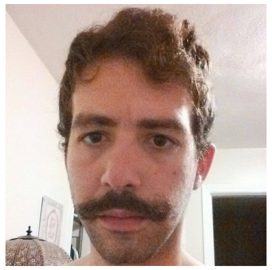
Black Hole Collapse in CFT

Julian Sonner

Iberian Strings
16 January 2017

BH Collapse in CFT



Tarek Anous (MIT/UBC)

Thomas Hartman (Cornell)

Antonin Rovai (Geneva)

[arXiv: 1603.04856 & to appear]

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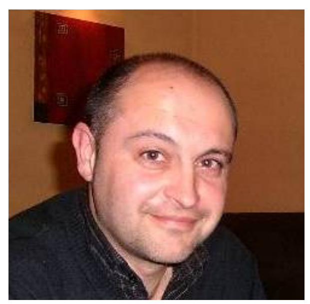
Fidelity Decay vs. Unitarity
(featuring: the Iberians)

Adolfo del Campo (UMass)



Javi Molina-Vilaplana (Cartagena)

Laura Garcia-Alvarez & QUTIS

[arXiv: 1607.08560 & 1701.XXYYZ]

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“introduction”

$$S = \frac{k_B c^3 A}{4 G_N \hbar}$$

black holes are thermodynamic systems

their entropy is proportional to the area of the event horizon

information loss paradox:

a BH formed from a pure state will evolve into a mixed state (of Hawking radiation)

holography:

a theory of quantum gravity should have information ~ area black holes are thermodynamic systems

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holography:

a theory of quantum gravity should have information $\sim$ area

General Plan

AdS/CFT relates gravity (often in AdS) to unitary field theory (often CFT)

Lots of progress gravity ➛ CFT (my favorite: AdS/CMT)

Less known about CFT ➛ (quantum) gravity

➙ despite developments in CFT, CMT:

- time evolution and spread of entanglement
- thermalization of closed quantum systems (e.g. via eigenstates)
- non-perturbative methods (e.g. bootstrap)

Thermalization ➛ BH formation (& evaporation)

Outline

1. Introduction

2. Part I: Unitarity vs. Gravity

3. Part II: The anti-information paradox

4. Conclusions

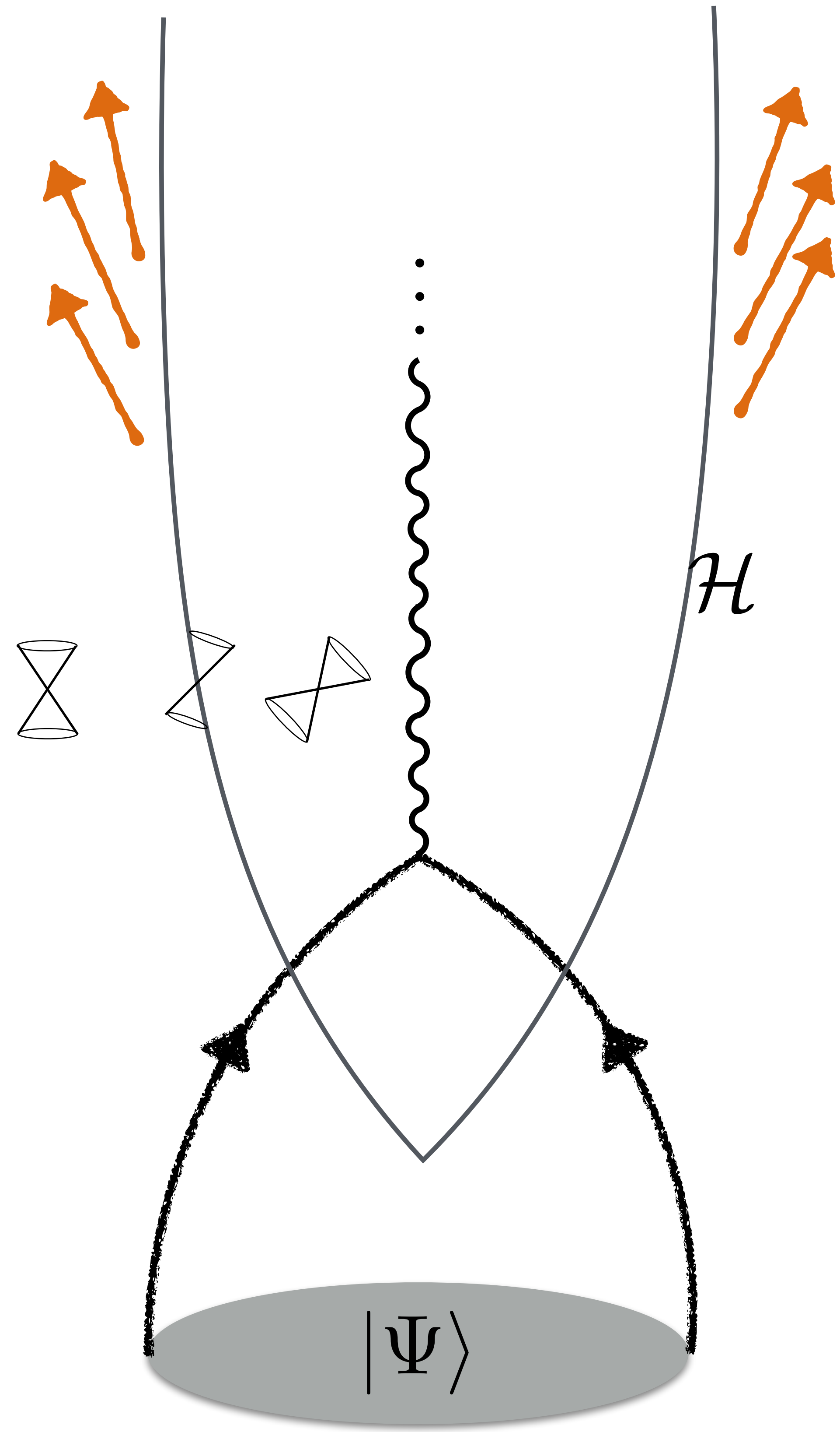
Part I: Unitarity vs. Gravity

“the trouble with black holes”

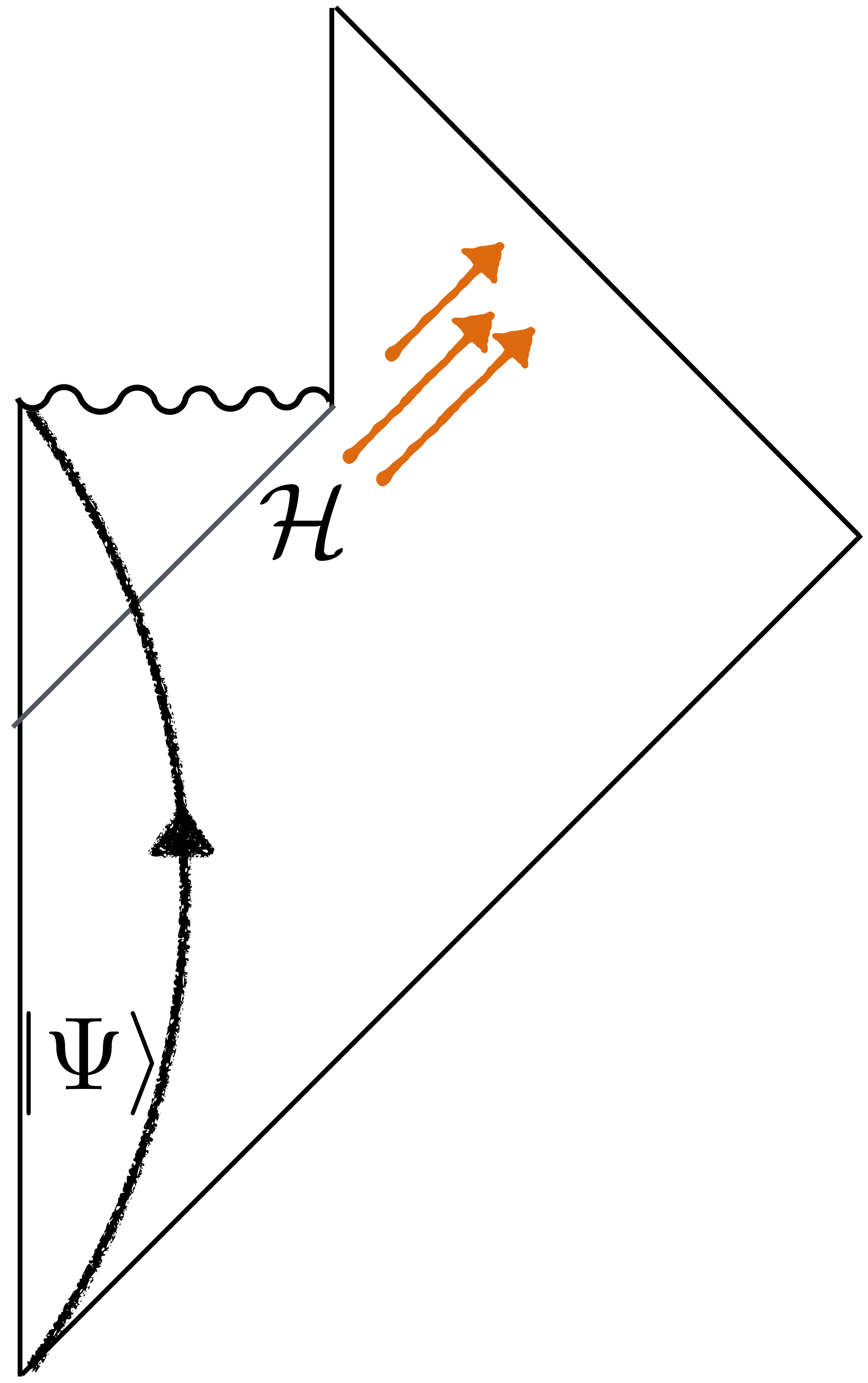
- outgoing **Hawking radiation** is thermal $\rho_{Gibbs}(T_H)$

