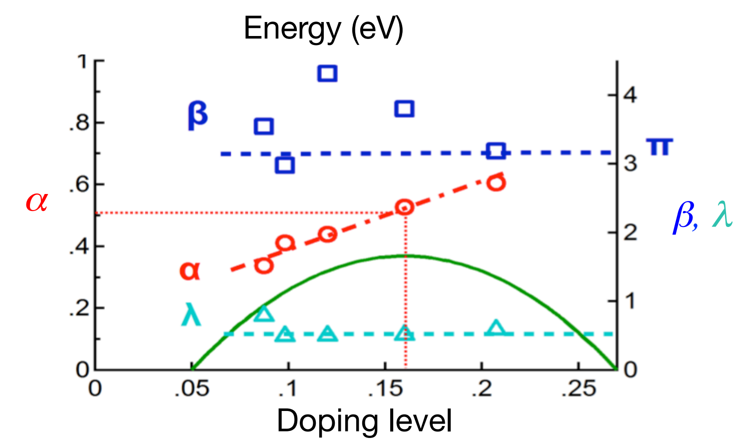
- Holographic prediction

$$\Sigma \sim \omega^{2\nu_{k_F}}$$

- Experimental fit to

$$\Sigma = \lambda(\omega^2 + \beta^2 T^2)^\alpha$$

(c)



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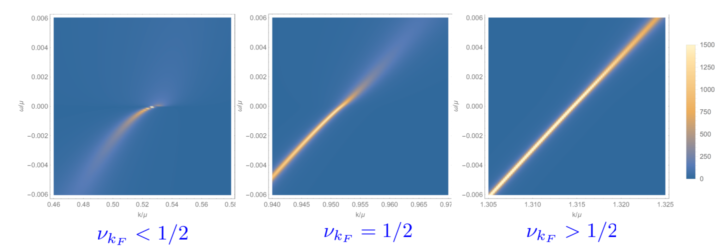

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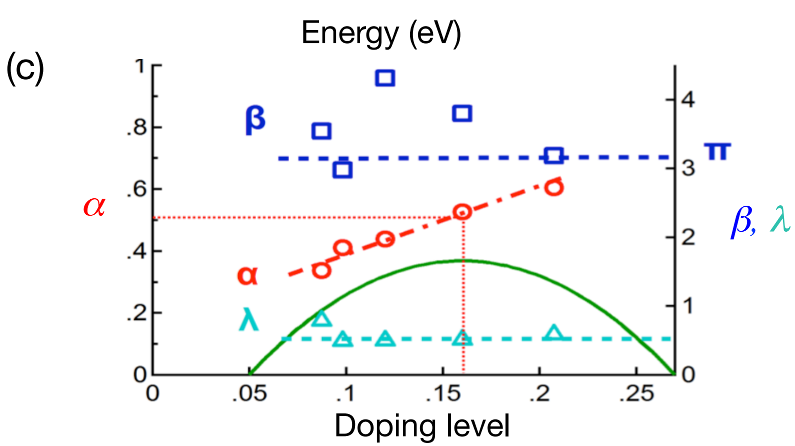
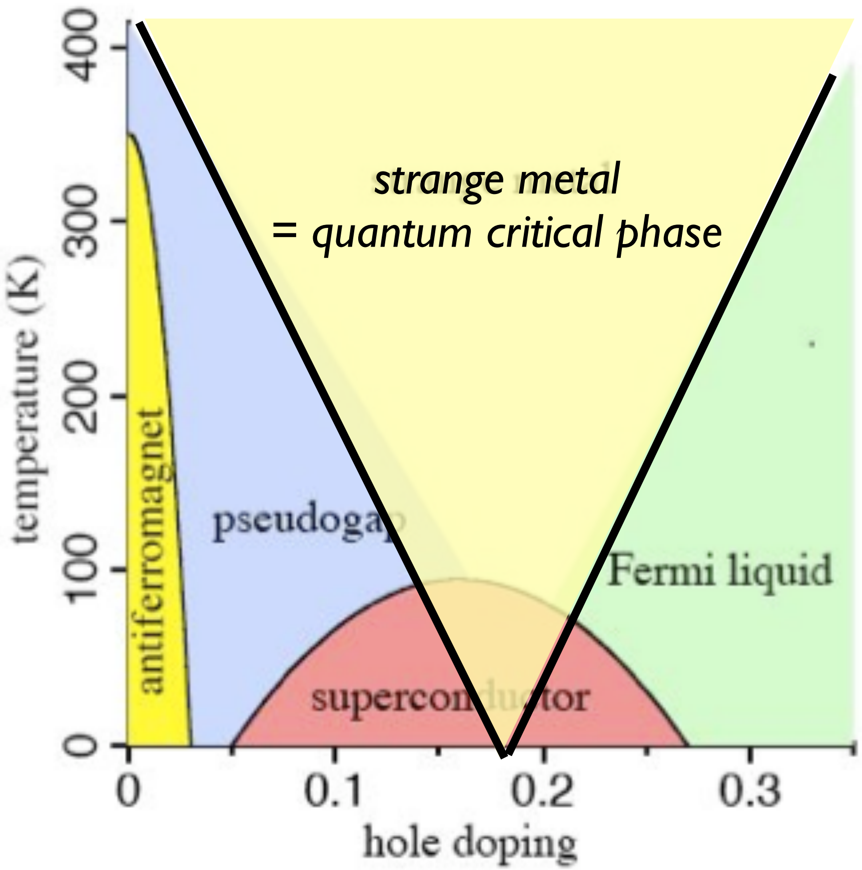


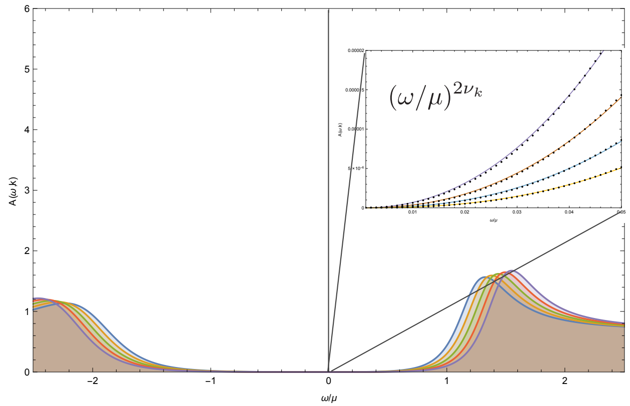
The strange metal in high Tc cuprates



Two specific predictions from holography

- Evidence of the quantum critical sector in the spectral function

Faulkner, Liu, McGreevy, Vegh
PRD 83 (2011) 125002,
Science 329 (2010) 1043



- Near $\omega = 0$ for $k \neq k_F$

$$\mathrm{Im}G(\omega, k) \sim \omega^{2\nu_k} \qquad \nu_k \sim \sqrt{\frac{1}{\xi^2} + k^2}$$

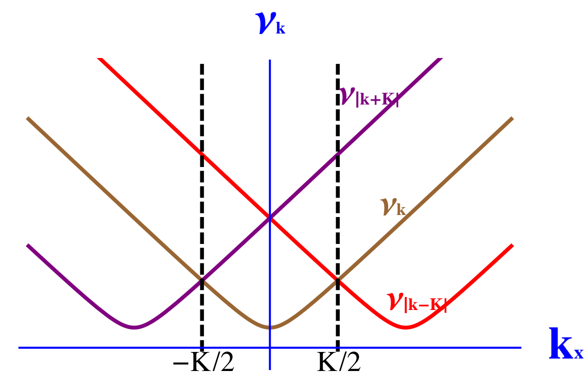
- The quantum critical contribution to the spectral function

$$\mathrm{Im}G(\omega,k) \sim \omega^{2\nu_k}$$

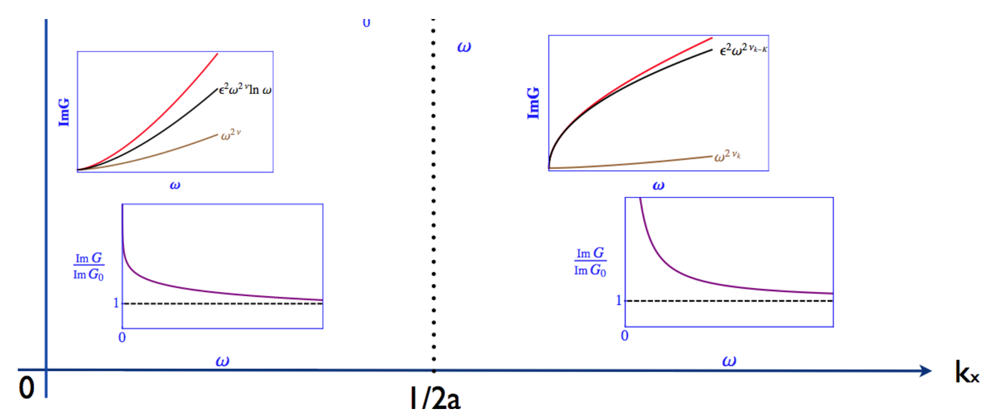
$$\nu_k \sim \sqrt{\frac{1}{\xi^2} + k^2}$$

