M-theory from the superpoint

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Prologue

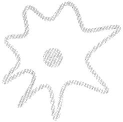


Figure : $\mathbb{R}^{0|1}$

Prologue

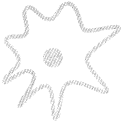


Figure : $\mathbb{R}^{0|1}$

$\mathbb{R}^{0|1}$ has a single odd coordinate θ, and $\theta^2 = 0$, so a power series terminates immediately:

$$f(\theta) = f(0) + f'(0)\theta.$$

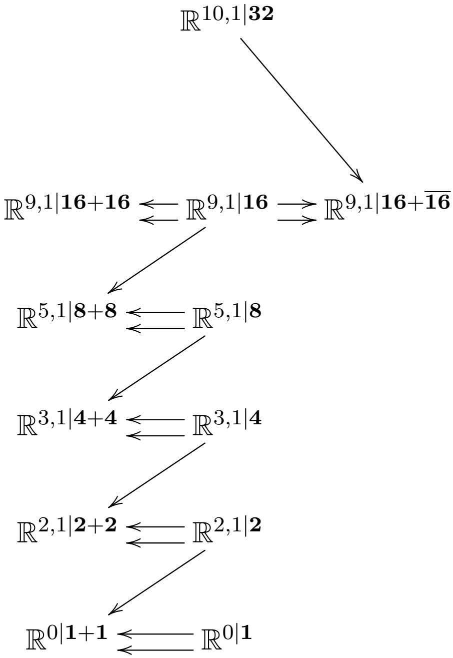
In essence, this means we should regard θ as infinitesimal. Thus $\mathbb{R}^{0|1}$ is a single point with an infinitesimal neighborhood, as depicted above.

Prologue

We will investigate the superpoint with mathematical tools.

Inside, we will find all the super-Minkowski spacetimes of string theory and M-theory, going up to dimension 11.

Then we will find the strings, Dp-branes and M-branes themselves, thanks to the brane bouquet of Fiorenza, Sati and Schreiber.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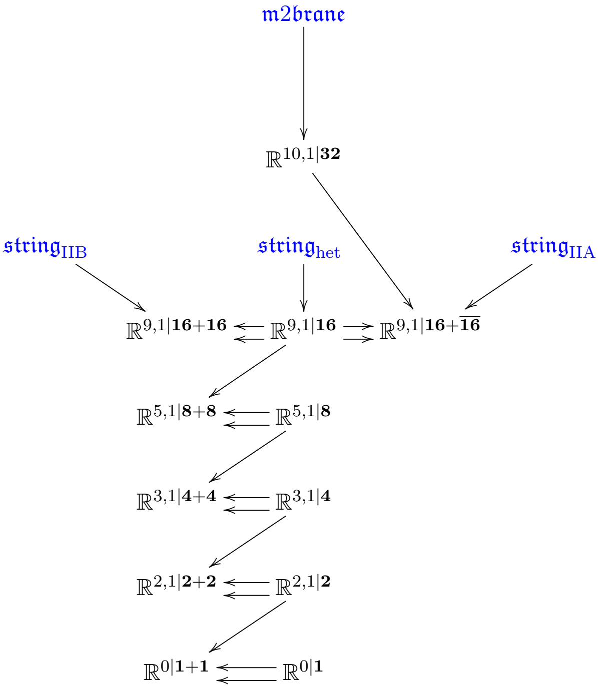