- a **horizon** $\mathcal{H}$ cloaks the singularity

- initial pure state $|\Psi\rangle$ of matter collapses inwards



black holes evaporate



The Paradox

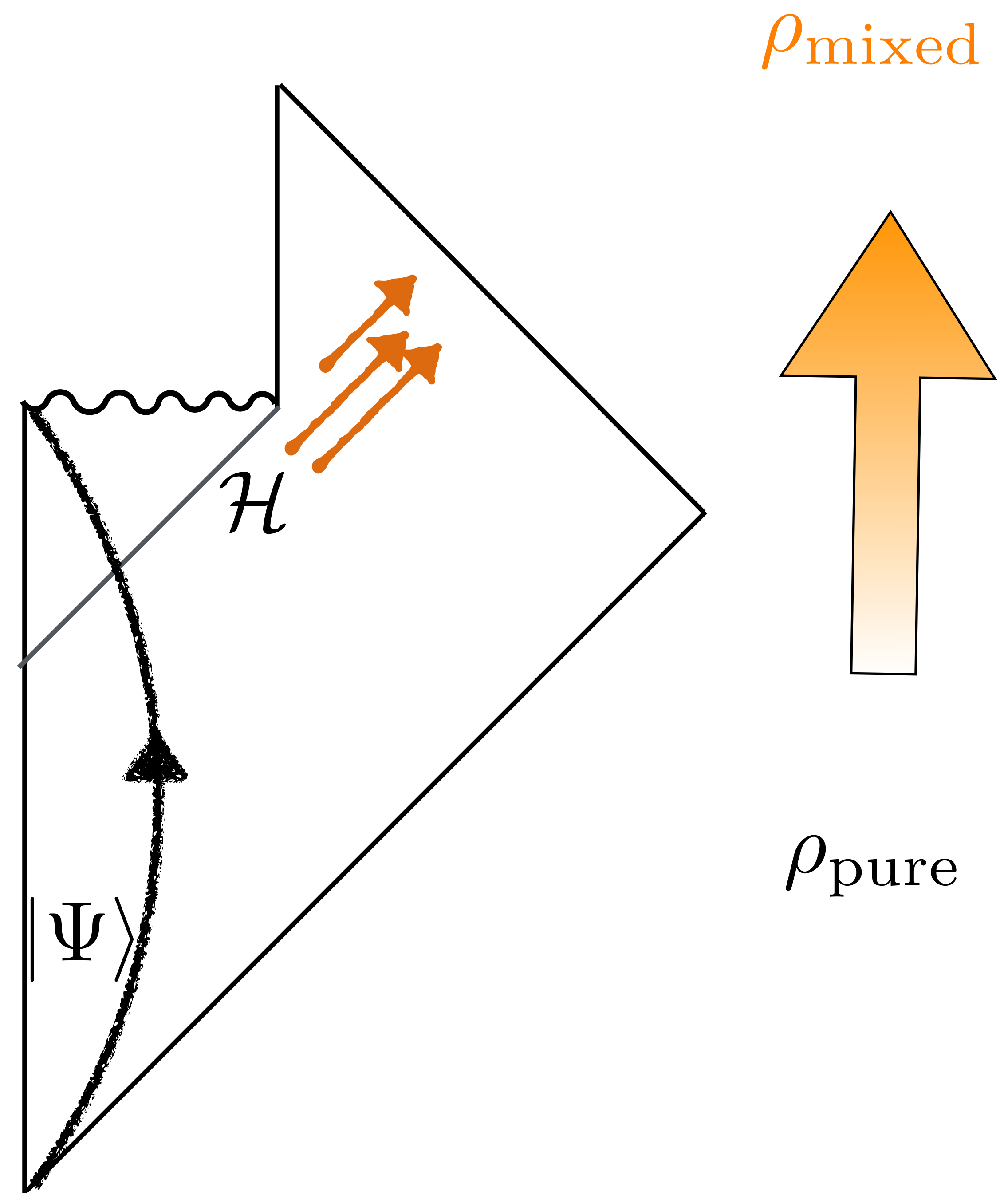
- gravity as an EFT implies pure to mixed evolution

- fundamentally incompatible with a unitary S-matrix

1. quantum gravity is non-unitary

2. gravity EFT makes no sense

3. (subtle) corrections to Hawking result



Holography:

quantum gravity = quantum field theory

hence AdS/CFT only allows for options 2 & 3.

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the anti-information loss paradox:

how does an obviously unitary theory lose information?

The Plan

(of a first-principles calculation in holographic CFT)

1. define an initial state in CFT which forms a black hole

2. understand time evolution in strong-coupling regime

3. diagnose signs of information loss & recovery

4. translate this into a consistent picture of bulk quantum gravity

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(of a first-principles calculation in holographic CFT)

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   - ➢ “quantum quench”

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   ➣ wouldn’t that be nice?

“unitarity constraints”

Unitarity vs Thermalization

(constraints on long-time correlations from unitarity)

Correlations in a closed quantum system, e.g.

$$G(t) = \mathrm{tr} \rho \mathcal{O}(t) \mathcal{O}(0)$$

Time average over a large time T cannot vanish by unitarity

$$\lim_{T \to \infty} \overline{|G(t)|^2} \neq 0$$

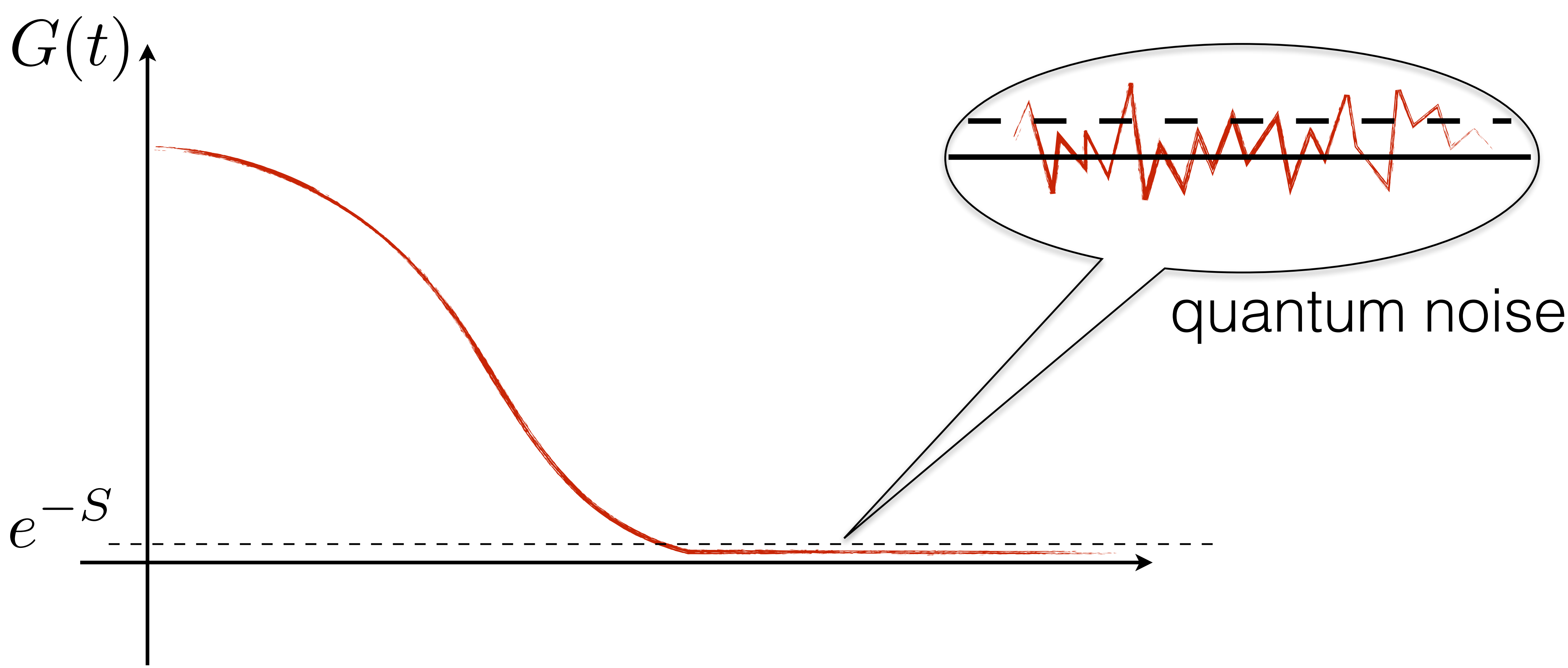
Need to assume spectrum is generic (no specific ordering principle)

