Holographic Equilibration in Confining Gauge Theories Under External Magnetic Fields

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States of strongly interacting matter:

i Confinement/Deconfinement transition:
$L(T) \sim \lim_{r\to\infty} \exp(-V(r)/T)$
- $T < T_c \Rightarrow L \to 0 \Rightarrow$ Conf.
- $T > T_c \Rightarrow L \neq 0 \Rightarrow$ Deconf.

ii Chiral symmetry restoration:
- $T < T_c \Rightarrow M_q \neq 0 \Rightarrow \chi SB$.
- $T > T_c \Rightarrow M_q \to 0 \Rightarrow \chi S$.

iii Diquark matter.

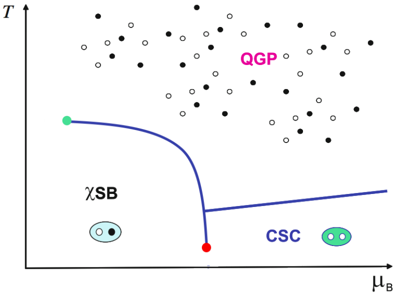


Figure 1: QCD phase diagram.

Lattice QCD: for $\mu_B = 0$, $T_c = 150 - 200 MeV$ and $n_c = 0.1 fm^{-3}$.

Heavy ion experiments and QGP:

- $\sim 10^{12} K$, $\sim 10 fm/c$, $\sim 10 fm$.
- RHIC:$\sqrt{s} = 200 GeV$,
  LHC:$\sqrt{s} = 2.76 TeV$.
- RHIC:$e|\overrightarrow{B}|/{m_\pi}^2 \approx 1-3$,
  LHC:$e|\overrightarrow{B}|/{m_\pi}^2 \approx 10-15$.
- Extremely small $\eta/s \approx 1/4\pi$.

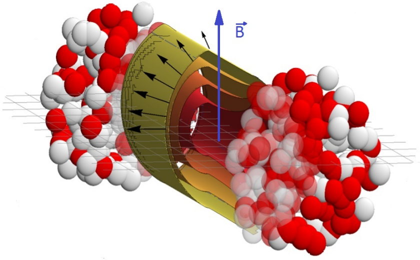


Figure 2: Heavy ion collisions.

QGP is in strong interaction regime and η/s ratio is in very good agreement with AdS/CFT result.

Holography:

- The glimpses:
  - i Holographic principle;)
  - ii Large-N_c theories;)
- Maldecena conjecture:

$$N = 4\text{SYM} \leftrightarrow \text{SUGRA on} AdS_5 \times S_5$$

$$Z_{CFT(N_c \gg \lambda \gg 1)} = Z_{AdS_5}$$

- In Nonequilibrium:

$$\langle \exp(i \int \phi^{(0)} O) \rangle = e^{\underline{S}[\phi|_{u=0}=\phi^{(0)}]}$$

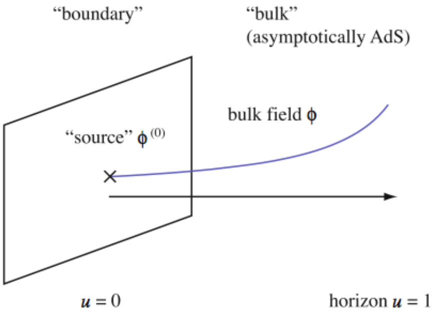


Figure 3: GKB-W relation in nonequilibrium situation.

QNMs:

- Introducing a fluctuation to constituive equation in hydrodynamics:

$$T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} + P\eta^{\mu\nu} + \tau^{\mu\nu}$$

results in dispersion relation which corresponds to poles of retarded Green function. This dispersion relation is related with transport coeffecients. e.g.:

$$\text{vector mode:} \quad \omega = -i\frac{\eta}{Ts}q^2$$

- Bulk field generally has the asymptotic behaviour:

$$\phi \sim Ar^{\Delta_-} + Br^{\Delta_+} \quad (r \to 0), \quad \Delta_+ > \Delta_-$$

by following AdS/CFT prescription, the location of poles:

$$G_R \propto \frac{B}{A} \Longrightarrow$$

i A vanishing slow fall off BC on boundary.

ii Incoming BC at horizon.

