Hierarchy of Relaxation Timescales in Local Random Liouvillians

David J. Luitz

Computational Quantum Many-Body Physics Group
Max Planck Institute for the Physics of Complex Systems

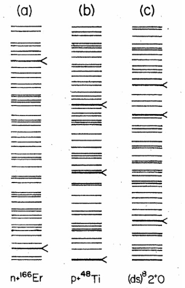
Work with:
Kevin Wang (Stanford)
Oscar-Emil Sommer (Cambridge)
Francesco Piazza (MPI-PKS)

Outline

Quantum many-body problem and random matrices

Quantum Many-Body Systems are tremendously complex

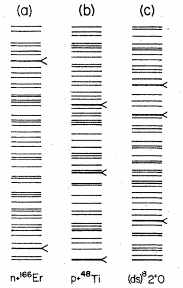
- Example: Atomic nuclei **strongly interacting fermions**
- Many resonances in neutron scattering



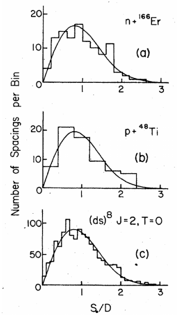
Quantum many-body problem and random matrices

Quantum Many-Body Systems are tremendously complex

- Example: Atomic nuclei **strongly interacting fermions**
- Many resonances in neutron scattering



- Spectra are so complicated that they share statistical properties of **random matrices** (Wigner)



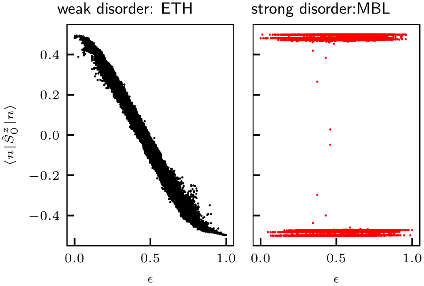
Eigenstate thermalization hypothesis

How can generic isolated quantum many-body systems reach thermal equilibrium?

- Local information about initial state is scrambled
- Due to special local strucutre of many-body eigenstates
- ETH [Deutsch 1991; Srednicki 1994, 1999; Rigol et al 2008]

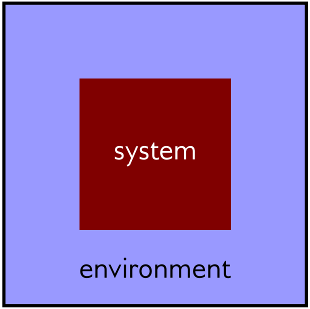
$$\langle n | O | m \rangle = O(E)\delta_{n,m} + \mathrm{e}^{-S(E)/2} f(E,\omega) R_{n,m}.$$

- Ex. in spin chain $H = \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \sum_i h_i S_i^z$.



Open Quantum Systems

Open quantum systems : Dynamics due to Hamiltonian H, and coupling to **environment**.



Loss of **purity** of the quantum state $\rightarrow$ mixed state $\rho(t)$.
Simplest description: Lindblad master equation
(neglects memory of the bath):

$$\partial_t \rho = -\mathrm{i}[H, \rho] + \sum_{n,m=1}^{N^2-1} K_{n,m} \left(L_n \, \rho \, L_m^\dagger - \frac{1}{2} \left\{ \rho, L_m^\dagger \, L_n \right\} \right)$$

L_n: Lindblad operators. $K_{n,m}$: Kossakowski matrix.

Liouvillian

Master equation is governed by Liouvillian (Lindbladian)

$$\partial_t \rho(t) = \mathcal{L}[\rho(t)]$$

Heisenberg picture, adjoint Liouvillian

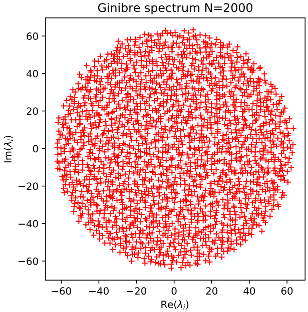
$$\partial_t \hat{O}(t) = \mathcal{L}_a[\hat{O}(t)]$$

What are generic features of many-body Liouvillians?

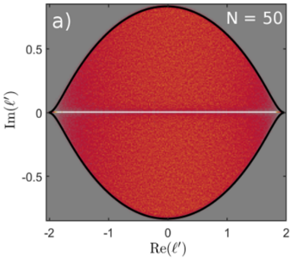
Random Matrix theory for Liouvillians

Is there a random matrix theory (similar to ETH) for generic Liouvillians?