- On a lattice

$$\mathrm{Im}G_{\mathrm{lattice}}(\omega,k) \sim \sum_{k \in \Lambda} \omega^{2\nu_k}$$

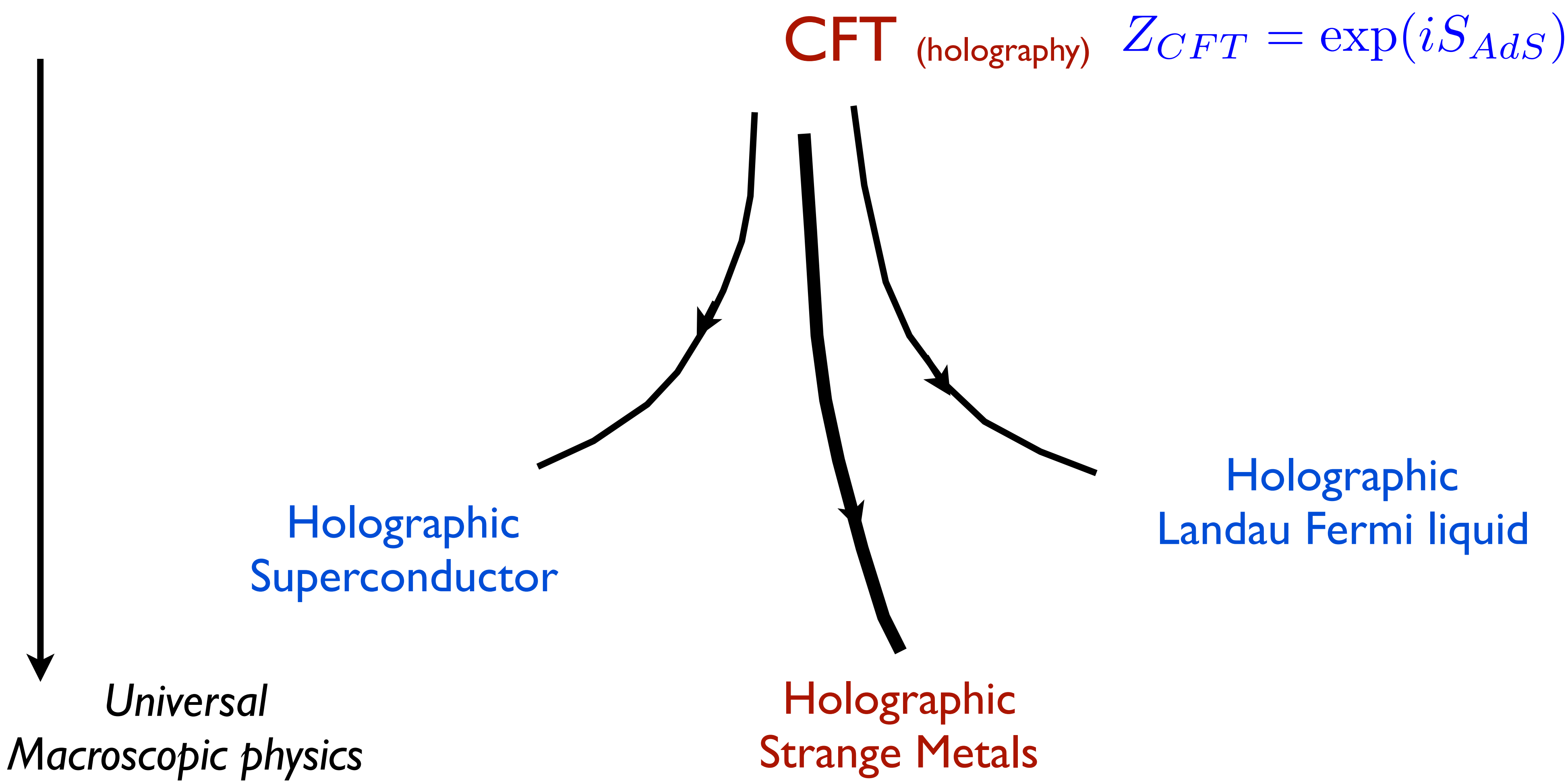


- The Green's function is no longer strictly periodic



Liu, Schalm, Sun, Zaanen, JHEP 1210 (2012) 036

The role of the quantum critical sector



Postdictions from holography

- Linear-in-T resistivity $\rho \equiv \frac{1}{\sigma} \sim T$ ☐
- Lots of Power law scaling $\sigma(\omega) \sim \omega^{-2/3}$ ☑

  quantum critical sector supported by an ordered state

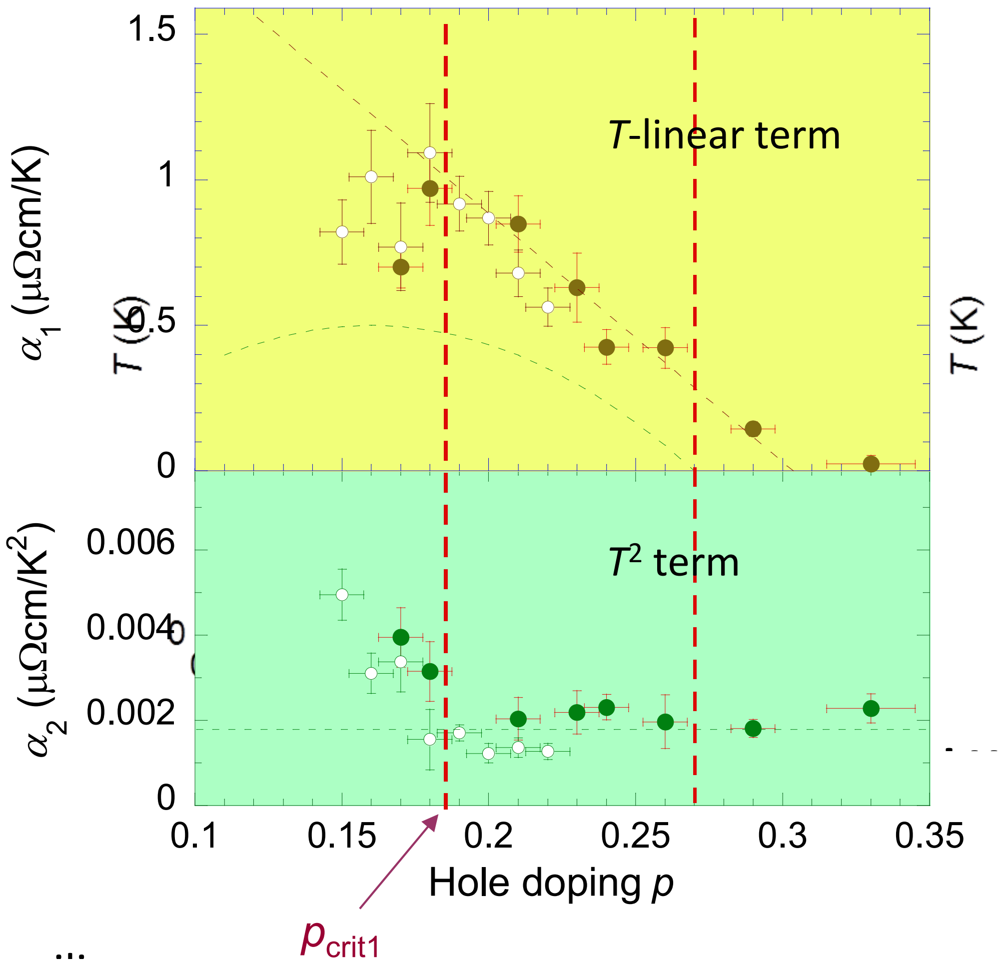
- Hall angle vs DC conductivity scaling $\theta = \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$ ☐
- Inverse Matthiessen law $\sigma \sim \sigma_I + \sigma_{II}$ ☑