The action:

$$S = S_g + S_f \tag{1}$$

- IHQCD with backreacting flavor branes with Veneziano limit:

$$N_c \to \infty, \quad N_f \to \infty \quad \text{and} \quad \frac{N_f}{N_c} \equiv x = \text{finite}$$

$$\lambda = g_{YM}^2 N_c = \text{fixed}$$

The glue sector:

$$S_g = M_p^3 N_c^2 \int d^5x \sqrt{-g} \left(R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi) \right) \tag{2}$$

The flavour sector:

$$S_f = -\frac{1}{2} M_p^3 N_c \mathbb{T}\mathrm{r} \int d^5x \left(V_f(\lambda, T^\dagger T)\sqrt{-\det \mathbf{A}_L} + V_f((\lambda, TT^\dagger)\sqrt{-\det \mathbf{A}_R} \right)$$

$$\mathbf{A}_{L\mu\nu} = g_{\mu\nu} + w(\lambda, T) F^L_{\mu\nu} + \frac{\kappa(\lambda, T)}{2} \left[(D_\mu T)^\dagger (D_\nu T) + (D_\nu T)^\dagger (D_\mu T) \right]$$

$$\mathbf{A}_{R\mu\nu} = g_{\mu\nu} + w(\lambda, T) F^R_{\mu\nu} + \frac{\kappa(\lambda, T)}{2} \left[(D_\mu T)(D_\nu T)^\dagger + (D_\nu T)(D_\mu T)^\dagger \right]$$

$$D_\mu T = \partial_\mu T + iTA^L_\mu - iA^R_\mu T \qquad V_f(\lambda, TT^\dagger) = V_{f0}(\lambda) e^{-a(\lambda) TT^\dagger}$$

$$T = \tau(r)\mathbb{I}_{N_f} \quad \text{and} \quad \tau(r) = 0 \quad \text{(chiral symmetric phase)}$$

$$\begin{aligned} S_f &= -M_p^3 N_c \mathbb{T}\mathrm{r} \int dx^5 V_f(\lambda)\sqrt{-g}\sqrt{\det(\delta^\mu_\nu + w(\lambda)^2 g^{\mu\rho} F_{\rho\nu})} \\ &= -M_p^3 N_c N_f \int dx^5 V_f(\lambda)\sqrt{-g}\left(1 + \frac{w(\lambda)^2}{4} F_{\mu\nu}F^{\mu\nu}\right) \end{aligned} \tag{3}$$

The Einsteins equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \left(\frac{4}{3}\partial_\mu\phi\partial_\nu\phi - \frac{2}{3}g_{\mu\nu}(\partial\phi)^2 + \frac{1}{2}g_{\mu\nu}V_{eff}\right) - x\frac{V_b}{2}\left(F_\mu{}^\rho F_{\nu\rho} - \frac{g_{\mu\nu}}{4}F_{\rho\sigma}F^{\rho\sigma}\right) = 0 \quad (4)$$

The Maxwell equations:

$$\partial_\mu\left(\sqrt{-g}V_b F^{\mu\nu}\right) = 0 \quad (5)$$

The dilaton equation:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) + \frac{3}{8}\frac{\partial V_{eff}}{\partial\phi} - \frac{3x}{32}\frac{\partial V_b}{\partial\phi}F^2 = 0 \quad (6)$$

Constant background magnetic field:

$$V_\mu = \left(0, -\frac{x_2 B}{2}, \frac{x_1 B}{2}, 0, 0\right) \quad (7)$$

Antsatz for the metric:

$$ds^2 = e^{2A(r)} \left(-e^{g(r)} dt^2 + dx_1^2 + dx_2^2 + e^{2W(r)} dx_3^2 + e^{-g(r)} dr^2\right) \quad (8)$$

$r \in [0, r_h]$: r_h is the location of the horizon where $g(r_h) = -\infty$.
The UV boundary is at $r = 0$, where we demand AdS_5 asymtotics ($A \to -\log(r), g \to 0, W \to 0$ as $r \to 0$).

The dilaton:

$$\lambda = \lambda(r) = e^{\phi}(r) \quad (9)$$

The Potentials:

$$V_b(\lambda) = V_f(\lambda) w(\lambda)^2, \qquad V_{eff}(\lambda) = V_g(\lambda) - x V_{f0} \tag{10}$$

i $V_g(\lambda) = \frac{12}{l^2}\left(1 + V_0\lambda + V_1\lambda^{4/3}\sqrt{\log(1 + V_2\lambda^{4/3} + V_3\lambda^2)}\right)$.