Nonhermitian random matrix



Purely dissipative Liouvillian



[Denisov PRL 2019; Sa arXiv:1905.02155]

Classical case: spindle [Timm PRE 2009]

Local random Liouvillians

What is the minimal structure in physical systems: Locality!

Consider purely dissipative spin $\frac{1}{2}$ system, $H = 0$.
k-local operators: Pauli strings with k nonidentities

$$\hat{S}_{\mathbf{x}} = \frac{1}{\sqrt{N}} \sigma_{x_1} \times \sigma_{x_2} \times \cdots \times \sigma_{x_\ell}, \qquad x_i \in \{0, 1, 2, 3\}$$

Allow only $k < k_{\mathrm{max}}$ local dissipation channels L_i

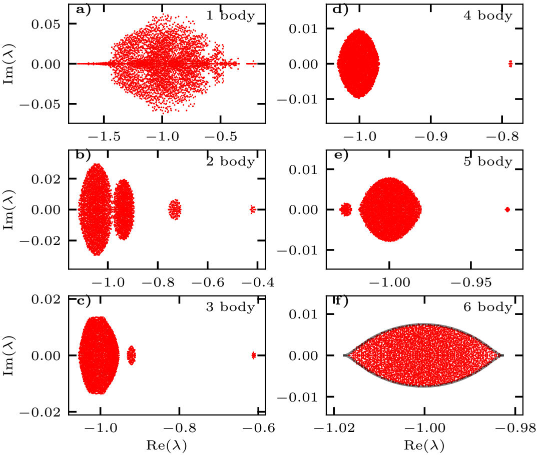
$$L(k=3) = \begin{array}{cccccc} \circ & \bullet & \circ & \circ & \bullet & \bullet \\ 1 & \sigma_x & 1 & 1 & \sigma_z & \sigma_y \end{array}$$

$$\mathcal{L}_D(\rho) = \sum_{i,j=1}^{N_L} K_{ij} [L_i \rho L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_i, \rho\}]$$

Spectrum of local random Liouvillians

$$\mathcal{L}_D\rho = \sum_{i,j=1}^{N_L} K_{ij}[L_i\rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_i, \rho\}]$$

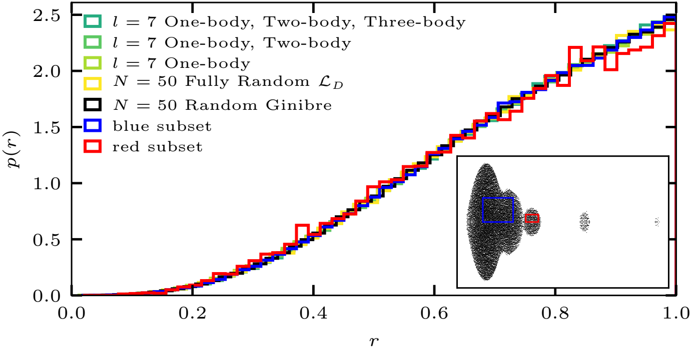
- L_i: full basis of up to k-local Pauli matrices
- Random pos. semidef. Kossakowski matrix K (Poisson stats.)
- Several eigenvalue clusters for local $\mathcal{L}$!



$\ell = 6$

[Wang, Piazza, Luitz, PRL 2020]

Spacing statistics

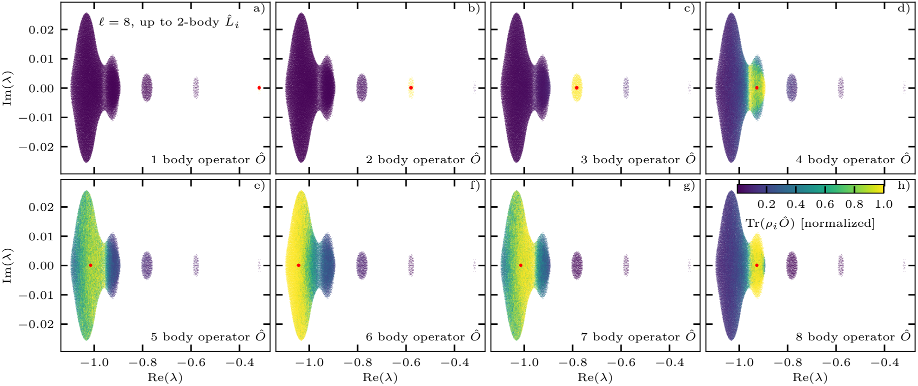


Statistics of complex spacings corresponds to Ginibre ensemble!

$$r = \frac{|\lambda_0 - \lambda_1|}{|\lambda_0 - \lambda_2|}$$

Eigenvalue clusters

$\ell = 8$, Lindblad operators up to 2 body.