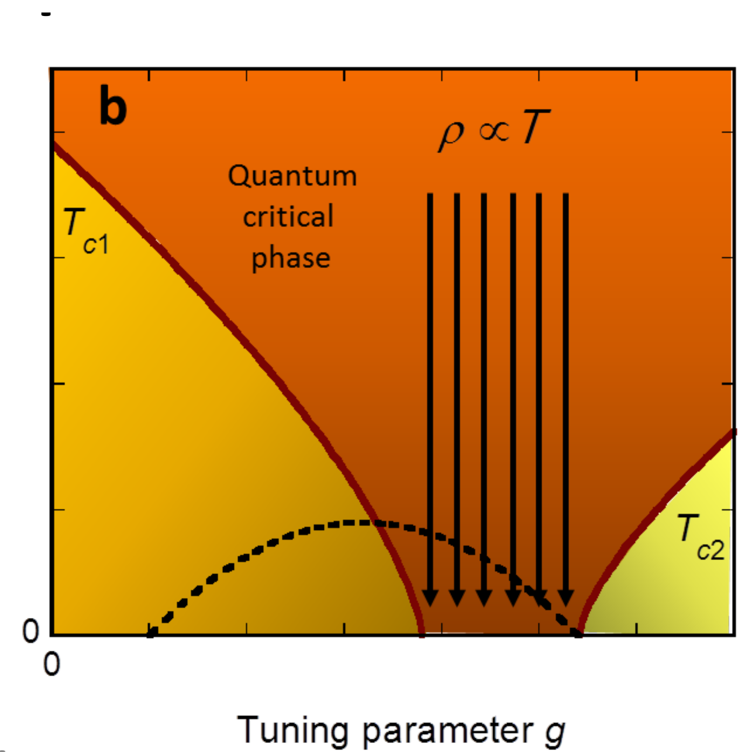
  quantum critical sector supported by an ordered state

A universal linear resistivity

- Linear resistivity in the High-Tc cuprates

Cooper, Hussey et al.
Science 323 (2009) 609



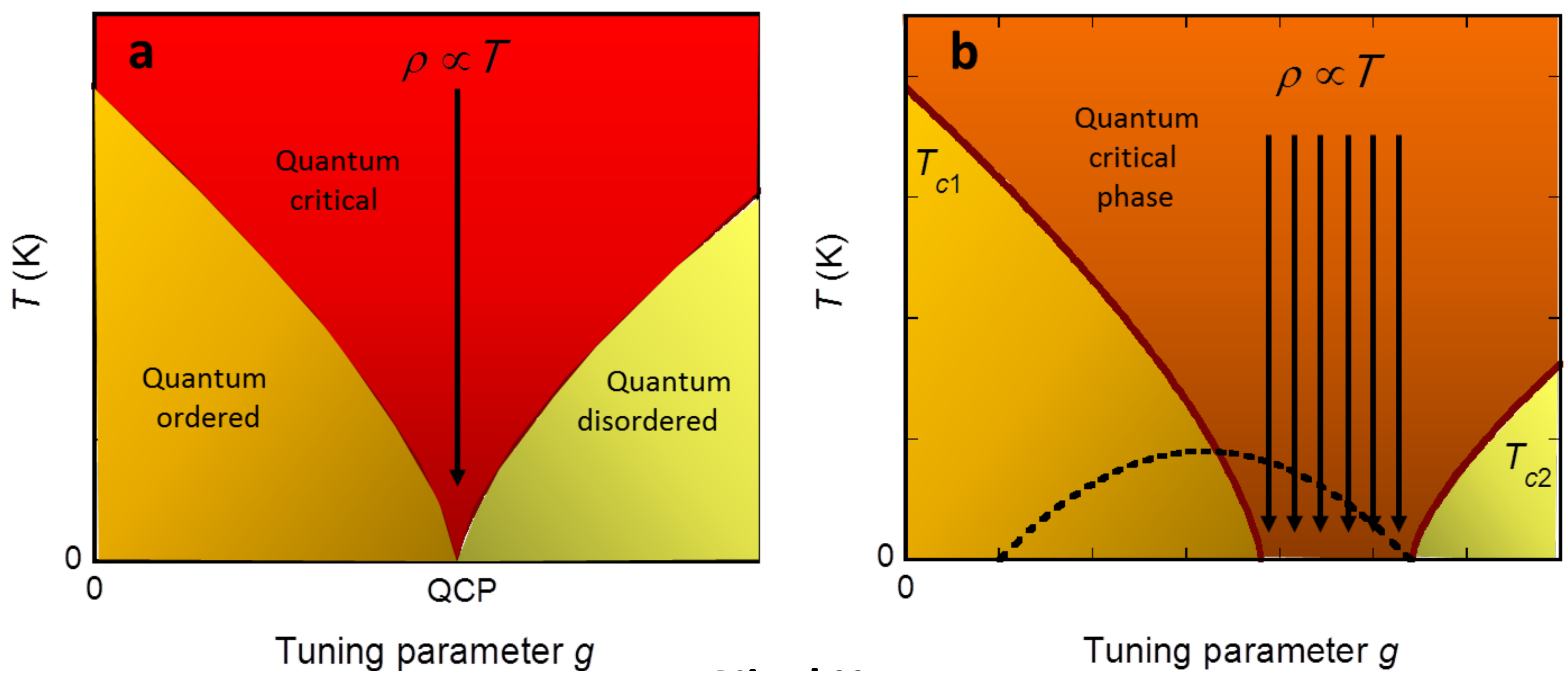


...

$$\rho = \alpha_1 T + \alpha_2 T^2$$

- Linear resistivity in the High-Tc cuprates

Cooper, Hussey et al.
Science 323 (2009) 609



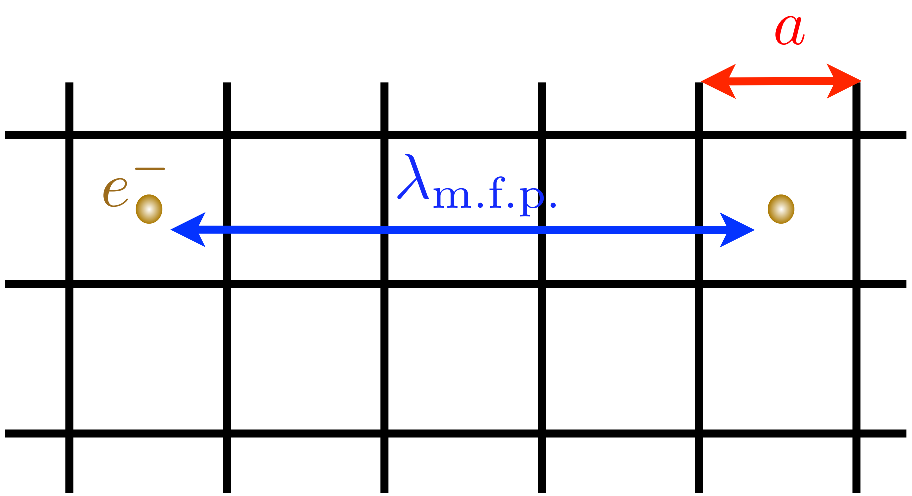
Inverse Matthiessen law also fits the data

$$\rho = \alpha_1 T + \alpha_2 T^2$$

$$\sigma = \alpha_1 T + \alpha_2 T^2$$

- Ordinary metals
- Momentum relaxes before collective behavior sets in

$$\tau_{\text{rel.}}^{-1} \sim \text{micro. physics}$$



- Ordinary metals
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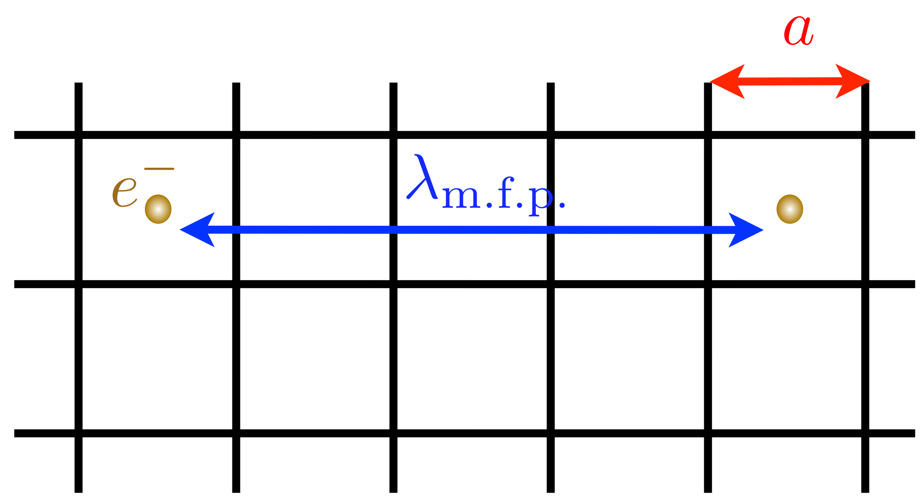
$\tau_{\rm rel.}^{-1} \sim \text{micro. physics}$



- Strongly correlated metals (no quasiparticles)