m2brane
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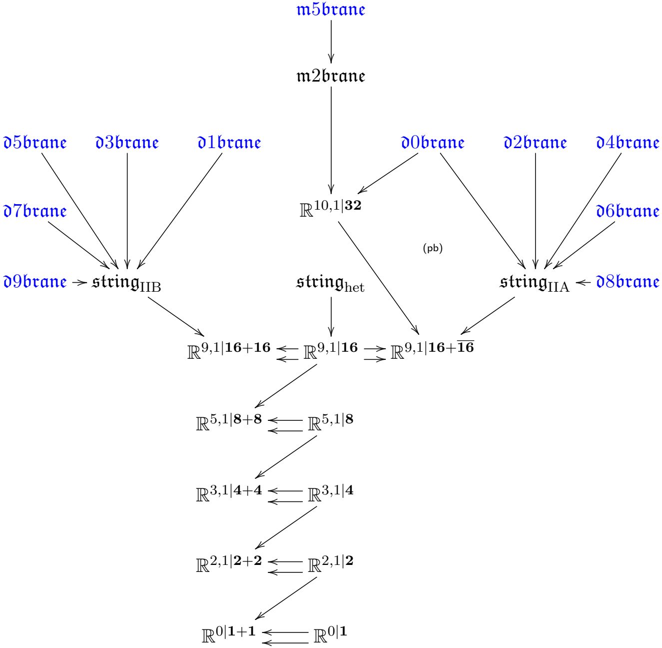

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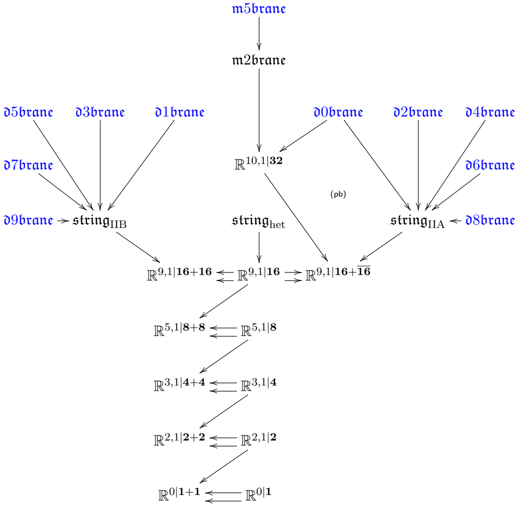

The brane bouquet.

Brane condensation

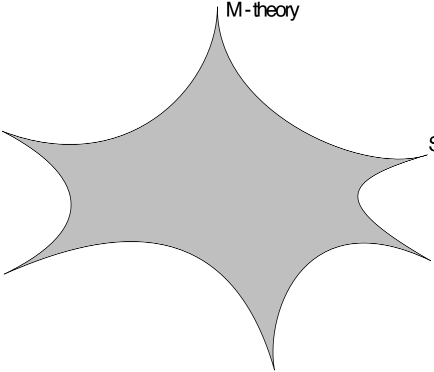

Figure : Cartoon by Polchinski.

Type IIA string theory contains D0-branes.

Brane condensation

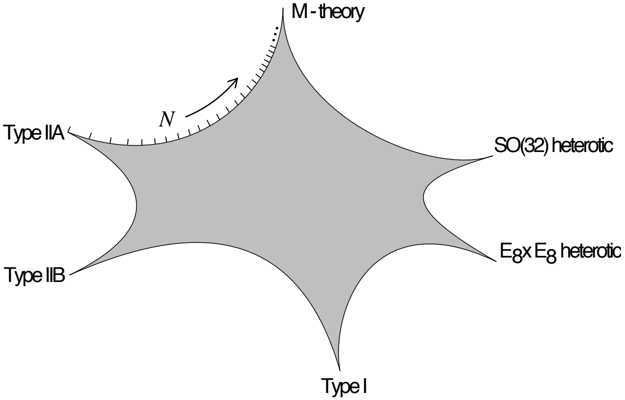

Figure : Cartoon by Polchinski.

As the number *N* of D0-branes grows large, type IIA string theory becomes M-theory.

Brane condensation

This means the 10-dimensional superspacetime where type IIA strings live "grows an extra dimension" to become the 11-dimensional superspacetime of M-theory.

Infinitesimally,

$$\mathbb{R}^{9,1|\mathbf{16}+\overline{\mathbf{16}}} \rightsquigarrow \mathbb{R}^{10,1|\mathbf{32}}.$$

Central extensions

Given

- $\mathfrak{g}$ a Lie superalgebra,
- $\omega\colon \Lambda^2\mathfrak{g} \to \mathbb{R}$ a 2-cocycle:

$$\omega([X, Y], Z) \pm \omega([Y, Z], X) \pm \omega([Z, X], Y) = 0,$$

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with one extra generator c, even and central, and modified Lie bracket:

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I'll write

$$\mathfrak{g}_\omega \to \mathfrak{g}$$

for the map setting c to zero, and often use this arrow to denote a central extension.