➢ fails for integrable theories
➢ connection with ETH

Unitarity vs Thermalization

(constraints on long-time correlations from unitarity)

$$\rho = e^{-\beta H}$$



see also [Barbon & Rabonivici]

Spectral Probes

(to appear with del Campo and Molina-Vilaplana)

Essence of 2-point function: dephasing at late times

$$G(t) = \frac{1}{Z}\sum_{i,j} e^{-\beta E_i - it(E_i - E_j)} |O_{ij}|^2$$

Conveniently captured in spectral form factor

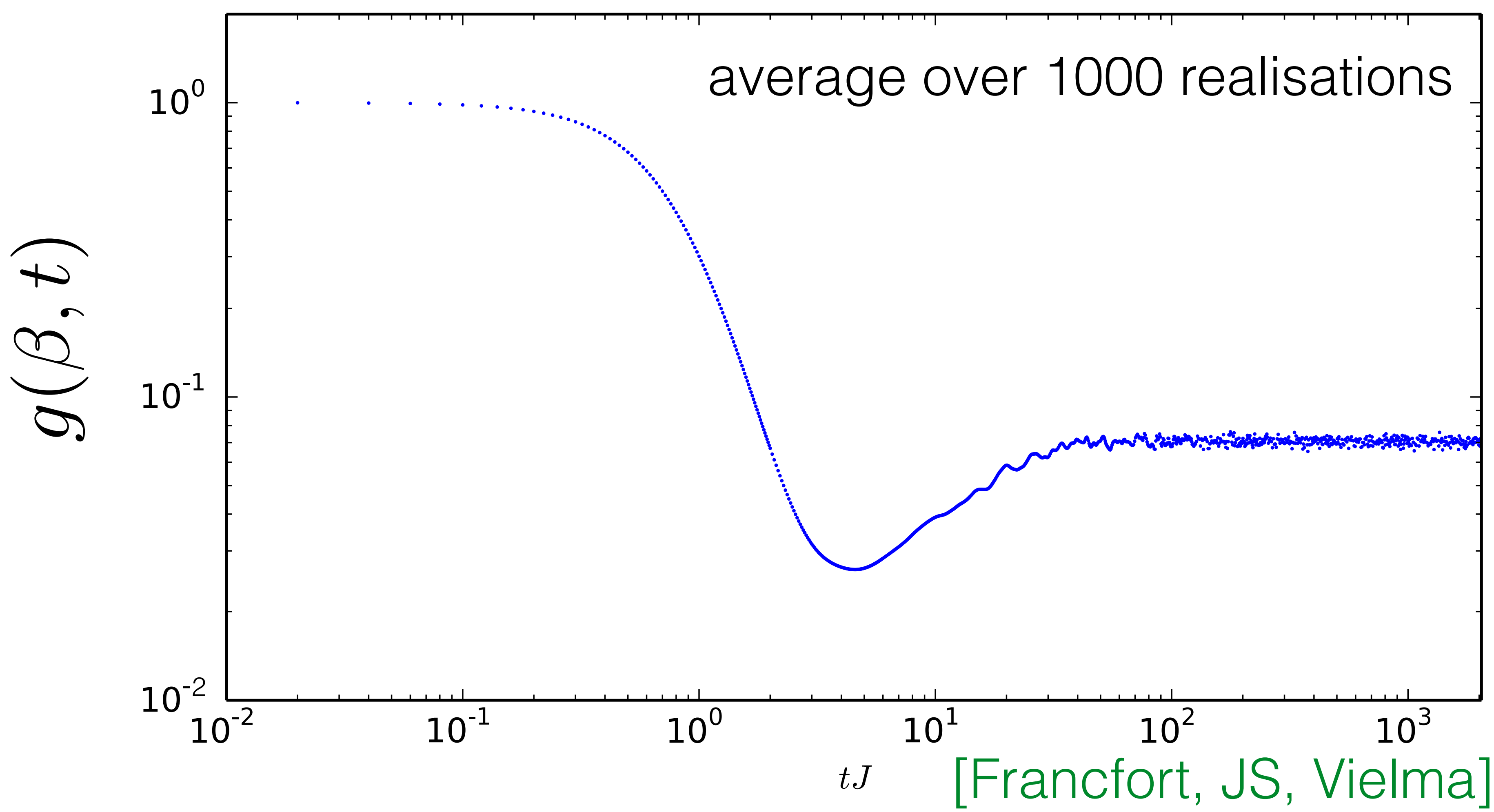
$$g(\beta, t) = \frac{1}{Z^2}|Z(\beta + it)|^2$$

Well-studied object in random matrix theory

Recently re-surfaced in context of SYK [Cotler et al.]
& 2D CFT [Dyer & Gur-Ari]

An Illustration (Dirac SYK)

also see [Cotler et al.]



[Francfort, JS, Vielma]

initial decay (~information loss), then “dip, ramp & plateau”

can we characterise these properties more generally?

Fidelity Decay

(a new perspective on spectral form factor)

Prepare the system in the “thermofield double” state on $\mathcal{H}^{\otimes 2}$

$$|\psi(\beta)\rangle := \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2} E_n} |n\rangle \otimes |n\rangle$$

Consider time evolution, U(t,0), with respect to

$$H = H_{\mathrm{s}} \otimes \mathbf{1}$$

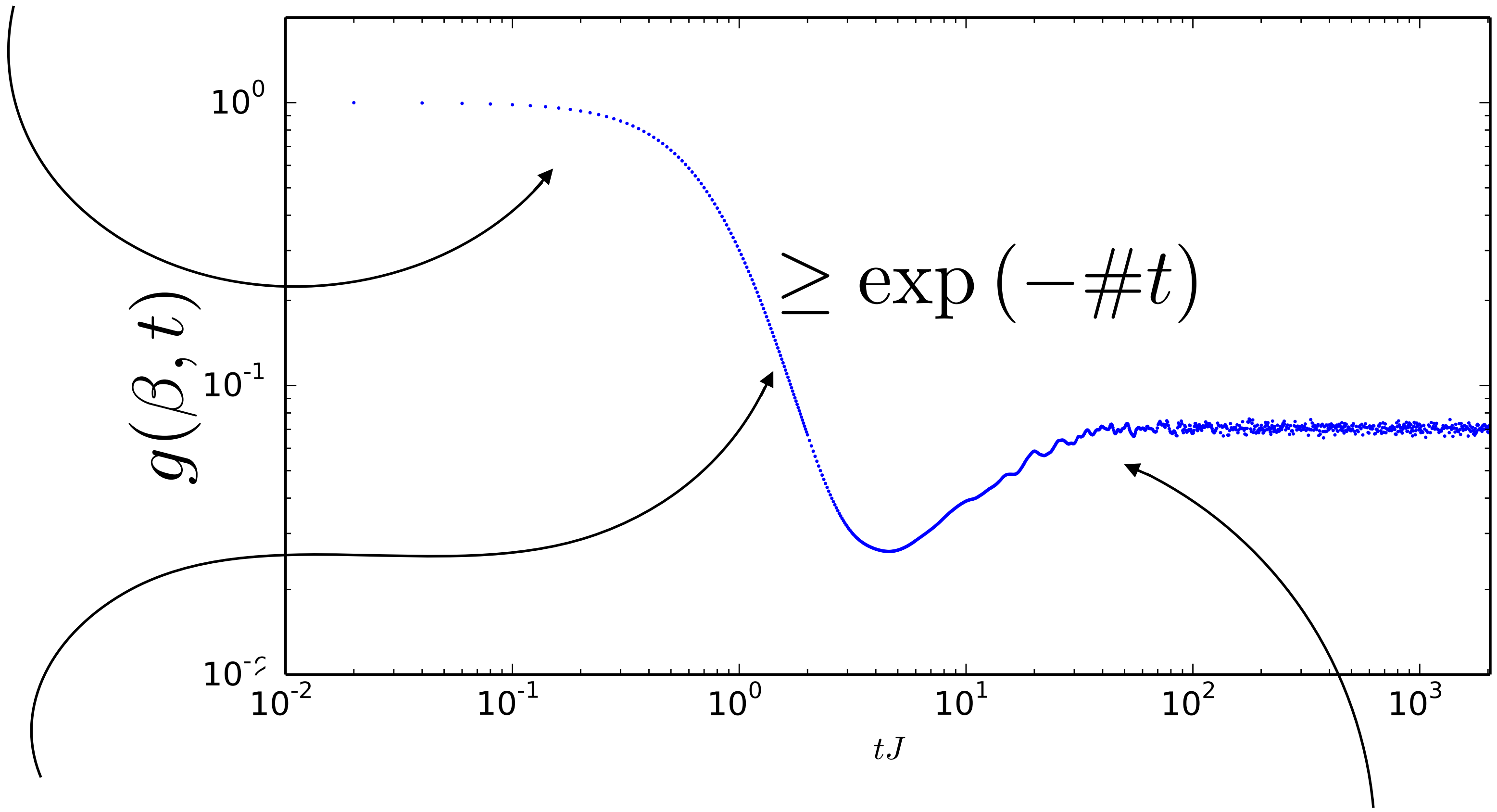
We can re-express the spectral form factor as the overlap

$$\mathcal{F}(t) = |\langle \psi(\beta)|U(t,0)|\psi(\beta),0\rangle|^2$$

⇒ Unitarity constraints on behaviour of “fidelity” $\mathcal{F}(t)$ (e.g QSL)