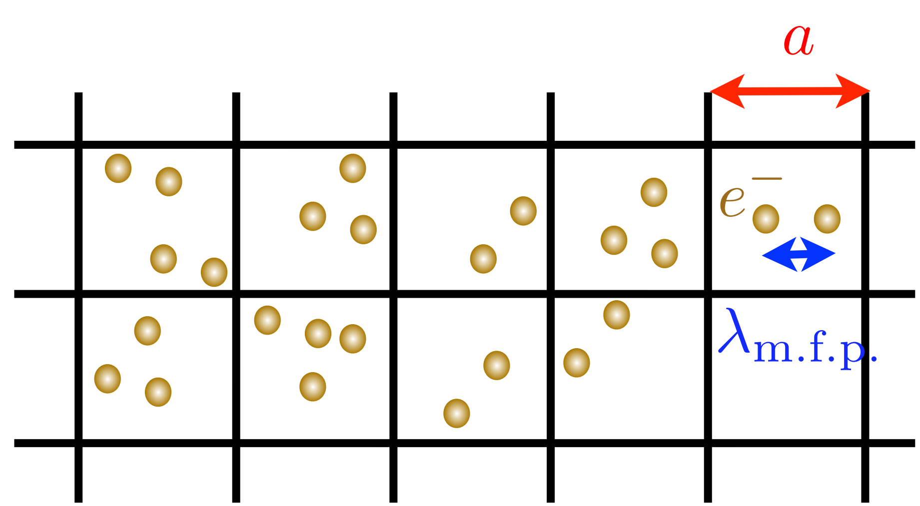
$\lambda_{\rm m.f.p.} \ll \text{external scales}$

- Hydro sets in when

$$\lambda_{\rm m.f.p.} \ll \frac{g_{\rm coupling}}{T}$$

- Momentum relaxes after collective behavior sets in

$\tau_{\rm rel.}^{-1} \sim \text{macro. physics}$



Resistivity and hydrodynamics

- Hydrodynamics is a universal LEET

Davison, Schalm, Zaanen PRB89 (2014) 245116
Andreev, Kivelson, Spivak PRL106 (2011) 256804

$$\rho_{DC} \sim \lim_{\omega\to 0} \int dk k^2 \frac{\mathrm{Im}\langle \mathcal{O}\mathcal{O}\rangle}{\omega}$$

- What choice for the impurity operator $\mathcal{O}$?
- Hydrodynamics: $T_{\mu\nu},\ J_\nu\ +\ \text{"irrelevant" ops}$
- For $\mathcal{O} = T^{00}$

$$\langle T^{00}T^{00}\rangle \sim \frac{1}{\omega^2 - k^2 + i\omega k^2 c_d \frac{\eta}{\epsilon+P}k^2 + \dots}$$

$$\rho_{DC} \sim \lim_{\omega\to 0}\int dk k(\eta k^2 + \dots) \sim s(T)$$

$$\eta = \frac{1}{4\pi}s$$

- Caveat: theory must be locally quantum critical $z \simeq \infty$

Lucas, Sachdev, Schalm, PRD89 (2014) 066018
Hartnoll, Mahajan, Punk, Sachdev PRB89 (2014) 155130

A universal mechanism for a linear resistivity

- Entropy density at low T

Davison, Schalm, Zaanen
PRB89 (2014) 245116

- *If* $s(T) \sim T + \ldots$ *Then* $\rho_{DC} \sim s(T) \sim T + \ldots$

Can be confirmed in a massive gravity model of a two-charge AdS black hole

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{3}{2} |\partial_\mu \Phi|^2 + \frac{6}{L^2} \cosh \Phi - \frac{1}{2} m^2 (\mathrm{Tr}(\mathcal{K})^2 - \mathrm{Tr}(\mathcal{K}^2)) \right)$$

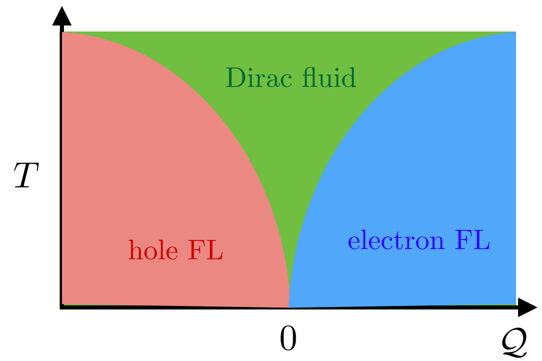
$$s_{BH} \sim T\mu + \ldots$$

- Universal linear-in-T resistivity from hydro + disorder

$$\rho_{DC} \sim T + \ldots$$

- Caveat: holography has many other "linear resistivity" scenarios

$$\epsilon_{a\sigma} = \hbar v_{\mathrm{F}} k$$
$$+$$
$$V_{\mathrm{int}} = \frac{\alpha_{\mathrm{eff}}}{r}$$



- marginally irrelevant $1/r$ Coulomb interactions:

$$\alpha_{\mathrm{eff}} = \frac{\alpha_0}{1 + (\alpha_0/4)\log((10^5\ \mathrm{K})/T)}, \quad \alpha_0 \approx \frac{1}{137}\frac{c}{v_{\mathrm{F}}\epsilon_{\mathrm{r}}} \sim 0.5.$$

- thermo/hydro nearly that of relativistic theory
- $\alpha_{\mathrm{eff}} \sim 0.3$ at $T = 100$ K

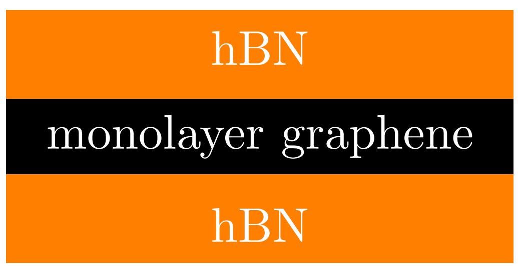
e.g. [Sheehy, Schmalian, *Physical Review Letters* **99** 226803 (2007)]
[Müller, Fritz, Sachdev, *Physical Review* **B78** 115406 (2008)]

Crossno, Kim et al. 1509.04713
Lucas, Crossno, Fong, Kim, Sachdev 1510.01738

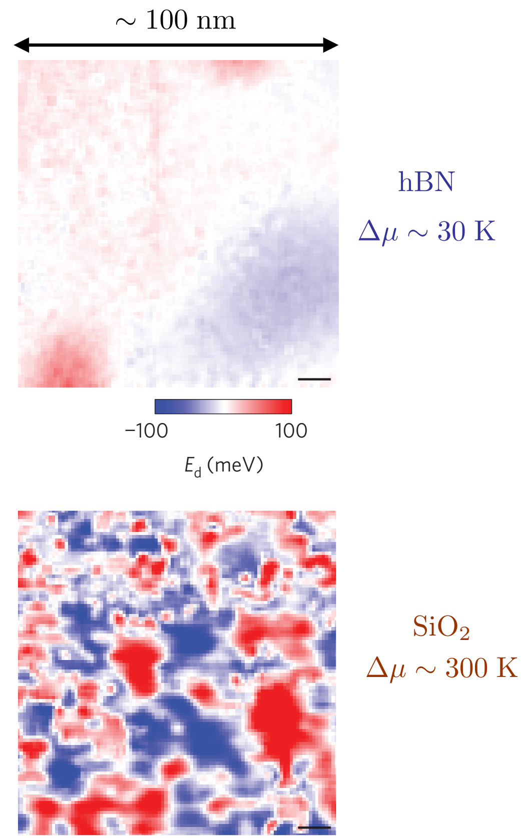
Slides from A. Lucas

Graphene: an Ideal Experimental Platform

- fabricating ultra pure monolayer graphene:
  [Dean *et al*, *Nature Nanotechnology* **5** 722 (2010)]



- weak disorder: charge puddles
  [Xue *et al*, *Nature Materials* **10** 282 (2011)]



