Stringy instantons, deformed geometry and axions.

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Motivation & Outline

- Motivation
  - **Axions** have very flat potentials $\Rightarrow$ Good for Inflation.
  - Axions developing a potential, i.e. through instantons, can always be described as a **3-Form eating up the 2-Form** dual to the axion.
  - There is no candidate 3-Form for stringy instantons. We will look for it in the **geometry deformed by the instanton**.
  - The instanton triggers a **geometric transition.**
- Outline
  1. Axions and 3-Forms.
  2. Backreacting Instantons.
  3. Flavor branes at the conifold.

Axions and 3-Forms

Axions

Axions are **periodic scalar** fields.

- This shift symmetry can be **broken to a discrete shift symmetry** by non-perturbative effects $\rightarrow$ Very flat potential, good for inflation.
- However, discrete shift symmetry is not enough to keep the corrections to the potential small. There is a dual description where the naturalness of the flatness is more manifest.
- For example, a quadratic potential with monodromy can be described by a Kaloper-Sorbo lagrangian (Kaloper & Sorbo, 2009),

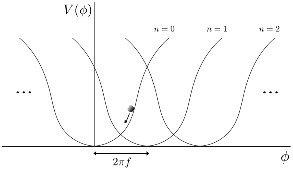
$$|\,\mathrm{d}\phi|^2 + n\phi F_4 + |F_4|^2 \tag{1}$$

- The potential is recovered using E.O.M,

$$V_0 \sim (n\phi - q)^2, \quad q \in \mathbb{Z} \tag{2}$$

A dual description.

⇒ Where the monodromy is given by the vev of F_4, different fluxes correspond to different branches of the potential.



- A dual description can be found using the hodge dual of ϕ, b_2,

$$|\,\mathrm{d}b_2 + nc_3|^2 + |F_4|^2\,, \quad F_4 = \mathrm{d}c_3 \tag{4}$$

The flatness of the axion potential is protected by gauge invariance,

$$c_3 \to c_3 + \mathrm{d}\Lambda_2\,, \quad b_2 \to b_2 - n\Lambda_2 \tag{5}$$

- Corrections to the potential must go as $\sim (|F_4|^2/M_p^4)^n \sim (V_0/M_p^4)^n$, hence making the potential naturally flat. This is not evident in the axion formulation.
- One can always use this formultaion. We will look for a three form c_3 coupling to the axion as $\sim \phi F_4$.

Example: Peccei-Quinn mechanism in 3-Form language.

- **Strong CP problem**: QCD has an anomalous $U(1)_A$ symmetry producing a *physical* θ-term.

$$\mathcal{L} \sim \frac{g^2\theta}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \tag{6}$$

  Where θ classifies topologically inequivalent vacua and is typically ~ 1. This term breaks CP and experimental data constrain $\theta \lesssim 10^{-10} \rightarrow$ **Fine Tuning.**

- **Solution**: PQ Mechanism. New anomalous spontaneously broken $U(1)_{PQ}$. So θ is promoted to pseudo-Goldstone boson, the axion. Non-perturbative effects give it a periodic potential of the form,

$$V(\theta) = 1 - cos(\theta) \tag{7}$$

  and fix $\theta = 0$.

Example: Peccei-Quinn mechanism in 3-Form language.

- The term can be rewritten as (Dvali, 2005),

$$\theta F\tilde{F} \sim \theta F_4 = \theta \, \mathrm{d}C_3, \quad C_{\alpha\beta\gamma} = \frac{g^2}{8\pi^2}\mathrm{Tr}\left(A_{[\alpha}A_{\beta}A_{\gamma]} - \frac{3}{2}A_{[\alpha}\partial_{\beta}A_{\gamma]}\right) \tag{8}$$

So, making θ-term small = How can I make the F_4 electric field small?

- **Solution**: Screen a field $\Rightarrow$ Higgs mechanism. Let C_3 eat a 2-Form b_2!
$\Rightarrow$ b_2 *is* Hodge dual of axion!
Potential for the axion $\Longleftrightarrow$ 3-Form eating up a 2-form.

