Fractional Instantons and Bions

Resurgence in Gauge and String Theories 2016
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Muneto Nitta
(Keio U.)

Collaborators:
Tatsu Misumi (Akita/Keio), Norisuke Sakai (Keio), Toshi Fujimori (Keio), Syo Kamata (Keio)

References

[1] Misumi, MN & Sakai, JHEP 1406 (2014)164 [arXiv:1404.7225]
[2] Misumi, MN & Sakai, PTEP (2015) 033B02 [arXiv:1409.3444]
[3] MN, JHEP 1503 (2015) 108 [arXiv:1412.7681]
[4] MN, JHEP 1508 (2015) 063 [arXiv:1503.06336]
[5] Misumi, MN & Sakai, in preparation

See also

[6] Misumi, MN & Sakai, JHEP 1509 (2015)157[arXiv:1507.00408]
Sakai's talk: resurgence in SG QM
[7] Misumi, MN & Sakai, JHEP 1605 (2016)057[arXiv:1604.00839]
Misumi's talk: non-BPS exact solutions in CP^N
[8] Fujimori, Kamata, Misumi, MN & Sakai, arXiv:1607.04205
(cancelled…) Fujimori's talk: complex bions in CP^N

Field Theory on Compactified Space $\mathbf{R}^{D-1} \times S^1$ with a Twisted Boundary Condition (TBC) along S^1

* Resurgence of quantum field theory
  (long history ….., Dunne & Unsal '12--)
* Gauge-Higgs unification (Hosotani mechanism)
* Large extra dimension

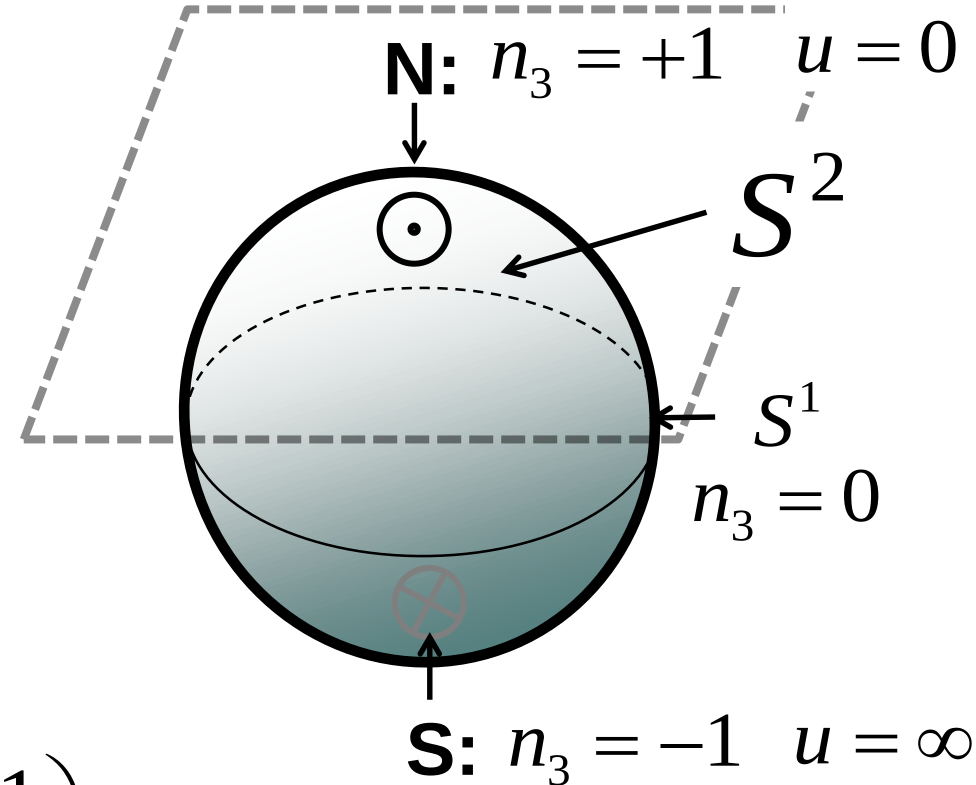
Topological solitons, instantons have *fractional topological charge*

Bions = composite of fractional instantons with zero instanton charge

O(3) model on $\mathbf{R}^2$

$$L = \frac{1}{g^2} \partial_\mu \mathbf{n} \cdot \partial^\mu \mathbf{n} \quad \mathbf{n} = (n_1, n_2, n_3) \quad \mathbf{n}^2 = 1$$

$$u \equiv \frac{n_1 + i n_2}{1 + n_3}$$ **Stereo graphic**



C*P*^1 model

$$L = \frac{1}{g^2} \frac{\partial_\mu u^* \partial^\mu u}{(1 + |u|^2)^2}$$

$$\mathbf{n} = \frac{1}{1 + |u|^2} \left(1, u^*\right) \boldsymbol{\sigma} \begin{pmatrix} 1 \\ u \end{pmatrix}$$

Sigma model instanton (lump) $\pi_2(S^2) \simeq \mathbf{Z}$ for $\mathbf{R}^2$

(skyrmion in cond-mat)

$$\mathbf{R}^2 \longrightarrow S^2$$

$$Q = \frac{1}{2\pi}\int d^2x\, i\varepsilon^{ij}\, \frac{\partial_i u \partial_j u^*}{(1+|u|^2)}$$

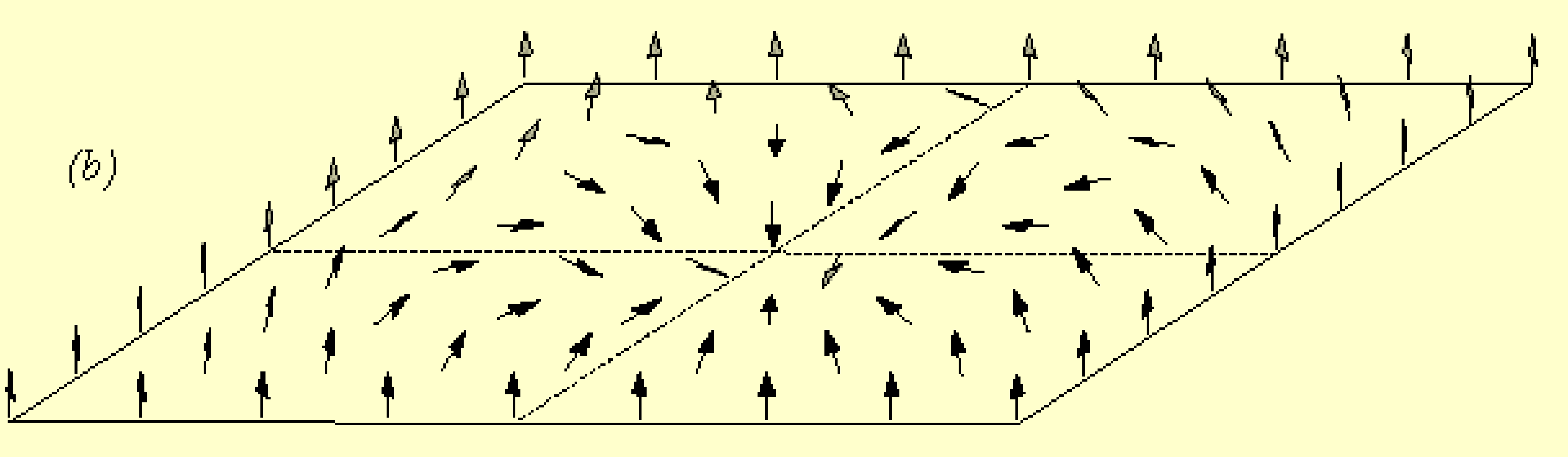
$$\sim \frac{1}{2\pi}\int d^2x\, \varepsilon^{ij}\, \mathbf{n}\cdot(\partial_i \mathbf{n} \times \partial_j \mathbf{n})$$

Belavin & Polyakov

$$u \equiv \frac{n_1 + i n_2}{1 - n_3} = \lambda(z - z_0)$$

$\lambda \in \mathbf{C}^*$ **Size, phase moduli**

$z_0 \in \mathbf{C}$ **Position moduli**



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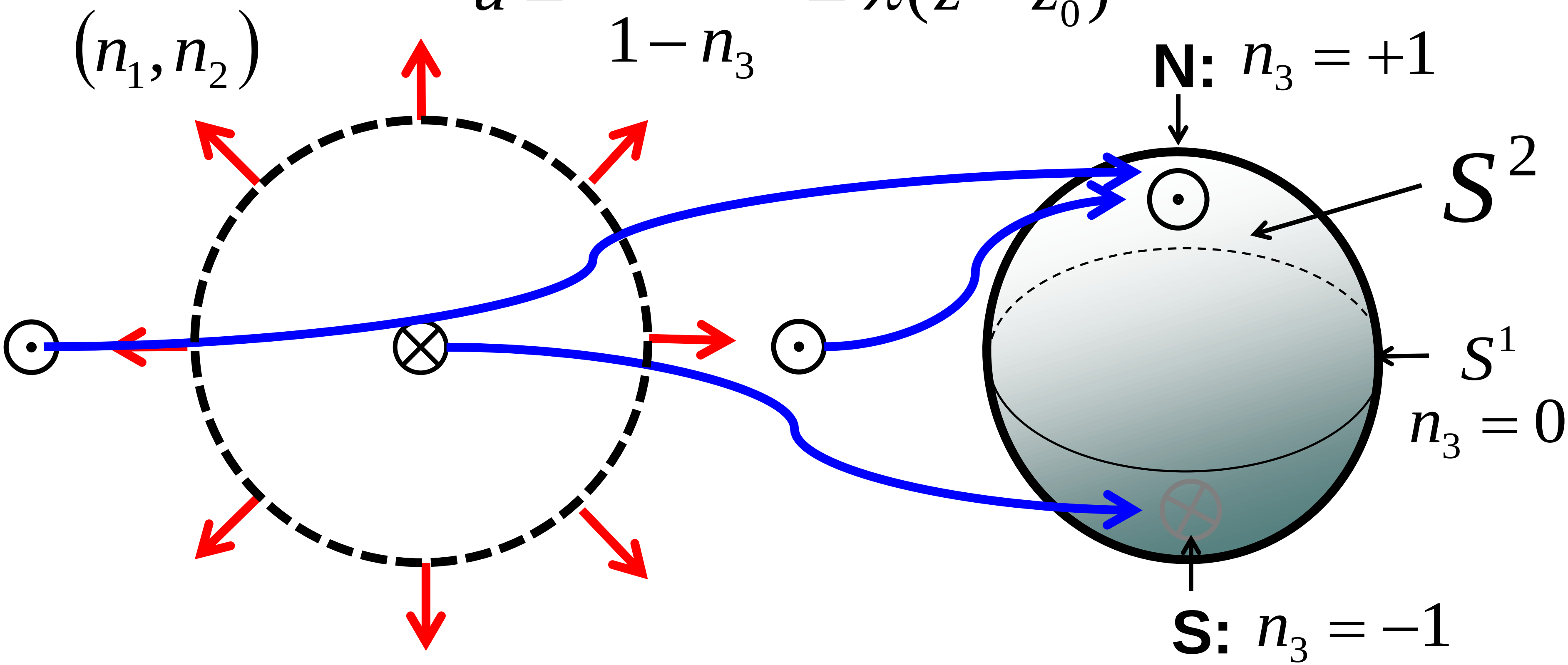
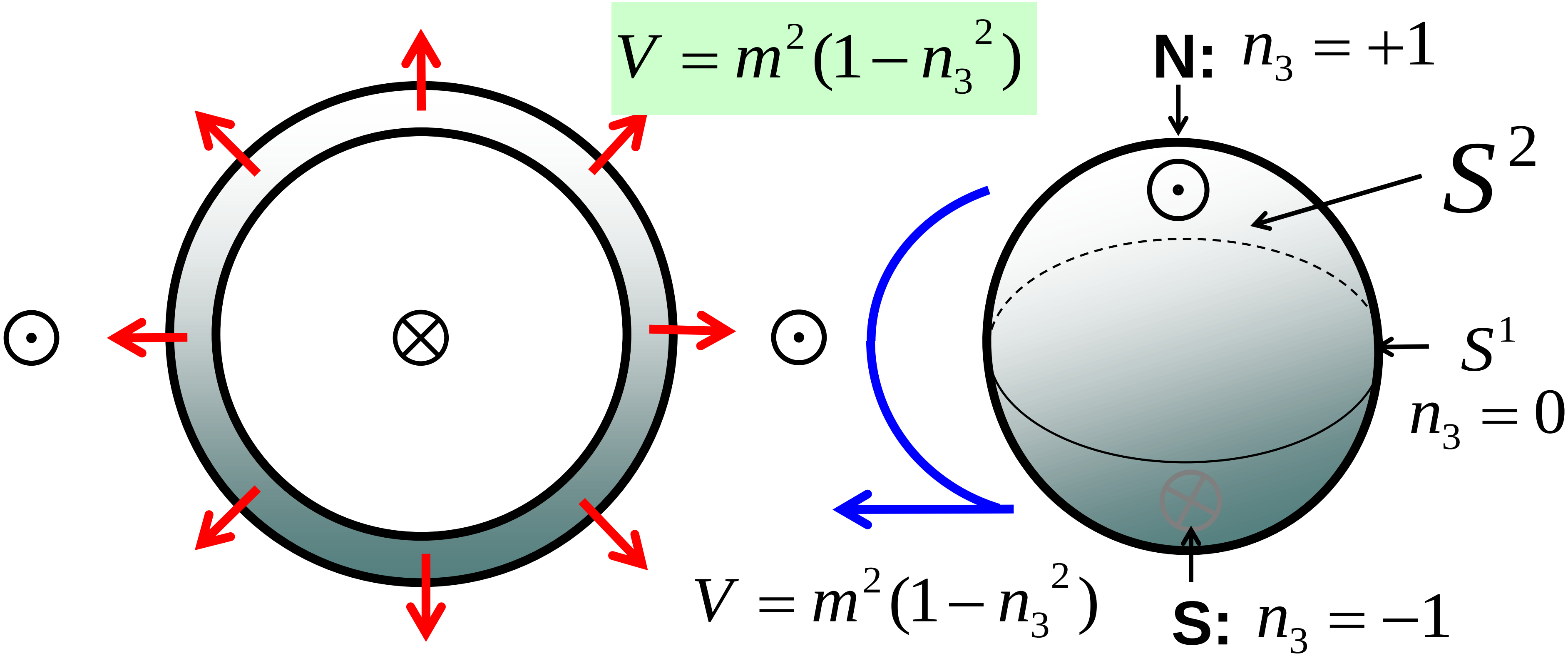


Sigma model instanton (lump) $\pi_2(S^2) \simeq \mathbf{Z}$ for $\mathbf{R}^2$
(skyrmion in cond-mat)

$$\mathbf{R}^2 \longrightarrow S^2$$

In the presence of a potential **Magnet with 1 easy axis**



A domain wall ring

Sigma model instanton
(skyrmion in cond-mat)

Eto, Isozumi, MN, Ohashi & Sakai ('05), PRD72 (2005) 025011 [hep-th/0412048]

$$\mathbf{R}^1 \times S^1 \longrightarrow S^2$$

Twisted boundary condition (tbc)

$$(n_1, n_2, n_3)(x^1, x^2 + R) = (-n_1, -n_2, n_3)(x^1, x^2)$$

$$u(x^1, x^2 + R) = -u(x^1, x^2)$$

R

Sigma model instanton
(skyrmion in cond-mat)

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$$u(x^1, x^2 + R) = -u(x^1, x^2)$$

R

Vacua are invariant under the Z_2 action

$$(n_1, n_2, n_3) \rightarrow (-n_1, -n_2, n_3)$$

$$n^1 = n^2 = 0 \leftrightarrow \quad n^3 = \pm 1$$

$$\pi_0 = \mathbf{Z}_2$$ **Domain wall**

Sigma model instanton
(skyrmion in cond-mat)

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PRD72 (2005) 025011 [hep-th/0412048]

$$\mathbf{R}^1 \times S^1 \longrightarrow S^2$$

Twisted boundary condition (tbc)

$$(n_1, n_2, n_3)(x^1, x^2 + R) = (-n_1, -n_2, n_3)(x^1, x^2)$$

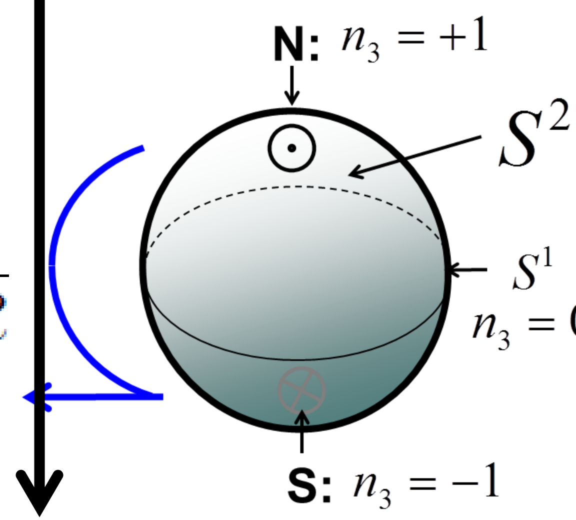
Scherk-Schwarz dimensional reduction

$$\begin{pmatrix} n_1(x^1, x^2) \\ n_2(x^1, x^2) \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{R}x^2 & -\sin\frac{\pi}{R}x^2 \\ \sin\frac{\pi}{R}x^2 & \cos\frac{\pi}{R}x^2 \end{pmatrix} \begin{pmatrix} \hat{n}_1(x^1) \\ \hat{n}_2(x^2) \end{pmatrix}$$

Effective potential (twisted mass)

$$V = \int_0^R dx^2 \left[(\partial_2 n_1)^2 + (\partial_2 n_2)^2 \right] \quad m^2 \equiv \frac{\pi^2}{4R}$$

$$= m^2(\hat{n}_1^2 + \hat{n}_2^2) = m^2(1 - \hat{n}_3^2)$$



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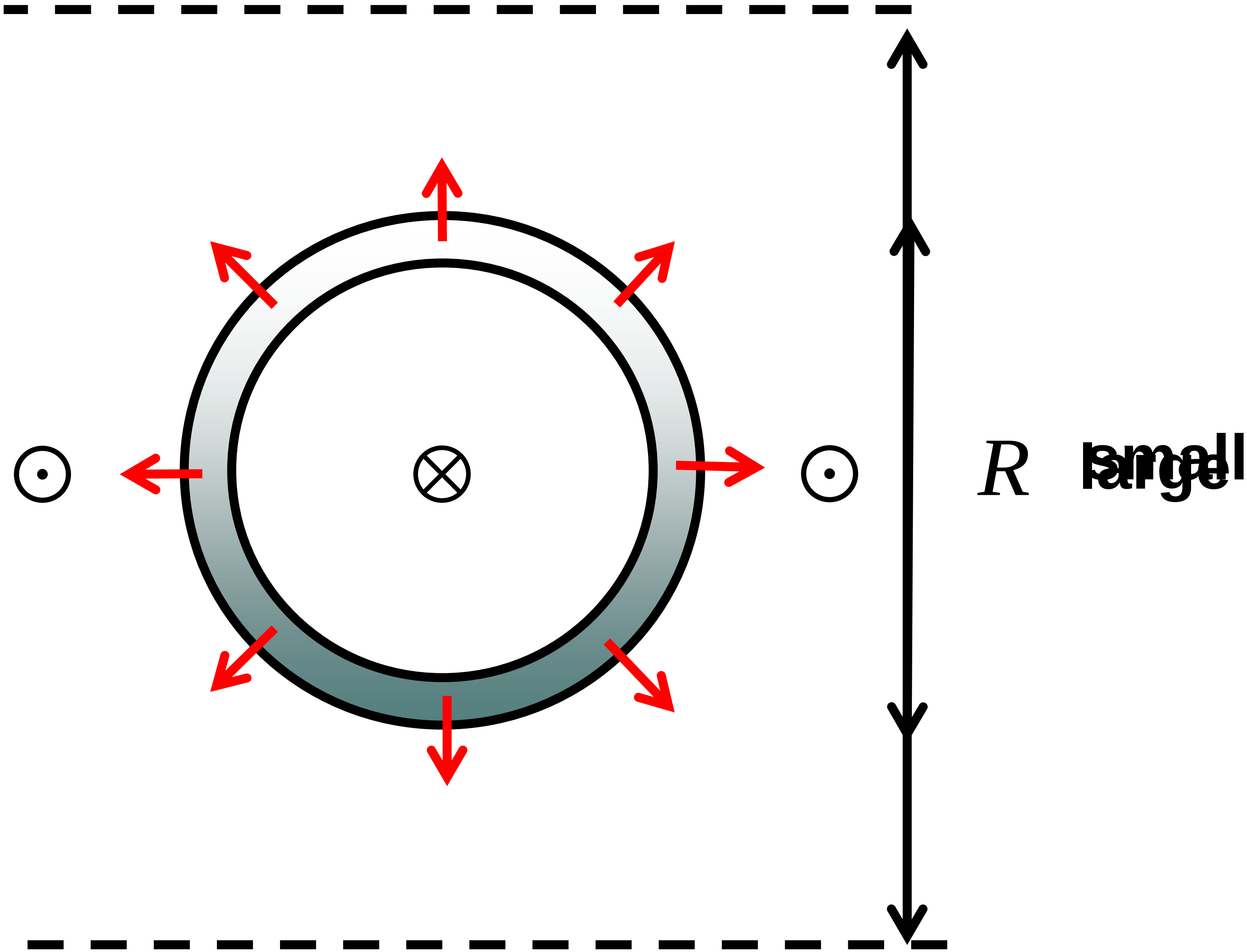