Crossno, Kim et al. 1509.04713
Lucas, Crossno, Fong, Kim, Sachdev 1510.01738

Slides from A. Lucas

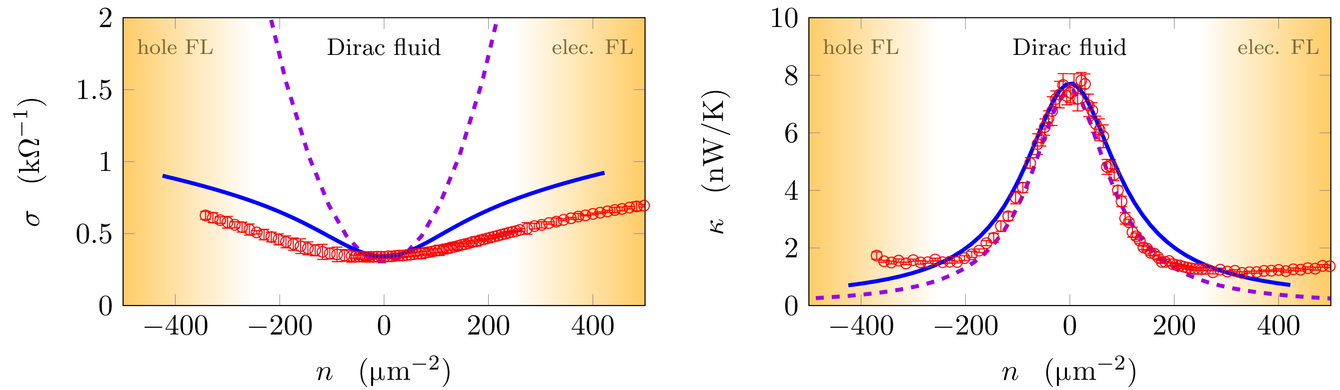
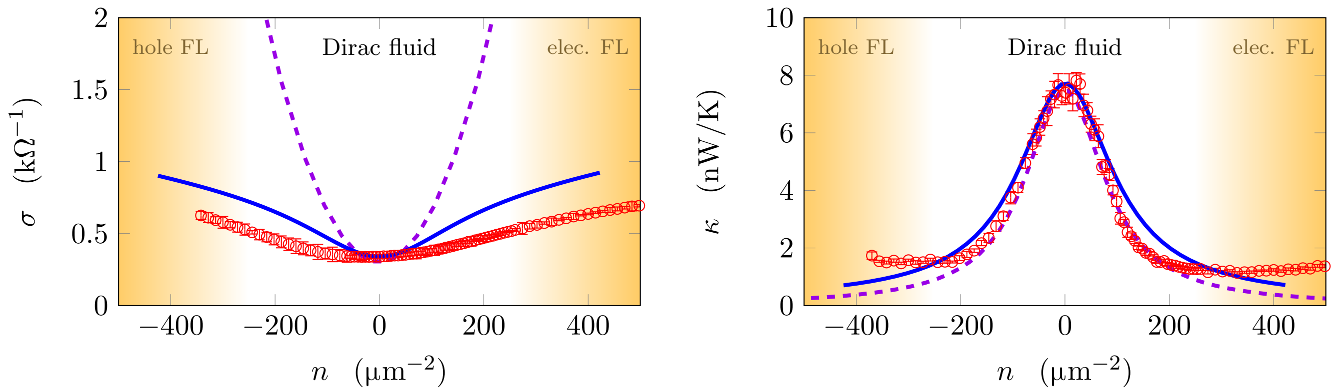


Figure 1: A comparison of our hydrodynamic theory of transport with the experimental results of [33] in clean samples of graphene at $T = 75$ K. We study the electrical and thermal conductances at various charge densities n near the charge neutrality point. Experimental data is shown as circular red data markers, and numerical results of our theory, averaged over 30 disorder realizations, are shown as the solid blue line. Our theory assumes the equations of state described in (27) with the parameters $C_0 \approx 11$, $C_2 \approx 9$, $C_4 \approx 200$, $\eta_0 \approx 110$, $\sigma_0 \approx 1.7$, and (28) with $u_0 \approx 0.13$. The yellow shaded region shows where Fermi liquid behavior is observed and the Wiedemann-Franz law is restored, and our hydrodynamic theory is not valid in or near this regime. We also show the predictions of (2) as dashed purple lines, and have chosen the 3 parameter fit to be optimized for $\kappa(n)$.

Crossno, Kim et al.
Lucas, Crossno, Fong, Kim, Sachdev



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Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]

[1] *Department of Physics, Harvard University, Cambridge, MA 02138, USA*
[2] *John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, USA*
[3] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
[4] *National Institute for Materials Science, Namiki 1-1, Tsukuba, Ibaraki 305-0044, Japan*
[5] *Raytheon BBN Technologies, Quantum Information Processing Group, Cambridge, Massachusetts 02138, USA*

(Dated: September 17, 2015)

Crossno, Kim et al.
Lucas, Crossno, Fong, Kim, Sachdev

Evidence for hydrodynamic electron flow in $PdCoO_2$

Philip J. W. Moll, [1,2,3] Pallavi Kushwaha, [3] Nabhanila Nandi, [3]
Burkhard Schmidt, [3] Andrew P. Mackenzie, [3,4*]

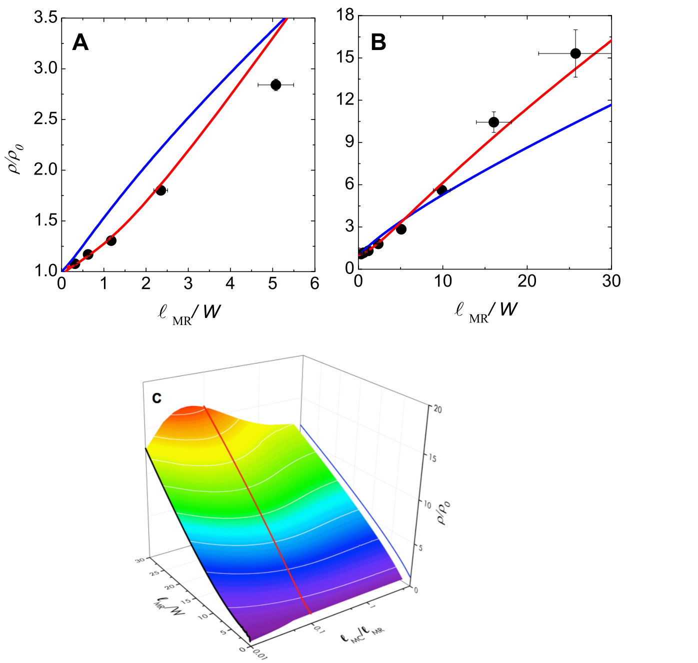


Fig. 4. Hydrodynamic effect on transport. (A, B) The measured resistivity of $PdCoO_2$ channels normalised to that of the widest channel (ρ_0), plotted against the inverse channel width $1/W$ multiplied by the bulk momentum- relaxing mean free path ℓ_{MR} (closed black circles). Blue solid line: prediction of a standard Boltzmann theory including boundary scattering but neglecting momentum-conserving collisions (Red line:prediction of a model that includes the effects of momentum-conserving scattering (see text). In (C) we show the predictions of the hydrodynamic theory over a wide range of parameter space.

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Autor
E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3]

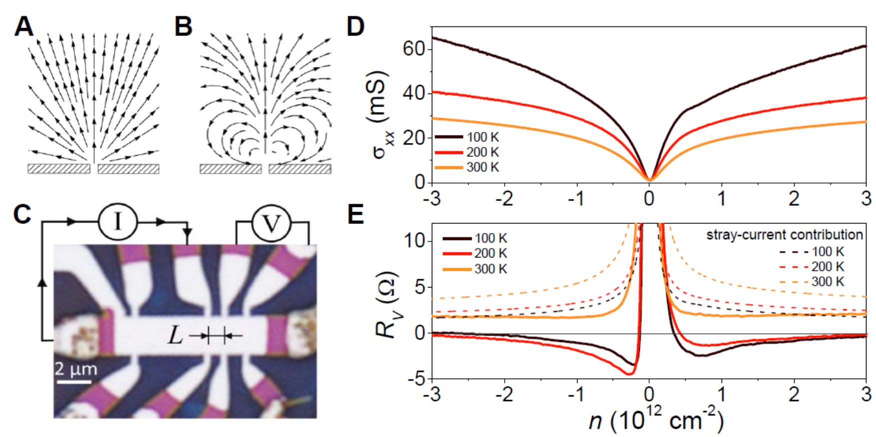
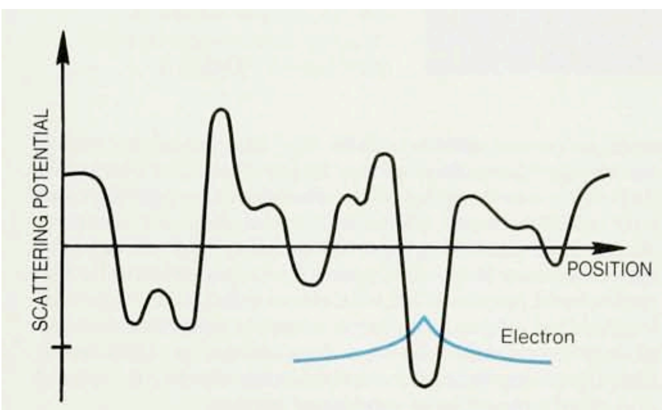


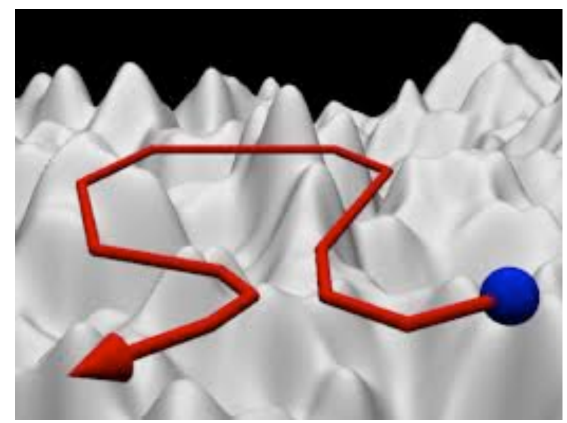
Fig. 1. Viscous backflow in doped graphene. (**A**,**B**) Calculated steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (A) and a viscous Fermi liquid (B). (**C**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**D**,**E**) Longitudinal conductivity σ_{xx} and R_V as a function of n induced by applying gate voltage. $I = 0.3$ µA; $L = 1$ µm. The dashed curves in (E) show the contribution expected from classical stray currents in this geometry (*18*).

