Hilbert Series and Mixed Branches of 3d, $\mathcal{N} = 4$ $T[SU(N)]$ theory

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Based on...

- F.C., Hirotaka Hayashi. 2016

Related background work:

- Dan Xie, Kazuya Yonekura. 2014
- Oscar Chacaltana, Jacques Distler, Yuji Tachikawa. 2012
- Davide Gaiotto, Edward Witten. 2008

Moduli Spaces of (SUSY) QFTs.

- In general the vacuum state of a QFT is not unique.
- Physics is different when the QFT lives on a different vacuum.
- Define the Moduli Space as the set of gauge inequivalent vacua. $\mathcal{M} = \{\text{all vacua}\}/G$
- Label different vacua by the vevs of the scalars.
- Geometrically $\mathcal{M}$ is an algebraic variety.
- Interesting object to study to understand IR dynamics of a QFT.

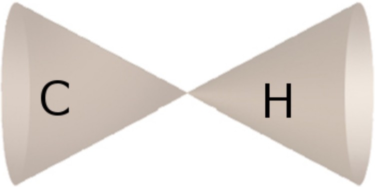
Moduli spaces for theories with 8 supercharges.

- In general classical $\mathcal{M} \neq$ quantum $\mathcal{M}$. Quantum corrections.
- To make the problem easier, consider the subset of SUSY QFTs.
- 4d QFT theory with 8 supercharges. (4d $\mathcal{N} = 2$)

We have the following multiplets:

- Hypermultiplet. $X = (Q, \tilde{Q}) = (q_\alpha, \varphi, \tilde{q}_{\dot{\beta}}, \sigma)$
- Vector multiplet. $V = (V_{\mathcal{N}=1}, \Phi) = (A_\mu, \psi_\alpha, \lambda_\beta, \phi)$

Moduli space splits into different zones, depending on which scalar takes a non-zero vev.



Generic Features of 3d $\mathcal{N} = 4$.

- Perform a dimensional reduction of the 4d $\mathcal{N} = 2$ theory.
- A_i is dual to a real scalar γ. Dual photon.
- γ can take vev.
  Coulomb branch is enlarged compared to 4d $\mathcal{N} = 2$.
- $*F = J$ is a conserved current. Extra $U(1)_J$ hidden symmetry
- $U(1)_J$ acts on γ by shifts $\gamma \to \gamma + a$
- Parametrize the directions opened up by $\langle\gamma\rangle$ by the vev of BPS monopole operators: disorder operators semiclassically given by $V \sim e^{(\frac{\sigma}{g^2} + i\gamma)}$

Higgs branch VS Coulomb branch.

Higgs branch

- Parametrized only by vevs of hypermultiplets.
- Hyperkahler variety $\mathcal{H}$.
- Classically exact.
- Gauge group generically completely broken.

Coulomb branch

- Parametrized only by vevs of vector multiplets (via monopole operators.)
- Hyperkahler variety $\mathcal{C}$.
- Heavy quantum corrections deform the geometry.
- Gauge group generically broken to $U(1)^r$.

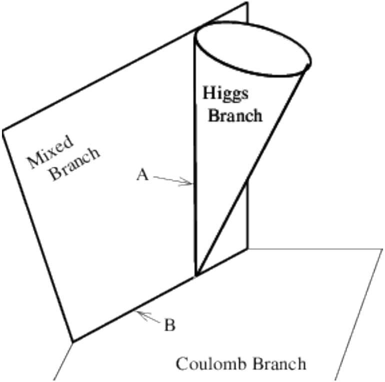
3d Mirror Symmetry swaps the two branches.

Mixed branches.

- Parametrized by both vevs of hypers and vectors.
- $\mathcal{M}_i \simeq \mathcal{H}_i \times \mathcal{C}_i$
- Needed to have a full picture of the moduli space.

$$\mathcal{M} = \bigcup_i \mathcal{M}_i = \bigcup_i \mathcal{H}_i \times \mathcal{C}_i$$

- Clearly not disjoint union: generically $\mathcal{M}_i \cap \mathcal{M}_j \neq \emptyset$.



Taken from Argyres '98

Hilbert Series as a tool to study the Moduli Space.