Sigma model instanton
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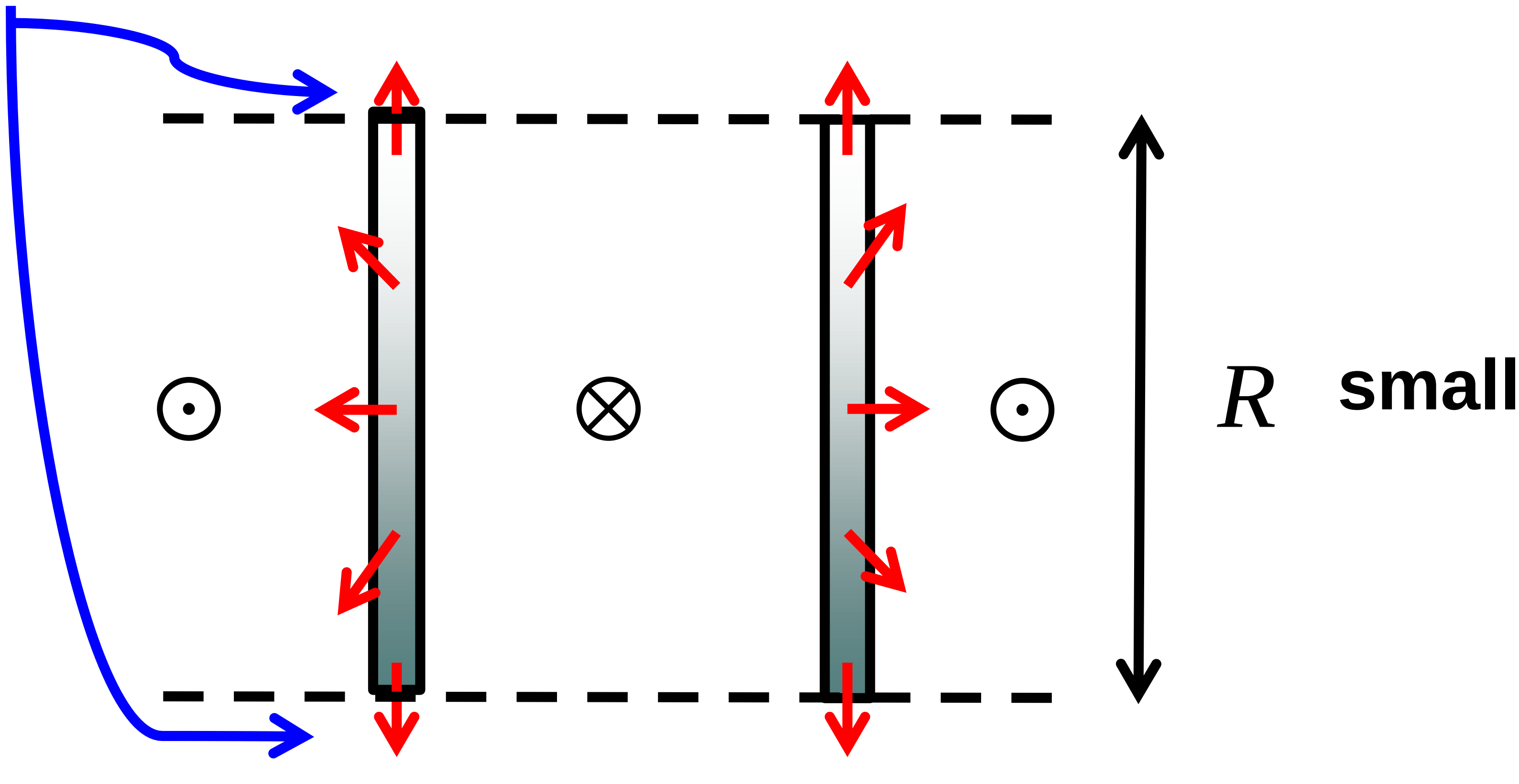
$$\mathbf{R}^1 \times S^1 \longrightarrow S^2$$

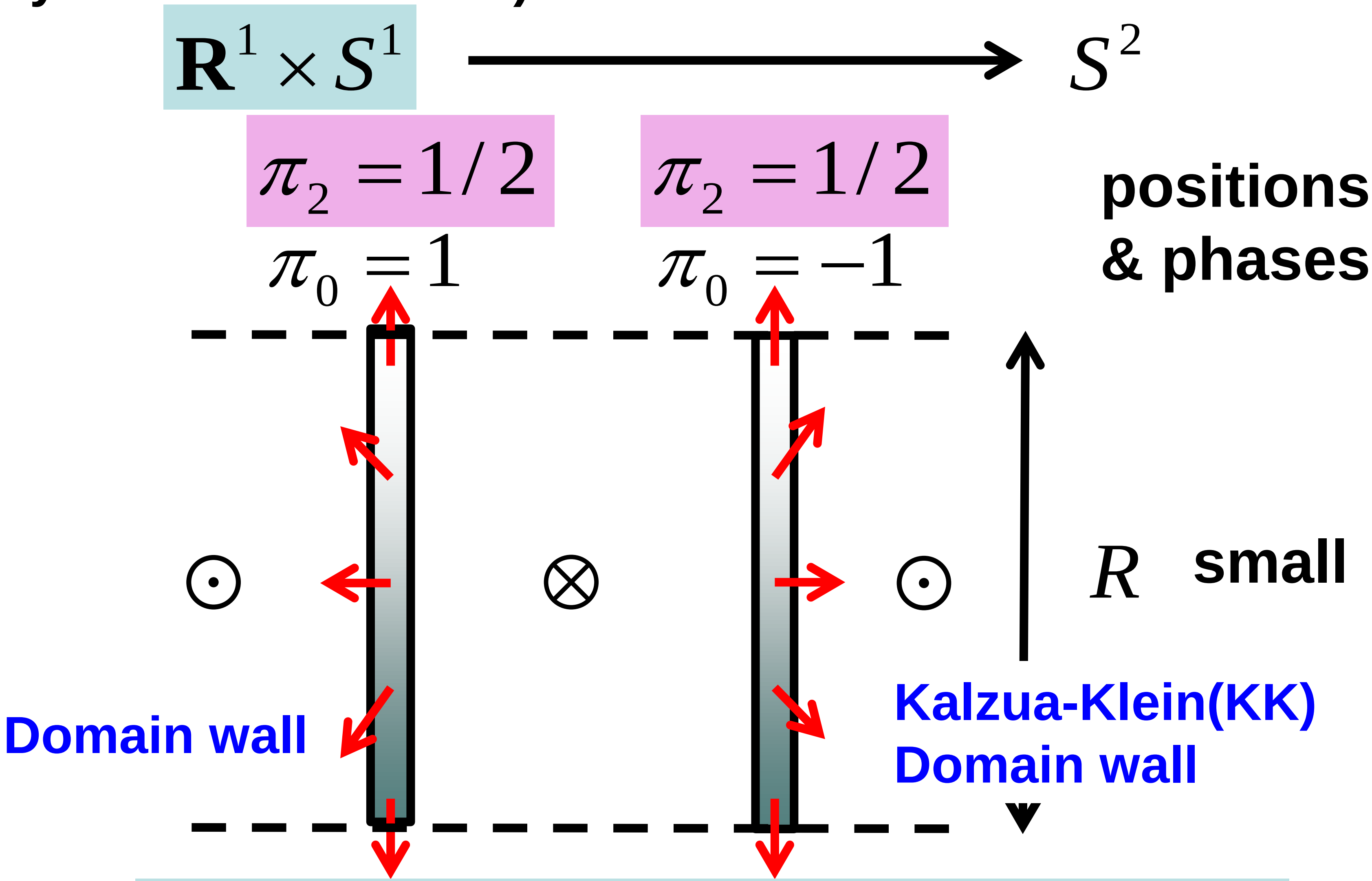
Identified because of TBC



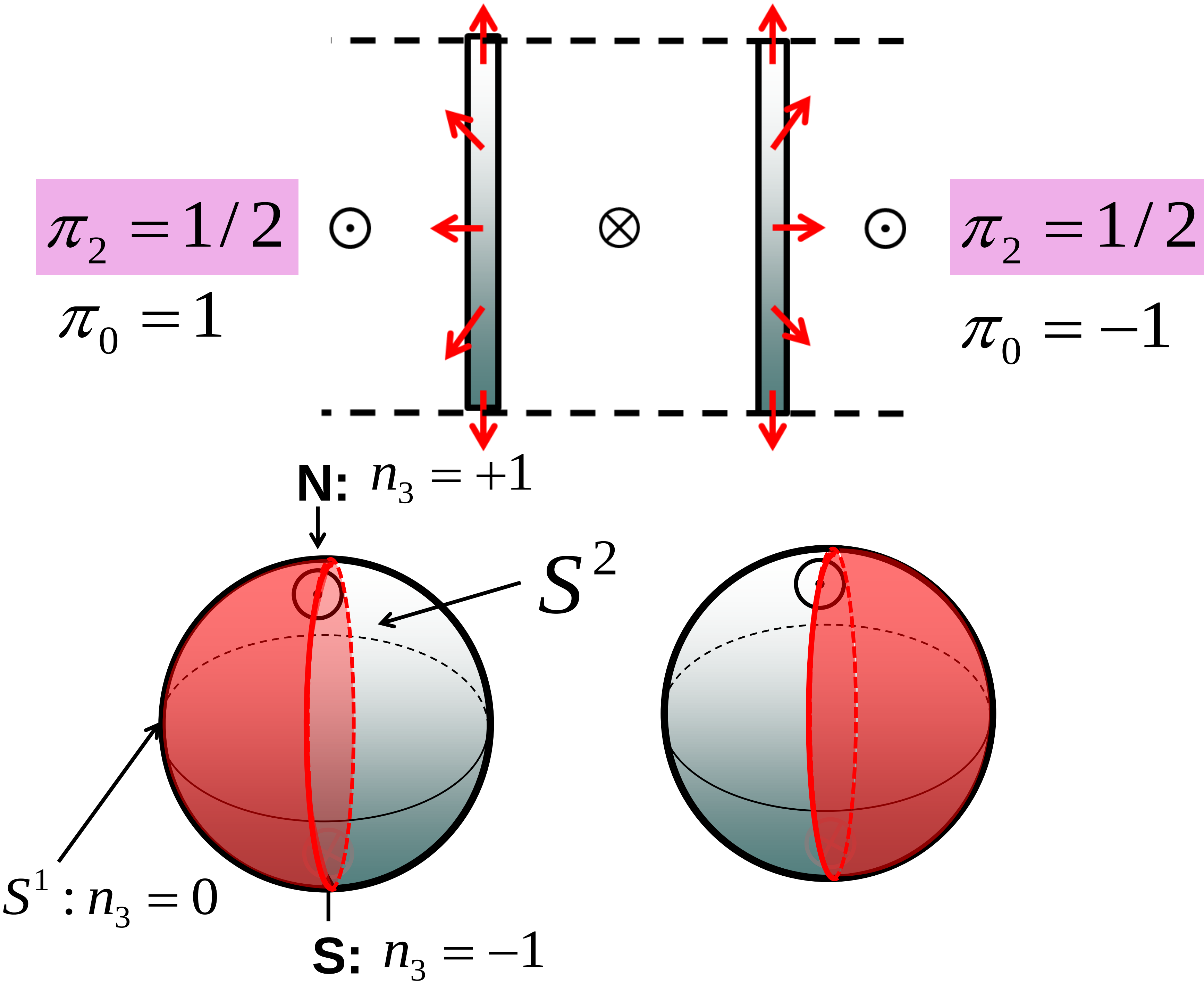
Reconnection occurs !!

Sigma model instanton
(skyrmion in cond-mat)

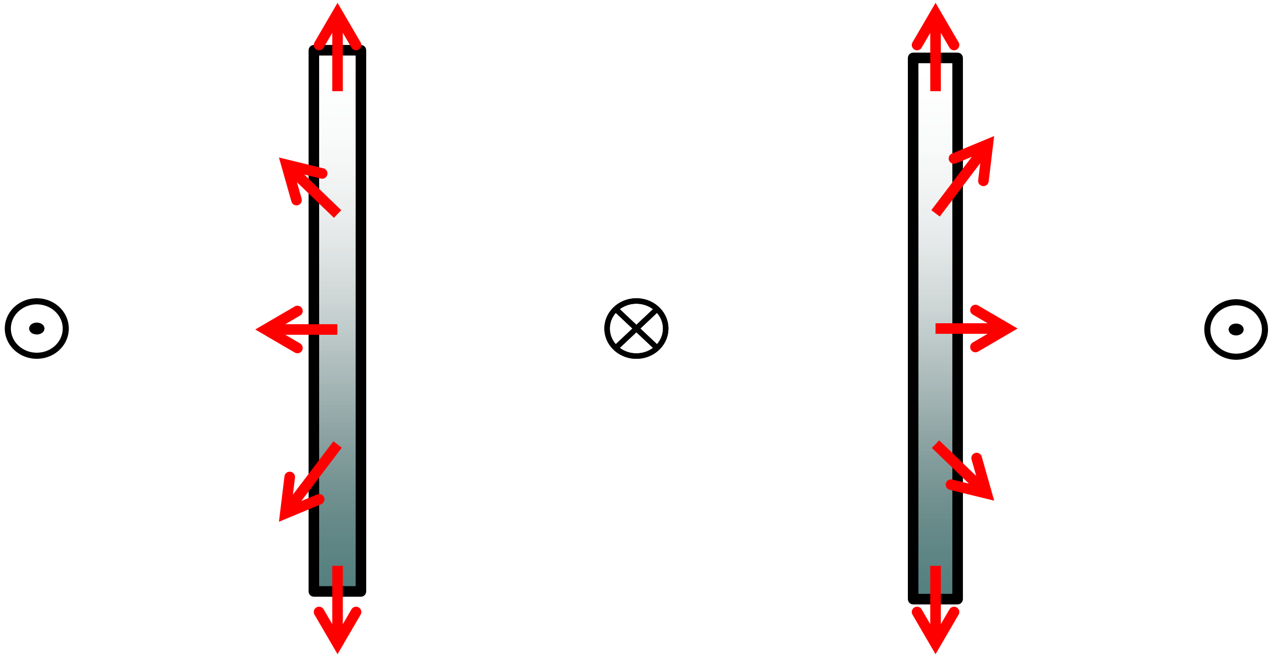
Eto, Isozumi, MN, Ohashi & Sakai ('05), PRD72 (2005) 025011 [hep-th/0412048]



Fractional instantons
= (anti-)domain walls with half SG kink



$\pi_2 = 1/2$
$\pi_0 = 1$
$\pi_2 = 1/2$
$\pi_0 = -1$
N: $n_3 = +1$
S^2
$S^1 : n_3 = 0$
S: $n_3 = -1$



How instanton moduli are translated to wall moduli

Size, phase moduli

$$\lambda = |\lambda| e^{i\alpha} \in \mathbf{C}^*$$

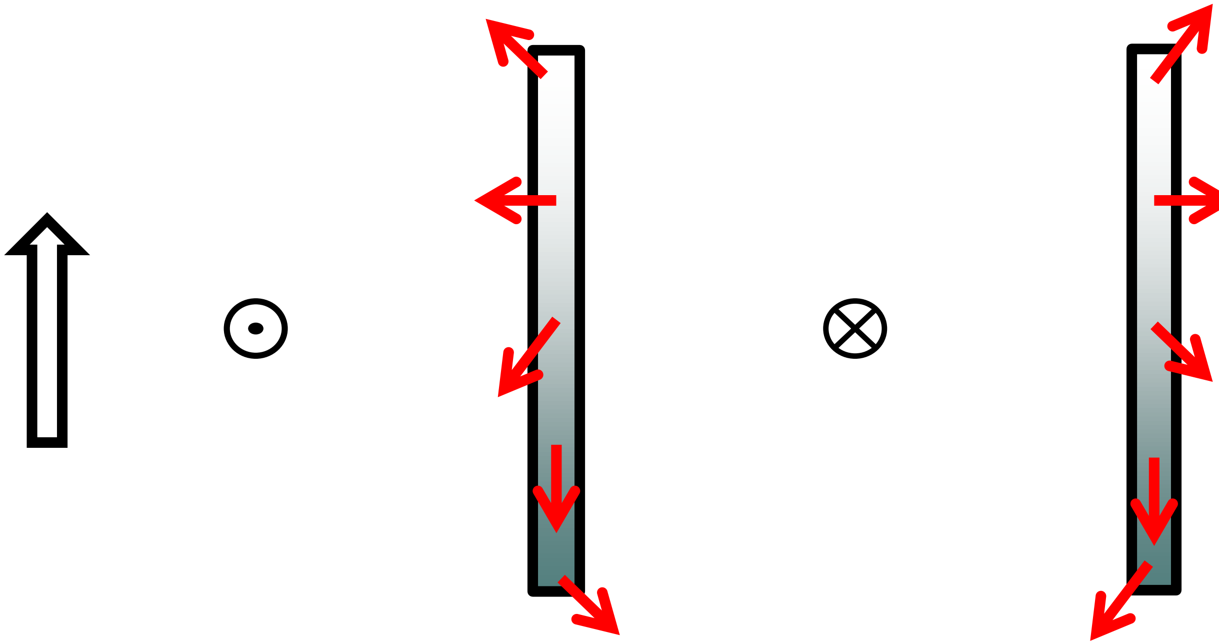
Position moduli

$$z_0 = x_0 + iy_0 \in \mathbf{C}$$

⇨

$\|\lambda\|$	**relative position**
$e^{i\alpha}$	**overall phase**
x_0	**overall position**
y_0	**relative phase**

space-time modulus tuned to internal modulus



How instanton moduli are translated to wall moduli

Size, phase moduli

$$\lambda = |\lambda| e^{i\alpha} \in \mathbf{C}^*$$

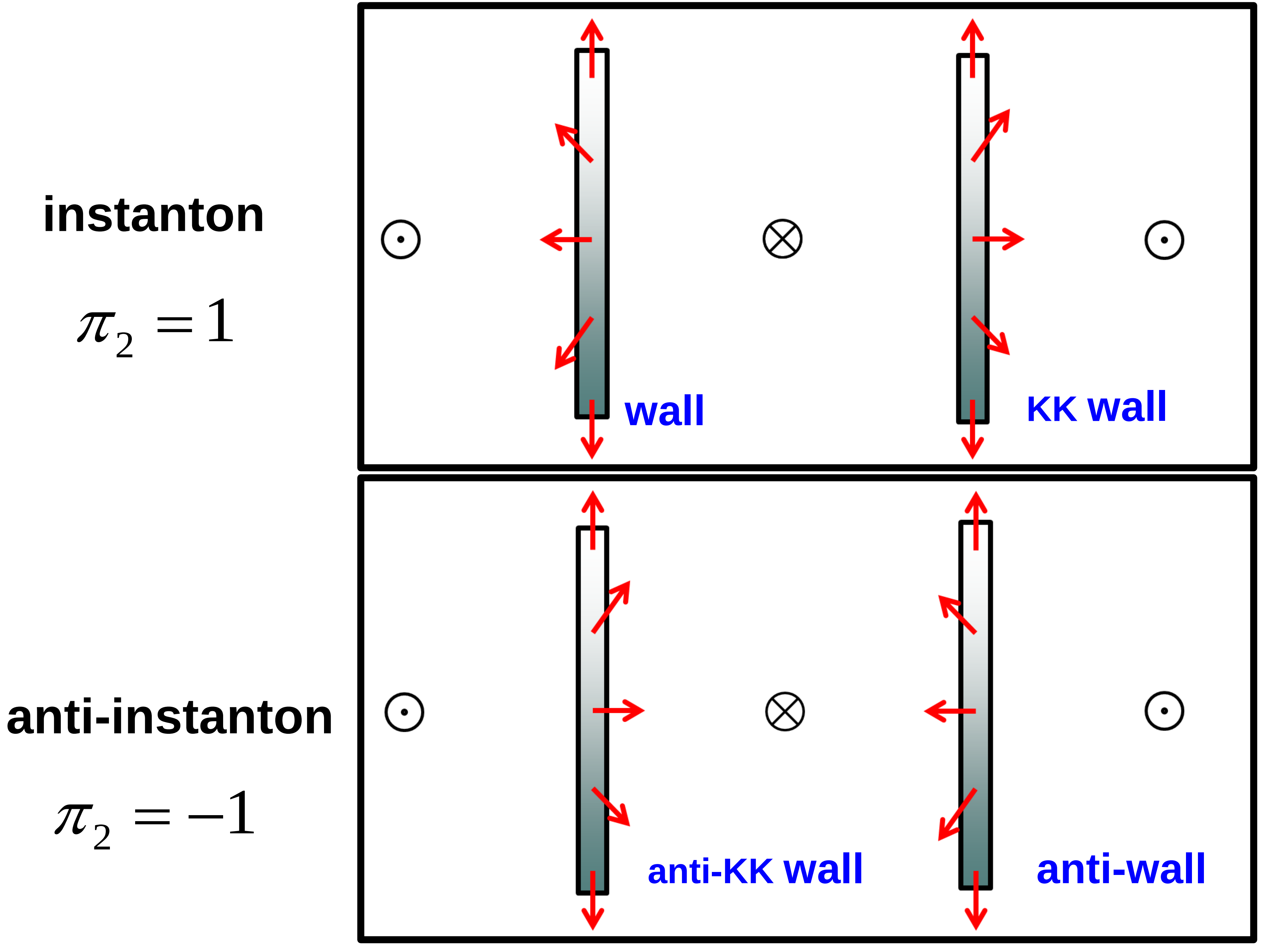
Position moduli

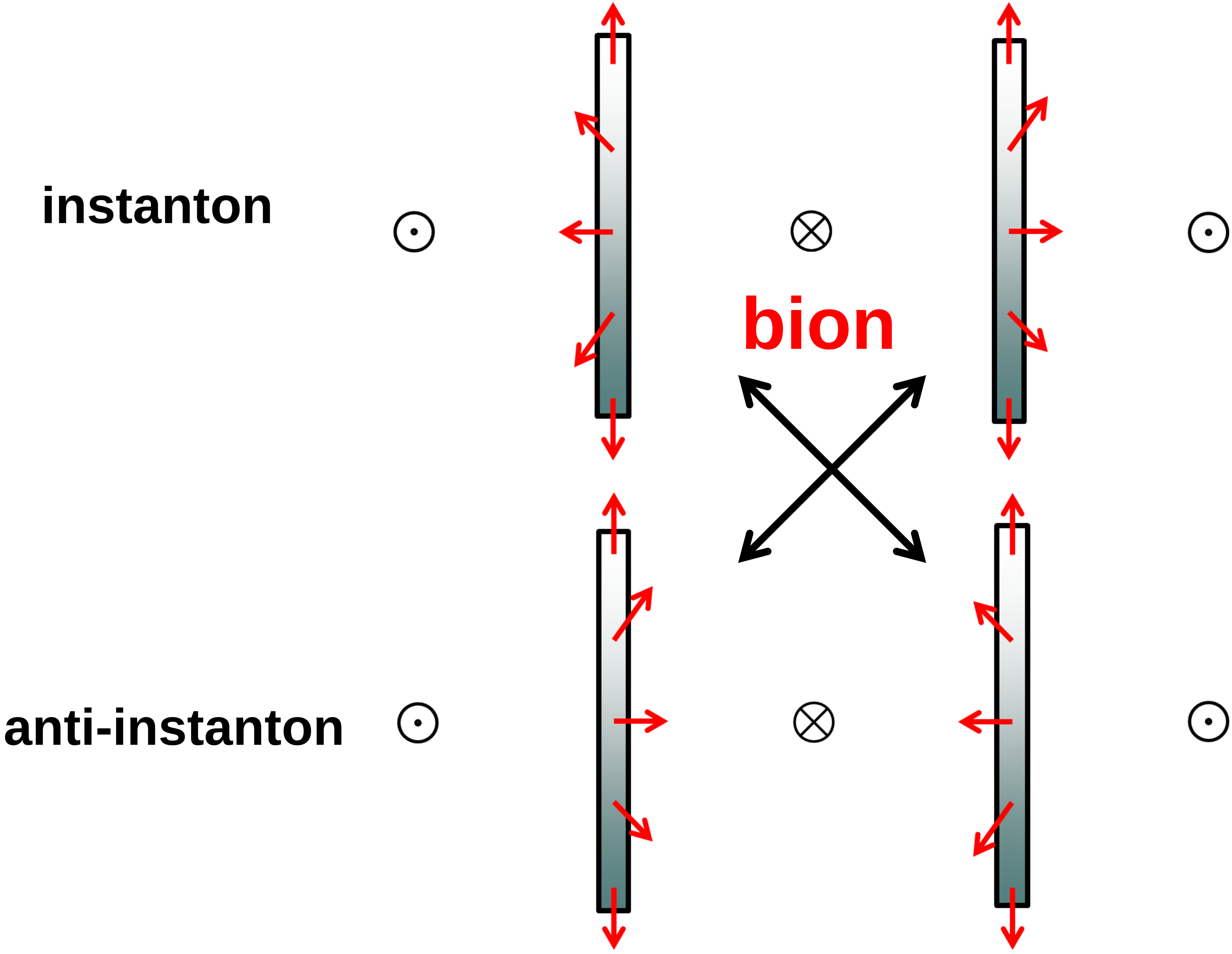
$$z_0 = x_0 + iy_0 \in \mathbf{C}$$

$\Rightarrow$

$\lvert\lambda\rvert$	**relative position**
$e^{i\alpha}$	**overall phase**
x_0	**overall position**
y_0	**relative phase**