Disorder and localization

- Strong disorder
- Anderson: disorder can localize charged excitations

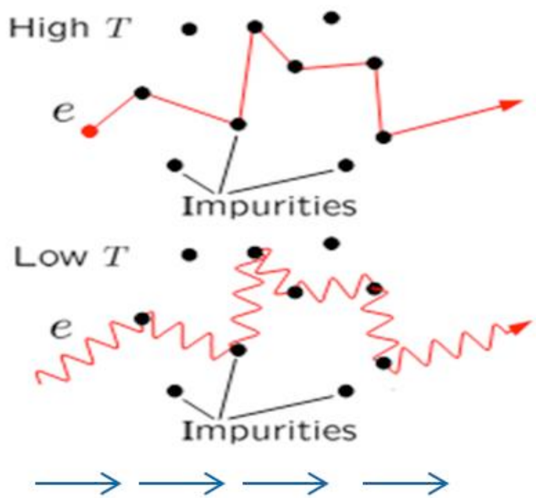
$$\text{free electron:} \quad \frac{\hat{p}^2}{2m}\Psi = E\Psi$$



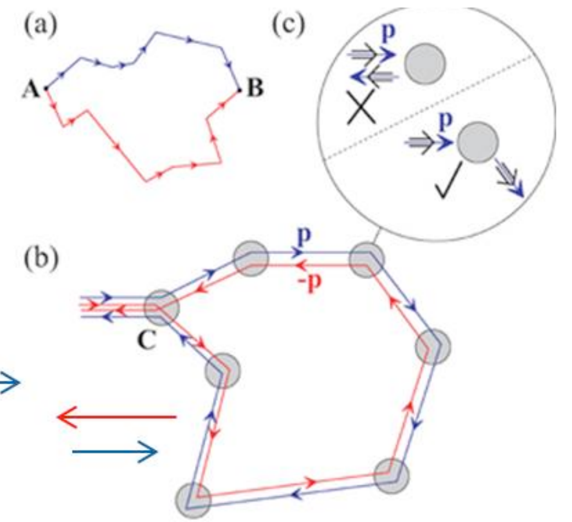


$$\text{localized electron:} \quad \frac{k\hat{x}^2}{2}\Psi = E\Psi$$

- Strong disorder
- Anderson: disorder can localize charged excitations



Anderson Localization



Einstein relation $\sigma = e^2 D\rho(E_F)$

$D = \frac{1}{2} v_F^2 \tau$

opposing interfering trajectories

constructive interference of backscattering

Strong disorder

- Strong disorder in weakly interacting systems
- Anderson: disorder can localize charged excitations

$\text{relevant: } d = 1, 2$ *groundstate is always an insulator*

$\text{marginal: } d = 3$

- Strong disorder in strongly interacting systems/many-body-theory
- Many-body-localization

Basko, Aleiner, Altschuler

Connected to quantum entanglement $S_{\text{ent};t=0\ |A\rangle\otimes|B\rangle} \sim \log(t)$

Failure to thermalize *(No eigenstate thermalization; no quantum chaos; "do not decohere" ... quantum computer)*

A lot of work in 1+1 dimensions

Ideal playground for holography

- Generic holographic disordered system has no disorder-driven insulating phase

Grozdanov, Lucas, KS, Sachdev

$$\sigma \geq \frac{1}{e^2} = 1$$

- Generic holographic disordered system has no disorder-driven insulating phase

Grozdanov, Lucas, KS, Sachdev

$$\sigma \geq \frac{1}{e^2} = 1$$

- Note: this is not a 1/N artifact. It is a strong coupling phenomenon.

- Can prove a similar bound for thermal conductivity. Grozdanov, Lucas, KS

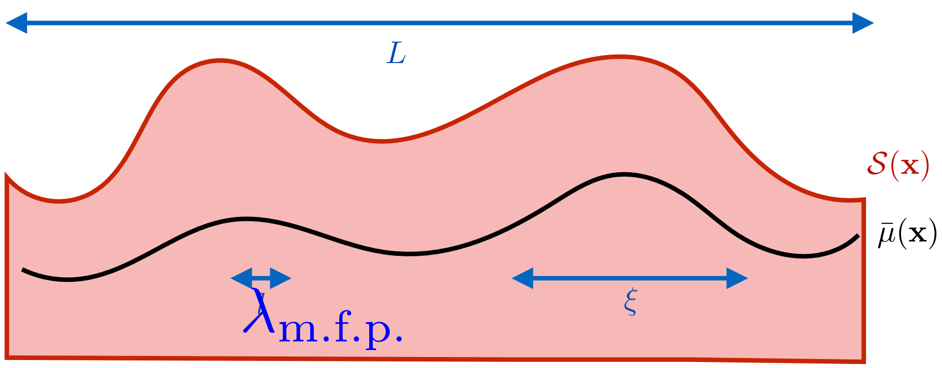
$$\kappa \geq 16\pi^3 \left(\frac{1}{1 - \frac{1}{2} V_{\min}} \right) \frac{T}{s} \quad d = 1$$

$$\kappa \geq \frac{4\pi^2}{3} \left(\frac{1}{1 - \frac{1}{6} V_{\min}} \right) T \quad d = 2$$

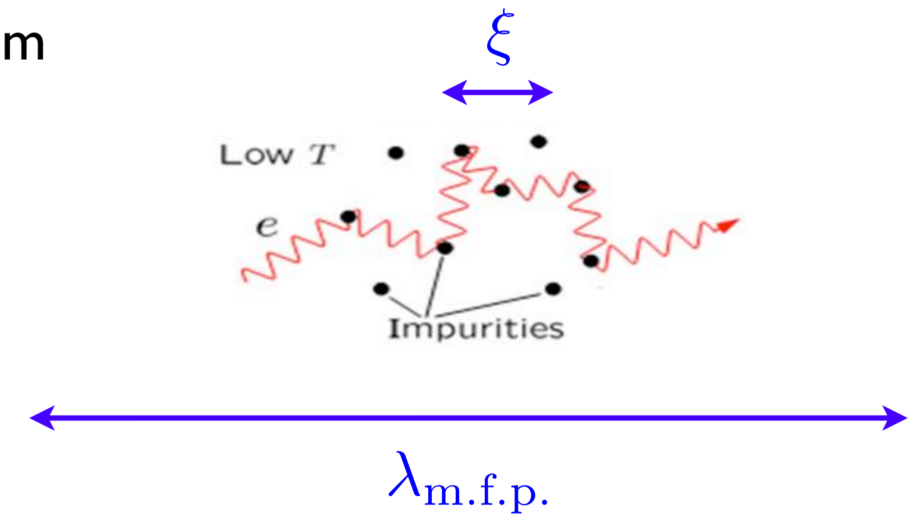
Bound follows from the fact that any Area a distance R from the horizon obeys

$$A_R \geq A_{\text{hor}}$$

- Classical gravity is infinitely strongly coupled system
- Hydrodynamics "always" applies
- No possibility for "random interference".

