Holographic Equilibration in Confining Gauge Theories Under External Magnetic Fields

Tuna Demircik
Sabanci University / Utrecht University
(arXiv:1605.08118v3 with U.Gursoy)

January 16, 2017
QGP

States of strongly interacting matter
Heavy ion experiments and QGP

Holography

Holography
QNMs

Gravity Setting

The glue sector
The flavour sector
Background at a finite magnetic field and temperature

QNMs

QNMs and Tensor decomposition
Shear channel
Scalar channel

Conclusion

Conclusion
States of strongly interacting matter:

i Confinement/Deconfinement transition:

\[ L(T) \sim \lim_{r \to \infty} \exp(-V(r)/T) \]

\[ T < T_c \Rightarrow L \to 0 \Rightarrow \text{Conf.} \]
\[ T > T_c \Rightarrow L \neq 0 \Rightarrow \text{Deconf.} \]

ii Chiral symmetry restoration:

\[ T < T_c \Rightarrow M_q \neq 0 \Rightarrow \chi_{SB}. \]
\[ T > T_c \Rightarrow M_q \to 0 \Rightarrow \chi_S. \]

iii Diquark matter.

Lattice QCD: for \( \mu_B = 0, \ T_c = 150 - 200 \text{MeV} \) and \( n_c = 0.1 \text{fm}^{-3} \).

Figure 1: QCD phase diagram.
Heavy ion experiments and QGP:

- \( \sim 10^{12} K, \sim 10 \text{fm}/c, \sim 10 \text{fm} \).
- RHIC: \( \sqrt{s} = 200 \text{GeV} \),
  LHC: \( \sqrt{s} = 2.76 \text{TeV} \).
- RHIC: \( e|\vec{B}|/m_\pi^2 \approx 1 - 3 \),
  LHC: \( e|\vec{B}|/m_\pi^2 \approx 10 - 15 \).
- Extremely small \( \eta/s \approx 1/4\pi \).

QGP is in strong interaction regime and \( \eta/s \) ratio is in very good agreement with AdS/CFT result.

Figure 2: Heavy ion collisions.
Holography:

- The glimpses:
  i. Holographic principle;
  ii. Large-$N_c$ theories;

- Maldecena conjecture:

$$N = 4\text{SYM} \leftrightarrow \text{SUGRA on AdS}_5 \times S_5$$

$$Z_{CFT}(N_c \gg \lambda \gg 1) = Z_{AdS_5}$$

- In Nonequilibrium:

$$\langle \exp(i \int \phi^{(0)} O) \rangle = e^{S[\phi|_{u=0}=\phi^{(0)}]}$$

**Figure 3:** GKB-W relation in nonequilibrium situation.
QNMs:

- Introducing a fluctuation to constitutive equation in hydrodynamics:
  \[ T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} + P\eta^{\mu\nu} + \tau^{\mu\nu} \]

  results in dispersion relation which corresponds to poles of retarded Green function. This dispersion relation is related with transport coefficients. e.g.:

  \[ \text{vector mode: } \omega = -i\frac{\eta}{T_s}q^2 \]

- Bulk field generally has the asymptotic behaviour:
  \[ \phi \sim A r^{\Delta -} + B r^{\Delta +} \quad (r \to 0), \quad \Delta_+ > \Delta_- \]

  by following AdS/CFT prescription, the location of poles:

  \[ G_R \propto \frac{B}{A} \implies \]

  - A vanishing slow fall off BC on boundary.

  - Incoming BC at horizon.
The action:

\[ S = S_g + S_f \]  (1)

- IHQCD with backreacting flavor branes with Veneziano limit:

\[ N_c \to \infty, \quad N_f \to \infty \quad \text{and} \quad \frac{N_f}{N_c} \equiv x = \text{finite} \]

\[ \lambda = g_{YM}^2 N_c = \text{fixed} \]