space-time modulus tuned to internal modulus

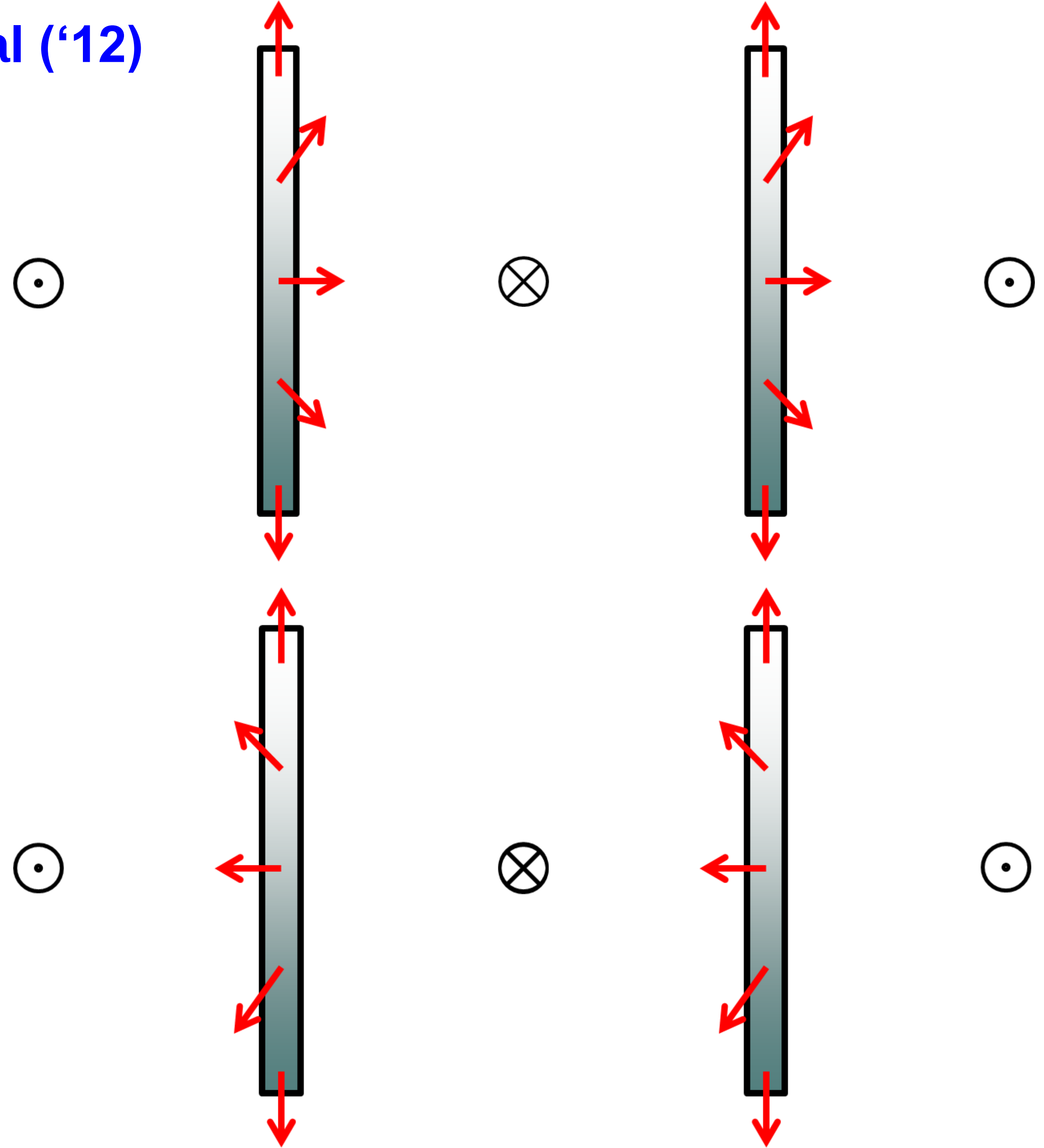

instanton
$\pi_2 = 1$
wall
KK wall
anti-instanton
$\pi_2 = -1$
anti-KK wall
anti-wall


instanton
bion
anti-instanton

Dunne & Unsal ('12)

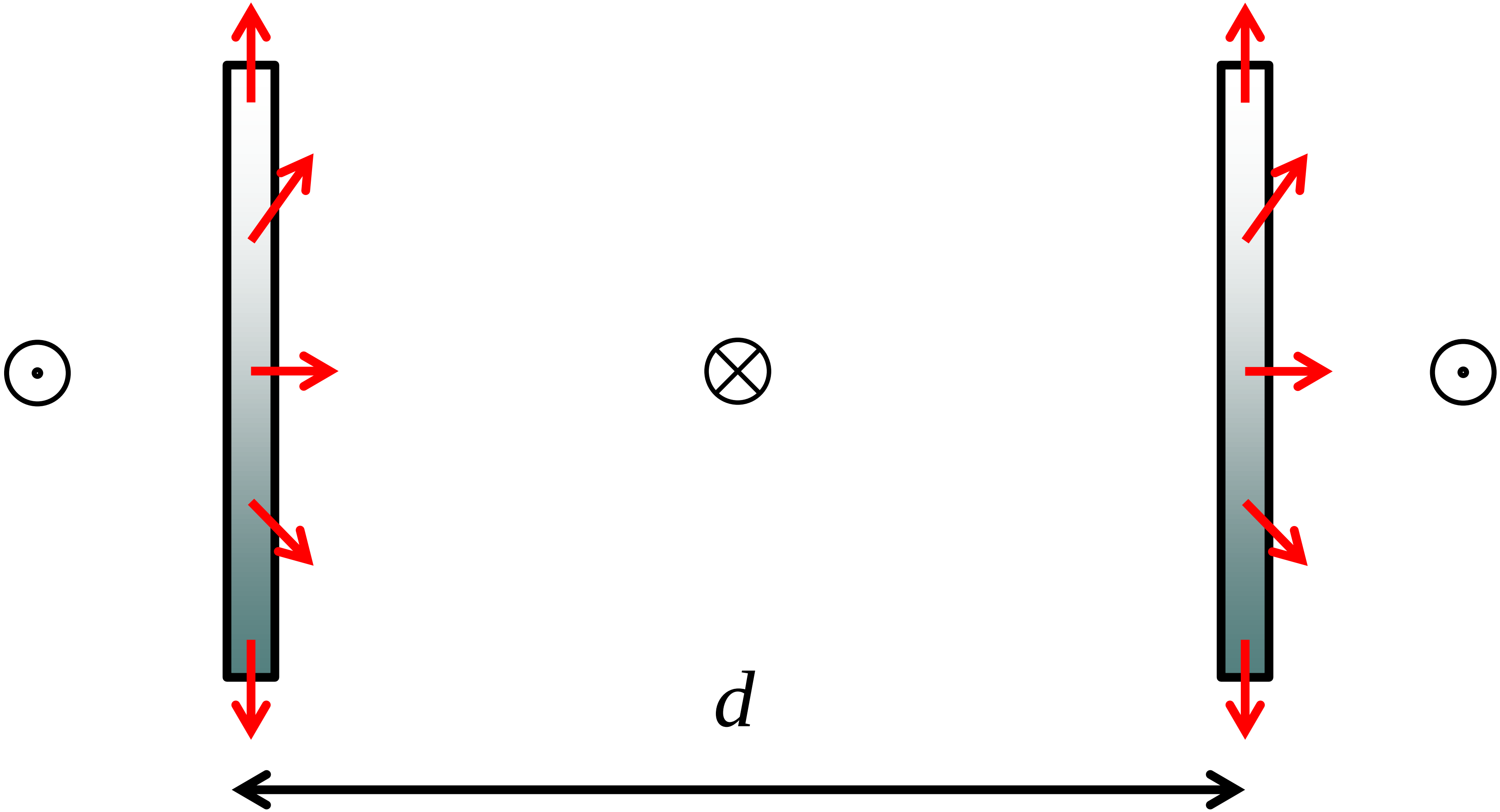
bion

$\pi_2 = 0$



bion

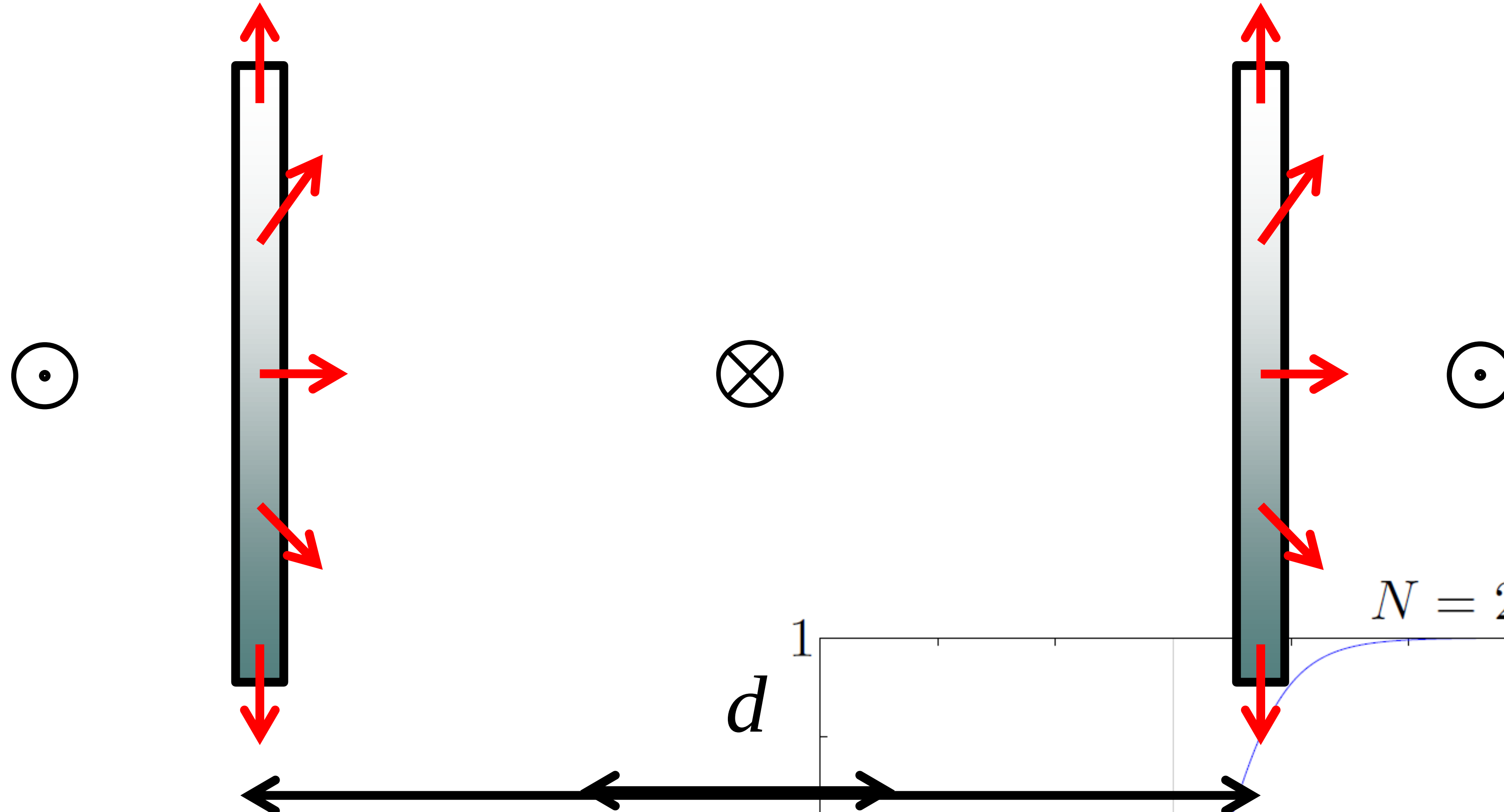
$\pi_2 = 0$

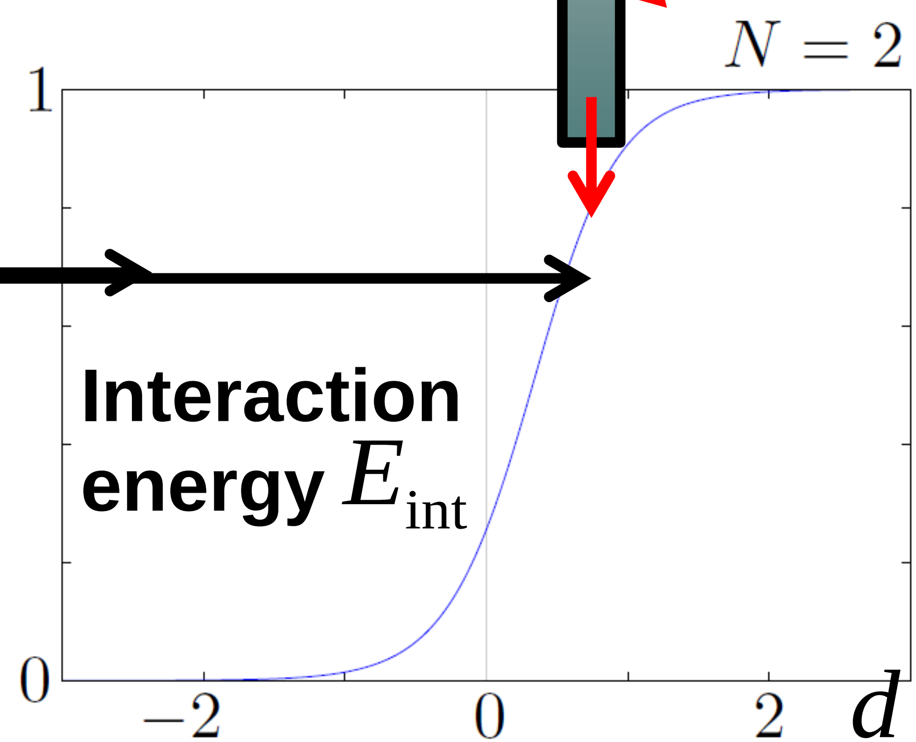


Interaction energy $E_{\text{int}} \sim -\frac{1}{g^2}\exp(-\xi d)$ **asymptotically**

Dunne & Unsal ('12)

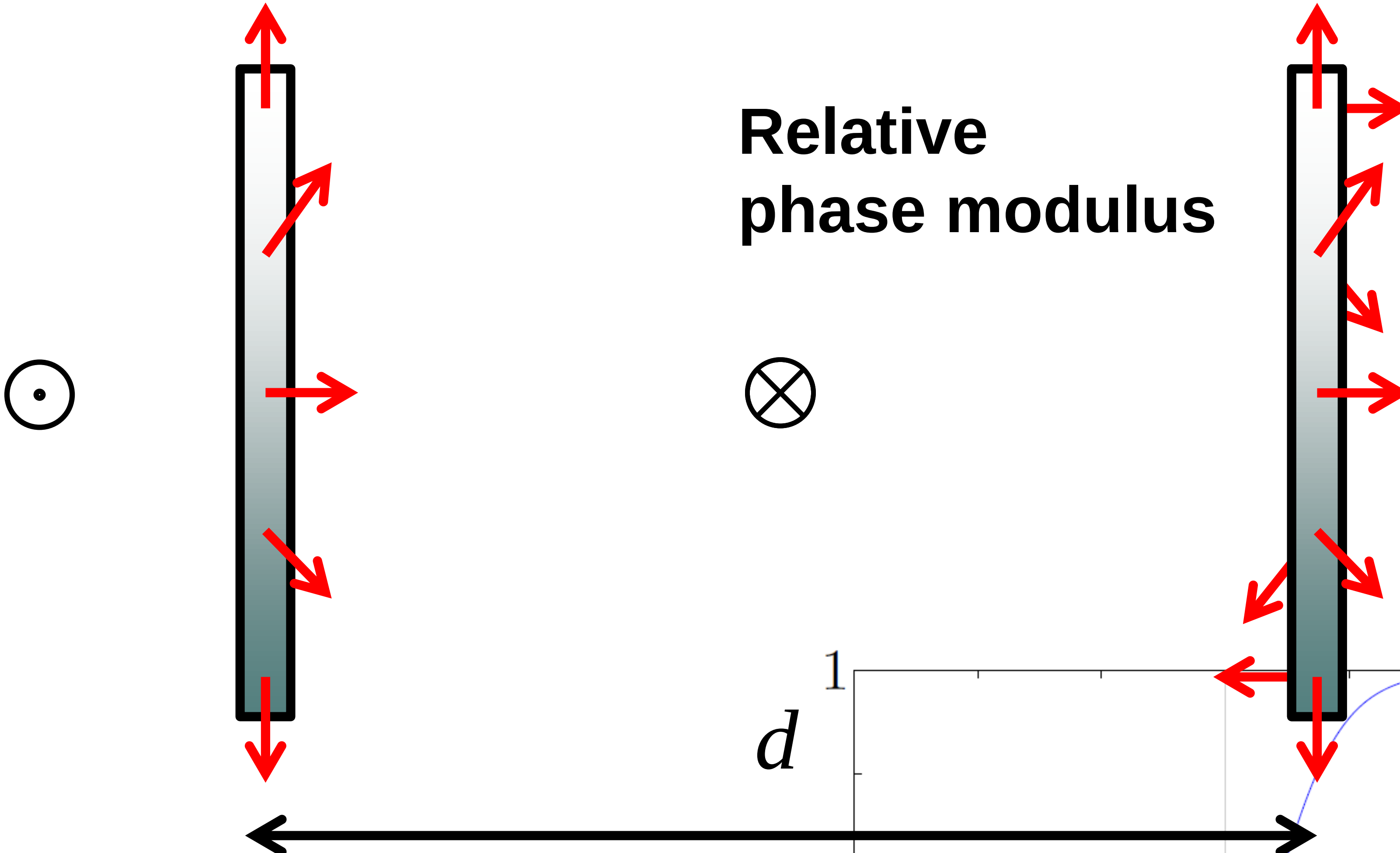
Resurgence through Bogomol'nyi Zin-Justin prescription

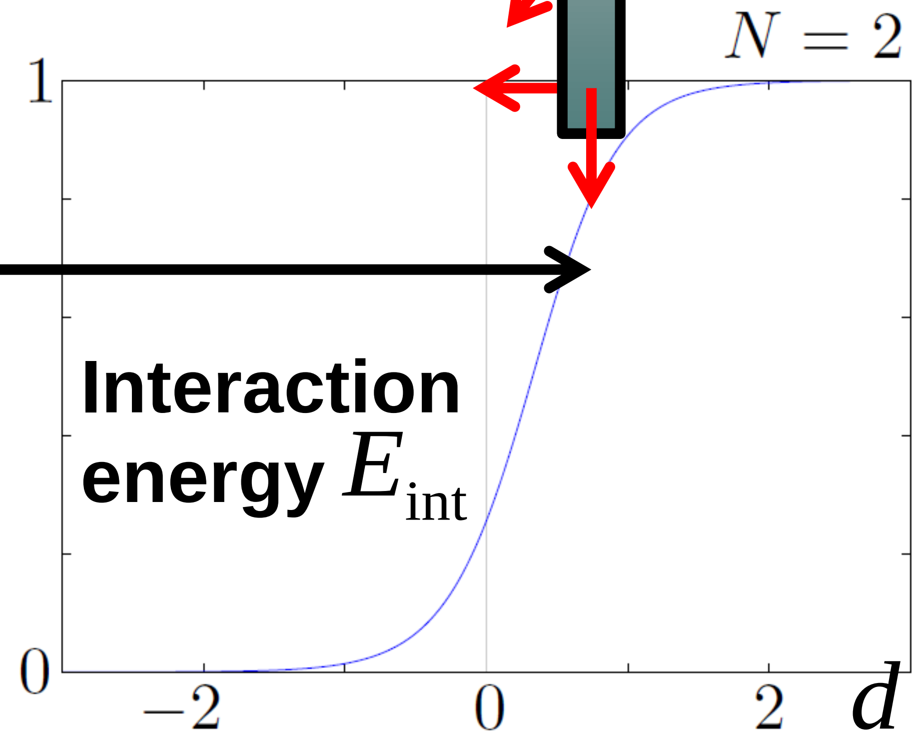




Explicit configuration of a bion at *arbitrary* distance

Misumi, MN & Sakai ('14)
JHEP 1406 (2014)164
[arXiv:1404.7225]