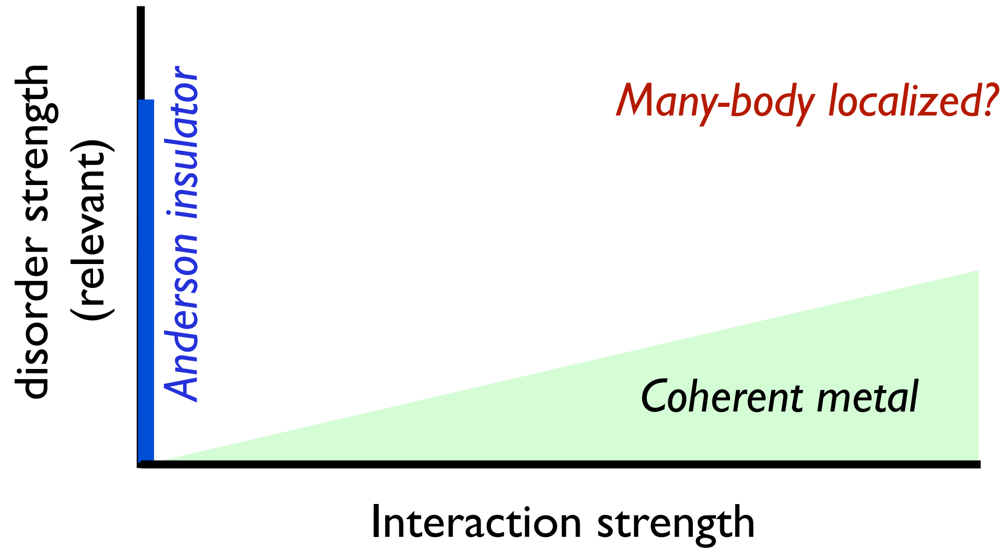


- A metal is a weakly coupled system
- Wave interference



Disorder and conductivity in holographic metals

- Disorder does not localize in ultra-strongly coupled systems
- There is a second incoherent metal phase



Lucas,
Grozdanov, Lucas, KS

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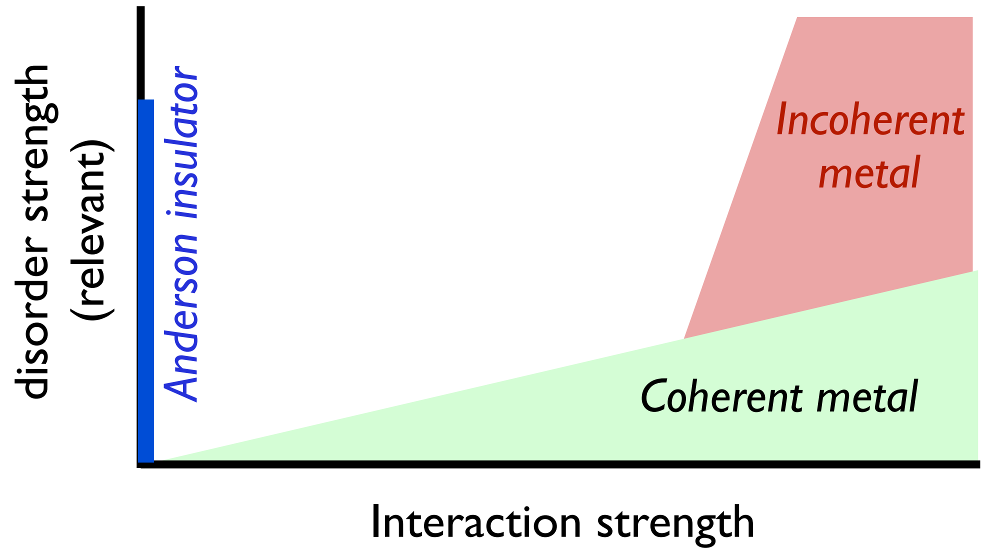


Lucas,
Grozdanov, Lucas, KS

- For localization in holography one has to go (far) beyond the classical approximation.

Postdictions from holography

- Linear-in-T resistivity $\rho \equiv \frac{1}{\sigma} \sim T$ ☑

  collective hydrodynamics plus disorder

- Lots of Power law scaling ☑

  quantum critical sector supported by an ordered state

- Hall angle vs DC conductivity scaling $\theta = \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$ ☐

- Inverse Matthiessen law $\sigma \sim \sigma_I + \sigma_{II}$ ☑

  quantum critical sector supported by an ordered state

The Hall Angle

Thermoelectric response and Momentum relaxation

Blake, Donos
PRL 114 (2015) 021601

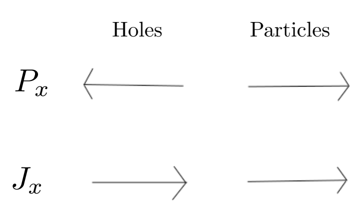
- Hall angle in "strange metals"

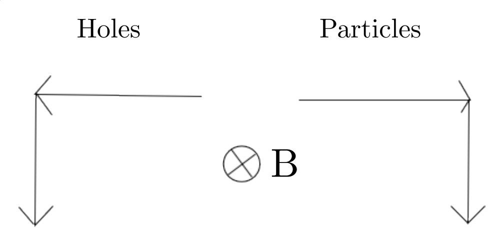
$$\sigma \sim \frac{1}{T} \qquad\qquad \theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$$

- Theory (e.g. Drude, memory matrix)

$$\sigma \sim \tau \qquad\qquad \theta_H \sim \tau$$

- Holography (no quasiparticles)





$$\sigma_{css} \text{ does not contribute to } \sigma_{xy}$$

$$\sigma = \sigma_{ccs} + \sigma_{relax} \qquad\qquad \sigma_{css} \sim \frac{1}{T}, \quad \sigma_{relax} \sim \frac{1}{T^2}$$

Postdictions from holography

- Linear-in-T resistivity $\rho \equiv \frac{1}{\sigma} \sim T$

  collective hydrodynamics plus disorder

- Lots of Power law scaling

  quantum critical sector supported by an ordered state

- Hall angle vs DC conductivity scaling $\theta = \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$

  quantum critical sector supported by an ordered state

- Inverse Matthiessen law $\sigma \sim \sigma_I + \sigma_{II}$

  quantum critical sector supported by an ordered state

Scaling in the cuprates

- Hyperscaling violating quantum critical theories

$$s \sim T^{(d-\theta)/z} \qquad\qquad A_t = Q\ r^{\zeta - z}$$

- ζ had not been considered before
- Non-zero ζ requires a CFT (no quasiparticles)

Phillips

- Parameters $d = 2,\ z = 4/3,\ \theta = 0,\ \zeta = -2/3$

Hartnoll, Karch
PRB89 (2015) 155126

- Fitting from Linear resistivity, Hall Angle, Hall Lorentz ratio
  - "postdicts" magnetoresistence and thermoelectric conductivity

    predicts Hall thermoelectric conductivity, and heat conductivity

    ...... in conflict with specific heat

What is the holographic strange metal?

- Holographic strange metal: Generalization of a Fermi liquid

Compressible $\frac{\partial}{\partial \mu} F \sim \mu^{\alpha}$

Exhibits Fermi Surfaces $G \sim \frac{1}{\omega - v_F k + \omega^{2\nu}}$

- Long range entangled

$$S_{EE} \sim (L k_F)^{d-1} \ln(L k_F)$$

Regular Fermi Liquid

Luttinger $Q \sim k^d$

$$S = Q^{\frac{d-1}{d}} A \ln(Q^{\frac{d-1}{d}} A)$$

Holography with $\theta = d - 1$

Huijse, Sachdev, Swingle,
PRB85 (2012) 035121

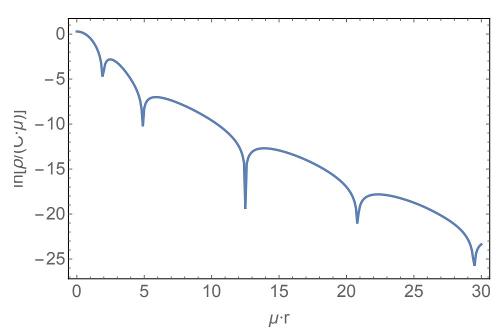
- Holographic strange metal: Generalization of a Fermi liquid

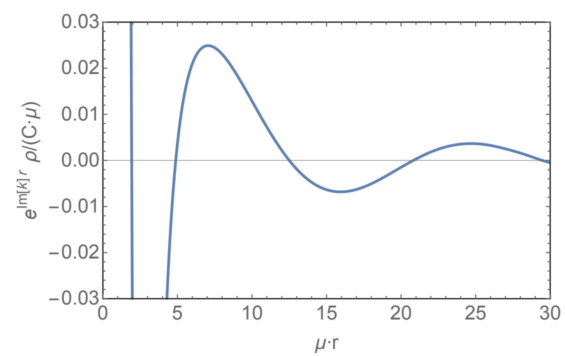
Compressible	$\frac{\partial}{\partial \mu} F \sim \mu^{\alpha}$
Exhibits Fermi Surfaces	$G \sim \frac{1}{\omega - v_F k + \omega^{2\nu}}$
Long range entangled	$S = Q^{\frac{d-1}{d}} A \ln(Q^{\frac{d-1}{d}} A)$

- Holographic Charge Oscillations

Blake, Donos, Tong,
JHEP 1504 (2015) 019

$$\chi(k) = \langle \rho(k) \rho(-k) \rangle$$





$$\delta\rho \sim \frac{e^{-r/\lambda}}{\sqrt{r}} \cos(r/\xi) \qquad r \gg R, T^{-1}, \mu^{-1}$$