Selected Results [del Campo, Molina-Vilaplana, JS]

1. Initial Gaussian decay governed by “Zeno Time” $\tau_Z = \langle \Delta E \rangle_\beta$



2. Intermediate decay slower than exponential

e.g. SYK & RMT $\sim 1/t^3$

3. Very late time: non-commuting limits & information loss

c.f. rest of talk!

Upshot

QFT correlations probe information loss

Unitarity demands non-trivial late-time behaviour $\mathcal{O}\left(e^{-S}\right)$
(assume finite-size system)

Fidelity of thermofield double state allows to map bounds on spectral form factor to QSL

Upshot

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Part II: the anti-information loss paradox

Looking for the Right Place

0D matrix models(IOP,…): connection to geometry?
2D black hole (CGHS): solvable but very different
SYK/2D black hole: Einstein dual,… ?

3D story shares salient features of 4D (and higher)
in fact central to micro-state counting success (D1-D5)

the trouble: no local degrees of freedom (Achucarro & Townsend):

$$S_{\rm 3D} = S_{\rm CS}[A] - S_{\rm CS}[\bar{A}]$$

other side of the coin: CFT_2 puts powerful tools at our disposal
also see [Fitzpatrick, Kaplan,…]

3D Gravity + Matter

➙ add matter: get local dof. BUT need new tools

focus on a universal sector, by defining a 1/c expansion:

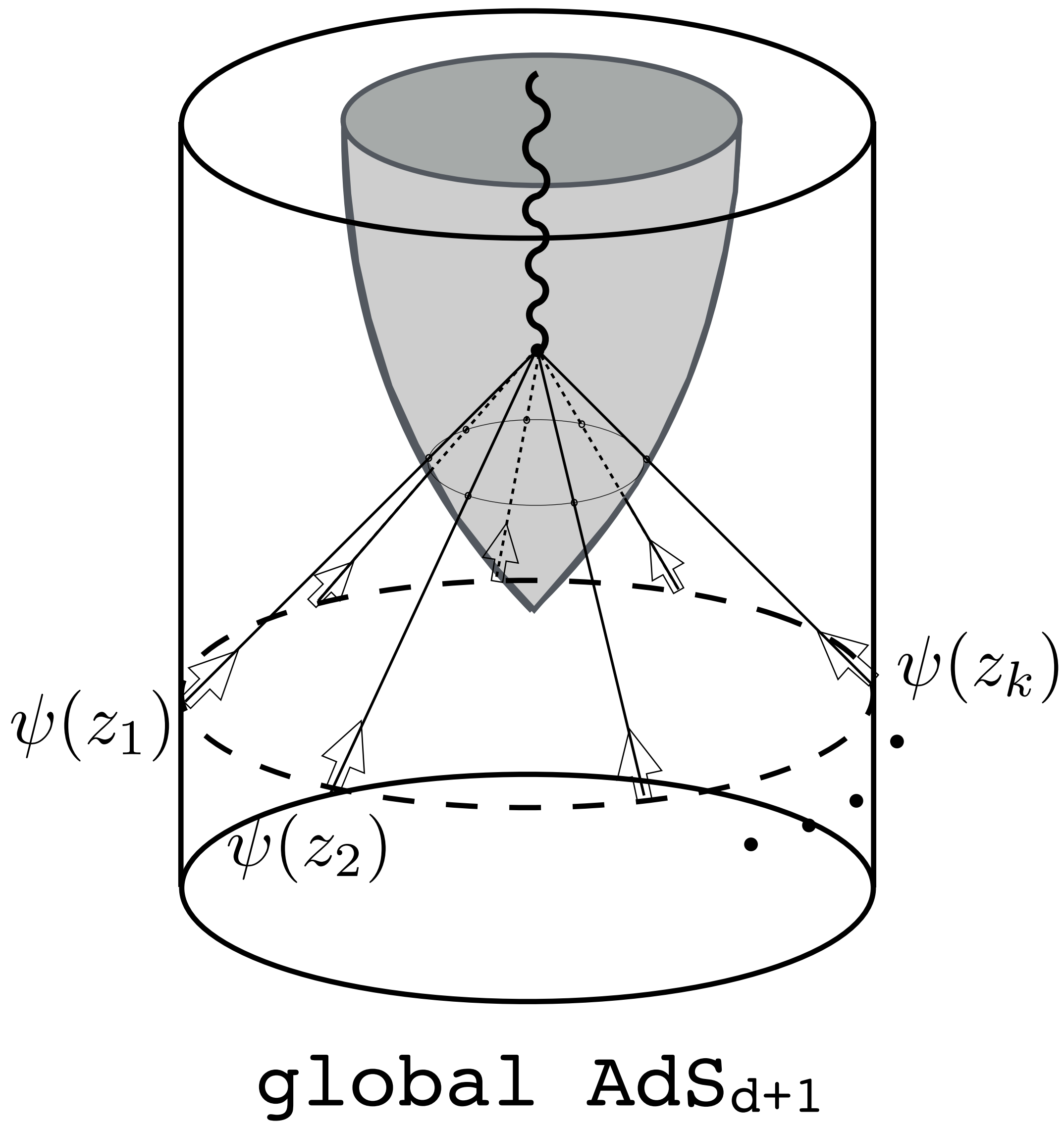
➙ any microscopic theory in this class defines some 3D quantum gravity theory (sparse spectrum)

3D gravity + matter non-trivial, but solvable
➙ ideal place to study BH puzzles!

From bulk point of view this is $1/G_N$ expansion

CFT_2 gives a non-perturbative definition of quantum gravity.

The Black Hole in the Tin Can



Throw in a shell of n dust particles

smooth limit: $n \to \infty$

BH collapse: Vaidya metric

Use light operators $\mathcal{Q}$ to probe geometry as function of t

remark: certain quantities such as entanglement entropy are sensitive to behind horizon physics (away from equilibrium)

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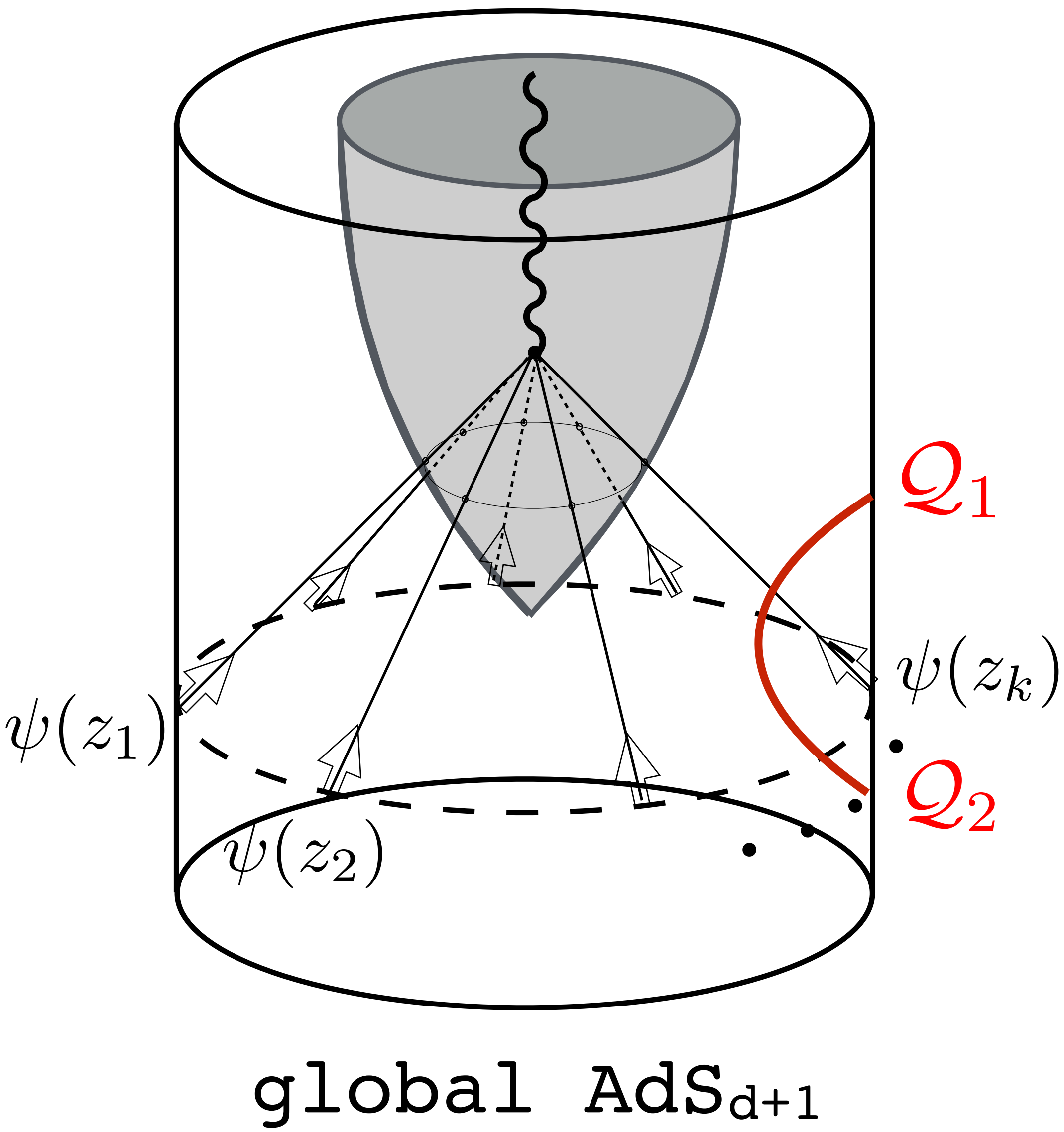