Axions in String Theory.

- Axions are ubiquitous in string theory. For instance upon compactification of p-forms on p-cycles of the internal space,

$$a(x) = \int_{\Pi_p} C_p \tag{9}$$

Where the shift symmetry arises from the gauge invariance of the RR form.

⇒ With potential arising non-pertubatively from euclidean strings/branes on the p-cycle.

Backreacting Instantons

D-Brane Instantons

- Consider a compactification to $\mathcal{M}_4 \times \mathrm{CY}_3$.
- Dp-Branes completely wrapped around internal euclidean $(p+1)$ cycles are instantons from the spacetime point of view. Hence dubbed D-brane instantons. There are two kinds:
  - Dp-Brane instanton inside D$(p+4)$-Brane spanning $\mathcal{M}_4 \to$ Gauge Instanton.
  - Dp-Brane instanton alone $\to$ **Stringy Instanton.**

  $\Rightarrow$ We study non-perturbative potentials for axions coupling to stringy instantons.
- For instance, consider type IIB String Theory with a D3-instanton wrapping a 4-cycle Π_4 in a CY_3 and producing a non-perturbative superpotential,

$$\mathcal{W}_{np} = A \exp\left(-Vol(\Pi_4)\right) \tag{10}$$

The puzzle

Axion potentials can be described by a 3-form eating up a 2-form (Additional 3-forms might be needed for non invertible potentials).For instance, for gauge D-brane instantons it is the CS 3-Form (Dvali, 2005).

- For stringy instantons $\rightarrow$ **No candidate 3-Form!**.
  In this example, for an axion $\phi = \int_{\Pi_4} C_4$, we would need something like $\phi \int_{\Pi_2} C_5$ and can't in IIB.
  $\Rightarrow$ Solution: look for the 3-Form in the *backreacted* geometry (Koerber & Martucci, 2007).
- In the backreacted geometry the **instanton disappears** and its open-string degrees of freedom are encoded into the geometric (closed strings) degrees of freedom. Both descriptions are related in an *"holographic"* way.

SU(3) × SU(3) structure manifolds.

- The backreacted geometry will, generically, be a non CY, SU(3) × SU(3) structure manifold. This structure has two globally well defined spinors $\eta_+^{(1)}, \eta_+^{(2)}$, with c.c. $\eta_-^{(1)}, \eta_-^{(2)}$.
  Define two polyforms (assume type IIB):

$$\Psi_\pm = -\frac{i}{||\eta^{(1)}||^2} \sum_l \frac{1}{l!} \eta_\pm^{(2)\dagger} \gamma_{m_1 \dots m_l} \eta_+^{(1)} \, dy^{m_l} \wedge \dots \wedge dy^{m_1} \tag{11}$$

- Organize 10d fields in holomorphic polyforms,

$$\mathcal{Z} \equiv e^{3A-\Phi} \Psi_2, \quad \mathcal{T} \equiv e^{-\Phi} \mathrm{Re}\, \Psi_1 + i \Delta C \tag{12}$$

- For an SU(3) structure manifold, $\eta^{(1)} \sim \eta^{(2)}$ and one recovers $\mathcal{Z} \sim \Omega$, $\mathcal{T} \sim e^{iJ}$

Backreacting a D3-instanton

- It can be shown from the condition of the D3-instanton being SUSY preserving, using the F-flatness condition for variations of $\mathcal{T}$, that its backreaction on the geometry is **encoded in a contribution to** $\mathcal{Z}$ (Koerber & Martucci, 2007):

$$\mathrm{d}(\delta\mathcal{Z}) \sim \mathcal{W}_{\mathrm{np}}\delta_2(\Pi_4) \tag{13}$$

$\Rightarrow$ So, the backreaction produces a 1-form $\mathcal{Z}_1$ that didn't exist in the original geometry.

- Note that $\mathcal{Z}_1$ is not closed and thus not harmonic.