Distinctive:
Fall-off remains exponential even at $T = 0$

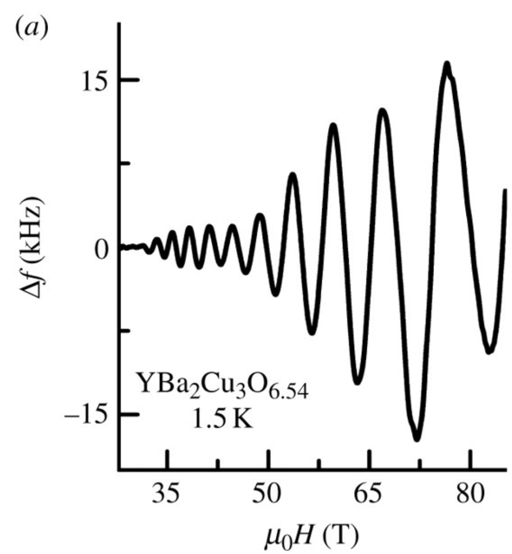
$$\mu = \mu_0 + C e^{-r^2/2R^2}$$

Quantum Oscillations

- Holographic strange metal: Generalization of a Fermi liquid
- Quantum Oscillations due to a conventional Fermi surface

$$\chi_{\rm osc} \sim \cos \frac{A_F}{B} \sum_{n=0}^{\infty} e^{-c_n \frac{T A_F}{\mu B}}$$

$$A_F = \pi k_F^2$$



Sebastian, Harrison, Lonzarich
PTRS A369 (2011) 1687

Quantum Oscillations

- Holographic strange metal: Generalization of a Fermi liquid
- Quantum Oscillations due to NFL Fermi surface without quasiparticles

$$\chi_{\rm osc} \sim \cos \frac{A_F}{B} \sum_{n=0}^{\infty} e^{-c_n \frac{T A_F}{\mu B} \left(\frac{T}{\mu}\right)^{2\nu - 1}}$$

Hartnoll, Hofman
PRB81 (2010) 155125

$$G_F \sim \frac{1}{\omega - v_F k + \omega^{2\nu}}$$

- Not been seen in the cuprates, so far.

The theory of a strange metal

The theory of a strange metal

- A quantum critical system --- a theory without quasiparticles, supported by an ordered state with transport characterized by collective behavior.

Faulkner, Polchinski;
Jensen;
Iqbal, Liu, Mezei

$$S = \int \mathrm{d}^d x \left[-(\partial\phi)^2 + \mu^2 \phi - \lambda \phi^4 - \phi \mathcal{O}_\phi \right] + S_{\mathrm{quant.crit}}[\mathcal{O}]$$

- Experiment: Excitations around the FS do not determine transport.
- Theory: framework provided by an holography

Postdictions from holography

- Linear-in-T resistivity $\rho \equiv \frac{1}{\sigma} \sim T$

  collective hydrodynamics plus disorder

- Lots of Power law scaling

  quantum critical sector supported by an ordered state

- Hall angle vs DC conductivity scaling $\theta = \frac{\sigma_{xy}}{\sigma_{xx}} \sim \frac{1}{T^2}$

  quantum critical sector supported by an ordered state

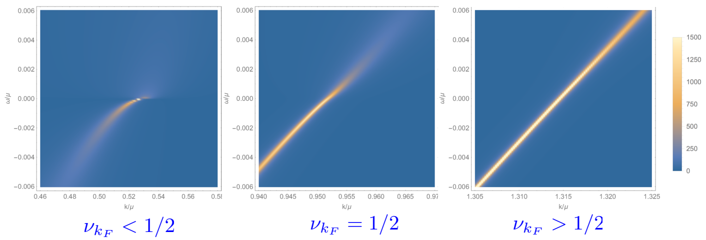
- Inverse Matthiessen law $\sigma \sim \sigma_I + \sigma_{II}$

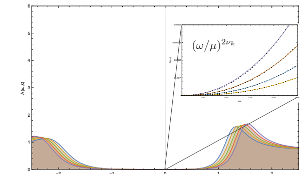
  quantum critical sector supported by an ordered state

Predictions for strange metals from holography.

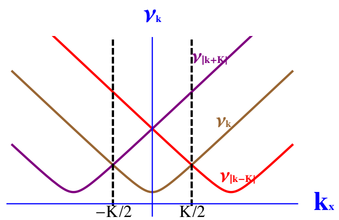
Predictions from holography

- ARPES
- NFL line-shapes

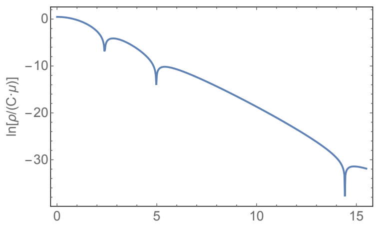




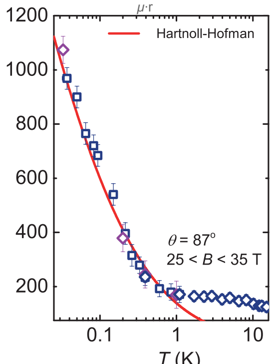
- Scaling near $\omega = 0 \ll E_F$
- Lattice dependence of the line-shape



- Charge oscillations
- Charge susceptibility: no $T = 0$ power-law



- Quantum oscillations
- Unconventional T dependence



- ARPES
- NFL line-shapes

no quasiparticles

- Scaling near $\omega = 0 \ll E_F$
- Lattice dependence of the line-shape

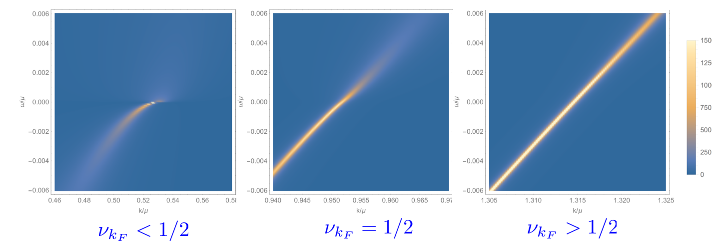
Evidence of the quantum critical sector

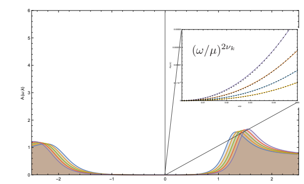
- Charge oscillations
- Charge susceptibility: no $T = 0$ power-law

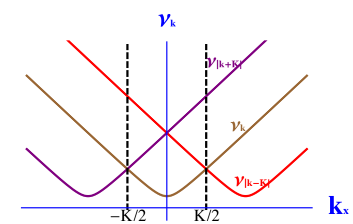
Evidence of the quantum critical sector

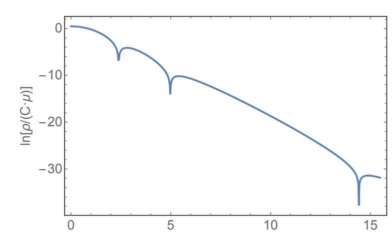
- Quantum oscillations
- Unconventional T dependence

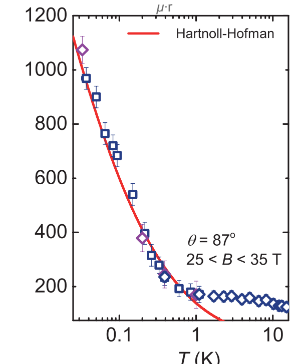
no quasiparticles











Predictions from holography

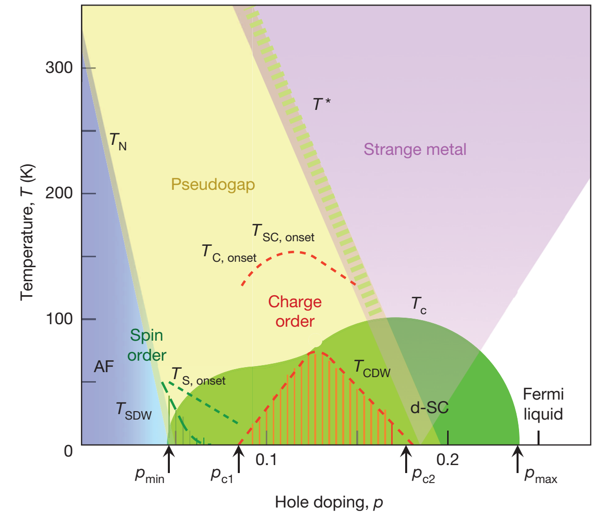
- Macroscopic signatures ....
- Two sectors contribute to transport

  - collective diffusive vs ballistic

1. Far superior method to compute *real time* finite temperature/density correlation functions

- Instabilities: superconductivity

1. Generating functional for new non-trivial *unknown* IR fixed points



1. onset
2. gap physics
3. exotic phases

Holography gives a consistent, predictive framework
that captures the right physics of experimental strange metals.

Holography gives a consistent, predictive framework
that captures the right physics of experimental strange metals.

One step remains: to write down the quantitative theory.

Thank you

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