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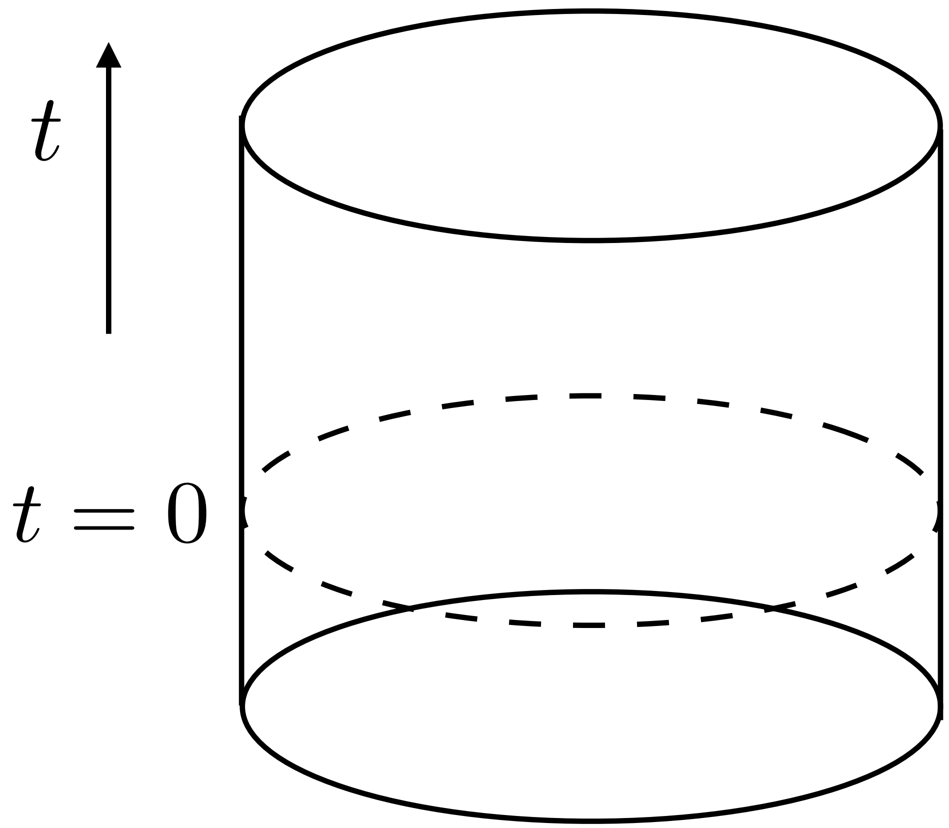
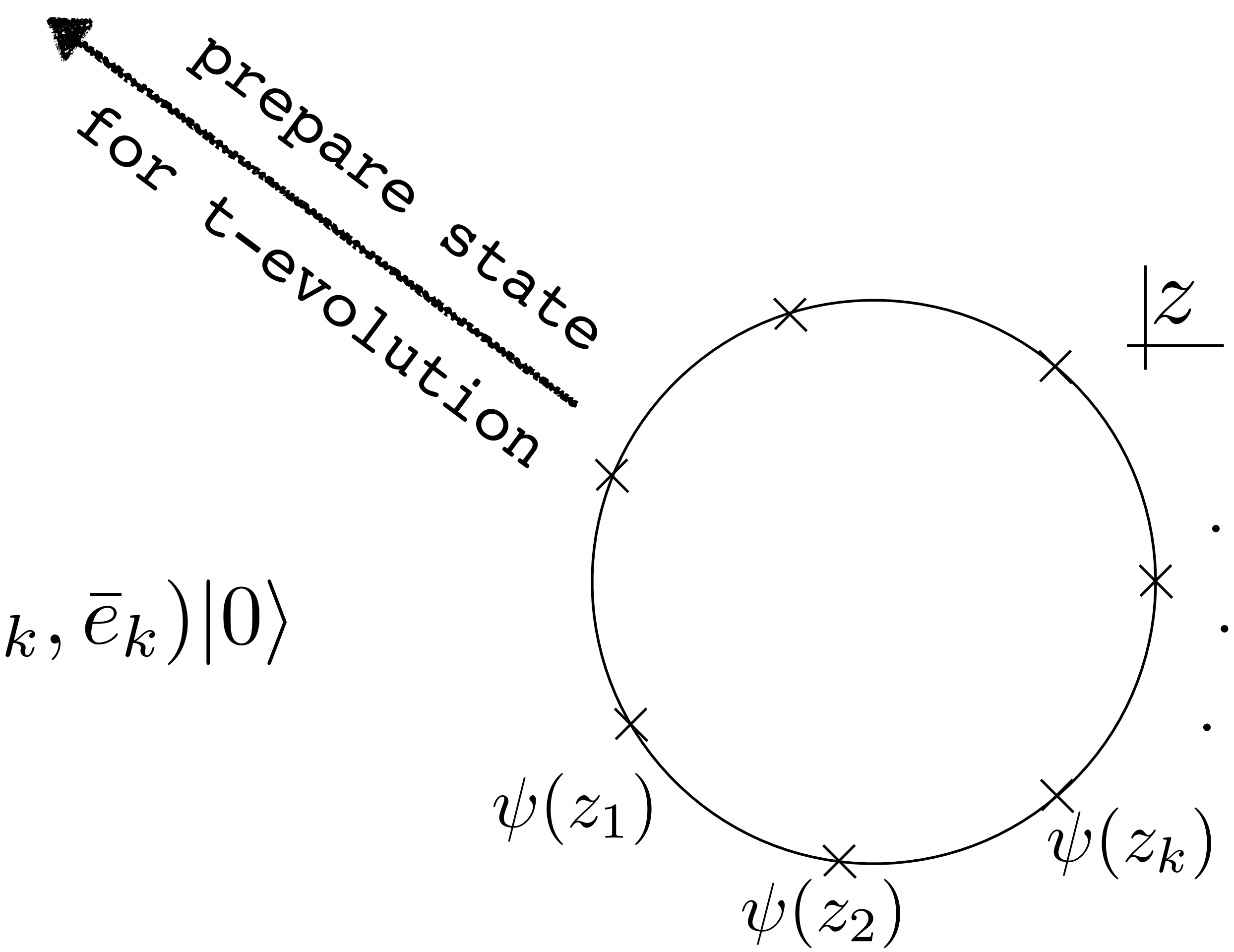
Translating to the CFT



start in excited state at t=0:

prepare by Euclidean path integral
➛ regulator σ



$$|\mathcal{V}\rangle = \frac{1}{\mathcal{N}} \prod_{k=1}^{n} \psi(e_k, \bar{e}_k)|0\rangle$$

Interrogating the CFT

Start probing the physics via 2n + p correlations

$$G(1,2,\dots p) = \langle \mathcal{V} | \mathcal{Q}_1 \,, \dots \, \mathcal{Q}_p | \mathcal{V} \rangle$$

we want to approach smooth, semi-classical gravity

$$c \to \infty$$

$$n \to \infty$$

$$\sigma \to 0$$

$$E \sim n h_\psi / \sigma \to \mathcal{O}(c)$$

infinite-point correlations in strongly-coupled CFT!