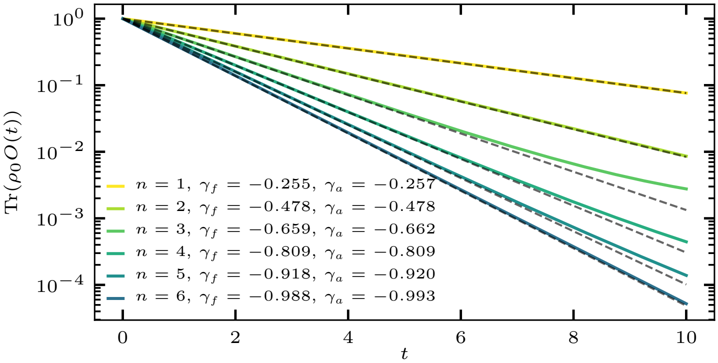
Highlight $\mathrm{Tr}(\rho_i \hat{O})$ for different k-local operators O: **Hierarchy!**

Hierarchy of relaxation timescales

Prepare $\rho_0 = |\sigma_1, \ldots \sigma_\ell\rangle\langle\sigma_1, \ldots \sigma_\ell|$. Calculate

$$\mathrm{Tr}(\rho_0 \hat{O}_n(t))$$

for **random** n-body operator $\hat{O}_n$.



$\ell = 10$, Lindblad operators up to 2 body.

Relaxation timescale corresponds to center of eigenvalue cluster.

Perturbation theory

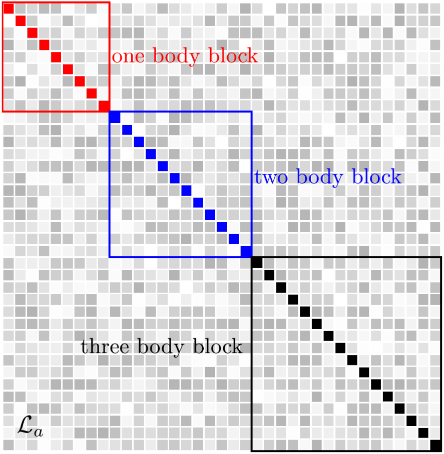
right eigenvectors of $\mathcal{L}$ organized by k-locality of operators: **consider adjoint Liouvillian in Pauli string basis**

$$\hat{S}_{\mathbf{x}} = \frac{1}{\sqrt{N}}\sigma_{x_1} \times \sigma_{x_2} \times \cdots \times \sigma_{x_\ell}, \qquad x_i \in \{0,1,2,3\}$$

$$\mathcal{L}_a[X] = \sum K_{nm}\left(L_m^\dagger X L_n - \frac{1}{2}\left\{L_m^\dagger L_n, X\right\}\right)$$

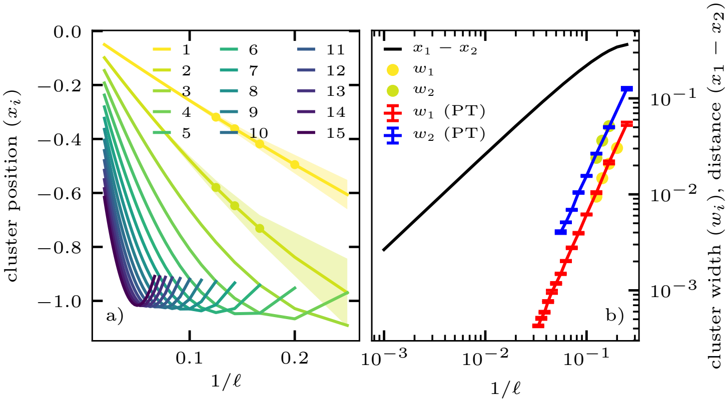
- Matrix elements $\mathrm{Tr}\left(\hat{S}_{\mathbf{x}}\mathcal{L}_a[\hat{S}_{\mathbf{y}}]\right)$.
- $K = U^\dagger D U$, mean$(K_{nn}) \propto N_L$, std$(K_{nm}) \propto N_L^{-3/2}$.
- Perturbation theory in offdiag parts of K: $K = \frac{N}{N_L}1 + K'$ (**pos. semidef**)