The brane bouquet, step 1

Note the parallels:

- M-theory spacetime has one more bosonic dimension than type IIA string theory spacetime.
- $\mathfrak{g}_\omega$ has one more bosonic dimension than $\mathfrak{g}$.

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The brane bouquet proposal, step 1

Brane condensation *is* central extension.

For this to make sense, super-Minkowski spacetime

$$\mathbb{R}^{D-1,1|\mathbf{S}}$$

must be a Lie superalgebra, and it is:

$$[Q_\alpha, Q_\beta] = -2\Gamma^{\mu}_{\alpha\beta} P_\mu$$

Moreover, on $\mathbb{R}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}$, the 2-form

$$\mu_{D0} = d\overline{\theta}\Gamma^{11} d\theta$$

must define a 2-cocycle, and it does:

- μ_{D0} is left invariant under translations in superspace.
- $d\mu_{D0} = 0$.

The superpoint

This prompts us to ask

Question

Are other dimensions of spacetime also the result of brane condensation/central extension?

At the most extreme end, let us start with the superpoint

$$\mathbb{R}^{0|1}$$

with a single odd coordinate θ.
This has exactly one 2-cocycle:

$$d\theta \wedge d\theta$$

Extending by this 2-cocycle gives $\mathbb{R}^{1|1}$, the superline, the worldline of the superparticle.

$$\mathbb{R}^{1|1} \to \mathbb{R}^{0|1}.$$

The superpoint

Let us a play a game with two moves:

- We can extend by 2-cocycles, satisfying a suitable invariance condition.
- We can double the number of spinors.

This will lead us from the superpoint up to 11 dimensions and beyond.

The superpoint

First, we will double the number of fermionic dimensions:

$$\mathbb{R}^{0|2}$$

We will write this operation as follows:

$$\mathbb{R}^{0|2} \leftleftarrows \mathbb{R}^{0|1}$$

Now, $\mathbb{R}^{0|2}$ has two odd generators, θ_1 and θ_2, and there are three 2-cocycles:

$$d\theta_1 \wedge d\theta_1, \quad d\theta_1 \wedge d\theta_2, \quad d\theta_2 \wedge d\theta_2.$$

Extending by all three we get:

$$\mathbb{R}^{3|2} \to \mathbb{R}^{0|2}.$$

Dimension 3

Now something remarkable happens: a metric appears!

$$\mathrm{Aut}_0(\mathbb{R}^{3|2}) = \mathbb{R}^+ \times \mathrm{Spin}(2,1).$$

We didn't put it in, but by looking at the automorphisms of the algebra, the three even generators in $\mathbb{R}^{3|2}$ transform under $\mathrm{Spin}(2,1)$ as vectors, and the two odd generators as spinors.

Thanks to this metric, we can look for $\mathrm{Spin}(2,1)$-invariant 2-cocycles on $\mathbb{R}^{2,1|2}$. There are none, because the only $\mathrm{Spin}(2,1)$-invariant map:

$$\mathbf{2} \otimes \mathbf{2} \to \mathbf{1}$$

is antisymmetric.

Dimension 4

Double the number of spinors again:

$$\mathbb{R}^{2,1|\mathbf{2}+\mathbf{2}} \leftleftarrows \mathbb{R}^{2,1|\mathbf{2}}$$

There is precisely one $\mathrm{Spin}(2,1)$-invariant 2-cocycle, and extending by this gives:

$$\mathbb{R}^{3,1|\mathbf{4}} \to \mathbb{R}^{2,1|\mathbf{2}+\mathbf{2}}$$

Dimension 4

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$$\mathbb{R}^{3,1|\mathbf{4}} \to \mathbb{R}^{2,1|\mathbf{2}+\mathbf{2}}$$

Again, the metric is not a choice:

$$\mathrm{Aut}_0(\mathbb{R}^{3,1|\mathbf{4}}) = \mathbb{R}^+ \times \mathrm{Spin}(3,1) \times \mathrm{U}(1).$$

$\mathrm{U}(1)$ is the R-symmetry group.