The glue sector:

\[ S_g = M_p^3 N_c^2 \int d^5x \sqrt{-g} \left( R - \frac{4}{3} (\partial \phi)^2 + V_g(\phi) \right) \]  (2)
The flavour sector:

\[ S_f = -\frac{1}{2} M_p^3 N_c \text{Tr} \int d^5 x (V_f(\lambda, T^T T) \sqrt{- \det A_L} + V_f((\lambda, TT^T) \sqrt{- \det A_R}) \]

\[ A_{L\mu\nu} = g_{\mu\nu} + w(\lambda, T) F_{\mu\nu}^L + \frac{\kappa(\lambda, T)}{2} [(D_{\mu} T)^\dagger (D_{\nu} T) + (D_{\nu} T)^\dagger (D_{\mu} T)] \]

\[ A_{R\mu\nu} = g_{\mu\nu} + w(\lambda, T) F_{\mu\nu}^R + \frac{\kappa(\lambda, T)}{2} [(D_{\mu} T)(D_{\nu} T)^\dagger + (D_{\nu} T)(D_{\mu} T)^\dagger] \]

\[ D_{\mu} T = \partial_{\mu} T + i T A_{\mu}^L - i A_{\mu}^R T \quad \quad V_f(\lambda, TT^T) = V_f(0) e^{-a(\lambda) TT^T} \]

\[ T = \tau(r) \mathbb{1}_{N_f} \quad \text{and} \quad \tau(r) = 0 \quad (\text{chiral symmetric phase}) \]

\[ S_f = -M_p^3 N_c \text{Tr} \int dx^5 V_f(\lambda) \sqrt{-g} \sqrt{- \det (\delta_{\mu\nu} + w(\lambda)^2 g^{\mu\rho} F_{\rho\nu})} \]

\[ = -M_p^3 N_c N_f \int dx^5 V_f(\lambda) \sqrt{-g} \left( 1 + \frac{w(\lambda)^2}{4} F_{\mu\nu} F^{\mu\nu} \right) \quad (3) \]
Background at a finite magnetic field and temperature

The Einsteins equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \left( \frac{4}{3} \partial_\mu \phi \partial_\nu \phi - \frac{2}{3} g_{\mu\nu} (\partial \phi)^2 + \frac{1}{2} g_{\mu\nu} V_{\text{eff}} \right)$$

$$- x \frac{V_b}{2} \left( F_{\mu}^{\ \rho} F_{\nu\rho} - \frac{g_{\mu\nu}}{4} F_{\rho\sigma} F^{\rho\sigma} \right) = 0 \quad (4)$$

The Maxwell equations:

$$\partial_\mu (\sqrt{-g} V_b F^{\mu\nu}) = 0 \quad (5)$$

The dilaton equation:

$$\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right) + \frac{3}{8} \frac{\partial V_{\text{eff}}}{\partial \phi} - \frac{3x}{32} \frac{\partial V_b}{\partial \phi} F^2 = 0 \quad (6)$$
Background at a finite magnetic field and temperature

**Constant background magnetic field:**

\[ V_\mu = \left( 0, -\frac{x_2 B}{2}, \frac{x_1 B}{2}, 0, 0 \right) \]  
(7)

**Ansatz for the metric:**

\[ ds^2 = e^{2A(r)} \left( -e^g(r) dt^2 + dx_1^2 + dx_2^2 + e^{2W(r)} dx_3^2 + e^{-g(r)} dr^2 \right) \]  
(8)

\[ r \in [0, r_h] : \quad r_h \quad \text{is the location of the horizon where} \quad g(r_h) = -\infty. \]

The UV boundary is at \( r = 0 \), where we demand \( AdS_5 \) asymptotics (\( A \to -\log(r), g \to 0, W \to 0 \) as \( r \to 0 \)).

**The dilaton:**

\[ \lambda = \lambda(r) = e^{\phi(r)} \]  
(9)
Background at a finite magnetic field and temperature

**The Potentials:**

\[ V_b(\lambda) = V_f(\lambda)w(\lambda)^2, \quad V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0} \]  \hspace{1cm} (10)

**i** \[ V_g(\lambda) = \frac{12}{l^2} \left( 1 + V_0\lambda + V_1\lambda^{4/3}\sqrt{\log(1 + V_2\lambda^{4/3} + V_3\lambda^2)} \right) . \]

- In the UV, It matches the perturbative large-$N_c$ $\beta$-function and corresponds to initial conditions of an RG flow with asymptotic freedom.
- In the IR, It guarantees that the dual field theory is confining with a gapped glueball spectrum.