Perturbation theory



- Unperturbed problem $K' = 0$, $\mathcal{L}_a$ is diagonal and splits in blocks of n-body operators: **Eigenvalues: Lemon centers**
- Leading order perturbation theory: Diagonalize blocks, lifting the degeneracy $\rightarrow$ lemon.

Cluster separation



- Combinatorical prediction of cluster position.
- Degenerate perturbation theory: diagonalize $\mathcal{L}_a$ in n-body blocks to predict cluster width.
- Perfect match with exact results: **Cluster separation persists in thermodynamic limit**

Experimental verification

Platform to implement a generic many-body dissipator?

Experimental verification

Platform to implement a generic many-body dissipator?

ARTICLE **OPEN**

Simulating quantum many-body dynamics on a current digital quantum computer

Adam Smith[1,2*], M. S. Kim[1], Frank Pollmann[2] and Johannes Knolle[1]

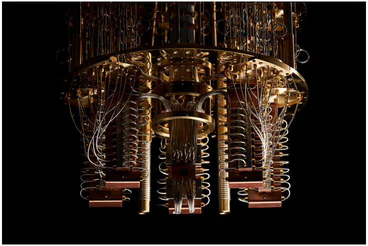
Universal quantum computers are potentially an ideal setting for simulating many-body quantum dynamics that is out of reach for classical digital computers. We use state-of-the-art IBM quantum computers to study paradigmatic examples of condensed matter physics—we simulate the effects of disorder and interactions on quantum particle transport, as well as correlation and entanglement spreading. Our benchmark results show that the quality of the current machines is below what is necessary for quantitatively accurate continuous-time dynamics of observables and reachable system sizes are small comparable to exact diagonalization. Despite this, we are successfully able to demonstrate clear qualitative behaviour associated with localization physics and many-body interaction effects.

npj Quantum Information (2019)5:106 ; https://doi.org/10.1038/s41534-019-0217-0

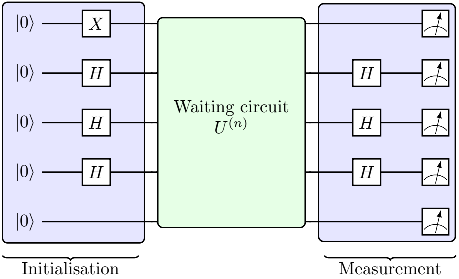
NISQC: Noisy, intermediate scale quantum computer.

IBM Quantum Computer

- Dissipation is ubiquitous in current quantum computers
- Big problem: Gate errors, measurement errors, generic coupling to environment
- **Turn problem into virtue!**
- IBM quantum experience: **cloud quantum computing $\rightarrow$ desktop experiment**

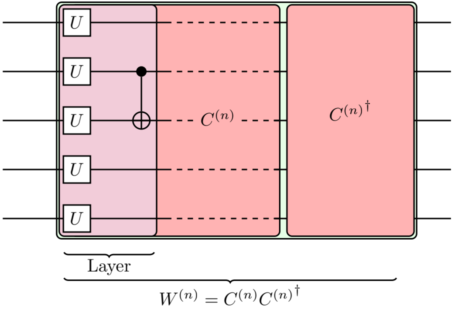


General strategy



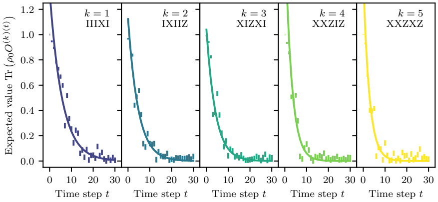
- Waiting circuit: **Identity** , characterize intrinsic machine Liouvillian
- Waiting circuit: **One qubit gates** , implement generic one body Liouvillian from gate errors
- Waiting circuit: **Two qubit gates** , implement generic two body Liouvillian from gate errors