- Correspondence between holomorphic maps on $\mathcal{M}$ and the chiral ring of BPS operators.
- Counting the BPS chiral operators in a graded way.
- Use the Hilbert series as a counting tool. In general

$$HS(t) = \sum_n a_n t^n$$

- For the full Coulomb branch we have

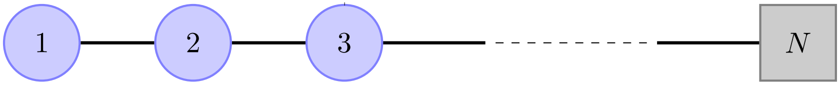
$$H_G(t,z) = \sum_{m \in \Gamma^*_{\hat{G}}/\mathcal{W}_{\hat{G}}} z^{J(m)} t^{\Delta(m)} P_G(t,m)$$

- The conformal dimension of monopole operators is

$$\Delta(m) = -\sum_{\alpha \in \Delta^+} |\alpha(m)| + \frac{1}{2} \sum_{i=1}^{n} \sum_{\rho_i \in \mathcal{R}_i} |\rho_i(m)|$$

Hanany, Cremonesi, Zaffaroni '13

$T[SU(N)]$ theory, as a quiver gauge theory.



- Circles represent gauge $U(N_i)$ factors of the gauge group.
- The square represents a flavour $SU(N)$ group.
- Lines represent bifundamental hypermultiplets.
- The lagrangian in 3d $\mathcal{N} = 4$ is fully determined by the matter content.
- The quiver defines in a unique way the theory.

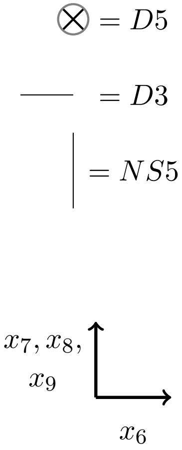
$T[SU(N)]$ theory, brane picture. Part 1.

- Consider Type IIB superstring theory.
- Take some $D3$-branes, $D5$-branes, $NS5$-branes and place them as in the following table.

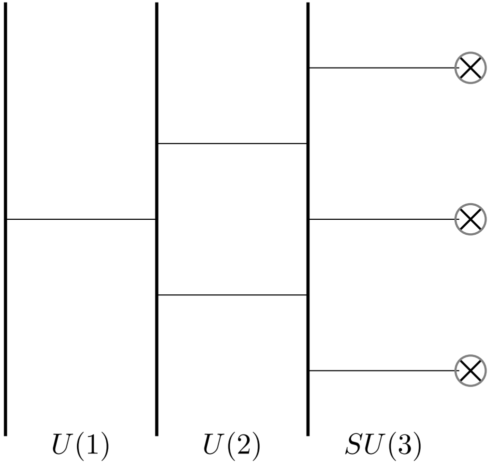
	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	-	-	-	x	x	x	-	x	x	x
D5	-	-	-	-	-	-	x	x	x	x
NS5	-	-	-	x	x	x	x	-	-	-

- Kaluza-Klein reduction on x_6
- Get a low energy EFT on the x_0, x_1, x_2
- HW cartoon. Hanany-Witten '96

$T[SU(N)]$ theory, brane picture. Part 2.



$T[SU(3)]$ example.



Full Coulomb branch of $T[SU(3)]$.

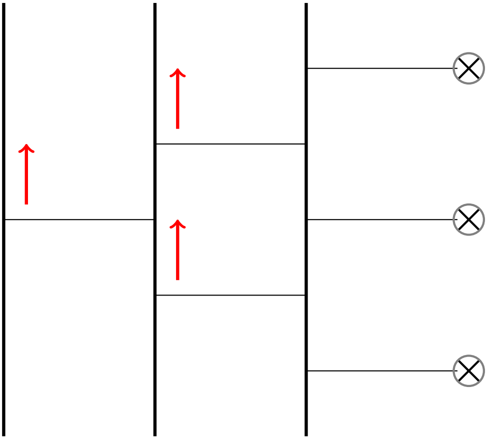


Figure: The brane picture for the branch $\rho = [1, 1, 1]$.

Full Higgs branch of $T[SU(3)]$.