- In the UV, It matches the perturbative large-N_c β-function and corresponds to initial conditions of an RG flow with asymptotic freedom.
- In the IR, It guarantees that the dual field theory is confining with a gapped glueball spectrum.

ii $V_{f0}(\lambda) = W_0(1 + W_1\lambda + W_2\lambda^2)$ and $w(\lambda) = \left(1 + \frac{3a_1}{4}\lambda\right)^{-\frac{4}{3}}$

- In the UV, It matches the perturbative anomalous dimension of the quark mass operator.
- In the IR, It satifies the requirement of chiral symmetry and the meson spectra.

Background at a finite magnetic field and temperature

- The Einstein's equations yield three 2nd order equations and one 1st order constraint equation.
- The solutions are only characterized by three parameters: B, T and Λ_{QCD}.
- The numerical solutions for $x = 0.1$ are constructed by UV matching procedure with $B = 0$ solutions:

  $T/T_c \in [1.00208, 1.84416]$ and
  $eB_{phys} \in [0.05978, 3.34753] GeV^2$.

Change in entropy density devided by T^3 in terms of T/T_c for different values of eB.

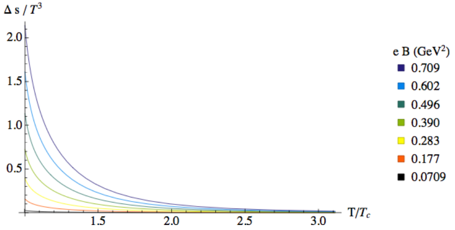


Figure 4: IHQCD+VQCD with $x = 0.1$

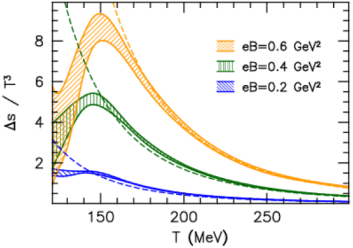


Figure 5: Lattice-QCD result.

Quasinormal modes:

The background metric has $SO(2)$ symmetry around the x_3-axis, because of the presence of a constant magnetic field along that direction, hence we introduce the fluctuations in the form of:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} \qquad V_\mu = V^{(0)}_\mu + V^{(1)}_\mu \tag{11}$$

where

$$g^{(1)}_{\mu\nu} = e^{i(kx_3 - \omega t)} h_{\mu\nu}(r) \qquad V^{(1)}_\mu = i e^{i(kx_3 - \omega t)} v_\mu(r) \tag{12}$$

After imposing the radial gauge:

$$h_{tr} = h_{x_3 r} = h_{rr} = h_{r\alpha} = 0 \qquad v_r = 0 \tag{13}$$

we end up with the classification:

$$\begin{aligned} &\text{Spin2} \quad (\text{scalar channel}): && h_{\alpha\beta} - \delta_{\alpha\beta} h/2 \\ &\text{Spin1} \quad (\text{shear channel}): && h_{t\alpha}, h_{x_3\alpha}, v_\alpha \\ &\text{Spin0} \quad (\text{sound channel}): && h_{tt}, h_{tx_3}, h_{x_3x_3}, h, v_t, v_{x_3}, \phi \end{aligned}$$

where $\alpha = x_1, x_2$ and $h = \sum_\alpha h_{\alpha\alpha}$.
We note that the gauge does not completely fix the diffeomorphism invariance. Under infinitesimal diffeomorphisms, $x^\mu \to x^\mu + \xi^\mu$ (where $\xi_\mu = e^{i(kx_3 - \omega t)} \zeta_\mu(r)$):

$$\begin{aligned} g^{(1)}_{\mu\nu} &\to g^{(1)}_{\mu\nu} - \nabla^{(0)}_\mu \xi_\nu - \nabla^{(0)}_\nu \xi_\mu \\ V^{(1)}_\mu &\to V^{(1)}_\mu - g^{(0)\tau\lambda} V^{(0)}_\tau \nabla^{(0)}_\mu \xi_\lambda - g^{(0)\tau\lambda} \xi_\lambda \nabla^{(0)}_\tau V^{(0)}_\mu \\ \phi^{(1)} &\to \phi^{(1)} - \xi^\mu \nabla^{(0)}_\mu \phi^{(0)} \end{aligned}$$