Benefits of 2D CFT

in the semi-classical limit (large c), get sum of exponentials

$$G(1,2,\ldots p) = \sum_{\text{blocks}} a_k e^{-\frac{c}{6} f_k^{(n)}(1,2,\ldots p)}$$

correlator approximated by largest term, the identity block

“it from **id**”

the dominant contribution comes from the identity Virasoro block, that is the unit operator **id** *and all its descendants*
T, ∂T, T^2 $T\partial T\ldots$, (multi-graviton exchange in bulk)

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T, ∂T, T^2 $T\partial T$…, (multi-graviton exchange in bulk)

subleading corrections exponentially suppressed in $e^{-c} \sim e^{-1/G}$

still need to calculate the semi-classical block:

CONFORMAL SCALAR FIELD ON THE HYPERELLIPTIC CURVE AND CRITICAL ASHKIN-TELLER MULTIPOINT CORRELATION FUNCTIONS

Al.B. ZAMOLODCHIKOV

Scientific Council of "Cybernetics", Academy of Sciences, USSR

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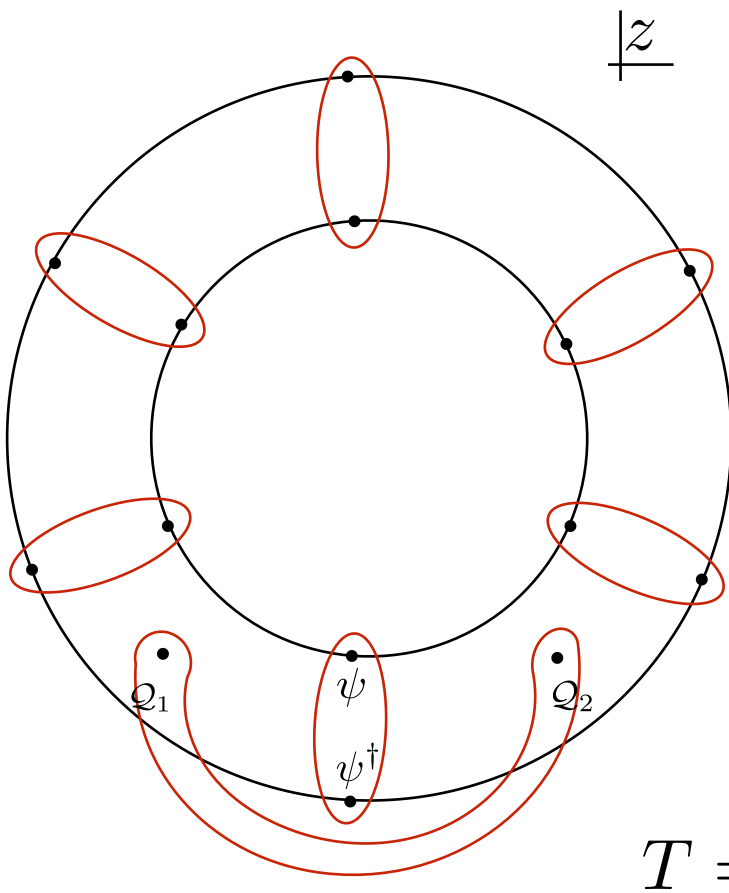
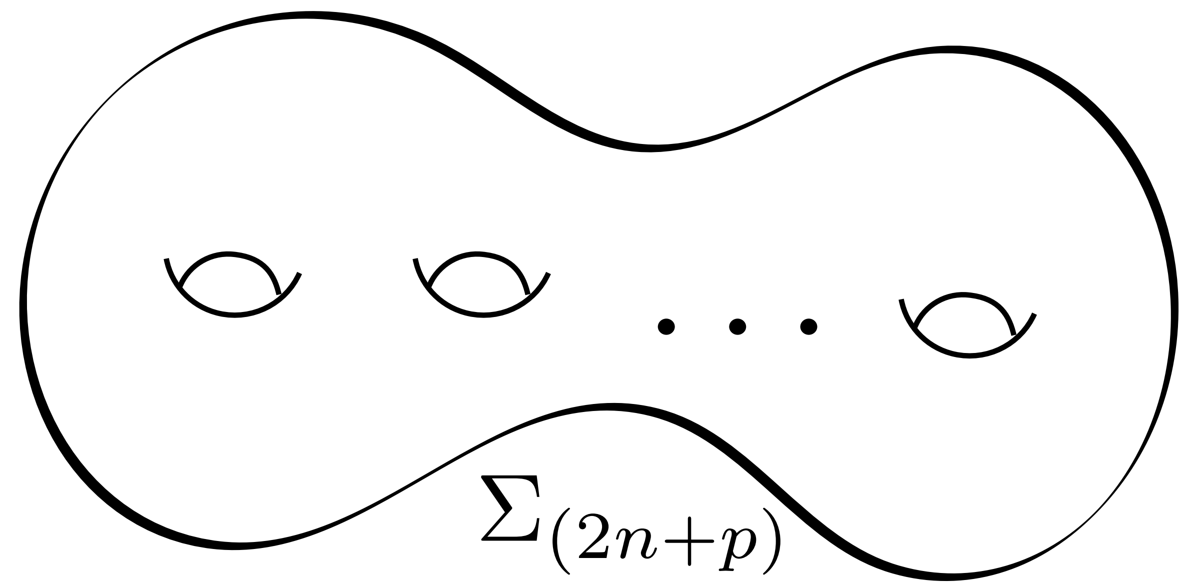
USSR: fighting hyper-intelligent Robot overlords with CFT?

The Monodromy Method

each contraction of operators in the plane defines a cycle



defines

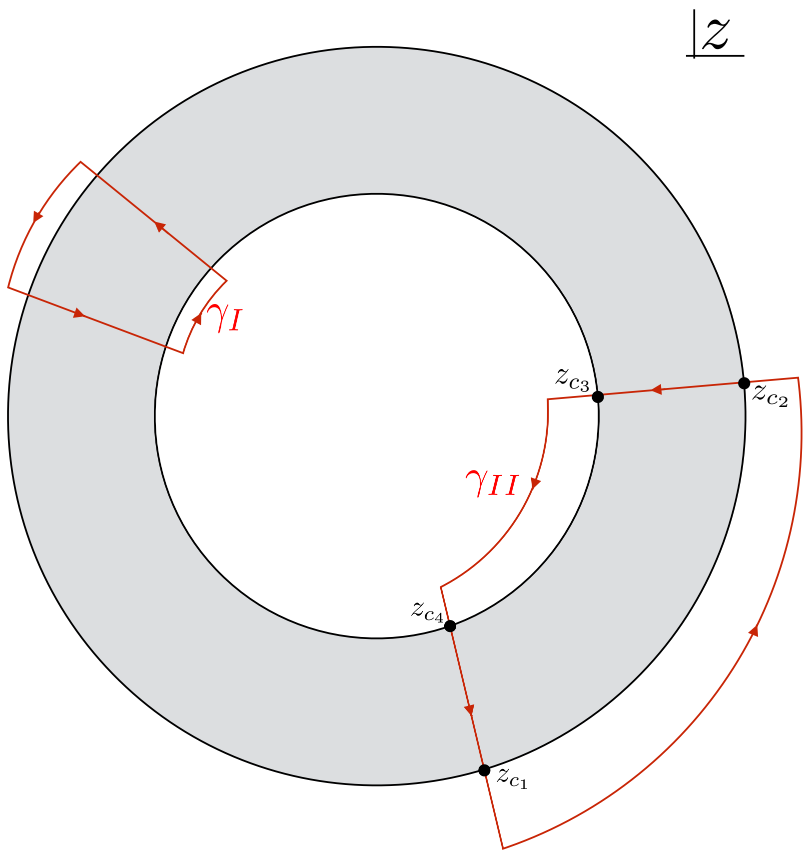


$$T = \sum_{k=1}^{2n+p} \left[\frac{6h/c}{(z-z_k)^2} + \frac{c_k}{z-z_k} \right]$$