Explicit configuration of a bion at *arbitrary* distance

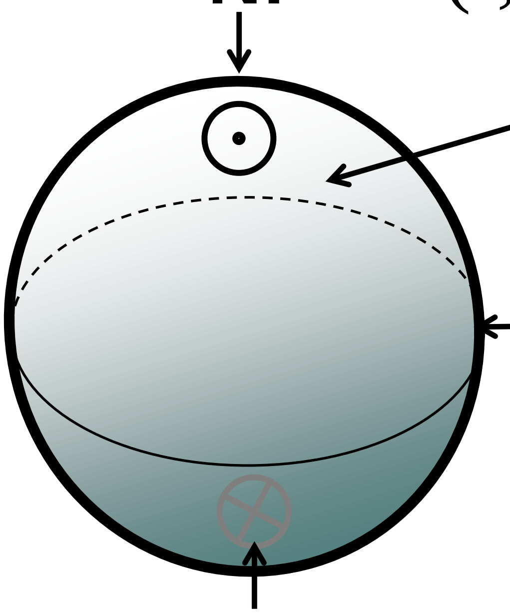
Misumi, MN & Sakai ('14)
JHEP 1406 (2014)164
[arXiv:1404.7225]

Homogenous coordinate of CP^1

$$\omega = (\omega_1, \omega_2) \sim \lambda(\omega_1, \omega_2) \quad \lambda \in \mathbf{C}^*$$

$$\mathbf{n} = \frac{\omega^\dagger(x)\vec{\sigma}\omega(x)}{\omega^\dagger(x)\omega(x)}$$

$$\omega = \frac{1}{\sqrt{1+|u|^2}}(1, u)$$



Twisted boundary condition (tbc)

$$(\omega_1, \omega_2)(x^1, x^2 + R) = (\omega_1, \omega_2)(x^1, x^2)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Bion ansatz

$$(\omega_1, \omega_2)(x^1, x^2 + R) = (\omega_1, \omega_2)(x^1, x^2)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(λ_1, θ_1) (λ_2, θ_2) **quasi-moduli**

$$\omega = \left(1 + \lambda_2 e^{i\theta_2} e^{\pi(z+\bar{z})},\ \lambda_1 e^{i\theta_1} e^{\pi z}\right)^T \qquad z = x^1 + ix^2$$

$\omega = (1,0)$, $x^1 \to -\infty$

$\omega = (0,1)$

$\omega = (1,0)$, $x^1 \to +\infty$

$$1 = \lambda_1 e^{\pi\tau_1} \qquad \lambda_1 e^{\pi\tau_2} = \lambda_2 e^{2\pi\tau_2}$$

$$\longrightarrow \tau_1 = \frac{1}{\pi}\log\lambda_1 \qquad \longrightarrow \tau_2 = \frac{1}{\pi}\log\frac{\lambda_1}{\lambda_2}$$

Bion ansatz

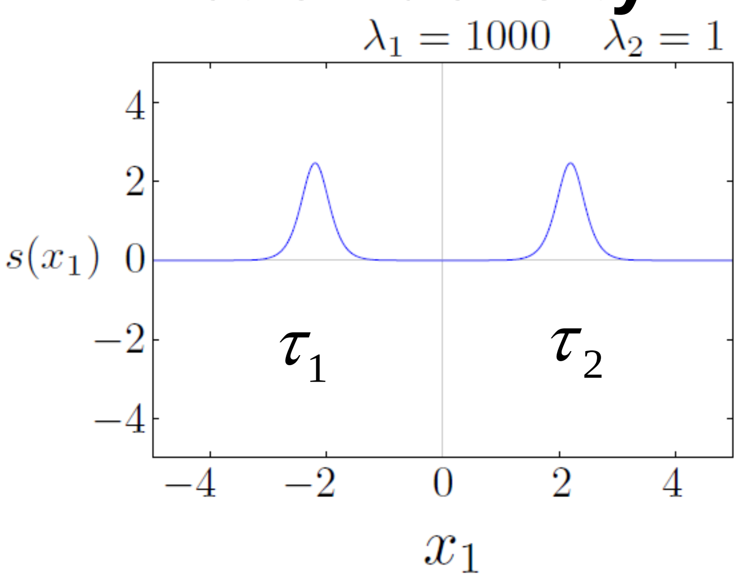
$$(\omega_1, \omega_2)(x^1, x^2 + R) = (\omega_1, \omega_2)(x^1, x^2)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

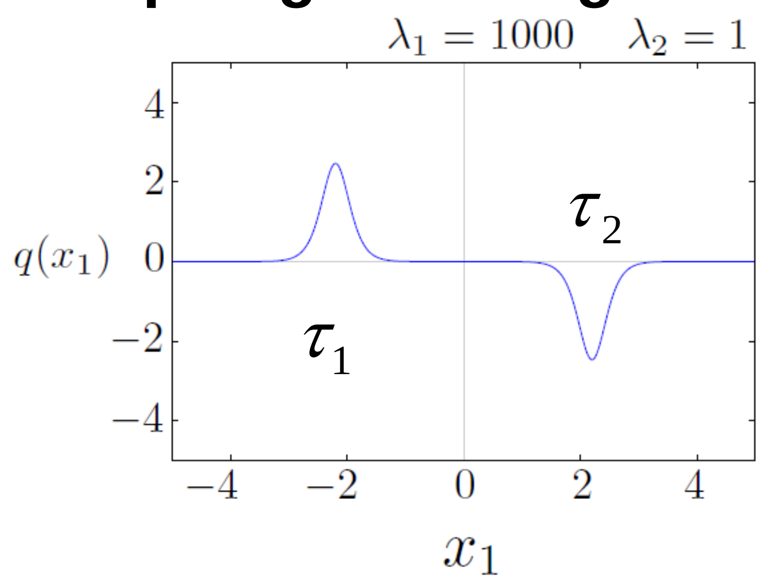
(λ_1, θ_1) (λ_2, θ_2) **quasi-moduli**

$$\omega = \left(1 + \lambda_2 e^{i\theta_2} e^{\pi(z+\bar{z})}, \ \lambda_1 e^{i\theta_1} e^{\pi z}\right)^T \qquad z = x^1 + i x^2$$

$\omega = (1,0)$ $\omega = (0,1)$ $\omega = (1,0)$

Action density **Topological charge density**



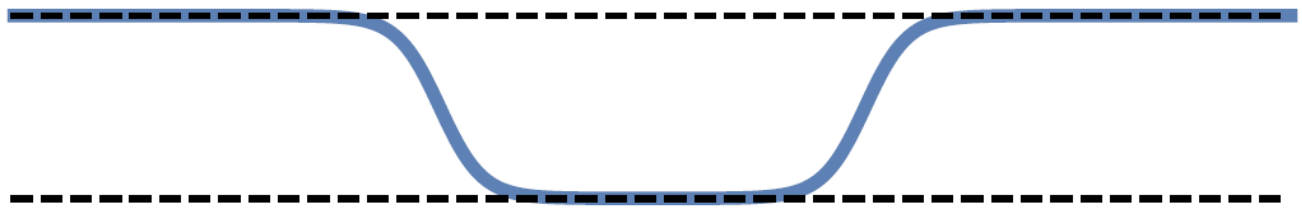


$$\Sigma(x_1) = \frac{1}{R}\int_0^R dx_2 A_2$$ **Wilson line**

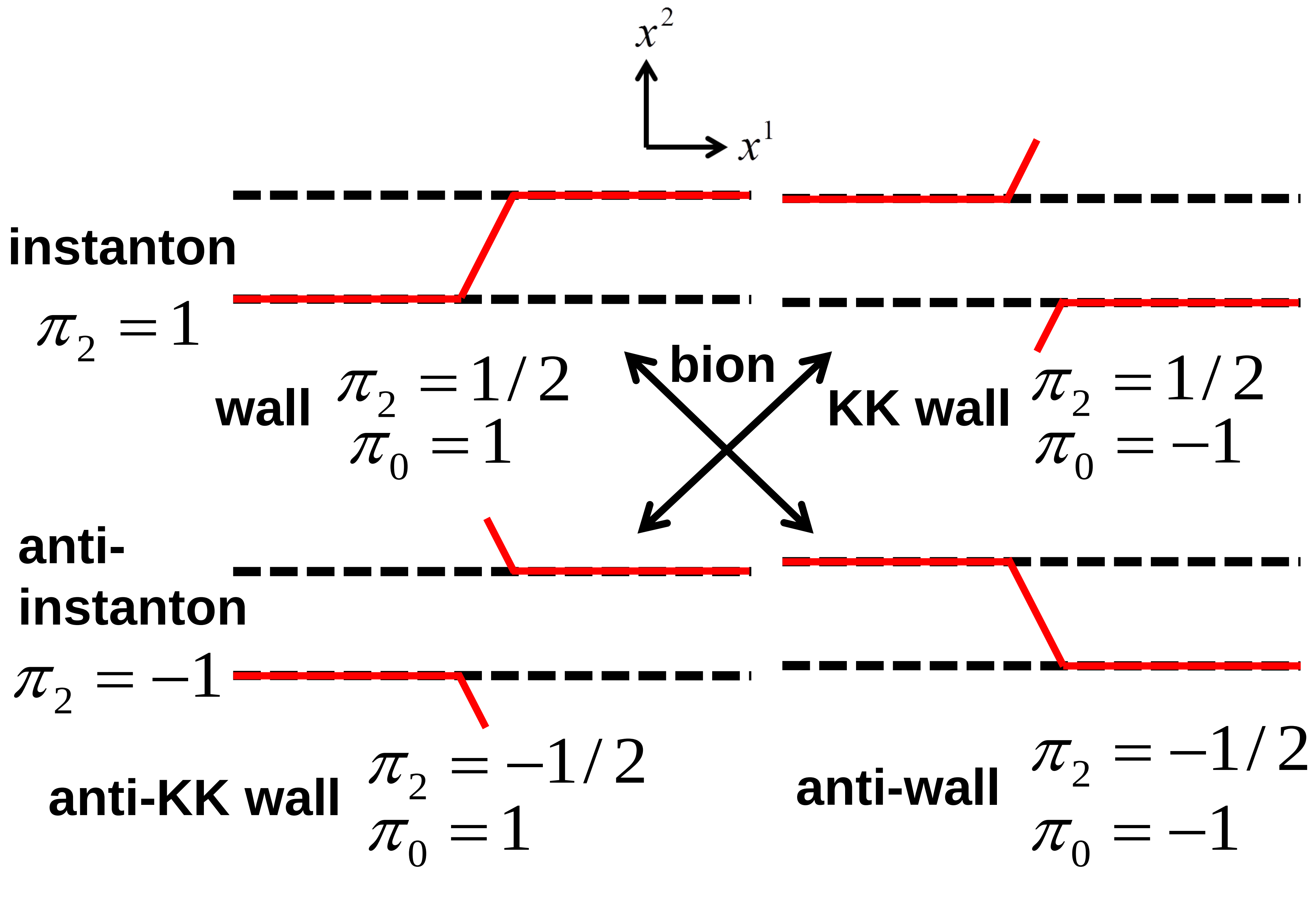
$$= \frac{1}{R}\int_0^R dx_2 \; i\left(\omega\partial_2\omega^\dagger - \partial_2\omega\omega^\dagger\right)/|\omega|^2$$

$\Sigma(x_1)$ **shows kink profile**

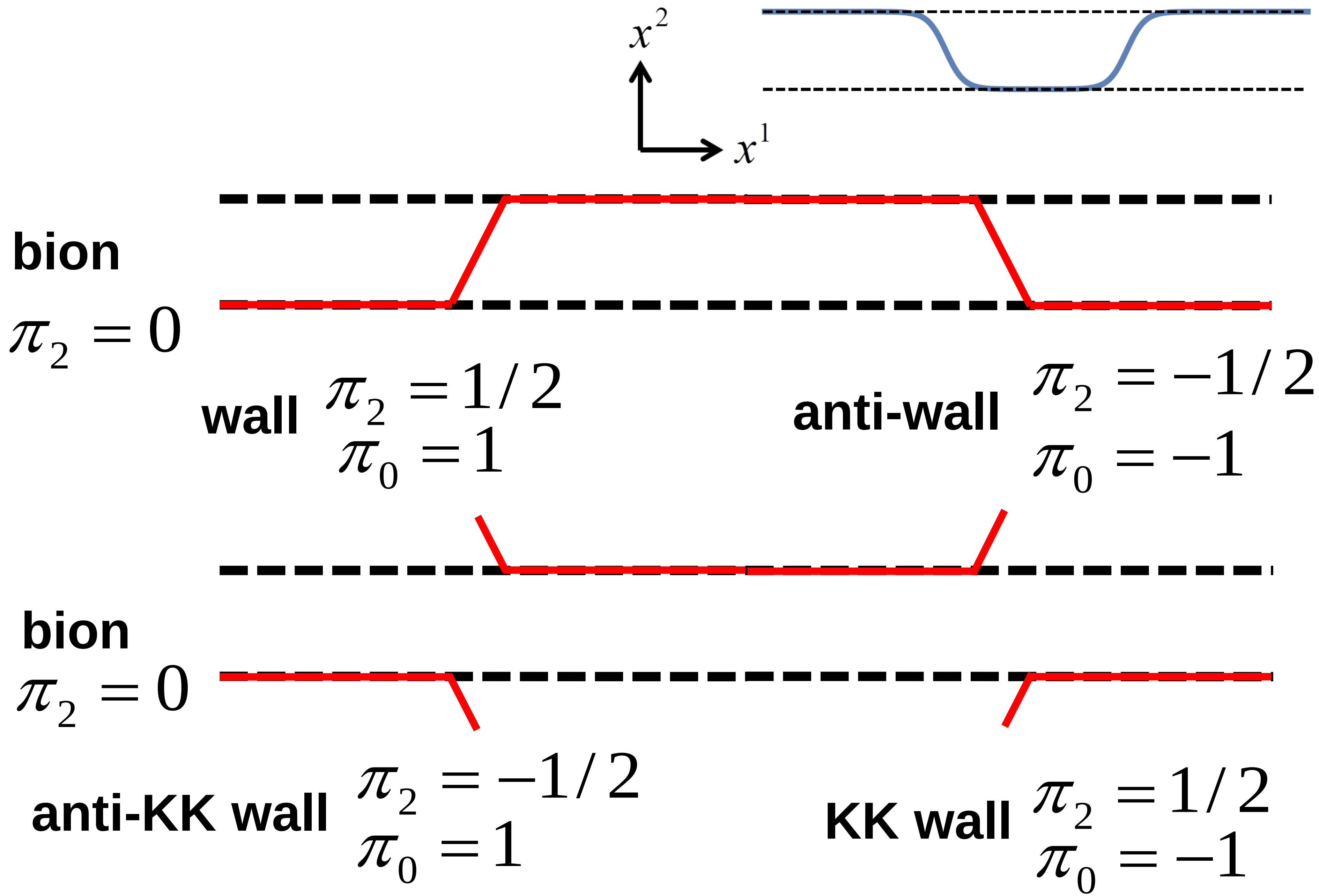
CP¹ bion



CP^1 fractional instanton and bion



CP^1 fractional instanton and bion



CP^{N-1} model

$$\mathbf{C}P^{N-1} = \frac{SU(N)}{SU(N-1)\times U(1)}$$

$$\omega = (\omega_1, \omega_2, \ldots, \omega_N) \sim \lambda(\omega_1, \omega_2, \ldots, \omega_N) \quad \lambda \in \mathbf{C}^*$$

Z_N Twisted boundary condition

$$(\omega_1, \ldots, \omega_N)(x^1, x^2 + R) = (\omega_1, \ldots, \omega_N)(x^1, x^2)W$$

$$W = \begin{pmatrix} 1 & & & \\ & e^{2\pi i/N} & & \\ & & \ddots & \\ & & & e^{2\pi(N-1)i/N} \end{pmatrix}$$

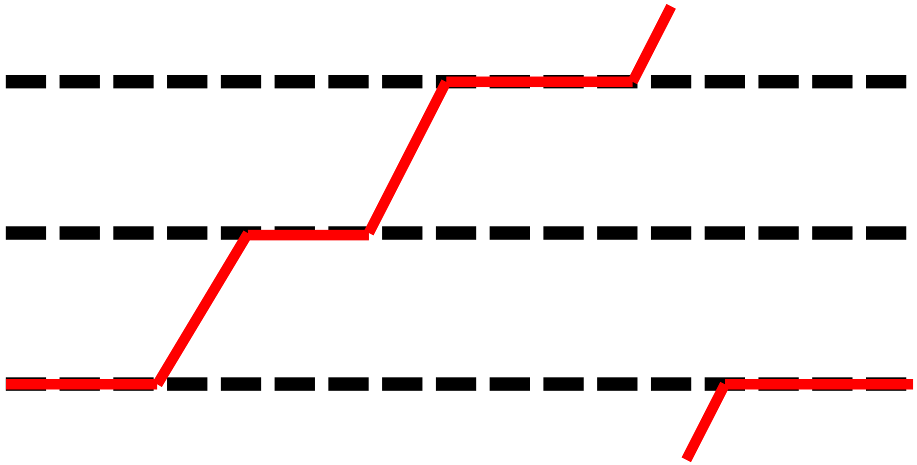
***N* vacua** $\omega W \sim \omega$

(1,0,..,0) (0,1,..,0) … (0,0,..,1)

Bion ansatz

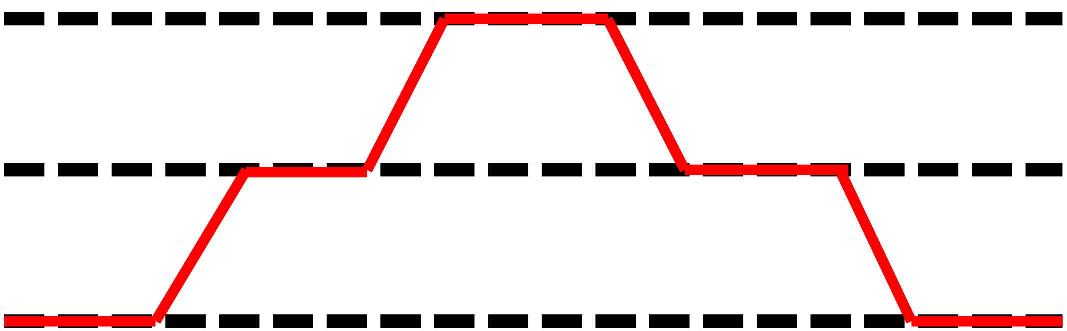
$$\omega = \left(0, \cdots, 0, 1 + \lambda_2 e^{i\theta_2} e^{2\pi(z+\bar{z})/N},\ \lambda_1 e^{i\theta_1} e^{2\pi z/N}, 0, \cdots, 0\right)$$

CP^2 fractional instanton and bion



CP^2 instanton $\pi_2 = 1$

(fractional instanton $\pi_2 = 1/3$)



CP^2 bion $\pi_2 = 0$

CP^{N_f-1} fractional instanton $\pi_2 = 1/N_f$

O(3) model [= $\mathbf{C}P^1$ model] on $\mathbf{R}^1 \times S^1$

Target space $M = S^2 \cong O(3)/O(2) \cong \mathbf{C}P^1 \cong SU(2)/U(1)$

Generalizations of *fractional instantons*

(1) $\mathbf{C}P^{N-1}$ model on $\mathbf{R}^1 \times S^1$

Eto, Isozumi, MN, Ohashi & Sakai, PRD72 (2005) 025011 [hep-th/0412048]

$$\mathbf{C}P^{N-1} \cong SU(N)/[SU(N-1)\times U(1)]$$

later by F.Bruckman ('08)

(2) Grassmann model on $\mathbf{R}^1 \times S^1$

Eto, Fujimori, Isozumi, MN, Ohashi, Ohta & Sakai, PRD73 (2006) 085008 [hep-th/0601181]

$$Gr_{N,M} \cong \frac{SU(N)}{SU(N-M)\times SU(M)\times U(1)}$$

$$\pi_2 = \mathbf{Z}$$

O(3) model [= CP^1 model] on $\mathbf{R}^1 \times S^1$

Target space $M = S^2 \cong O(3)/O(2) \cong \mathbf{C}P^1 \cong SU(2)/U(1)$

Generalizations of *bions*

(1) C$P^{N\text{-}1}$ model on $\mathbf{R}^1 \times S^1$

$$\mathbf{C}P^{N-1} \cong SU(N)/[SU(N-1) \times U(1)]$$

Dunne & Unsal,
JHEP1211(2012)170
PRD87(2013)025015

(2) Grassmann model on $\mathbf{R}^1 \times S^1$

$$Gr_{N,M} \cong \frac{SU(N)}{SU(N-M) \times SU(M) \times U(1)}$$

$$\pi_2 = \mathbf{Z}$$

Misumi, MN & Sakai,
PTEP(2015)033B02[arXiv:1409.3444]
Dunne & Unsal, arXiv:1505.07803

(3) O(*N*) model on $\mathbf{R}^{N-2} \times S^1$ $\quad \frac{O(N)}{O(N-1)} \cong S^{N-1} \quad \pi_{N-1} = \mathbf{Z}$

O(4) model [=SU(2) principal chiral model] on $\mathbf{R}^2 \times S^1$

MN, JHEP 1503 (2015) 108 [arXiv:1412.7681]

(4) SU(*N*) principal chiral model on $\mathbf{R}^2 \times S^1$

$$M = SU(N) \text{ or } G \quad \pi_3(M) = \mathbf{Z}$$

MN, JHEP 1508 (2015) 063 [arXiv:1503.06336]