There are no further $\mathrm{Spin}(3,1)$-invariant 2-cocycles.

Dimension 6

Double the number of spinors again:

$$\mathbb{R}^{3,1|\mathbf{4}+\mathbf{4}} \underset{\longleftarrow}{\longleftarrow} \mathbb{R}^{3,1|\mathbf{4}}$$

Now there are two $\mathrm{Spin}(3,1)$-invariant 2-cocycles.

$$\mathbb{R}^{5,1|\mathbf{8}} \to \mathbb{R}^{3,1|\mathbf{4}+\mathbf{4}}.$$

Dimension 6

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Now there are two $\mathrm{Spin}(3,1)$-invariant 2-cocycles.

$$\mathbb{R}^{5,1|\mathbf{8}} \to \mathbb{R}^{3,1|\mathbf{4}+\mathbf{4}}.$$

Again, the metric is not a choice:

$$\mathrm{Aut}_0(\mathbb{R}^{5,1|\mathbf{8}}) = \mathbb{R}^+ \times \mathrm{Spin}(5,1) \times \mathrm{Sp}(1).$$

$\mathrm{Sp}(1)$ is the R-symmetry group.

There are no further $\mathrm{Spin}(5,1)$-invariant 2-cocycles.

Dimension 10

Now we have a choice of two different ways to double the spinors, a type IIA and type IIB:

$$\mathbb{R}^{5,1|\mathbf{8}+\bar{\mathbf{8}}} \leftleftarrows \mathbb{R}^{5,1|\mathbf{8}}$$

and

$$\mathbb{R}^{5,1|\mathbf{8}+\mathbf{8}} \leftleftarrows \mathbb{R}^{5,1|\mathbf{8}}$$

There are no $\mathrm{Spin}(5,1)$-invariant 2-cocycles in type IIB, but on type IIA there are four:

$$\mathbb{R}^{9,1|\mathbf{16}} \to \mathbb{R}^{5,1|\mathbf{8}+\bar{\mathbf{8}}}.$$

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$$\mathrm{Aut}_0(\mathbb{R}^{9,1|\mathbf{16}}) = \mathbb{R}^+ \times \mathrm{Spin}(9,1).$$

There are no further $\mathrm{Spin}(9,1)$-invariant 2-cocycles.

Dimension 11

Again, we have a choice of two different ways to double the spinors, a type IIA and type IIB:

$$\mathbb{R}^{9,1|\mathbf{16}+\overline{\mathbf{16}}} \leftleftarrows \mathbb{R}^{9,1|\mathbf{16}}$$

and

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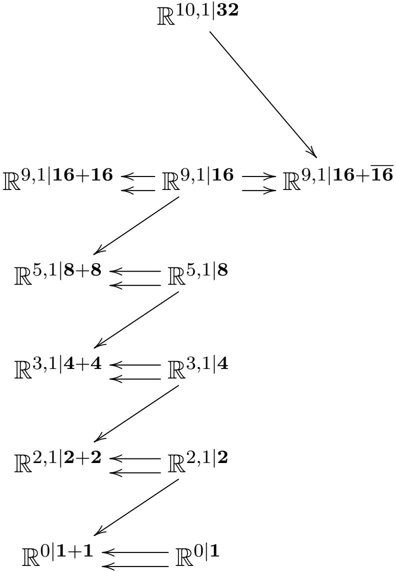
There are no $\mathrm{Spin}(9,1)$-invariant 2-cocycles in type IIB, but on type IIA there is one, the one we started with:

$$\mathbb{R}^{10,1|\mathbf{32}} \to \mathbb{R}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}.$$

In summary:



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$\mathbb{R}^{0|\mathbf{1}}$

Lie algebra cohomology

What does the 2-cocycle

$$\mu_{D0} = d\bar{\theta}\Gamma^{11} d\theta$$

have to do with the D0-brane?