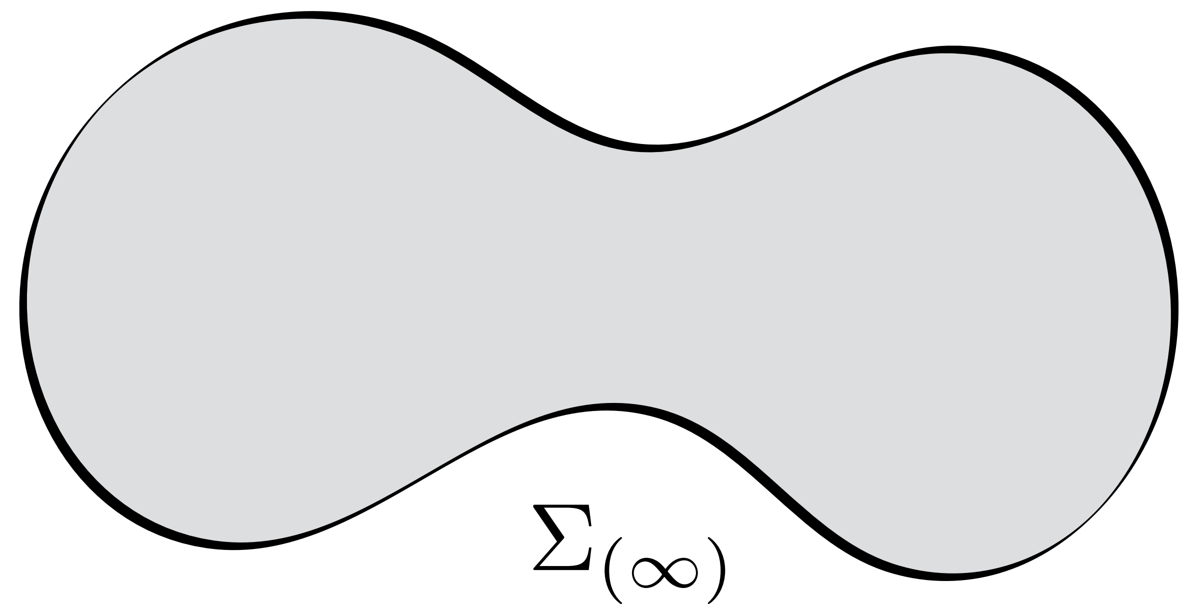
fix monodromies of $y''(z) + T y(z) = 0 \xrightarrow{c_k} f_k^{(n)}(1, 2, \ldots p)$

Taking the smooth Limit

generally a hard problem, big simplification occurs for n $\longrightarrow \infty$



defines



stress tensor $\longmapsto$ distribution on $\Sigma_{(\infty)}$

continuum monodromy method $f_0^{\infty}(1,2,\ldots,p)$	$\longleftrightarrow$	3D semi-classical gravity $\mathcal{L}_{\text{geo}}(1,2,\ldots p)$

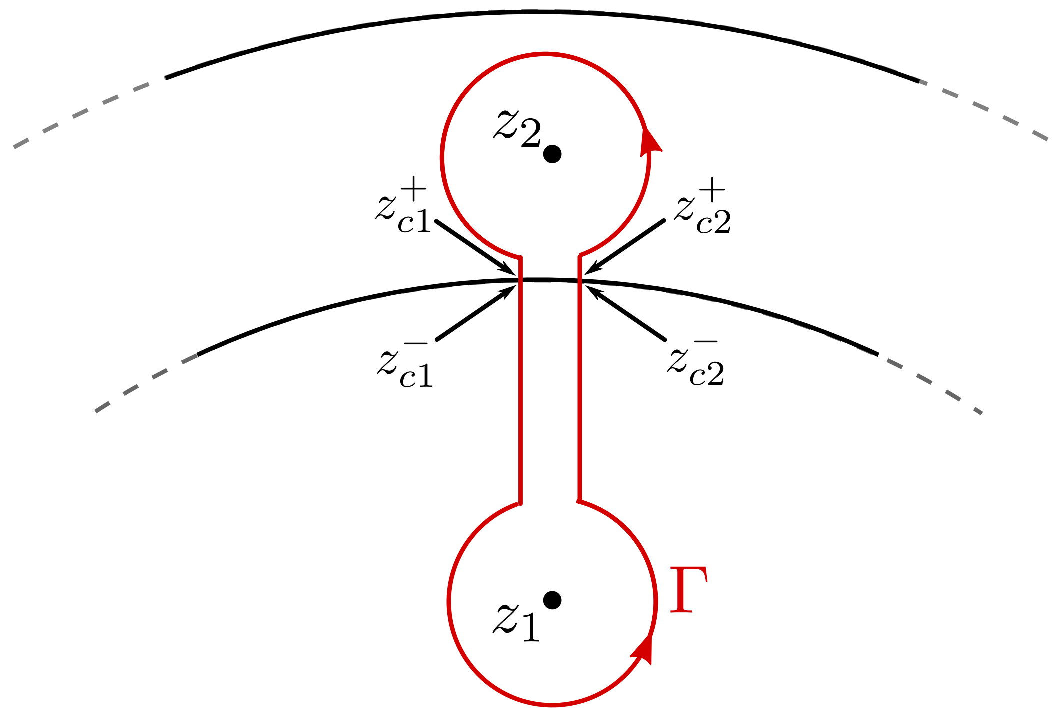
Two-point Autocorrelation

let us now return to the black hole and compute

$$G(t) = \mathrm{tr}\rho\mathcal{O}(t)\mathcal{O}(0)$$

in the collapse state $|\mathcal{V}\rangle$

$$G(1,2) \quad =$$



can be done analytically:

$$G(t_1,t_2) = \left(\frac{1}{\pi T}\cos\left(\frac{t_1}{2}\right)\sinh\left(\pi T t_2\right) - 2\sin\left(\frac{t_1}{2}\right)\cosh\left(\pi T t_2\right)\right)^{-2\Delta^{\mathcal{Q}}}$$

Physical Consequences

not (yet) known from gravity (but matches known limits)
$\Longrightarrow$ CFT prediction for 3D gravity

The correlation function decays without bound at large time

$$G(t_1, t_2) \sim \exp(-\frac{2\pi\Delta^{\mathcal{Q}} t}{\beta})$$

Manifestly in conflict with unitarity: **CFT loses information!**

Can also compute entanglement entropy of interval A

$$S(A) \to S_{\text{Gibbs}}(A;T) \longrightarrow \rho(A) = \rho_{\text{Gibbs}}(A;T)$$

Restoring Unitarity

This is the anti-information paradox: what happened to unitarity?

$$|G(t)| = \left|\sum_{n,k} e^{i(E_n - E_k)t} \Psi_n^*(\mathcal{V}) \langle n|\mathcal{Q}|k\rangle \langle k|\mathcal{Q}|\mathcal{V}\rangle\right| \neq 0$$

➙ correlations cannot become arbitrarily small in $|\mathcal{V}\rangle$

Neglected contributions exponentially suppressed at t=0 (must be present due to crossing symmetry)

$$\sum_{k\neq \mathrm{vac}} a_k e^{-\frac{c}{6} f_k^{\infty}(1,2,\ldots p)} \sim e^{-S}$$

restore unitary at large time ➙ non-perturbative effects in $1/G_N$

Conclusions

time-dependent 3D quantum gravity with matter in 1/c expansion
'it from id' ➙ ideal arena to think about quantum BHs

unitarity constraints on 2-point functions, spectral form factor,… can be mapped to fidelity decay and quantum speed limits (QSL)
➙ bounds on scrambling exponent?

CFT correlation functions seemingly violate unitarity (naïve).
non-perturbative corrections in c restore unitarity

on gravity side these correspond to non-perturbative effects in G_N.
geometric interpretation? bulk interpretation?

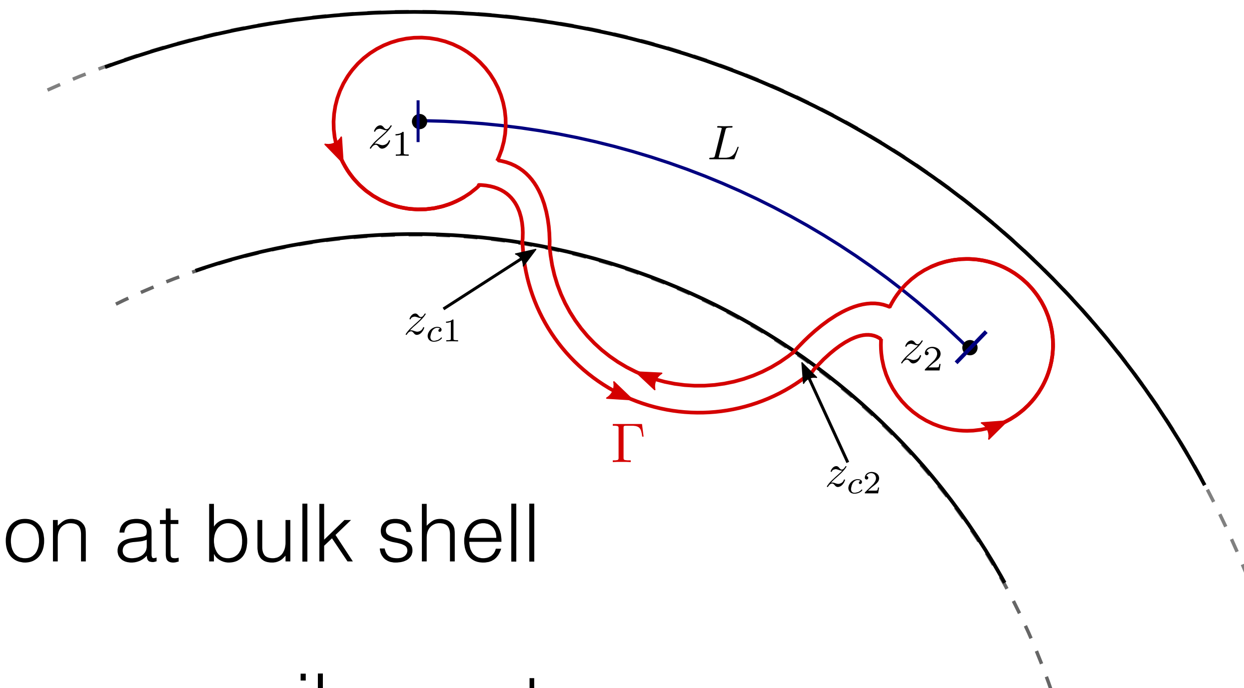
are the corrections universal? make a black hole that evaporates,…

thank you!