§ Grassmann models

(1) CP^{N-1} model on $\mathbf{R}^1 \times S^1$

$$\mathbf{C}P^{N-1} \cong SU(N)/[SU(N-1)\times U(1)]$$

(2) Grassmann model on $\mathbf{R}^1 \times S^1$

$$Gr_{N,M} \cong \frac{SU(N)}{SU(N-M)\times SU(M)\times U(1)}$$

$M=1$

$\pi_2 = \mathbf{Z}$

U(*M*) gauge theory with complex *M* x *N* matrix H

$$H \to g_C H g_F \quad g_C \in U(N_C = M) \quad g_F \in SU(N_F = N)_F$$

$$\mathcal{L}_{\text{gauge}} = \text{Tr}\left[\frac{1}{2g^2}F_{\mu\nu}F_{\mu\nu} + \mathcal{D}_\mu H\left(\mathcal{D}_\mu H\right)^\dagger\right] + \text{Tr}\left[\frac{g^2}{4}\left(v^2\mathbf{1}_{N_C} - HH^\dagger\right)^2\right]$$

Twisted b.c.(Z_{Nf} symmetric)

$$H(x^1, x^2 + R) = H(x^1, x^2)\exp\left[\frac{2\pi i}{N_f}\text{diag}(1,2,\ldots,N_f - 1)\right]$$

Promote this to supersymmetric theory (8 SUSY)

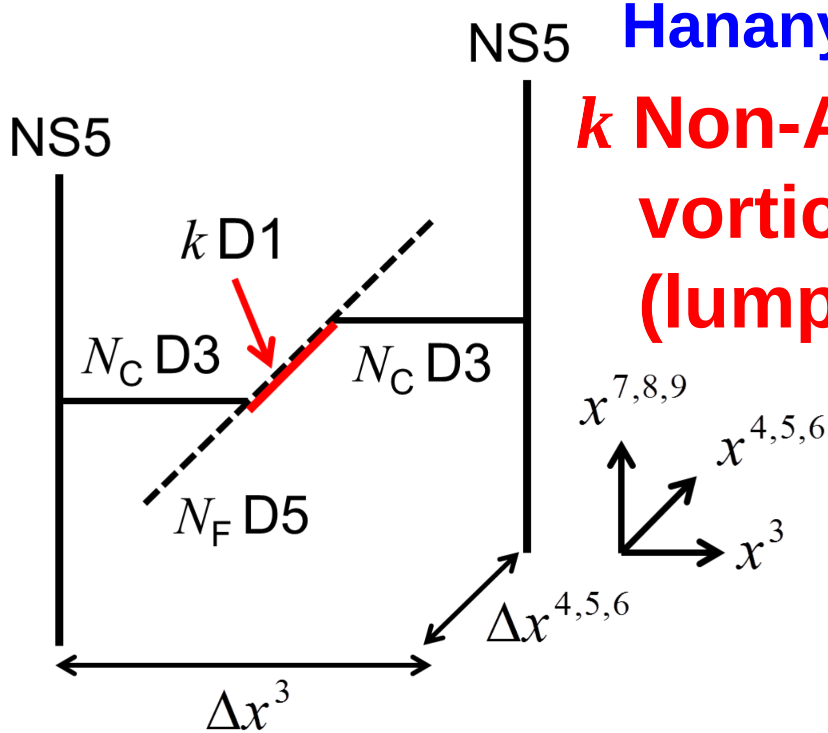
$\left(H, \tilde{H}(=0)\right)$ ***Nf* hypermulets** $\left(A_\mu, \Sigma\right)$ **U(*Nc*) gauge multiplets**

Vacuum moduli = $T^*\left[\frac{SU(N_F)}{SU(N_F - N_C)\times SU(N_C)\times U(1)]}\right]$

Embed it to a D-brane configuration for usefulness

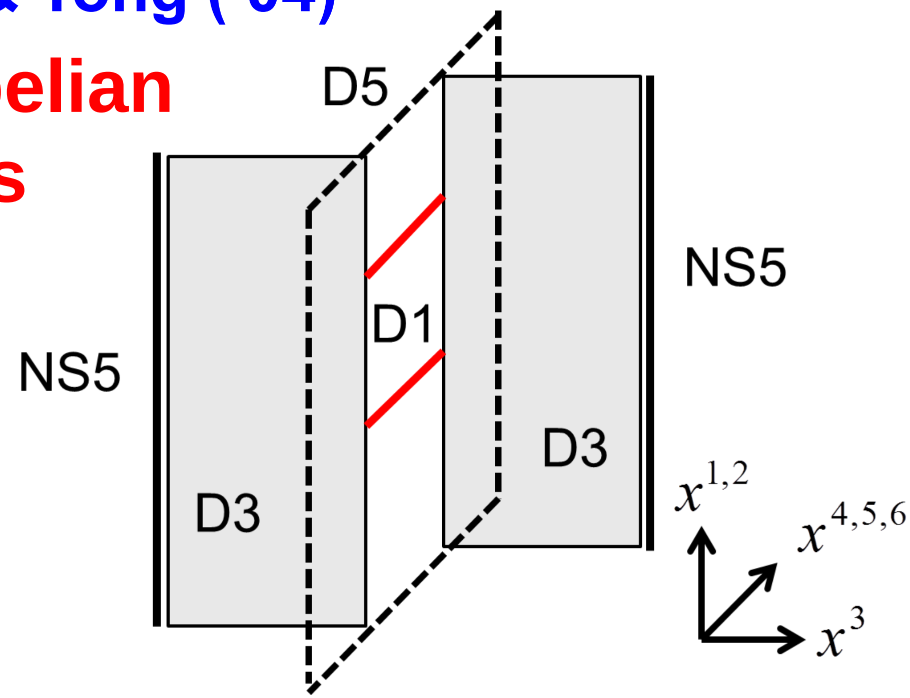
Hanany-Witten setup + Hanany-Tong's vortices

D3 world-volume = 2+1d $U(N_c)$ gauge + N_f fund matter



Hanany & Tong ('04)

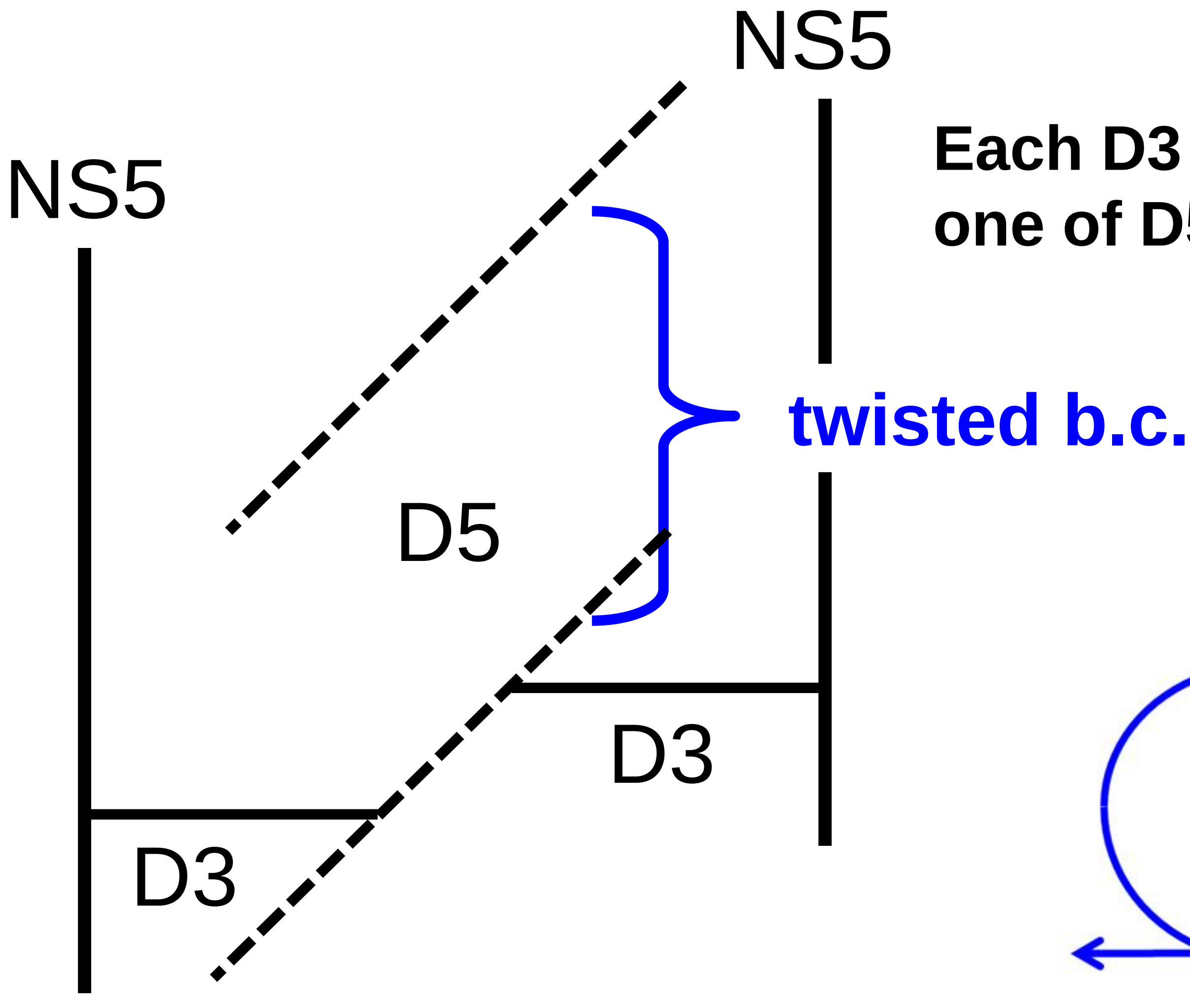
k Non-Abelian vortices (lump)



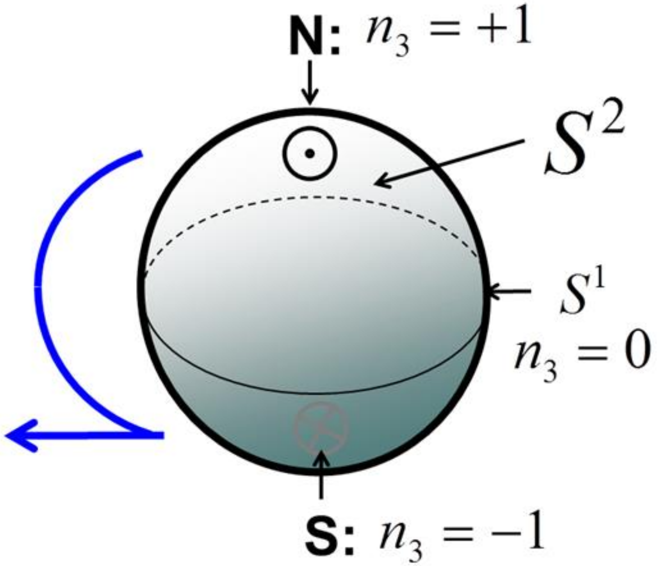
	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
N_C D3	○	○	○	○	—	—	—	—	—	—
N_F D5	○	○	○	—	○	○	○	—	—	—
2 NS5	○	○	○	—	—	—	—	○	○	○
k D1	○	×	×	—	○	—	—	—	—	—

Compactification along x^2

Nontrivial Wilson line on D5

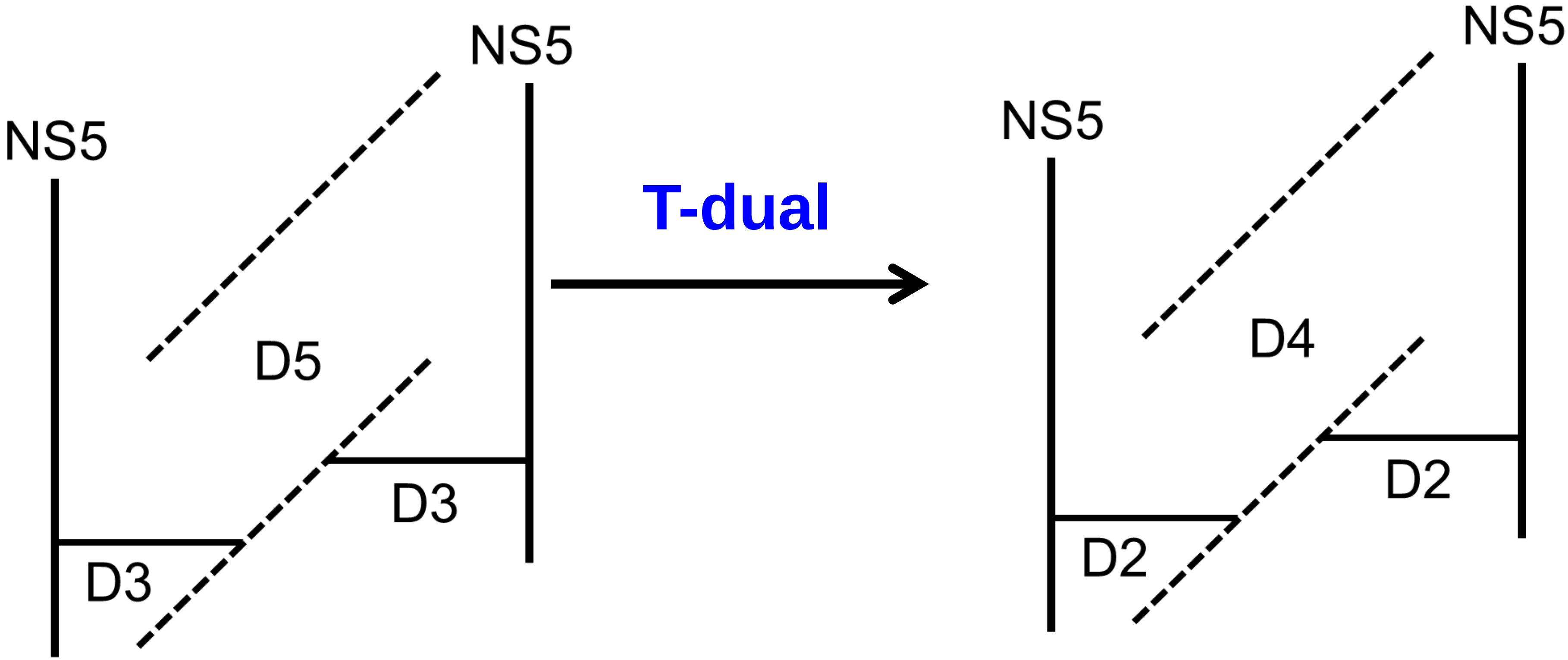


Each D3 ends on one of D5s (s-rule)



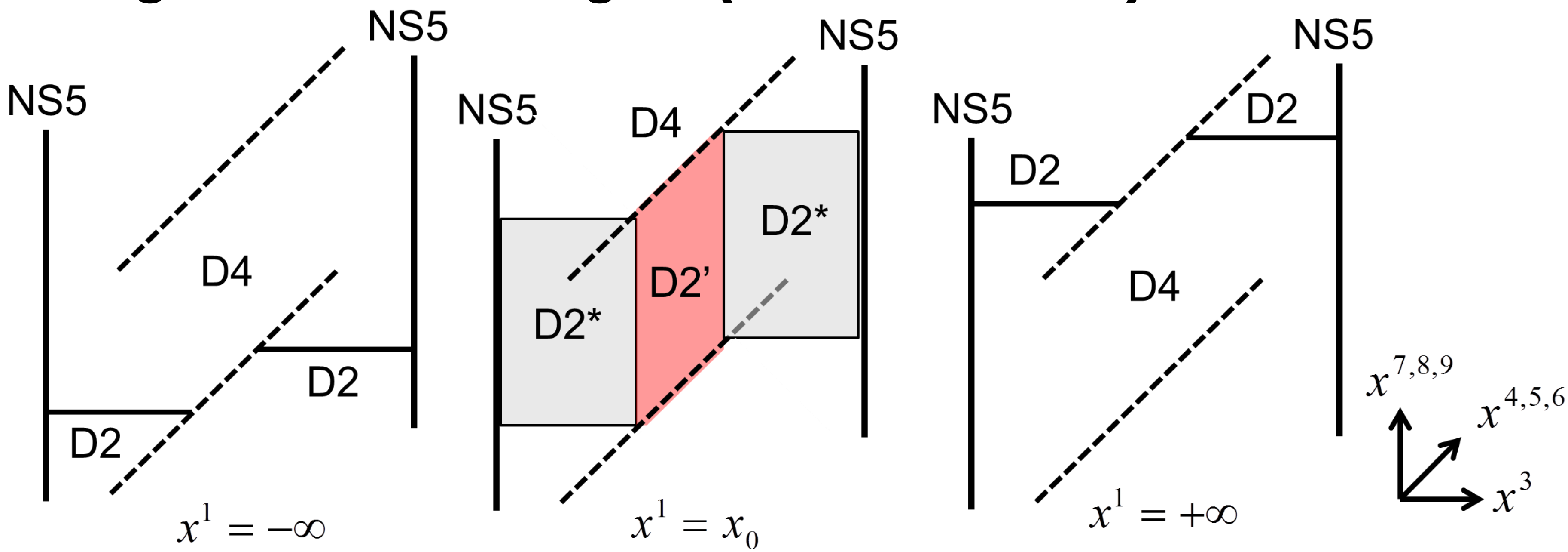
For $Gr_{Nf,Nc}$, Nf!/Nc!(Nf-Nc)! vacua

Taking a T-dual along x^2 (without vortices)

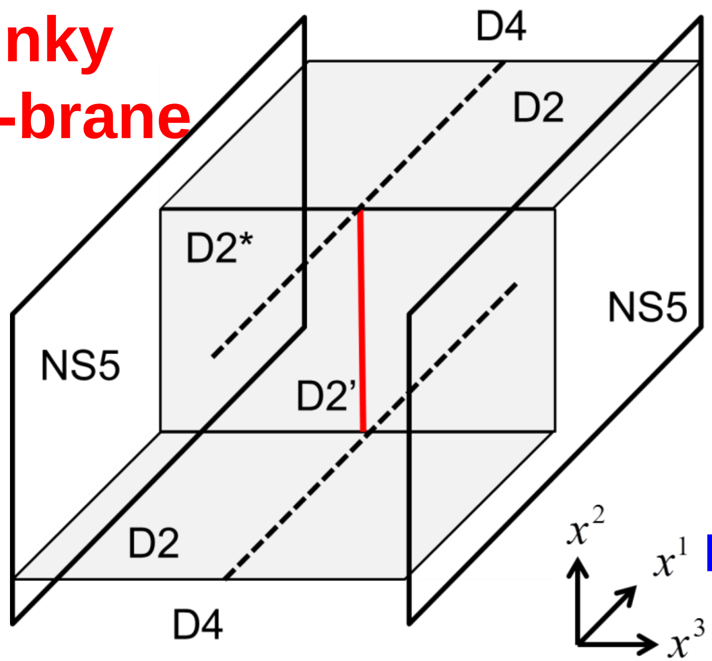


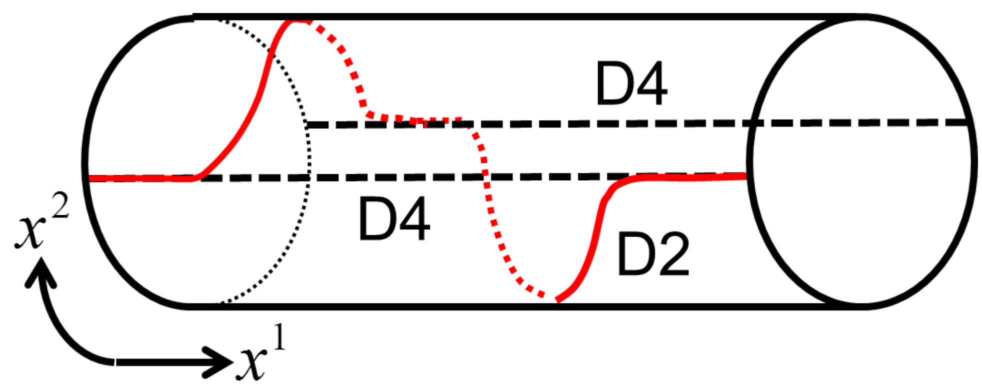
	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
N_C D2	○	○	—	○	—	—	—	—	—	—
N_F D4	○	○	—	—	○	○	○	—	—	—
2 NS5	○	○	○	—	—	—	—	○	○	○
k D2′	○	×	○	—	○	—	—	—	—	—
k D2*	○	×	○	○	—	—	—	—	—	—

Taking a T-dual along x^2 (with vortices)



kinky D-brane

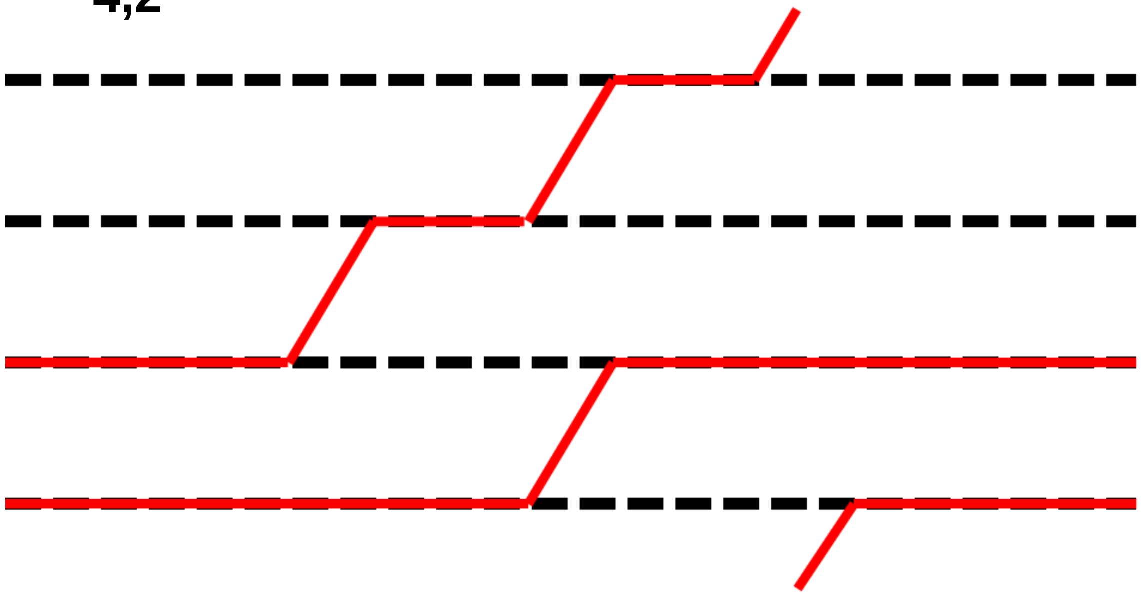




wall **KK wall**

Eto, Fujimori, Isozumi, MN, Ohashi, Ohta & Sakai
PRD73 (2006) 085008 [hep-th/0601181]

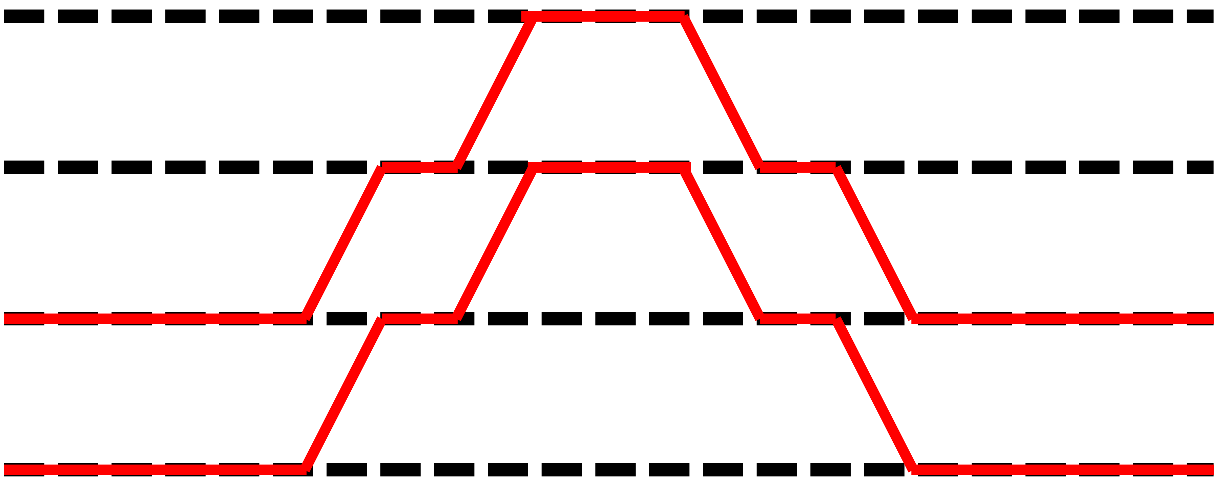
$Gr_{4,2}$ fractional instanton and bion



$Gr_{4,2}$ instanton

$$\pi_2 = 1$$

(**fractional instanton** $\pi_2 = 1/4$)



$Gr_{4,2}$ bion
(the maximum)

$Gr_{Nf,Nc}$ fractional instanton $\pi_2 = 1/N_f$

For any given config, we gave ansatz with arbitrary positions.
--- Quasi-moduli integrals are possible.