**ii** \[ V_{f0}(\lambda) = W_0(1 + W_1\lambda + W_2\lambda^2) \text{ and } w(\lambda) = \left( 1 + \frac{3a_1}{4}\lambda \right)^{-\frac{4}{3}} \]

- In the UV, It matches the perturbative anomalous dimension of the quark mass operator.
- In the IR, It satifies the requirement of chiral symmetry and the meson spectra.
The Einstein’s equations yield three 2nd order equations and one 1st order constraint equation.

The solutions are only characterized by three parameters: $B$, $T$ and $\Lambda_{QCD}$.

The numerical solutions for $x = 0.1$ are constructed by UV matching procedure with $B = 0$ solutions:

$$T/T_c \in [1.00208, 1.84416] \quad \text{and} \quad eB_{phys} \in [0.05978, 3.34753] \text{GeV}^2.$$
Change in entropy density divided by $T^3$ in terms of $T/T_c$ for different values of $eB$.

**Figure 4:** IHQCD+VQCD with $x = 0.1$

**Figure 5:** Lattice-QCD result.
Quasinormal modes:

The background metric has $SO(2)$ symmetry around the $x_3$-axis, because of the presence of a constant magnetic field along that direction, hence we introduce the fluctuations in the form of:

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + g_{\mu\nu}^{(1)} \quad V_\mu = V_\mu^{(0)} + V_\mu^{(1)}$$  \hspace{1cm} (11)

where

$$g_{\mu\nu}^{(1)} = e^{i(kx_3 - \omega t)} h_{\mu\nu}(r) \quad V_\mu^{(1)} = i e^{i(kx_3 - \omega t)} v_\mu(r)$$  \hspace{1cm} (12)

After imposing the radial gauge:

$$h_{tr} = h_{x_3 r} = h_{rr} = h_{r\alpha} = 0 \quad v_r = 0$$  \hspace{1cm} (13)
we end up with the classification:

Spin2 (scalar channel): \( h_{\alpha \beta} - \delta_{\alpha \beta} \frac{h}{2} \)

Spin1 (shear channel): \( h_t\alpha, h_{x_3 \alpha}, v_{\alpha} \)

Spin0 (sound channel): \( h_{tt}, h_{tx_3}, h_{x_3 x_3}, h, v_t, v_{x_3}, \phi \)

where \( \alpha = x_1, x_2 \) and \( h = \sum_{\alpha} h_{\alpha \alpha} \).

We note that the gauge does not completely fix the diffeomorphism invariance. Under infinitesimal diffeomorphisms, \( x^\mu \rightarrow x^\mu + \xi^\mu \) (where \( \xi^\mu = e^{i (kx_3 - \omega t)} \zeta^\mu (r) \)):

\[
\begin{align*}
g_{\mu \nu}^{(1)} & \rightarrow g_{\mu \nu}^{(1)} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu \\
V_\mu^{(1)} & \rightarrow V_\mu^{(1)} - g^{(0) \tau \lambda} V_{\tau}^{(0)} \nabla_\mu^{(0)} \xi_\lambda - g^{(0) \tau \lambda} \xi_\lambda \nabla_{\tau}^{(0)} V_\mu^{(0)} \\
\phi^{(1)} & \rightarrow \phi^{(1)} - \xi_\mu \nabla_\mu^{(0)} \phi^{(0)}
\end{align*}
\]
Shear channel fluctuation:

\[ g^{(1)}_{tx_2} = e^{i(kx_3 - \omega t)} e^{2A(r)} H_{tx_2}(r) \]
\[ g^{(1)}_{x_2x_3} = e^{i(kx_3 - \omega t)} e^{2A(r)} H_{x_2x_3}(r) \] (14)

\[ V^{(1)}_{x_1} = i e^{i(kx_3 - \omega t)} v_{x_1}(r) \] (15)