CNOT error circuit



- Each layer: random single qbit unitaries U, CNOT to random pair
- **Apply inverted circuit** ${C^{(n)}}^\dagger$
- Net effect $C^{(n)}{C^{(n)}}^\dagger = 1$ for perfect gates. **Gate errors!**
- **Measure "time" evolution of $\mathrm{Tr}(\rho_0 O_n(t))$ for "all" Pauli strings O**

Reconstruction of the spectrum



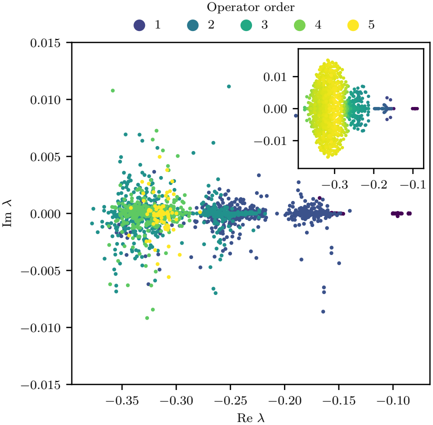
- Measure time evolution of many Pauli string observables

$$\mathrm{Tr} \rho_0 O^{(k)}(t)$$

- Fit

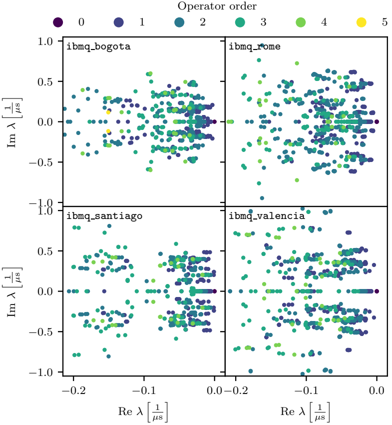
$$\sum_n c_n \mathrm{e}^{\lambda_n t}$$

How well does it work?



Intrinsic spectrum

Identity waiting circuit

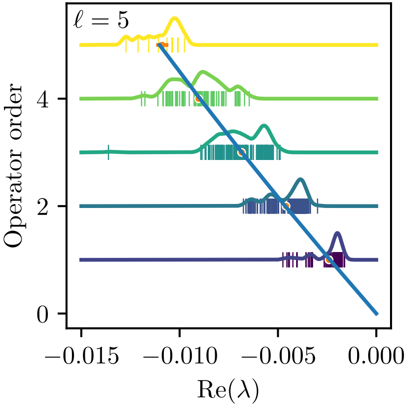


Matches theory for $k = 1$!

Need to include qubit Hamiltonian in theory.

One body waiting circuit

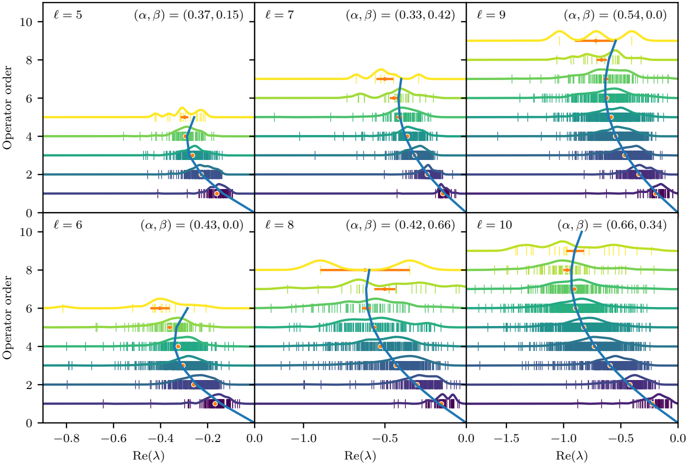
1 body jump operators



CNOT circuit Liouvillian spectrum

"CNOT waiting circuit"

two body Liouvillian



Conclusion

- Local random Liouvillians: random matrix spectral statistics
- Hierarchy of timescales determined by observable locality
- Essential features confirmed on IBM Quantum Computer
- Intrinsic dissipation: 1-body

K. Wang, F. Piazza and D. J. Luitz *"Hierarchy of relaxation timescales in local random Liouvillians"*, Phys. Rev. Lett **124**, 100604 (2020)

O.-E. Sommer, F. Piazza and D. J. Luitz *"Many-body Hierarchy of Dissipative Timescales in a Quantum Computer"*, arXiv:2011.08853

Thank you!

DFG SFB 1143

IBM

MAX-PLANCK-GESELLSCHAFT mpipks