§ O(*N*) model and SU(N) Principal chiral model

O(4) model = SU(2) principal chiral model on $\mathbf{R}^{3,1}$

= Skyrme model (if 4 deriv term added)

$$U = \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix} \in SU(2) \quad |\phi_1|^2 + |\phi_2|^2 = 1 \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$L = \frac{1}{g^2} \mathrm{tr}\left(U^\dagger \partial_\mu U\right)^2 = -\frac{1}{g^2} \mathrm{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) = -\frac{2}{g^2} \partial^\mu \phi^\dagger \partial_\mu \phi$$

$$\pi_3\left[SU(2)\right] \simeq \mathbf{Z}$$

Instantons exist in $\mathbf{R}^3$, $\mathbf{R}^2 \times S^1$ as Skyrmions

$$B = -\frac{1}{24\pi^2} \int d^3x \varepsilon^{ijk} \mathrm{tr}\left(U^\dagger \partial_i U U^\dagger \partial_j U U^\dagger \partial_k U\right)$$

$i, j, k = 1,2,3$

$$= -\frac{1}{4\pi^2} \int d^3x \varepsilon^{ijk} \phi^\dagger \partial_i \phi \phi^\dagger \partial_j \phi \phi^\dagger \partial_k \phi$$

“Baryon number”

O(4) model = SU(2) principal chiral model on $\mathbf{R}^{3,1}$

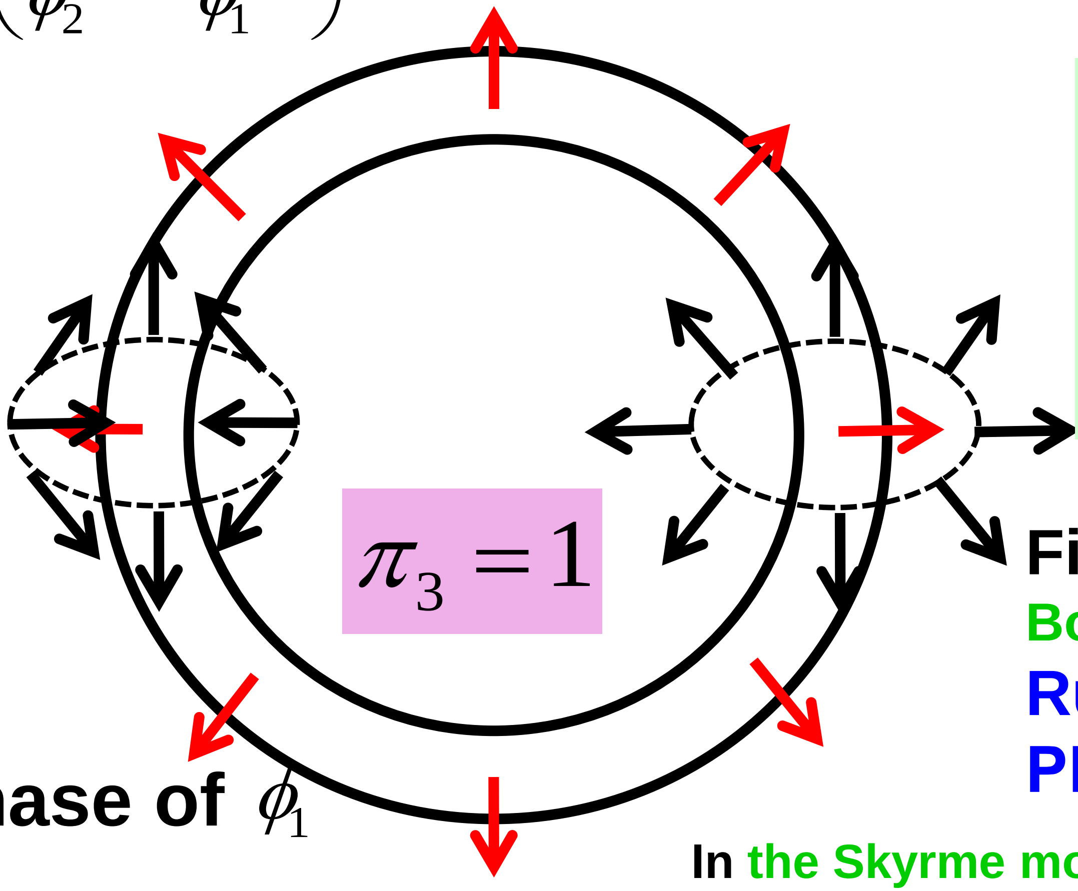
= Skyrme model (if 4 deriv term added)

$$U = \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix} \in SU(2) \quad |\phi_1|^2 + |\phi_2|^2 = 1$$

$$\pi_3[SU(2)] \cong \mathbf{Z}$$

Skyrmion =

A vortex ring of ϕ_1 with phase of ϕ_2 twisted once (vorton)



↑ (black) **phase of ϕ_1**

↑ (red) **phase of ϕ_2**

First found in
Bose-Einstein Condensates
Ruostekoski & Anglin
PRL ('01)

In the Skyrme model, Gudnason & MN,
Phys.Rev.D90(2014)085007[arXiv:1407.7210]
Phys.Rev.D94(2016)025008[arXiv:1606.00336]

O(4) model = SU(2) principal chiral model on $\mathbf{R}^2 \times S^1$

= Skyrme model (if 4 deriv term added)

twisted b.c $U(x+R) = WU(x)W^\dagger \quad W = \sigma_3 = \mathrm{diag.}(1,-1)$

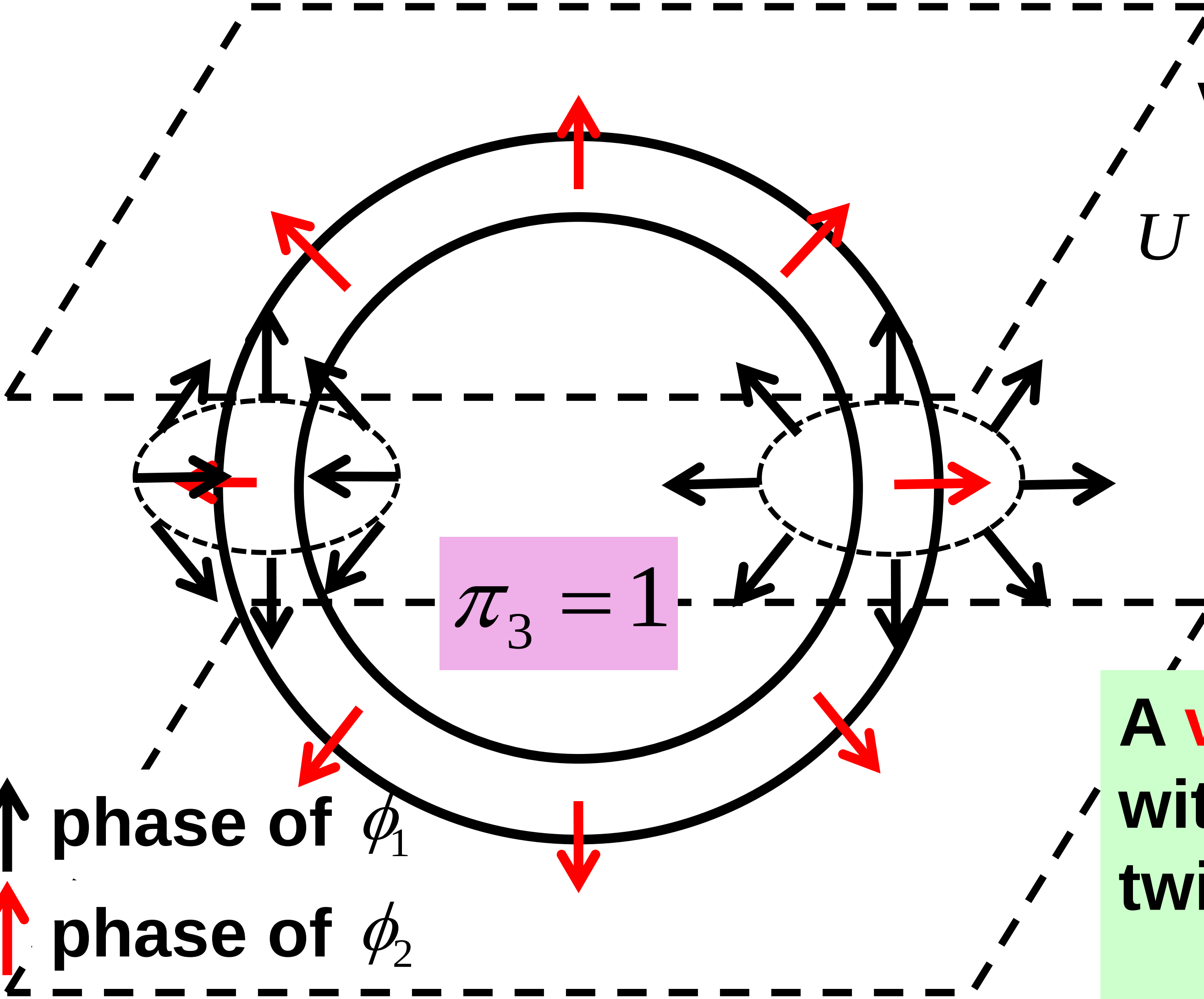
vacua $[U, W] = 0$

$$U = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_1^* \end{pmatrix} \in U(1)$$

$$|\phi_1|^2 = 1$$

$\pi_1[U(1)] \simeq \mathbf{Z}$

vortex



A vortex ring of ϕ_1 with phase of ϕ_2 twisted once (vorton)

O(4) model = SU(2) principal chiral model on $\mathbf{R}^2 \times S^1$

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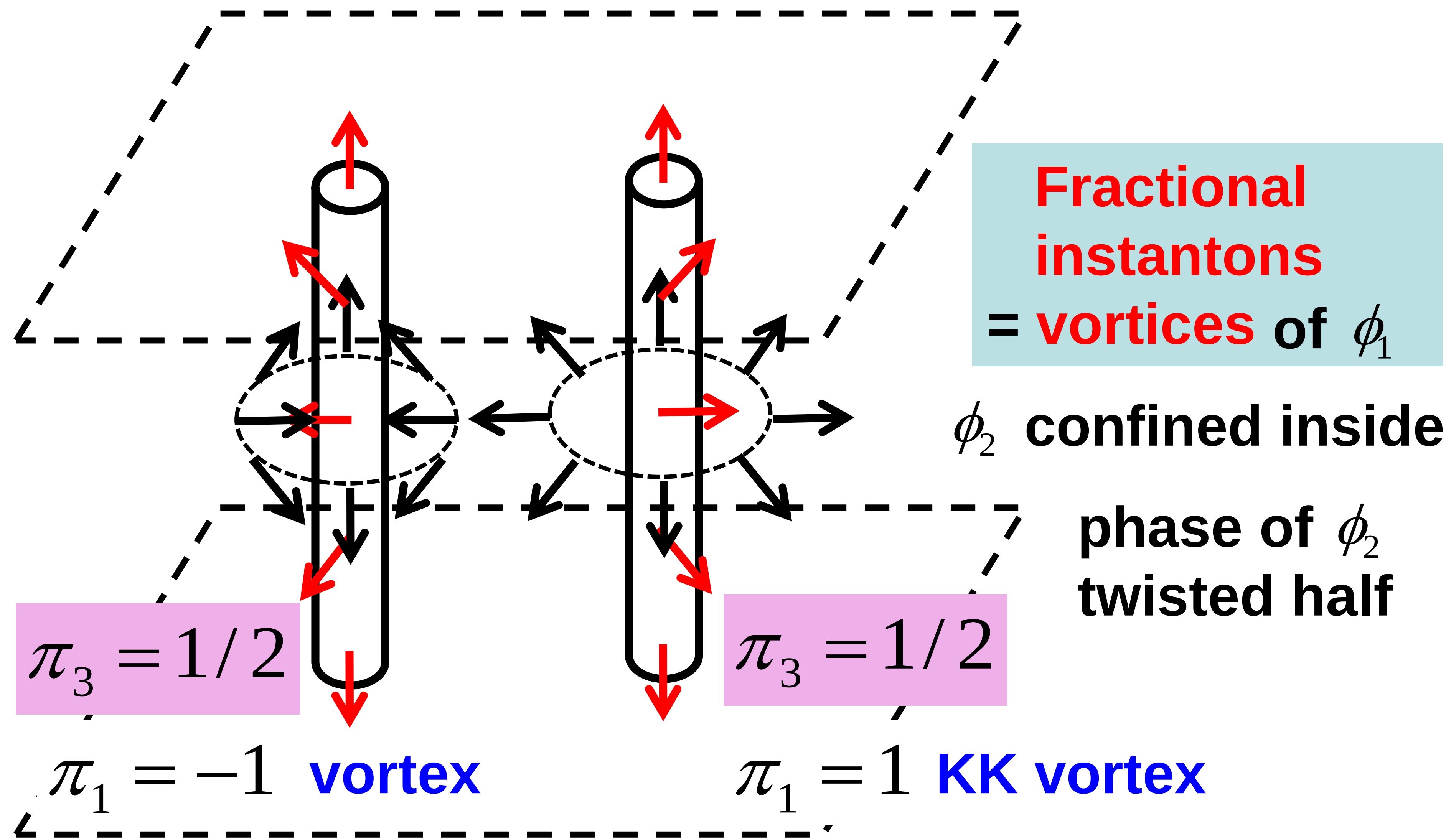
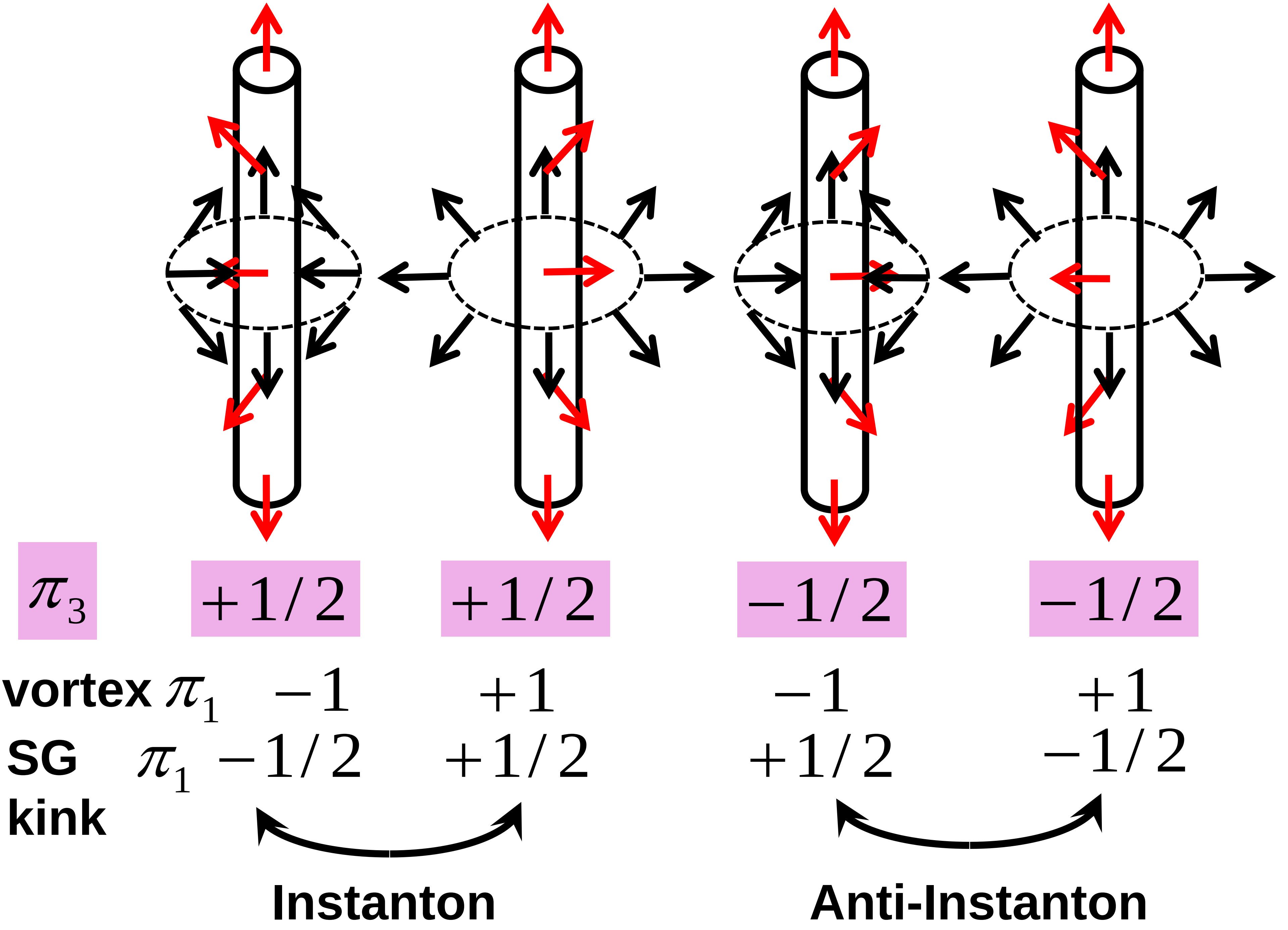
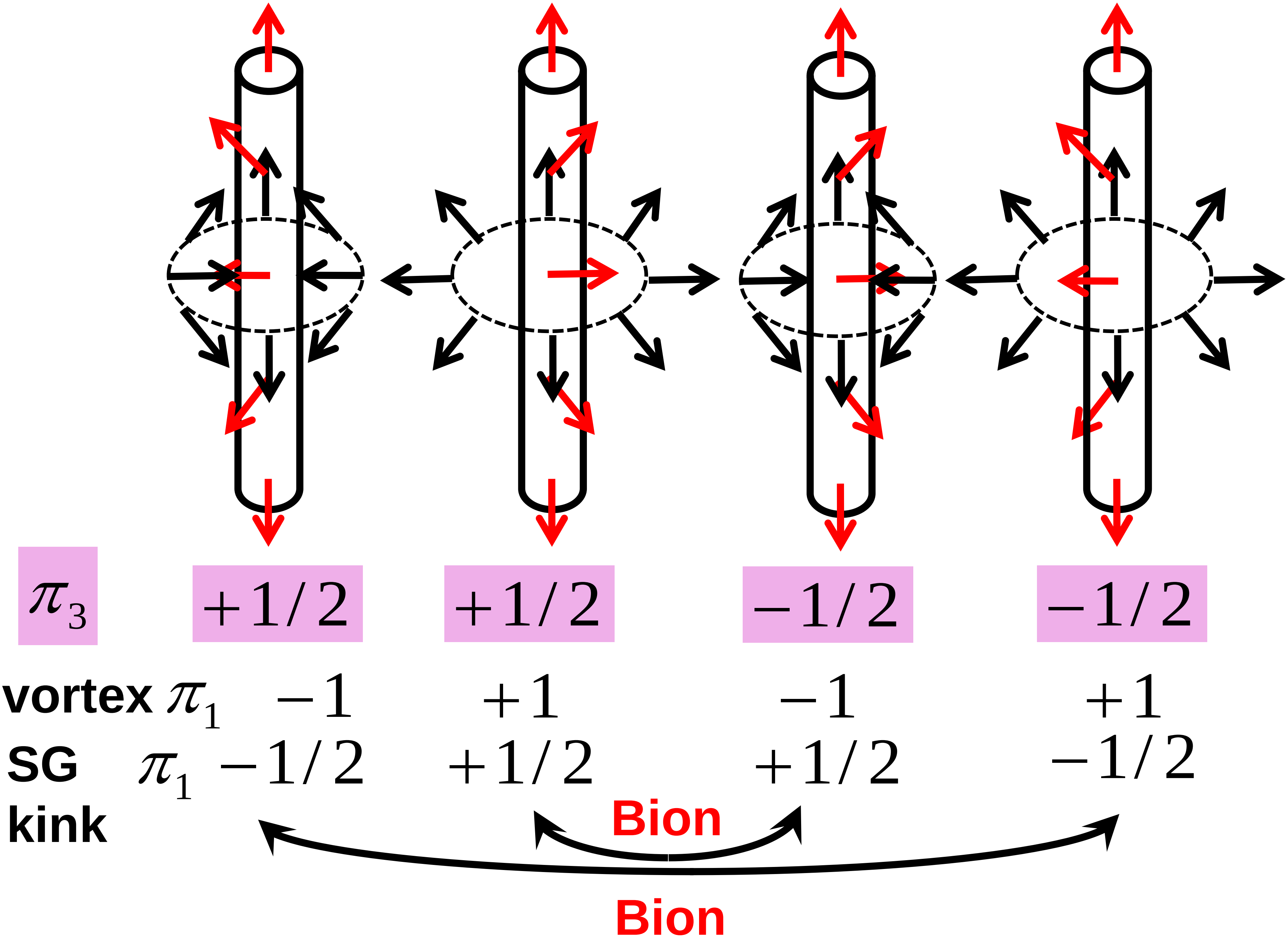
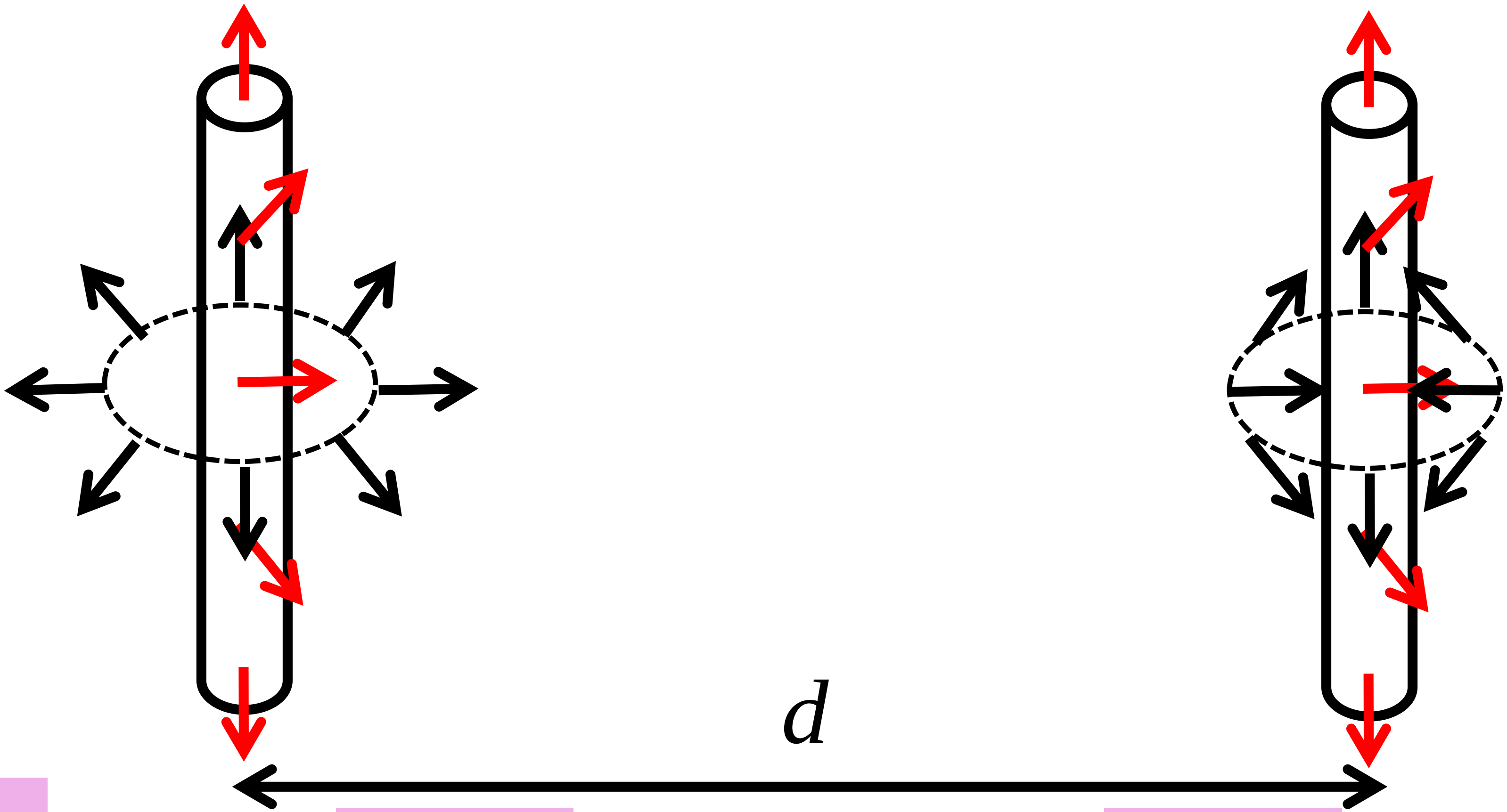
twisted b.c $U(x+R) = WU(x)W^\dagger \quad W = \sigma_3 = \mathrm{diag.}(1,-1)$