It gives rise to the D0-brane's WZW term:

$$S_{D0} = -m \int \sqrt{-\Pi_0 \cdot \Pi_0}\, d\tau - m \int \bar{\theta}\Gamma^{11} \dot{\theta}\, d\tau.$$

Lie algebra cohomology

In the general, the Lie algebra cohomology of $\mathbb{R}^{D-1,1|\mathbf{S}}$ gives rise to the WZW terms for Green–Schwarz actions. To compute this, write a basis of left-invariant 1-forms on super-Minkowski:

$$e^\mu = dx^\mu - \overline{\theta}\Gamma^\mu\theta, \quad d\theta^\alpha.$$

Find the Lorentz-invariant combinations, such as:

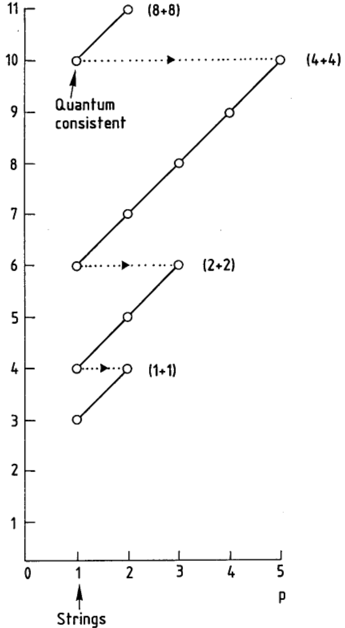
$$\mu_p = e^{\nu_1} \wedge \cdots \wedge e^{\nu_p} \wedge d\overline{\theta}\Gamma_{\nu_1\cdots\nu_p} d\theta.$$

This a $(p+2)$-cocycle if and only if it is closed:

$$d\mu_p = 0.$$

This happens only for special values of $D, \mathcal{N}$ and p.

The brane scan



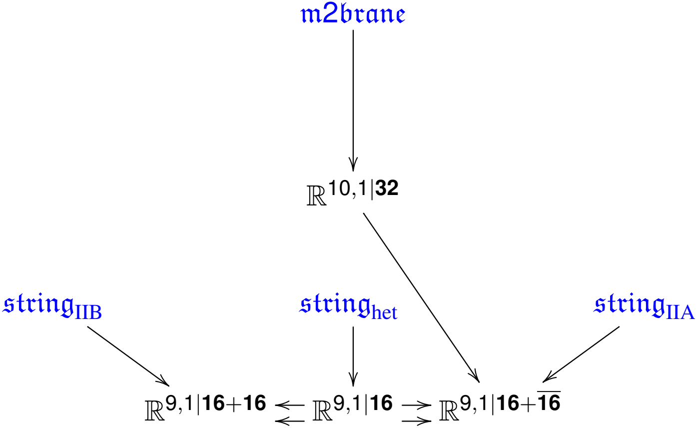
The brane bouquet

- These cocycles really determine the theory.
- Schreiber has a mathematical machine that takes cocycles and produces action functionals.

$$\text{Lie algebra cocycle on } \mathfrak{g} \quad \rightsquigarrow \quad \text{WZW term on } G$$

- Centrally extending by these cocycles, we get new algebras.

The brane bouquet



Beyond Lie algebras

But these are not 2-cocycles, so:

- $\mathfrak{string}_{\mathrm{het}}$, $\mathfrak{string}_{\mathrm{IIA}}$, $\mathfrak{string}_{\mathrm{IIB}}$ and $\mathfrak{m2brane}$ are not Lie algebras!
- Instead, they are L_∞-algebras.

L_∞-algebras

An L_∞-algebra $\mathfrak{g}$ is like a Lie algebra, defined on a chain complex:

$$\mathfrak{g}_0 \xleftarrow{\partial} \mathfrak{g}_1 \xleftarrow{\partial} \cdots \xleftarrow{\partial} \mathfrak{g}_n \xleftarrow{\partial} \cdots$$

But the Jacobi identity *does not hold*:

$$[[X, Y], Z] \pm [[Y, Z], X] \pm [[Z, X], Y] \neq 0.$$

Instead, it holds up to boundary terms:

$$[[X, Y], Z] \pm [[Y, Z], X] \pm [[Z, X], Y] = \partial[X, Y, Z].$$

Where this new, trilinear bracket:

$$[-, -, -] \colon \mathfrak{g}^{3\otimes} \to \mathfrak{g},$$

in turn satisfies an identity like Jacobi *up to boundary terms* controlled by a 4-linear bracket ...