The 3-form and the KS coupling

- Let us define $\alpha_1 \equiv \mathcal{Z}_1$ and $\beta_2 \equiv d\alpha_1$.
- In type IIB string theory there is a RR 4-form C_4. We may expand it as,

$$C_4 = \alpha_1(y) \wedge c_3(x) + \beta_2(y) \wedge b_2(x) + \ldots \tag{14}$$

$\Rightarrow$ So we have a 3-form and a 2-form dual to an scalar.

- We see that,

$$F_5 = (1 + *)\, dC_4 = (1 + *)(\beta_2 \wedge (c_3 + db_2) - \alpha_1 \wedge F_4) \tag{15}$$

Which describes a 3-Form eating up a 2-Form, as we wanted! Furthermore,

$$\int_{10d} F_5 \wedge *F_5 = -\int_{10d} C_4 \wedge d{*F_5} \to \int_{10d} C_4 \wedge \beta_2 \wedge F_4 = \int_{4d} \phi F_4 \tag{16}$$

Which is the KS coupling we were looking for.

Toroidal examples.

- For a 4-cycle defined by the equation $f = 0$ the 1-form is $\mathcal{Z}_{(1)} \sim \mathrm{d}f\tilde{W}_{np}$.
- **Example 1**: Factorisable 6-torus, $\mathcal{M}^4 \times \mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$ with local coordinates z_1, z_2, z_3 and take the cycle $f = z_3 = 0$,

$$\mathcal{Z}_{(1)} \sim e^{-T}\,\mathrm{d}z_3 \tag{17}$$

$\Rightarrow$ already in the original geometry, because $\mathbf{T}^6$ has trivial holonomy.

- **Example 2**: Orbifold $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ with action,

$$\theta : (z_1, z_2, z_3) \to (-z_1, z_2, -z_3), \quad \omega : (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \tag{18}$$

Taking local coordinates $u_i = z_i^2$, $t = z_1 z_2 z_3$, $t^2 = u_1^2 u_2^2 u_3^2$, and cycle $f \sim u_3 = 0$ the backreaction is,

$$\mathcal{Z}_{(1)} \sim \mathrm{d}u_3 \sim z_3\,\mathrm{d}z_3 \tag{19}$$

D-brane gauge Instanton backreaction

- D3-brane instantons wrapping the same 4-cycle as N spacetime-filling D7-branes are gauge instantons from the SU(N) point of view.
- In the gauge (open string) description a CS 3-Form is available to couple to the axion. And backreaction?
- We can backreact the D7's together with their non-perturbative effects to find a dual description in terms of closed string degrees of freedom. **Again a 1-Form arises** when taking the non-perturbative effects into account (Koerber & Martucci, 2007),

$$\mathrm{d}\mathcal{Z} = i l_s \langle S \rangle \, \delta_2(\Pi_4) \tag{20}$$

  Where S is the gaugino condensate describing the non-perturbative effects in the gauge theory.
- There are two descriptions and so **2 different 3-forms that do not coexist.**

Generalization

- The F-flatness equations for IIA are more involved, because of the form of $\mathcal{W}$, but one can use mirror symmetry.
- **Mirror symmetry** is a symmetry between IIA and IIB living in mirror CY_3's. In the large complex structure limit it equal to 3 T-dualities.
- The mirror dual to our setup consists on a D2-brane wrapping a 3-cycle. The backreaction gives rise to,

$$\delta\mathcal{T} = \mathcal{T}_{(2)} + \mathcal{T}_{(4)} \tag{21}$$

$\Rightarrow$ so we can obtain the 3-Form ,

$$C = \delta\mathcal{T} \wedge c_3 \tag{22}$$

Where C is the RR polyform.