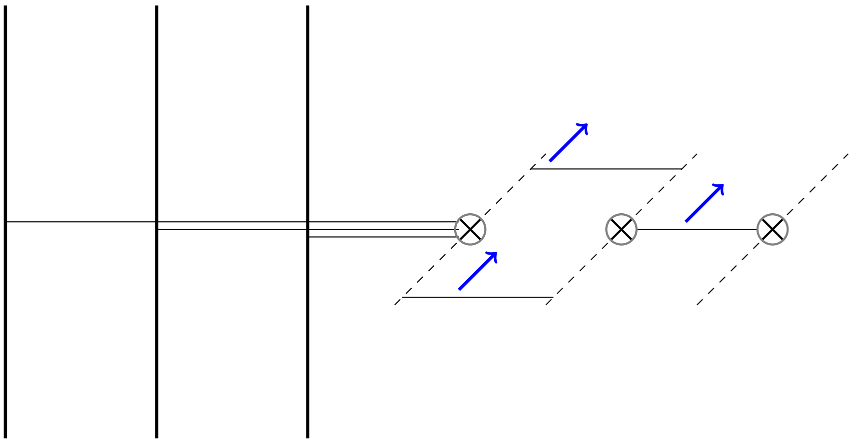


Figure: The brane picture for the branch $\rho = [3]$.

Mixed branch of $T[SU(3)]$.

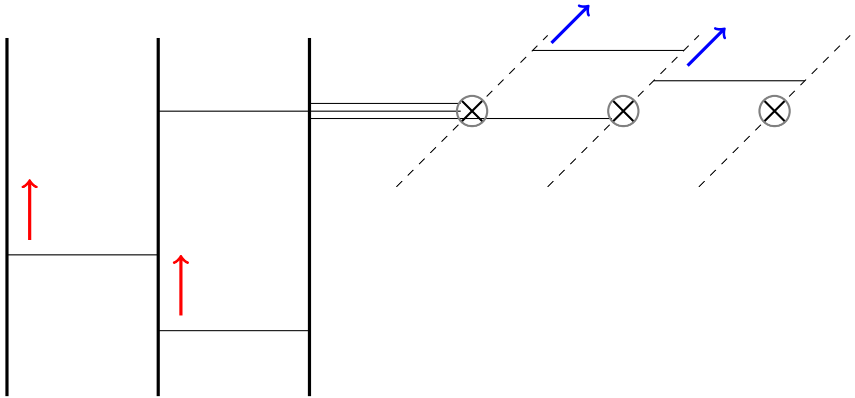


Figure: The brane picture for the mixed branch $\rho = [2, 1]$. Note the S-rule at work.

The restriction formula.

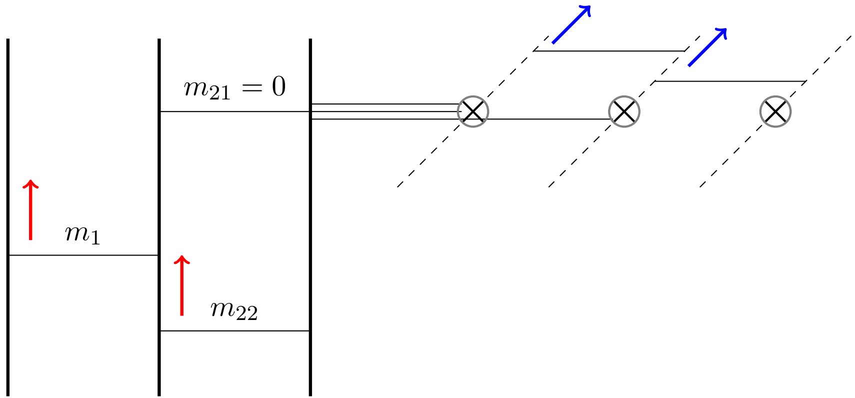
- From the quantization of monopole operators

$$\sigma \sim m$$

  with σ the adjoint scalar in the vector multiplet.
- **Discretized brane positions $\sim$ magnetic charges** (main conceptual result of the paper).
- Then the S-rule will tell us how to restrict the summation in the fill HS, to ge the HS of the (coulomb branch part of the) mixed branch.
- **Simply put to zero the frozen brane positions**, in

$$H_G(t, z) = \sum_{m \in \Gamma^*_{\hat{G}} / \mathcal{W}_{\hat{G}}} z^{J(m)} t^{\Delta(m)} P_G(t, m)$$

The restriction rule for $T[SU(3)]$.



Conclusions

- We give an interpretation of the magnetic charges of monopole operators in terms of brane positions in type IIB.
- We propose a restriction rule on the HS of the full Coulomb Branch, to get the HS of the Coulomb branch part of a mixeb branch.
- We can then compute the HS of any mixed branch of $T[SU(N)]$, by using the explicit restriction and mirror symmetry.

Thank you for your attention.