Shear channel fluctuation:

$$g^{(1)}_{tx_2} = e^{i(kx_3-\omega t)}e^{2A(r)}H_{tx_2}(r) \qquad g^{(1)}_{x_2x_3} = e^{i(kx_3-\omega t)}e^{2A(r)}H_{x_2x_3}(r) \quad (14)$$

$$V^{(1)}_{x_1} = ie^{i(kx_3-\omega t)}v_{x_1}(r) \quad (15)$$

- For convenience:

$$H_{tx_2} = h^t_{x_2} \qquad H_{x_2x_3} = h^{x_3}_{x_2}$$

$$Z(\phi) = xV_f(\phi)w^2(\phi)$$

Linear fluctuation equations:

$$H''_{tx_2}(r) + \left[3A'(r) + W'(r)\right] H'_{tx_2}(r) + \left[\frac{-e^{2W(r)}k^2}{e^{g(r)}} - \frac{4B^2 e^{-2A(r)} Z(\phi)}{e^{g(r)}}\right] H_{tx_2}(r) - \frac{e^{-2W(r)}k\omega}{e^{g(r)}} H_{x_3x_2} - \frac{4Be^{2A(r)}\omega Z(\phi)}{e^{g(r)}} v_{x_1}(r) = 0$$

$$H''_{x_3x_2}(r) + \left[3A'(r) + g'(r) - W'(r)\right] H'_{tx_2}(r) + \left[\frac{\omega^2}{e^{2g(r)}} - \frac{4B^2 e^{-2A(r)} Z(\phi)}{e^{g(r)}}\right] H_{x_3x_2}(r) + \frac{k\omega}{e^{g(r)}} H_{tx_2} + \frac{4Be^{2A(r)}kZ(\phi)}{e^{g(r)}} v_{x_1}(r) = 0$$

$$v''_{x_1}(r) + \left(A'(r) + g'(r) + W'(r) + \frac{Z'(\phi)\phi'(r)}{Z(\phi)}\right) v'_{x_1}(r) + \left(\frac{\omega^2}{e^{2g(r)}} - \frac{e^{2W(r)}k^2}{e^{g(r)}}\right) v_{x_1}(r) + \frac{B\omega}{e^{2g(r)}} H_{tx_2}(r) + \frac{Be^{2W(r)}k}{e^{g(r)}} H_{x_3x_2}(r) = 0$$

Constraint equation:

$$\frac{k}{2} H'_{x_3x_2}(r) + \frac{e^{2W(r)}\omega}{2e^{g(r)}} H'_{tx_2}(r) + 2Be^{2(W(r)-A(r))} Z(\phi) v'_{x_1}(r) = 0$$

Gauge invariant combinations:

$$Z_2(r) = kH_{tx_2}(r) + \omega H_{x_3x_2}(r) \tag{16}$$

$$Z_3(r) = v_{x_1} + \frac{B}{2k\omega}[kH_{tx_2}(r) - \omega H_{x_3x_2}(r)] \tag{17}$$

Gauge invariant fluctuation equations:

$$Z_2''(r) + C_1 Z_2'(r) + C_2 Z_2(r) + C_3 Z_3'(r) = 0 \tag{18}$$

$$Z_3''(r) + D_1 Z_3'(r) + D_2 Z_3(r) + D_3 Z_2'(r) = 0 \tag{19}$$

Boundary conditions:

i Infalling BC at the horizon:

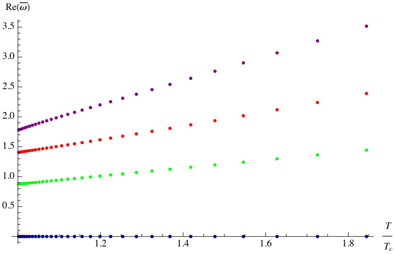
$$Z_2(r_h - r) = (r_h - r)^{-\frac{i\omega}{4\pi T}}\left[b_0 + b_1(r_h - r) + \ldots\right]$$

$$Z_3(r_h - r) = (r_h - r)^{-\frac{i\omega}{4\pi T}}\left[c_0 + c_1(r_h - r) + \ldots\right]$$