For convenience:

\[ H_{tx_2} = h^t_{x_2} \]
\[ H_{x_2x_3} = h^x_{x_2} \]

\[ Z(\phi) = x V_f(\phi) w^2(\phi) \]
Linear fluctuation equations:

\[ H''_{txx} (r) + \left[ 3A'(r) + W'(r) \right] H'_{tx2} (r) + \left[ \frac{-e^2W(r) k^2}{eg(r)} - \frac{4B^2 e^{-2A(r)} Z(\phi)}{eg(r)} \right] H_{tx2} (r) \]

\[- \frac{e^{-2W(r) k\omega}}{eg(r)} H_{x3x2} - \frac{4Be^2A(r) \omega Z(\phi)}{eg(r)} v_{x1} (r) = 0 \]

\[ H''_{x3x2} (r) + \left[ 3A'(r) + g'(r) - W'(r) \right] H'_{tx2} (r) + \left[ \frac{\omega^2}{e^2g(r)} - \frac{4B^2 e^{-2A(r)} Z(\phi)}{eg(r)} \right] H_{x3x2} (r) \]

\[ + \frac{k\omega}{eg(r)} H_{tx2} + \frac{4Be^2A(r) kZ(\phi)}{eg(r)} v_{x1} (r) = 0 \]

\[ v''_{x1} (r) + \left( A'(r) + g'(r) + W'(r) + \frac{Z'(\phi)\phi'(r)}{Z(\phi)} \right) v'_{x1} (r) + \left( \frac{\omega^2}{e^2g(r)} - \frac{e^2W(r) k^2}{eg(r)} \right) v_{x1} (r) \]

\[ + \frac{B\omega}{e^2g(r)} H_{tx2} + \frac{Be^2W(r) k}{eg(r)} H_{x3x2} (r) = 0 \]

Constraint equation:

\[ \frac{k}{2} H'_{x3x2} (r) + \frac{e^2W(r) \omega}{2eg(r)} H'_{tx2} (r) + 2Be^2(W(r) - A(r)) Z(\phi) v'_{x1} (r) = 0 \]
Shear channel

**Gauge invariant combinations:**

\[ Z_2(r) = kH_{tx_2}(r) + \omega H_{x_3x_2}(r) \]  
\[ Z_3(r) = v_{x_1} + \frac{B}{2k\omega} [kH_{tx_2}(r) - \omega H_{x_3x_2}(r)] \]

**Gauge invariant fluctuation equations:**

\[ Z_2''(r) + C_1 Z_2'(r) + C_2 Z_2(r) + C_3 Z_3'(r) = 0 \]  
\[ Z_3''(r) + D_1 Z_3'(r) + D_2 Z_3(r) + D_3 Z_2'(r) = 0 \]

**Boundary conditions:**

i. Infalling BC at the horizon:

\[ Z_2(r_h - r) = (r_h - r)^{-\frac{i\omega}{4\pi T}} [b_0 + b_1(r_h - r) + ...] \]
\[ Z_3(r_h - r) = (r_h - r)^{-\frac{i\omega}{4\pi T}} [c_0 + c_1(r_h - r) + ...] \]

ii. Dirichlet BC on the boundary:

\[ \lim_{r \to r_c} \det[H] = \lim_{r \to r_c} [Z_2^{(1)} Z_3^{(2)} - Z_2^{(2)} Z_3^{(1)}] = 0 \]
Shear channel

**T-dependence:**

![Graph (a)](image1.png)

![Graph (b)](image2.png)

**Figure 6:** T-dependence of quasinormal frequencies in the shear channel for temperature values varying between $T / T_c \in [1.0021, 1.8442]$ and for fixed $eB_{phys} = 0.2391\text{GeV}^2$, $k = 1$: (a) the real parts and (b) imaginary parts of the three lowest and the two purely imaginary modes. Blue curve is the hydrodynamic mode.
**B-dependence:**

![Figure 7](image-url)

**Figure 7:** B-dependence of quasinormal frequencies in the shear channel for magnetic field values varying between $eB_{phys} \in [0.1196, 3.3475]\text{GeV}^2$ and for fixed $T/T_c = 1.0221$, $\bar{k} = 1$: (a) the real and (b) the imaginary parts the three lowest and the two purely imaginary mode. Blue curve corresponds to the hydrodynamic mode. We observe that the hydrodynamic approximation breaks down at $eB_{phys} = 2.9291\text{GeV}^2$. 