π_3
$+1/2$
$+1/2$
$-1/2$
$-1/2$
vortex π_1
-1
$+1$
-1
$+1$
SG π_1
$-1/2$
$+1/2$
$+1/2$
$-1/2$
kink
Instanton
Anti-Instanton


π_3
$+1/2$
$+1/2$
$-1/2$
$-1/2$
vortex π_1
-1
$+1$
-1
$+1$
SG π_1
kink
$-1/2$
$+1/2$
$+1/2$
$-1/2$
Bion
Bion



	π_3	$+1/2$		$-1/2$
vortex	π_1	$+1$		-1
SG	π_1	$+1/2$	**Bion**	$+1/2$

kink **Interaction energy** $E_{\text{int}} \sim \frac{1}{g^2}\log(d/\xi)$ **asymptotically**

They are "*confined*" unlike CP^{N-1} kinks

SU(N) principal chiral model on $\mathbf{R}^2 \times S^1$

= SU(N) Skyrme model (if 4 deriv term added)

Z_N twisted b.c

$$U(x+R) = W U(x) W^\dagger \qquad W = \mathrm{diag}(1, \omega, \omega^2, \ldots, \omega^{N-1})$$

$$\omega = \exp(2\pi i / N)$$

Vacua: $[U, W] = 0$

$U(1)^{N-1}$ **Cartan subalgebra of SU(N)**

$$\pi_1\left[U(1)^{N-1}\right] \cong \mathbf{Z}^{N-1} \quad \left(\longleftrightarrow \quad \pi_2\left[\frac{SU(1)}{U(1)^{N-1}}\right] \cong \mathbf{Z}^{N-1} \right)$$

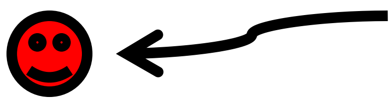
Monopole charge

Fractional instantons = global vortices

N **constituents with** $\pi_3 = 1/N$

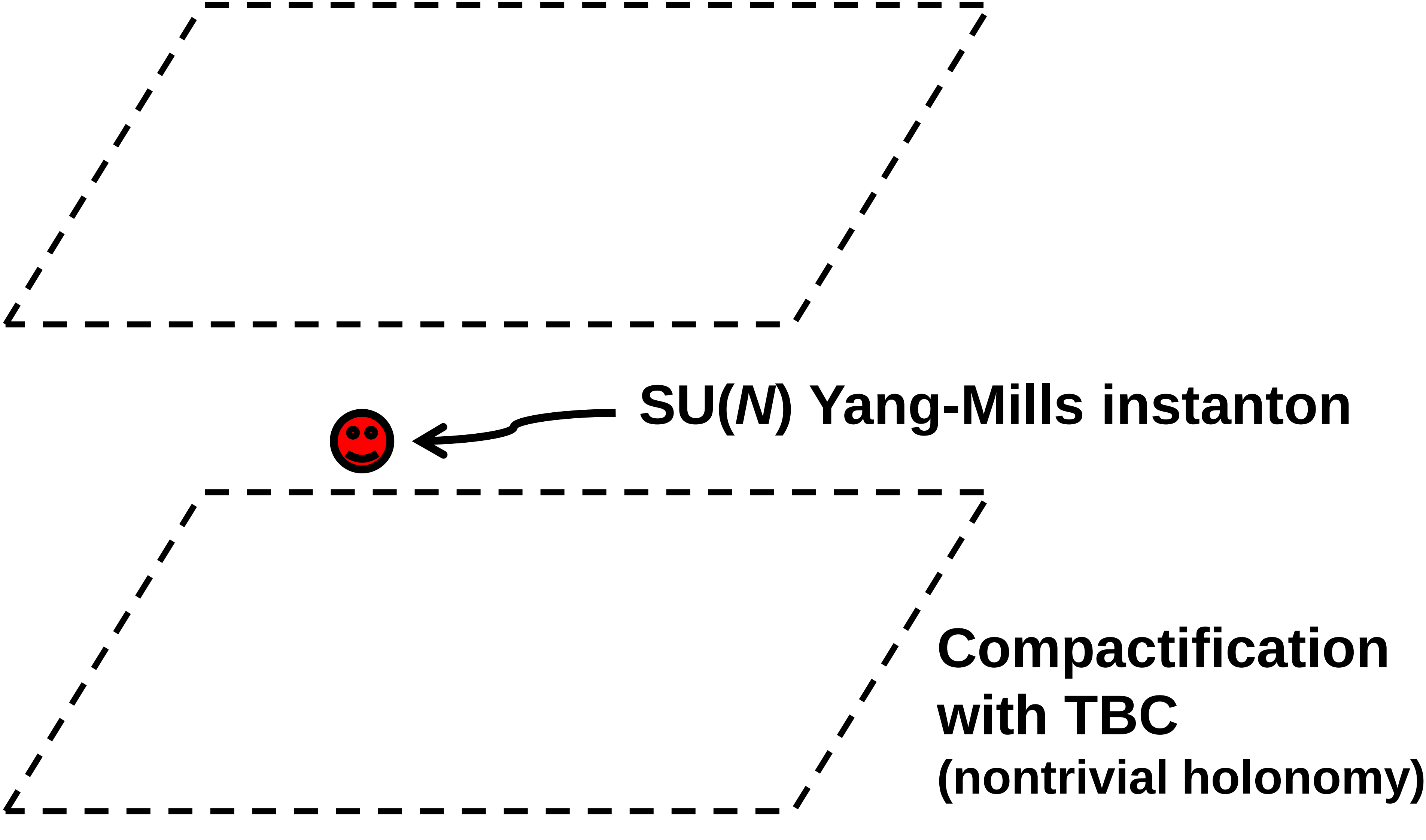
§ Relations among 4d,3d,2d bions

What relations with Yang-Mills instantons, bions?

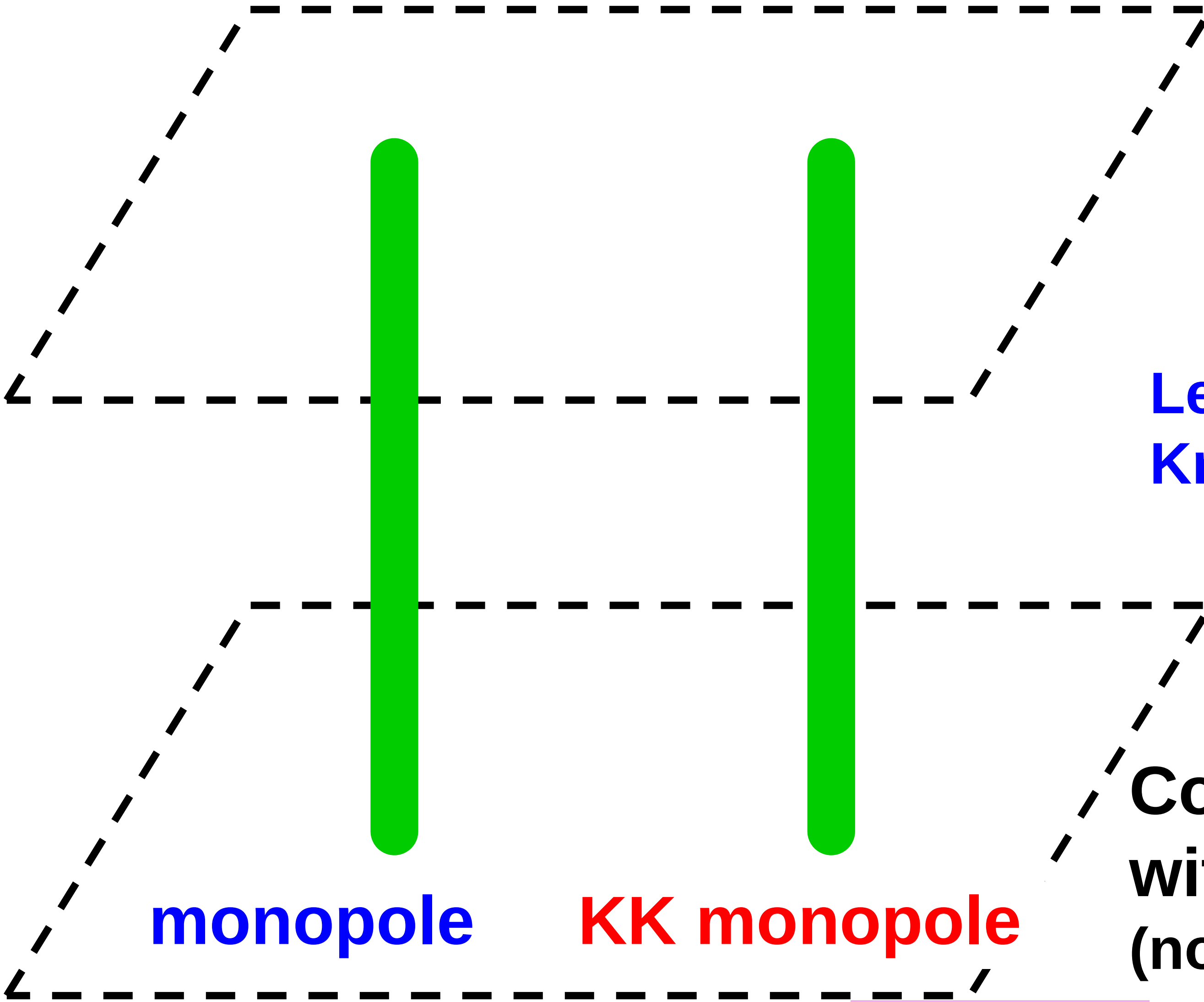


SU(N) Yang-Mills instanton

What relations with Yang-Mills instantons, bions?



What relations with Yang-Mills instantons, bions?



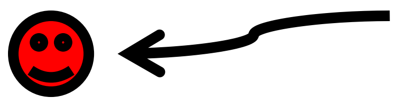
$\pi_2 = 1$ $\pi_3 = 1/2$ $\pi_2 = -1$ $\pi_3 = 1/2$

Lee & Yi ('97)
Kraan & Baal ('98)

Compactification with TBC
(nontrivial holonomy)

$\pi_2 = 0$
$\pi_3 = 1$

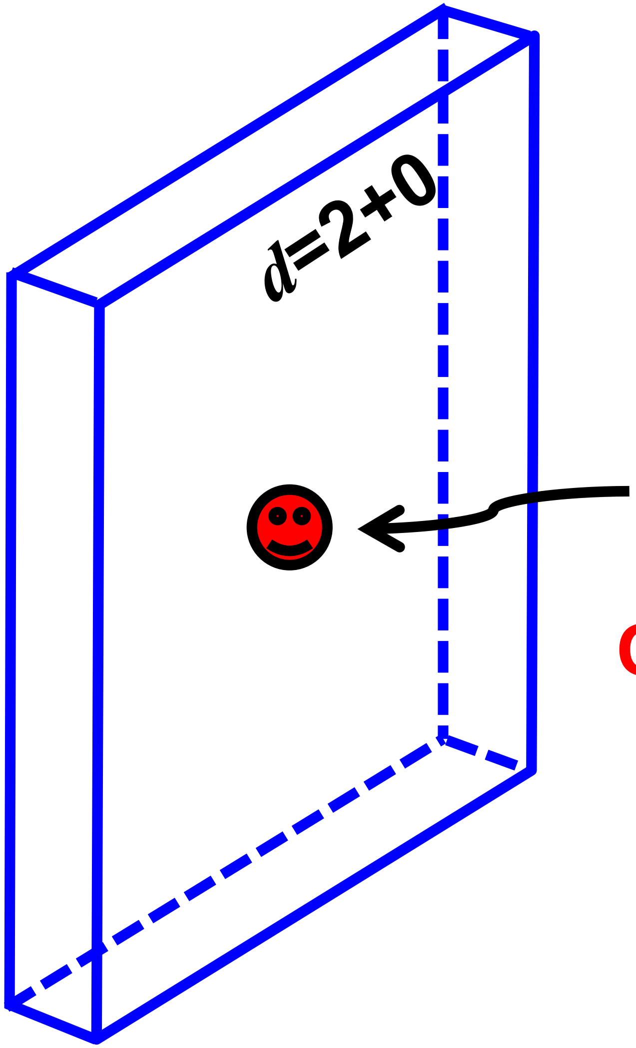
What relations with Yang-Mills instantons, bions?



SU(*N*) Yang-Mills instanton

What relations with Yang-Mills instantons, bions?

Hanany & Tong ('04)



Higgsing

SU(*N*) Yang-Mills instanton

=

C*P*$^{N\text{-}1}$ sigma model instanton

$$\pi_2 = 1$$

U(*N*) Non-Abelian vortex

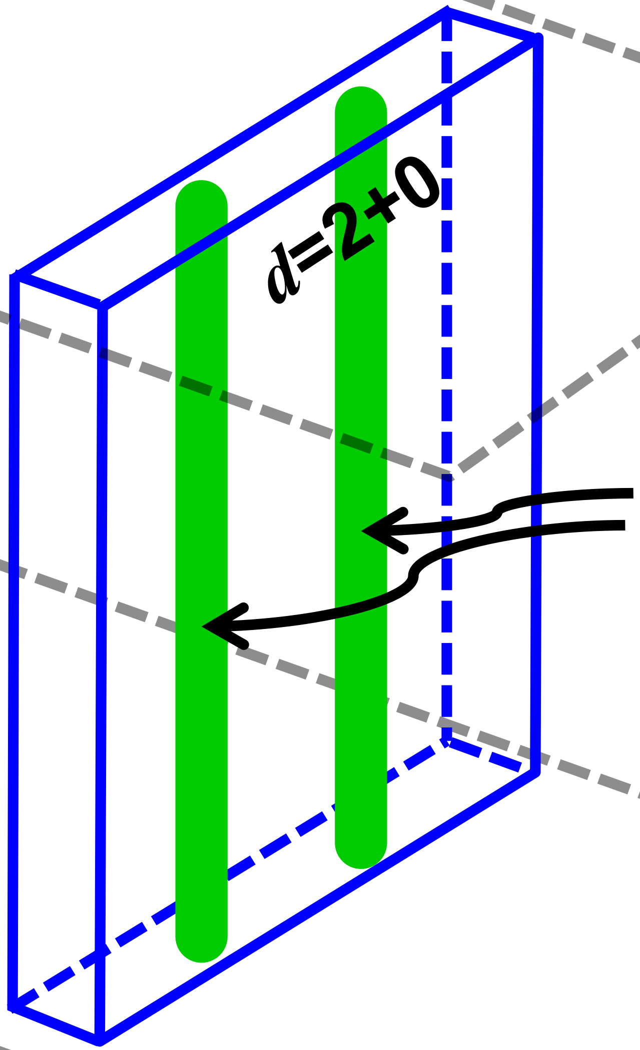
Eff theory = C*P*$^{N\text{-}1}$ model

Auzzi et.al ('04)

Hanany & Tong ('04)

What relations with Yang-Mills instantons, bions?

Eto, Isozumi, MN, Ohashi & Sakai ('05)
PRD72 (2005) 025011 [hep-th/0412048]



Higgsing

Monopole, KK monopole

=

CP^{N-1} sigma model kinks
with $1/N$ charges $\pi_2 = 1/N$

U(N) Non-Abelian vortex
Eff theory = CP^{N-1} model

Auzzi et.al ('04)
Hanany & Tong ('04)

What relations with Yang-Mills instantons, bions?