L_∞-algebras: examples

A Lie algebra is an L_∞-algebra concentrated in degree 0:

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Given any $(p+2)$-cocycle $\omega\colon \Lambda^{p+2}\mathfrak{g} \to \mathbb{R}$, we can construct an L_∞-algebra $\mathfrak{g}_\omega$ as follows:

$$\mathfrak{g} \longleftarrow 0 \longleftarrow \cdots \longleftarrow \mathbb{R}$$

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where

- $\mathfrak{g}$ is in degree 0, $\mathbb{R}$ is in degree p.
- $[-,-]$ is the Lie bracket.
- The $(p+2)$-linear bracket, $[-,\cdots,-] = \omega$, is the cocycle.
- All other brackets are 0.

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All of this generalizes to superalgebras in a straightforward way. This is how we construct $\mathfrak{string}_{\mathrm{het}}$, $\mathfrak{string}_{\mathrm{IIA}}$, $\mathfrak{string}_{\mathrm{IIB}}$ and $\mathfrak{m2brane}$ from $\mathbb{R}^{D-1,1|\mathbf{S}}$.

Dp-branes and the M5-brane

Thanks to $\mathfrak{string}_{\mathrm{het}}$, $\mathfrak{string}_{\mathrm{IIA}}$, $\mathfrak{string}_{\mathrm{IIB}}$ and $\mathfrak{m2brane}$, we can find the branes missing from the brane scan.

Fact

The left-invariant forms on $\mathfrak{g}_\omega$ are generated by the left-invariant forms on $\mathfrak{g}$ with one additional $(p+1)$-form b such that $db = \omega$.

For example:

- On $\mathfrak{string}_{\mathrm{IIA}} = \mathbb{R}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}_{\mu_{\mathrm{IIA}}}$, the left-invariant forms are
- from $\mathbb{R}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}$:

$$e^\nu = dx^\nu - \overline{\theta}\Gamma^\nu d\theta, \quad d\theta^\alpha$$

- and a 2-form F such that

$$dF = \mu_{\mathrm{IIA}}.$$

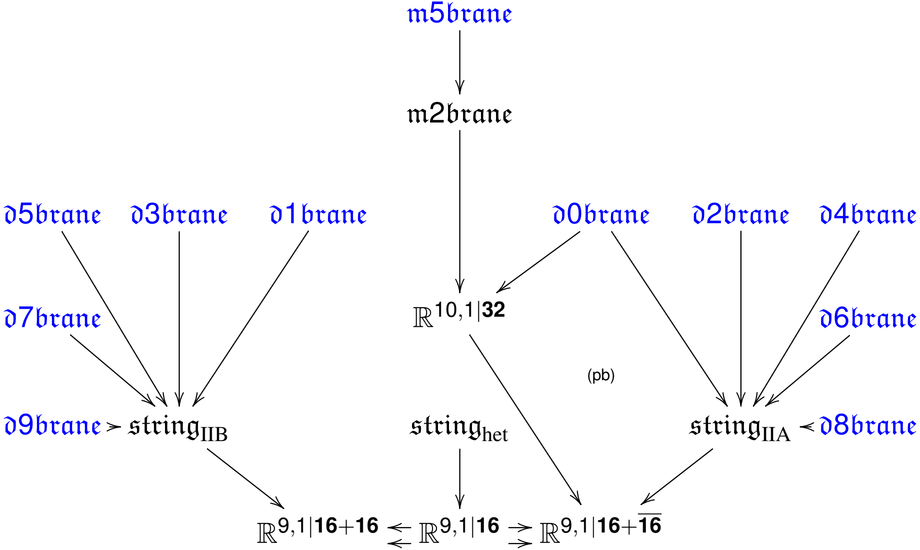
Dp-branes and the M5-brane

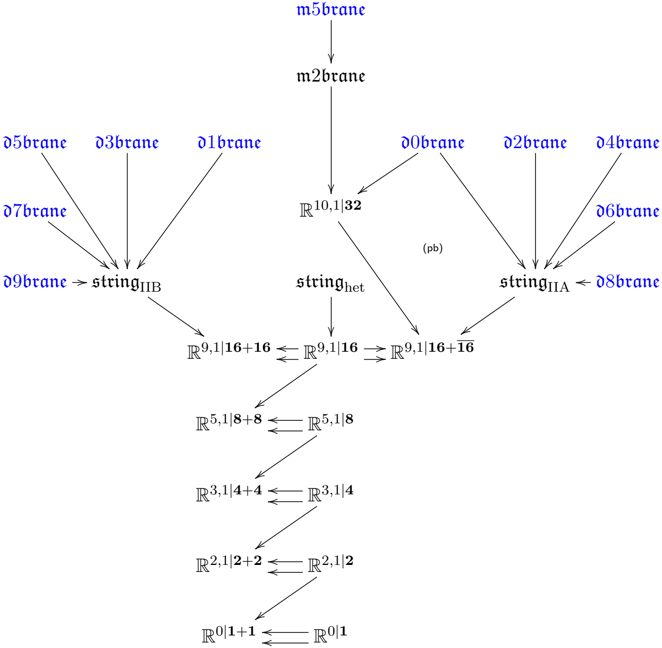
Thanks to F, there are new cocycles on $\mathfrak{string}_{\mathrm{IIA}}$.

$$\mu_{\mathrm{D}p} = \sum_{k=0}^{(p+2)/2} c_k^p e^{\nu_1} \wedge \cdots \wedge e^{\nu_{p-2k}} \wedge d\bar{\theta} \wedge \Gamma_{\nu_1 \cdots \nu_{p-2k}} d\theta \wedge F \wedge \cdots \wedge F.$$

- c_k^p are some coefficients chosen to make $d\mu_{\mathrm{D}p} = 0$.
- Applying Schreiber's machine to this cocycle gives the Dp-brane action.
- Similarly, we can find a cocycle for the M5-brane on $\mathfrak{m2brane}$.

The brane bouquet





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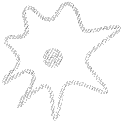

Figure : $\mathbb{R}^{0|1}$

OBRIGADO

References I

The use of L_∞-algebras in physics originates with the work of D'Auria and Fré, who call them 'free differential algebras'.

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The connection between Lie algebra cohomology and Green–Schwarz p-brane actions is due to de Azcárraga and Townsend:

- J. A. de Azcárraga and P. K. Townsend, Superspace geometry and the classification of supersymmetric extended objects, *Phys. Rev. Lett.* **62** (1989), pp. 2579–2582.

The discovery that the WZW terms for Dp-branes and the M5-branes live on the 'extended superspacetimes' $\mathfrak{string}_{\mathrm{IIA}}$, $\mathfrak{string}_{\mathrm{IIB}}$ and $\mathfrak{m2brane}$ appears in two articles. The case of the type IIA Dp-branes and the M5-brane is in:

- C. Chryssomalakos, J. de Azcárraga, J. Izquierdo, and C. Pérez Bueno, The geometry of branes and extended superspaces, *Nucl. Phys.* **B 567** (2000), pp. 293-330, arXiv:hep-th/9904137.

while the type IIB Dp-branes are in section 2 of:

- M. Sakaguchi, IIB-branes and new spacetime superalgebras, *JHEP* 04 (2000), pp. 019, arXiv:hep-th/9909143.

References III

Later, Fiorenza, Sati and Schreiber placed this into the context of the homotopy theory of L_∞-algebras, discovering the brane bouquet:

- D. Fiorenza, H. Sati, U. Schreiber, Super Lie n-algebra extensions, higher WZW models, and super p-branes with tensor multiplet fields, *Intern. J. Geom. Meth. Mod. Phys.* **12** (2015), 1550018 (35 pages). arXiv:1308.5264.

Finally, Schreiber and I derive the brane bouquet from the superpoint.

- J. Huerta and U. Schreiber, M-theory from the superpoint. In preparation.