Tuna Demircik
Sabanci University / Utrecht University (arXiv:1605.08118v3 with U.Gursoy)
Holographic Equilibration in Confining Gauge Theories Under External Magnetic Fields
Figure 8: $\bar{k}$-dependence of quasinormal frequencies in the shear channel for momentum values varying between $\bar{k} \in [0, 2.2700]$ and for fixed $T/T_c = 1.0221$ and $eB_{phys} = 0.2391\, GeV^2$: (a) the real parts and (b) the imaginary parts of the three lowest modes and the two purely imaginary modes. Blue curve corresponds to the hydrodynamic mode. We observe that the hydrodynamic approximation breaks down at $\bar{k}_c \approx 1.3995$. 

Tuna Demircik
Sabanci University / Utrecht University (arXiv:1605.08118v3 with U.Gursoy)
Holographic Equilibration in Confining Gauge Theories Under External Magnetic Fields
Scalar channel fluctuations:

\[ g_{x_1x_2}^{(1)} = e^{i(kx_3 - \omega t)}e^{2A(r)}H_{x_1x_2}(r) \]  

(20)

For convenience:

\[ H_{x_1x_2} = h_{x_1}^{x_2} = Z_1(r) \]

Gauge invariant fluctuation equations:

\[ Z_1''(r) + [3A'(r) + g'(r) + W'(r)] Z_1'(r) + \left[ \frac{\omega^2}{e^{2g(r)}} - \frac{k^2}{e^{2W(r)+g(r)}} \right] Z_1(r) = 0 \]  

(21)

Boundary conditions:

i  Infalling BC at the horizon:

\[ Z_1(r_h - r) = (r_h - r)^{-\frac{i\omega}{4\pi T}} [a_0 + a_1(r_h - r) + ...] \]

ii  Dirichlet BC on the boundary: \[ Z_1(r_c) = 0 \]
T-dependence:

Figure 9: T-dependence of quasinormal frequencies in the scalar channel for temperature values varying between $T/T_c \in [1.0021, 1.8442]$ and for fixed $eB_{phys} = 0.2391\text{GeV}^2$, $k = 1$: (a) the real parts and (b) the imaginary parts of the four lowest modes and the purely imaginary mode. We observe crossing between the three lowest lying modes at $T/T_c = 1.1371$ and $T/T_c = 1.4185$. 
B-dependence:

![Graph showing B-dependence of quasinormal frequencies in the scalar channel.](image)

**Figure 10:** B-dependence of quasinormal frequencies in the scalar channel magnetic field values varying between $eB_{phys} \in [0.1196, 3.3475] GeV^2$ and for fixed $T/T_c = 1.0221$, $\bar{k} = 1$: (a) the real parts and (b) the imaginary parts the four lowest and the purely imaginary mode.
**k-dependence:**

![Graphs showing k-dependence](image)

**Figure 11:** $\bar{k}$-dependence of quasinormal frequencies in the scalar channel for momentum values varying between $\bar{k} \in [0, 2.93]$ and for fixed $T/T_c = 1.0221$ and $eB_{phys} = 0.2391 \text{GeV}^2$: (a) the real parts and (b) the imaginary parts of the four lowest modes and the purely imaginary mode. We observe various crossings between these modes in (b).
Conclusion:

- Two classes of QNMs are observed:
  
  1. $\text{Re}|\omega| \neq 0$ and $\text{Im}|\omega| \neq 0$.
  2. $\text{Re}|\omega| = 0$ and $\text{Im}|\omega| \neq 0$.

  The latter is observed for the first time for magnetic black brane.

- For most of the QNM: $B \uparrow \Rightarrow \text{Re}|\omega| \uparrow$ and $\text{Im}|\omega| \uparrow (\tau \downarrow)$.

- Several crossing phenomena are observed with changing $\bar{k}$, $T$ and $B$. This means that dominant mode that controls equilibration process depends on these parameters.

- Hydro breakdown observed with changing $\bar{k}$ and $B$.
  
  1. $\bar{k}_c \approx 1.4$.
  2. $eB_{\text{phys}} \approx 2.93\text{GeV}^2$. 
Thank you for your attention.