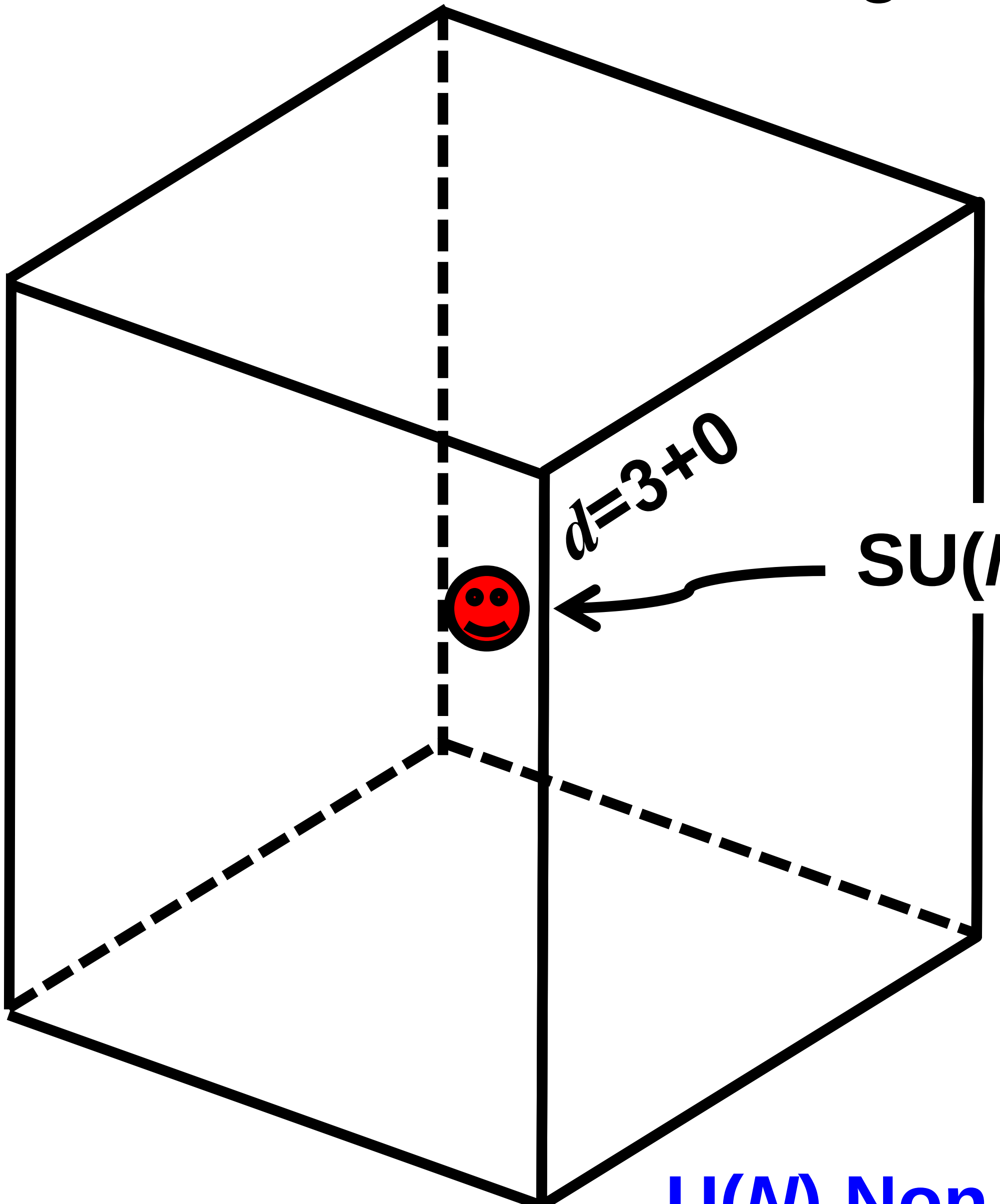
Eto,MN,Ohashi & Tong

Phys.Rev.Lett. 95 (2005) 252003

[hep-th/0508130]

(another)

Higgsing



SU(*N*) Yang-Mills instanton

(Josephson instanton)

||

Instanton (Skyrmion)

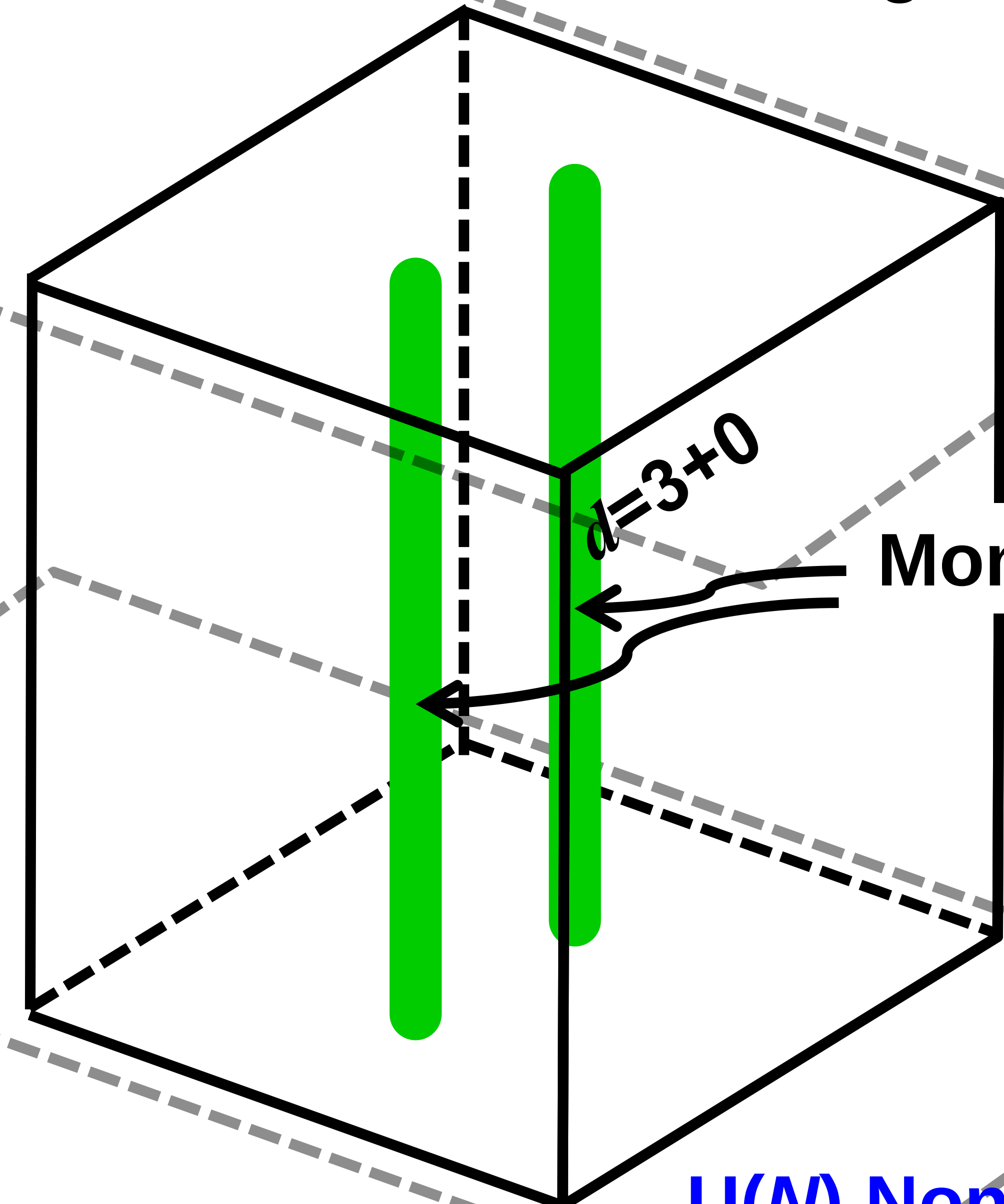
in SU(*N*) PCM $\pi_3 = 1$

U(*N*) Non-Abelian domain wall

Eff theory = SU(*N*) PCM

What relations with Yang-Mills instantons, bions?

MN, Phys.Rev. D92 (2015) 045010
[arXiv:1503.02060]



(another)
Higgsing

Monopole, KK monopole

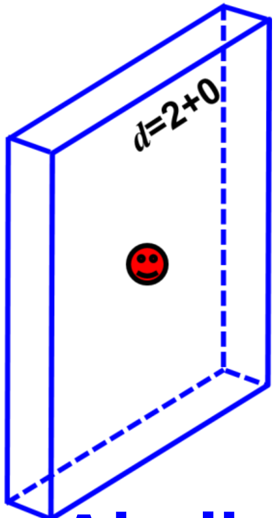
(Josephson monopole)

=

Vortices with 1/*N* charge in SU(*N*) PCM $\pi_3 = 1/N$

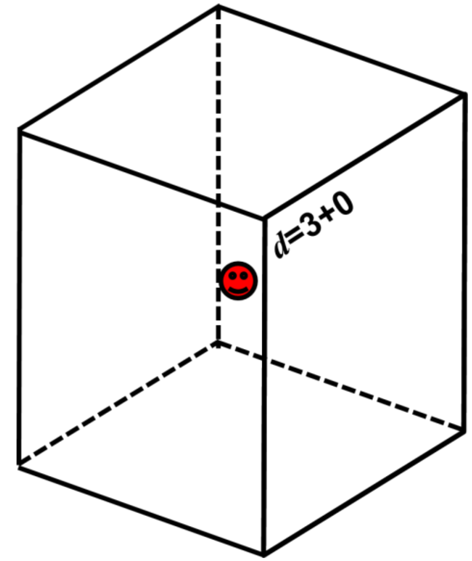
U(*N*) Non-Abelian domain wall
Eff theory = SU(*N*) PCM

Summary



SU(N) Yang-Mills in 4dim

SU(N) YM instanton
Monopole-instanton
SU(N) YM bion



Non-Abelian vortex

1/N instanton charge

Non-Abelian domain wall

C$P^{N\text{-}1}$ model in 2dim

C$P^{N\text{-}1}$ instanton
Domain wall-instanton
C$P^{N\text{-}1}$ bion

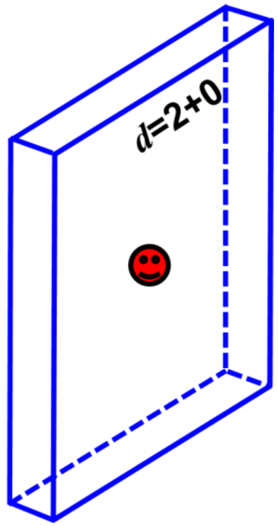
SU(N) PCM in 3dim

SU(N) PCM instanton
Vortex-instanton
SU(N) PCM bion

	4d	3d	2d
force	$1/d^2$	$1/d$	$\exp(-md)$

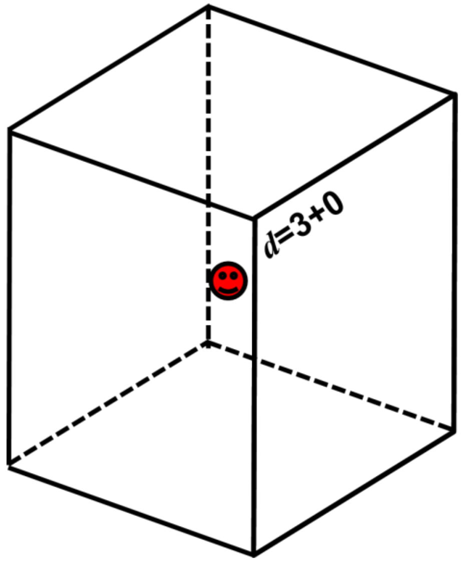
Discussion: Relations among resurgence in 4d, 3d, 2d

Summary



SU(N) Yang-Mills in 4dim

- SU(N) YM instanton
- Monopole-instanton
- SU(N) YM bion



un-Higgsing

1/N instanton charge

un-Higgsing

CP^{N-1} model in 2dim

- CP^{N-1} instanton
- Domain wall-instanton
- CP^{N-1} bion

SU(N) PCM in 3dim

- SU(N) PCM instanton
- Vortex-instanton
- SU(N) PCM bion

	4d	3d	2d
force	$1/d^2$	$1/d$	$\exp(-md)$

Discussion: Relations among **resurgence** in 4d, 3d, 2d

§ CP^1 quantum mechanics

CP¹ quantum mechanics with fermion

$\varphi \to u$ **in previous slides**

potential **fermion** ψ

$$L = \frac{1}{g^2} G\left[\partial_t\varphi\partial_t\bar{\varphi} - m^2\varphi\bar{\varphi} + i\bar{\psi}\mathcal{D}_t\psi + \epsilon m(1+\varphi\,\partial_\varphi \log G)\bar{\psi}\psi\right]$$

$$G = \frac{1}{(1+\varphi\bar{\varphi})^2} \qquad \mathcal{D}_t\psi = \left[\partial_t + \partial_t\varphi\,\partial_\varphi \log G\right]\psi$$

$\varepsilon = 1$ **Supersymmetric QM**

Integrating out fermion

$$L = \frac{1}{g^2}\frac{\partial_t\varphi\partial_t\bar{\varphi}}{(1+\varphi\bar{\varphi})^2} - V(\varphi\bar{\varphi})$$

$$V(\varphi\bar{\varphi}) \equiv \frac{1}{g^2}\frac{m^2\varphi\bar{\varphi}}{(1+\varphi\bar{\varphi})^2} - \epsilon m\frac{1-\varphi\bar{\varphi}}{1+\varphi\bar{\varphi}}$$

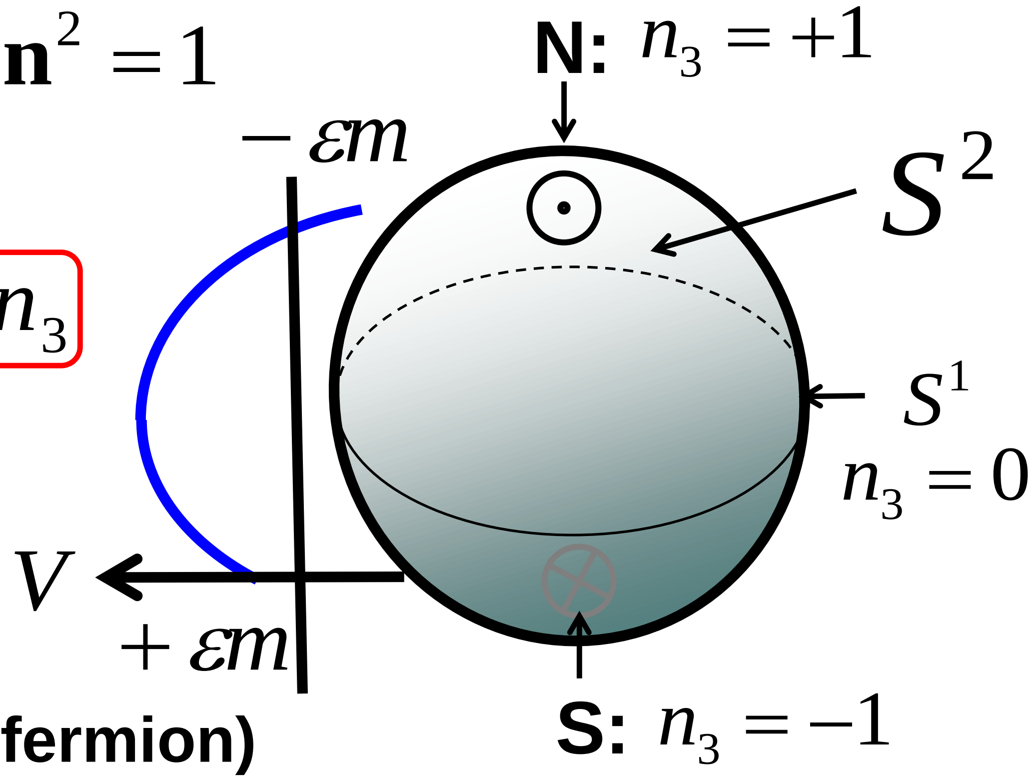
O(3) QM

$$L = \frac{1}{g^2}\partial_t \mathbf{n}\cdot\partial_t \mathbf{n} - V$$

$$\mathbf{n} = (n_1, n_2, n_3)$$

$$\mathbf{n}^2 = 1$$

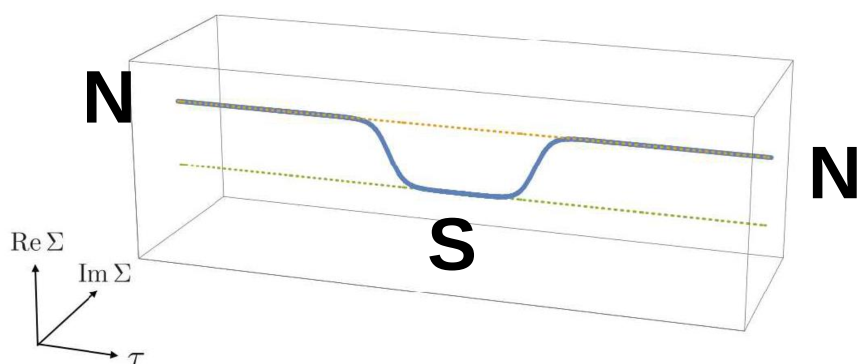
$$V = \frac{m^2}{4g^2}(1 - n_3{}^2) - \varepsilon m n_3$$



Real bion $\lim_{\tau\to\pm\infty}\varphi = \lim_{\tau\to\pm\infty}\bar{\varphi} = 0$

(=kink-antikink stabilized by fermion)

$$\varphi = e^{i\phi_0}\sqrt{\frac{\omega^2}{\omega^2 - m^2}}\frac{1}{i\sinh\omega(\tau-\tau_0)} \qquad \omega \equiv m\sqrt{1+\frac{2\epsilon g^2}{m}}$$



$$h = (1, a_+e^{\omega\tau} + a_-e^{-\omega\tau})$$

$$a_+ = e^{-\omega\tau_+ - i\phi_+}, \qquad a_- = e^{\omega\tau_- - i\phi_-}$$

See Tin's talk; Misumi's & Sakai's talks

complexification

$$(\varphi, \bar{\varphi}) = (\varphi_R + i\varphi_I \, , \, \varphi_R - i\varphi_I) \quad \longrightarrow \quad (\varphi_R^{\mathbb{C}} + i\varphi_I^{\mathbb{C}} \, , \, \varphi_R^{\mathbb{C}} - i\varphi_I^{\mathbb{C}})$$

$$\bar{\varphi} \;\to\; \tilde{\varphi} \;\neq\; \text{complex conjugate of } \varphi.$$

Complex CP¹ Action

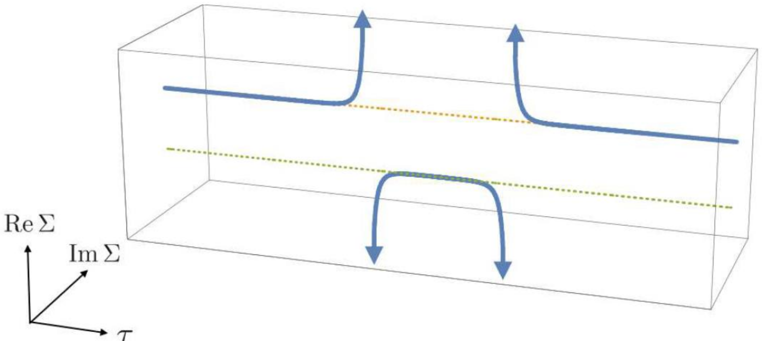
$$S[\varphi, \tilde{\varphi}] \;=\; \int d\tau \left[\frac{1}{g^2}\frac{\partial_\tau\varphi\partial_\tau\tilde{\varphi}}{(1+\varphi\tilde{\varphi})^2} + V(\varphi\tilde{\varphi})\right]$$

$$\varphi \;=\; e^{i\phi_0}\sqrt{\frac{\omega^2}{\omega^2 - m^2}}\frac{1}{i\sinh\omega(\tau-\tau_0)}$$

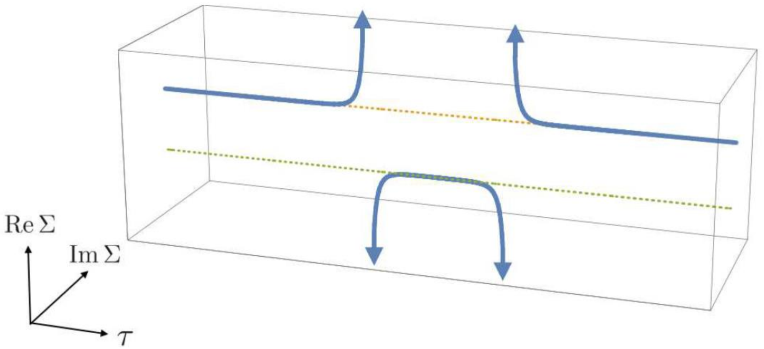
real bion

$$\tau_0 \;\to\; \tilde{\tau}_0 = \tau_0 + \frac{1}{\omega}\frac{\pi i}{2}$$

$$\varphi = e^{i\phi_0}\sqrt{\frac{\omega^2}{\omega^2 - m^2}}\frac{1}{\cosh\omega(\tau-\tau_0)}, \qquad \tilde{\varphi} = -e^{-i\phi_0}\sqrt{\frac{\omega^2}{\omega^2 - m^2}}\frac{1}{\cosh\omega(\tau-\tau_0)}.$$



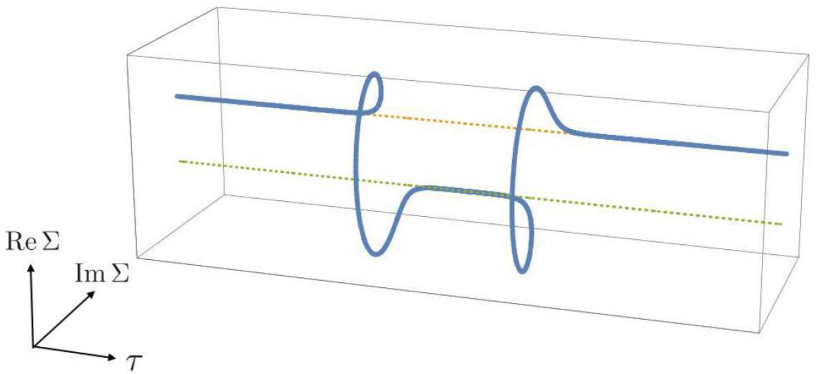
Singular complex bion



Singular complex bion

$$\theta \equiv \arg g^2$$

Small imarginary



regular complex bion

Thank you for your attention!!

References

[1] Misumi, MN & Sakai, JHEP 1406 (2014)164 [arXiv:1404.7225]

[2] Misumi, MN & Sakai, PTEP (2015) 033B02 [arXiv:1409.3444]

[3] MN, JHEP 1503 (2015) 108 [arXiv:1412.7681]

[4] MN, JHEP 1508 (2015) 063 [arXiv:1503.06336]

[5] Misumi, MN & Sakai, in preparation

See also

[6] Misumi, MN & Sakai, JHEP 1509 (2015)157[arXiv:1507.00408]

Sakai's talk: resurgence of SG QM

[7] Misumi, MN & Sakai, JHEP 1605 (2016)057[arXiv:1604.00839]

Misumi's talk: non-BPS exact sol in CP(N)

[8] Fujimori, Kamata, Misumi, MN & Sakai, arXiv:1607.04205

(cancelled) Fujimori's talk: complex bions

Keio U. has started Topological Science Project.

Looking for 1 or 2 postdocs starting in Oct.

contact: nitta (at) phys-h.keio.ac.jp