Flavor Branes at the conifold

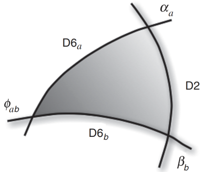
Field Operators from Instantons

- Consider (Ibanez & Uranga, 2012) CY_3 compactification with n $D6_A$-branes wrapping 3-cycles Π_A and a D2-brane instanton wrapping a 3 cycle Π_s. Let us take some of them, $D6_a$ with intersection number $I_{a,s} = 1$, and the rest, $D6_b$, with $I_{b,s} = -1$.
- The D2 instanton contribution to the field theory is, upon integrating over fermion zero modes,

$$e^{-S_{cl.}} \int \mathrm{d}\alpha_a \, \mathrm{d}\beta_b e^{-c(\alpha \Phi_{ab} \beta)} = e^{-S_{cl}} \det(\Phi_{ab}) \tag{23}$$

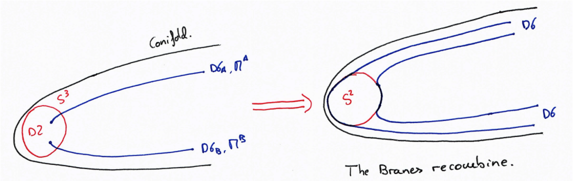
  Where $a, b = 1 \ldots n$. The Φ_{ab} are 4d chiral multiplets in the bifundamental at $D6_a$-$D6_b$ intersections and α_a, β_a are the fermionic zero modes of the instanton needed to preserve the gauge symmetry. These come from a WS instanton stretching between the D2 and the flavor D6's.

⇒ This operator **breaks a U(1)** subgroup of the flavor group. Can this breaking of the symmetry be seen geometrically as a transition triggered by the instanton?



Geometric transition at the conifold

- D2 instanton at S^3 gives rise to new 2-form $d\delta\mathcal{T}_2 = \delta_3(S^3)$ whose hodge dual ω_4 is related to the dual cycle to S^3, B, as $d\delta_3(B) = \omega_4$.
  $\Rightarrow$ This means that there is a new 2-cycle poincaré dual to ω_4 and it is boundary to B. This cycle must be S^2.
- So, $\Pi_3^a = B + ...$, $\Pi_3^b = -B + ...$ develop boundaries S^2 and $-S^2$, respectively. Thus **the two flavor branes recombine.**



- The geometric transition is reproducing the required symmetry breaking. But, can we generalize this to non-abelian setups?

Conclusions & Outlook

- **Axions** are a useful tool for inflation and are easily realised in String Theory.
- Axion physics can be described in a dual language where a **3-form eats up a 2-form.**
- For stringy instantons **no known 3-form** to couple to the 2-form was known. We have showed that it only appears when the **"backreaction"** is taken into account.
- The geometry ceases to be CY and new forms (and cycles) that were not there arise.
- **Further work:**
  - Use this mechanism as a self consistency test for the Weak Gravity Conjecture for axions and 3-Forms.
  - Study the backreaction of particular instantons in specific setups $\Rightarrow$ $\Rightarrow$ Conifolds...

Thank You!

The Eq. in more detail.

- F-flatness equation

$$\partial_\phi \mathcal{W} - 3(\partial_\phi \log \mathcal{N})\mathcal{W} = 0 \tag{24}$$

  Applying to $\mathcal{T}$ one finds,

$$\delta_\mathcal{T} \mathcal{W} - 3(\delta_\mathcal{T} \log \mathcal{N})\mathcal{W} = 0 \Rightarrow d_H \mathcal{Z} = 2iW_0 e^{2A} \text{Im} t \tag{25}$$

  While for $\mathcal{Z}$ one finds,

$$\int_M \langle \delta\mathcal{Z}, d_H \text{Re} t - iF - \frac{\mathcal{W}}{2\mathcal{N}} e^{-4A} \tilde{\mathcal{Z}} \rangle = 0 \tag{26}$$

  Which is more complicated.

- Adding a non-perturbative contribution and neglecting the first term one finds,

$$d_H \mathcal{Z} \simeq 2i\mathcal{W}_{np} j_{np} = -2i\mathcal{W}_{np} \delta^{(2)}(\Pi) \tag{27}$$

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