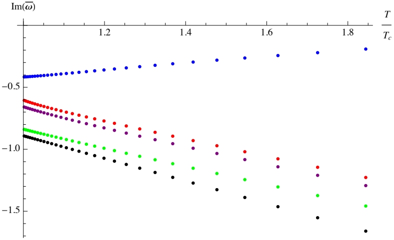
ii Dirichlet BC on the boundary:

$$\lim_{r\to r_c} \det[H] = \lim_{r\to r_c}[Z_2^{(1)}Z_3^{(2)} - Z_2^{(2)}Z_3^{(1)}] = 0$$

T-dependence:



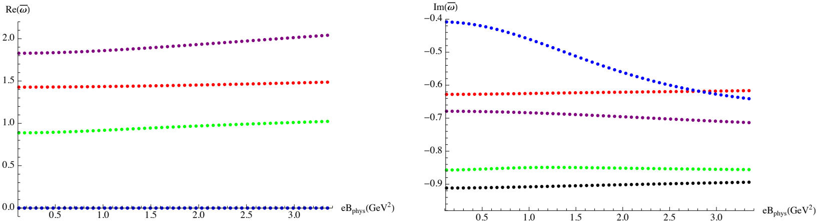
(a)



(b)

Figure 6: T-dependence of quasinormal frequencies in the shear channel for temperature values varying between $T/T_c \in [1.0021, 1.8442]$ and for fixed $eB_{phys} = 0.2391 GeV^2$, $\bar{k} = 1$: (a) the real parts and (b) imaginary parts of the three lowest and the two purely imaginary modes. Blue curve is the hydrodynamic mode.

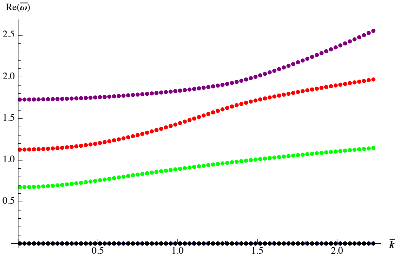
B-dependence:



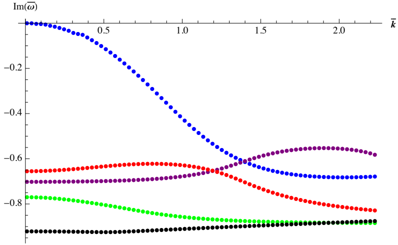
(a) (b)

Figure 7: B-dependence of quasinormal frequencies in the shear channel for magnetic field values varying between $eB_{phys} \in [0.1196, 3.3475] GeV^2$ and for fixed $T/T_c = 1.0221$, $\bar{k} = 1$: (a) the real and (b) the imaginary parts the three lowest and the two purely imaginary mode. Blue curve corresponds to the hydrodynamic mode. We observe that the hydrodynamic approximation breaks down at $eB_{phys} = 2.9291 GeV^2$.

k-dependence:



(a)



(b)

Figure 8: $\bar{k}$-dependence of quasinormal frequencies in the shear channel for momentum values varying between $\bar{k} \in [0, 2.2700]$ and for fixed $T/T_c = 1.0221$and $eB_{phys} = 0.2391 GeV^2$: (a) the real parts and (b) the imaginary parts of the three lowest modes and the two purely imaginary modes. Blue curve corresponds to the hydrodynamic mode. We observe that the hydrodynamic approximation breaks down at $\bar{k}_c = 1.3995$.

Scalar channel fluctuations:

$$g^{(1)}_{x_1x_2} = e^{i(kx_3-\omega t)}e^{2A(r)}H_{x_1x_2}(r) \tag{20}$$

- For convenience:

$$H_{x_1x_2} = h^{x_1}_{x_2} = Z_1(r)$$

Gauge invariant fluctuation equations:

$$Z_1''(r) + \left[3A'(r) + g'(r) + W'(r)\right] Z_1'(r) + \left[\frac{\omega^2}{e^{2g(r)}} - \frac{k^2}{e^{2W(r)+g(r)}}\right] Z_1(r) = 0 \tag{21}$$

Boundary conditions:

i Infalling BC at the horizon:

$$Z_1(r_h - r) = (r_h - r)^{-\frac{i\omega}{4\pi T}}\left[a_0 + a_1(r_h - r) + ...\right]$$

ii Dirichlet BC on the boundary: $Z_1(r_c) = 0$

T-dependence:

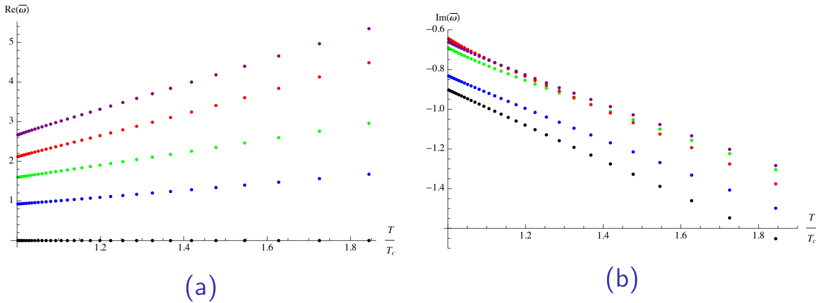


Figure 9: T-dependence of quasinormal frequencies in the scalar channel for temperature values varying between $T/T_c \in [1.0021, 1.8442]$ and for fixed $eB_{phys} = 0.2391 GeV^2$, $\bar{k} = 1$: (a) the real parts and (b) the imaginary parts of the four lowest modes and the purely imaginary mode. We observe crossing between the three lowest lying modes at $T/T_c = 1.1371$ and $T/T_c = 1.4185$.

B-dependence:

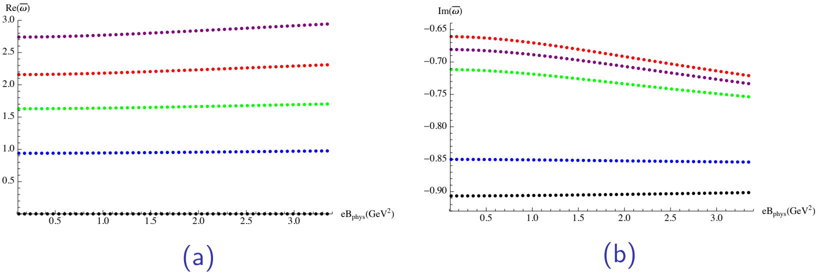


Figure 10: B-dependence of quasinormal frequencies in the scalar channel magnetic field values varying between $eB_{phys} \in [0.1196, 3.3475] GeV^2$ and for fixed $T/T_c = 1.0221$, $\bar{k} = 1$: (a) the real parts and (b) the imaginary parts the four lowest and the purely imaginary mode.

k-dependence:

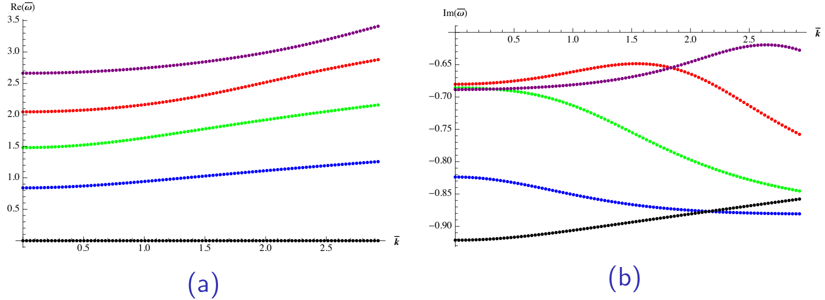


Figure 11: $\bar{k}$-dependence of quasinormal frequencies in the scalar channel for momentum values varying between $\bar{k} \in [0, 2.93]$ and for fixed $T/T_c = 1.0221$ and $eB_{phys} = 0.2391 GeV^2$: (a) the real parts and (b) the imaginary parts of the four lowest modes and the purely imaginary mode. We observe various crossings between these modes in (b).

Conclusion:

- Two classes of QNMs are observed:
  - i $Re|\omega| \neq 0$ and $Im|\omega| \neq 0$.
  - ii $Re|\omega| = 0$ and $Im|\omega| \neq 0$.

  The latter is observed for first time for magnetic black brane.
- For most of the QNM: $B \uparrow \Rightarrow Re|\omega| \uparrow$ and $Im|\omega| \uparrow (\tau \downarrow)$.
- Several crossing phenomena are observed with changing $\bar{k}$, T and B. This means that dominant mode that controls equilibration process depends on these parameters.
- Hydro breakdown observed with changing $\bar{k}$ and B.
  - i $\bar{k}_c \cong 1.4$.
  - ii $eB_{phys} \cong 2.93 GeV^2$.

Conclusion

Thank you for your attention.