entanglement entropy

Q-type operators ➙ twist insertions: $G_q(t) = \langle \mathcal{V} | \sigma_q(t, \ell_1) \tilde{\sigma}_q(t, \ell_2) | \mathcal{V} \rangle$

$$S(A) = \lim_{q \to 1} \frac{1}{1-q} G_q(t)$$



crossing points z_{c1} & z_{c2} ↔ refraction at bulk shell

it from id ➙ require trivial monodromy on smile contour

write $z_1 = e^{i\theta_1}, z_2 = e^{i(\theta_1 + L)}$ & continue to Lorentzian time $\theta_1 = t$

maximize S(A) over crossing points ➙ parametric equation for S(t)

entanglement entropy

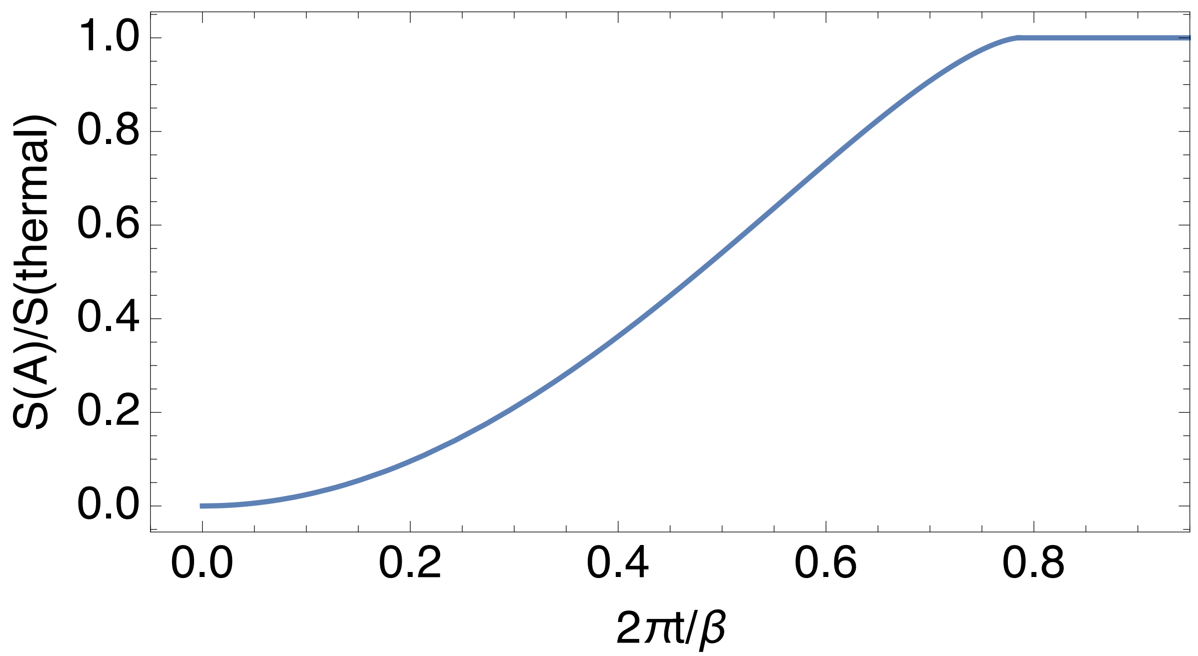
Implicit formula for growth of entanglement entropy:

$$t = \frac{\beta}{2\pi}\cosh^{-1}\left\{\cosh\left(2\pi T q\right) + 2\pi T \tan\left(\frac{L}{2} - q\right)\sinh\left(2\pi T q\right)\right\}$$

$$S_{EE} = \frac{c}{3}\log\left\{\frac{\sin\left(\frac{L}{2}-q\right)\cosh\left(2\pi T q\right) + \frac{1}{2\pi T}\left[1+\frac{1}{2}\left\{1+4\pi^2T^2\right\}\tan^2\left(\frac{L}{2}-q\right)\right]\cos\left(\frac{L}{2}-q\right)\sinh\left(2\pi T q\right)}{\epsilon_{UV}/2}\right\}$$

matches **exactly** global AdS_3 Vaidya:

- thermal at late time
- EE growth = change of channel
- sees **beyond horizon**

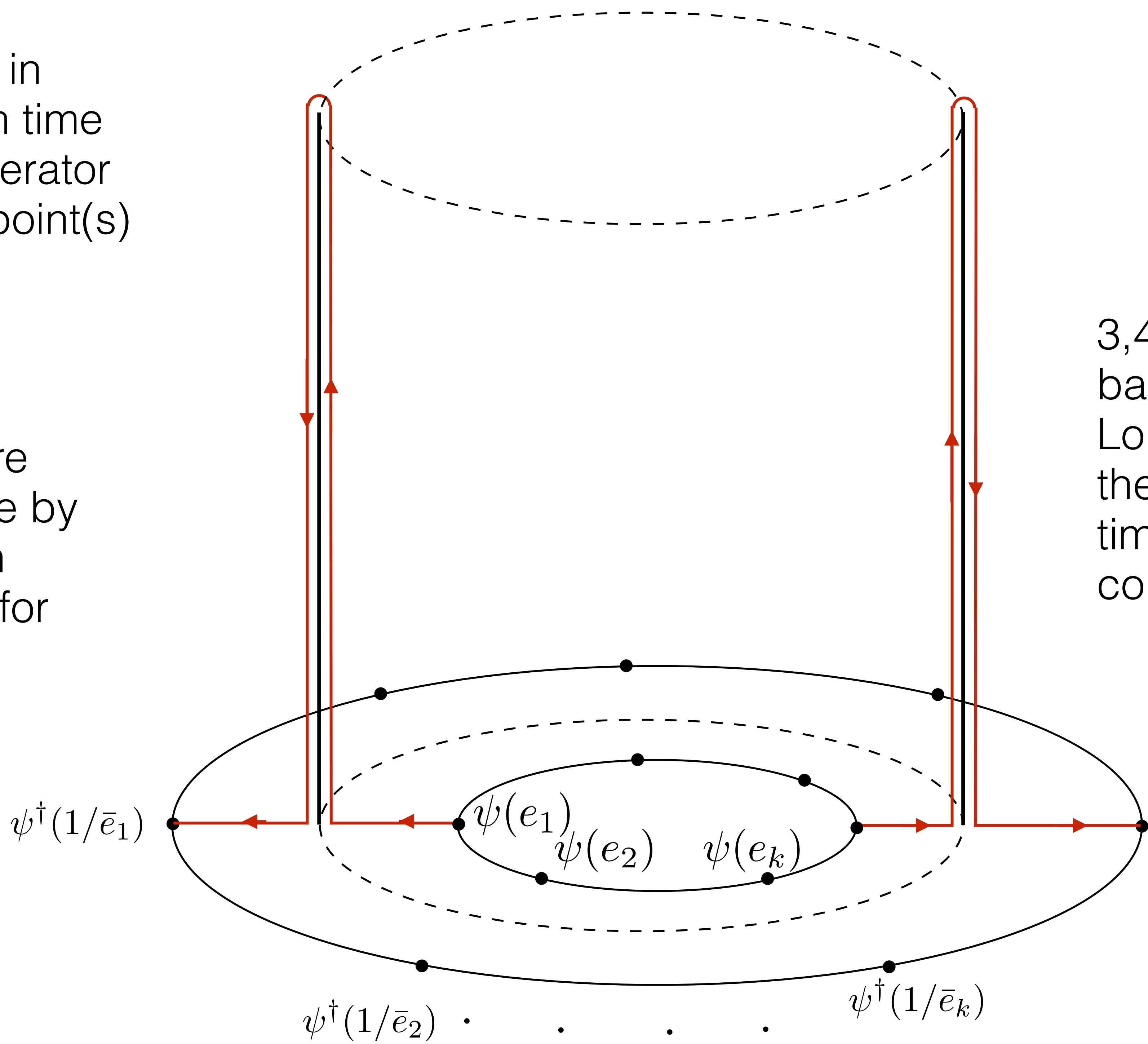


CFT calculation shows that purity of state is preserved: $S(A) = S(A^c)$

alternative picture: IN-IN computation

2.) evolve in Lorentzian time until Q-operator insertion point(s)

1.) prepare initial state by Euclidean evolution for time σ



3,4.) evolve back in Lorentzian time, then Euclidean time to form conjugate