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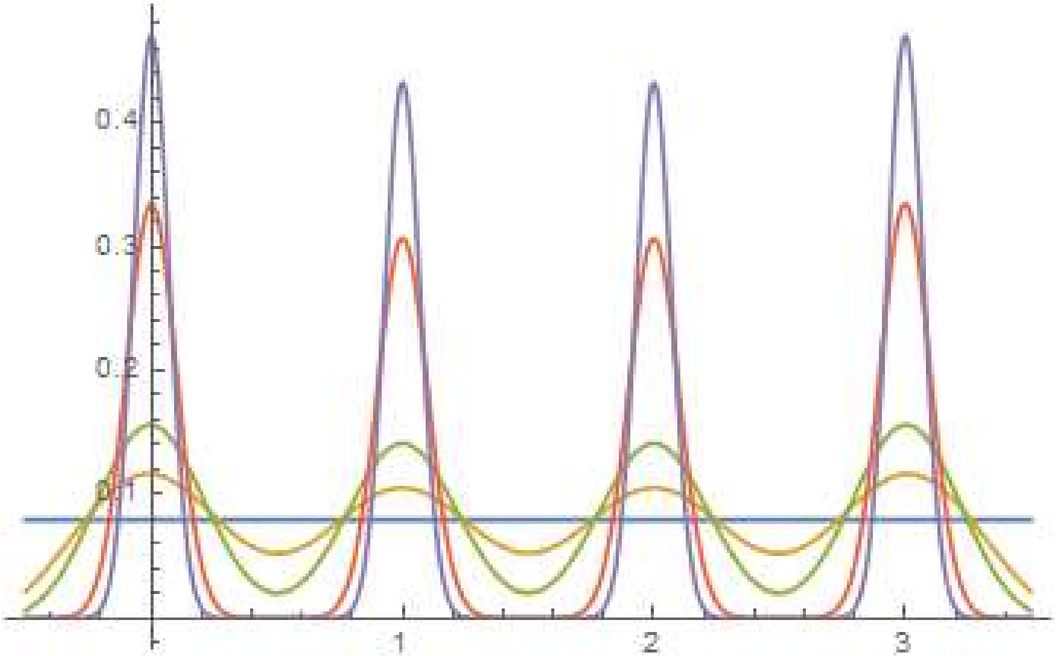

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Imaginary time flow deformations of Laughlin states

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Laughlin states

We are interested in understanding 2-dimensional systems of many electrons under the influence of a magnetic field.

In 1983, Robert Laughlin proposed explicit expressions for ground state wave functions of charged particles under the influence of an uniform magnetic field.

Hamiltonian mechanics

In Hamiltonian mechanics, equations of motion are given, in canonical coordinates (x, p), by

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{\partial H}{\partial p} \\ \dfrac{dp}{dt} = -\dfrac{\partial H}{\partial x} \end{cases}$$

where H is a function in phase space known as the Hamiltonian, often representing the total energy of the system.

Hamiltonian mechanics

We can rewrite Hamilton's equations as the flow of the vector field X_H.

$$\frac{d(x,p)}{dt} = X_H \qquad X_H = \left(\frac{\partial H}{\partial p}, -\frac{\partial H}{\partial x}\right)$$

Symplectic Geometry

Provides a straightforward generalization of Hamiltonian mechanics.

$$\omega = \sum_{i=1}^{n} dp_i \wedge dx_i \qquad \omega\left(X_H, \cdot\right) = -dH\left(\cdot\right)$$

Symplectic Geometry

Provides a straightforward generalization of Hamiltonian mechanics.

A symplectic manifold is a pair (M, ω) where M is a smooth manifold and ω is a closed nondegenerate 2-form on M.

Quantization

Phase space → Hilbert space

Points in phase space → Quantum states

Functions → Linear operators

Quantization on Euclidean space

The Euclidean space with canonical symplectic form can be quantized by defining the quantum states to be the position (or momentum) dependant square-integrable functions $\psi(x)$ in phase space.

$$x \;\rightarrow\; \mathrm{X}(\psi) = x\,\psi$$

$$\mathrm{p} \;\rightarrow\; \mathrm{P}(\psi) = -i\hbar\frac{\partial\psi}{\partial x}$$

Polarizations

Definition 2.4.2 (Polarization)**.** If (M, ω) is a symplectic manifold, a polarization is a distribution P on the complexification of the tangent bundle $T^{\mathbb{C}}M$ such that

- it is Lagrangian, i.e., $\forall x \in M \quad \dim P_x = n \quad \text{and} \quad \forall X, Y \in P_x \quad \omega_x(X, Y) = 0$;
- it is involutive, i.e., $[P, P] \subset P$;
- The dimension of $P_x \cap \overline{P_x}$ is constant;
- $P + \overline{P}$ is also involutive.

Kähler geometry

If (M, ω) is a complex, symplectic manifold such that ω is locally given by

$$\omega = i\partial\overline{\partial}h$$

where h is a local smooth function and

$$\gamma(\cdot,\cdot) := \omega(\cdot, J\cdot)$$

is a Riemannian metric. Then the triple (M, ω, J) is called a Kähler manifold and γ the Kähler metric.

Deformation of Kähler structures

We pick some hamiltonian H and consider the diffeomorphism defined by

$$z^j \mapsto z_s^j := e^{-isX_H} z^j$$

thus obtaining a deformation of the Kähler structure

$$(M, \omega, J) \to (M, \omega, J_s)$$

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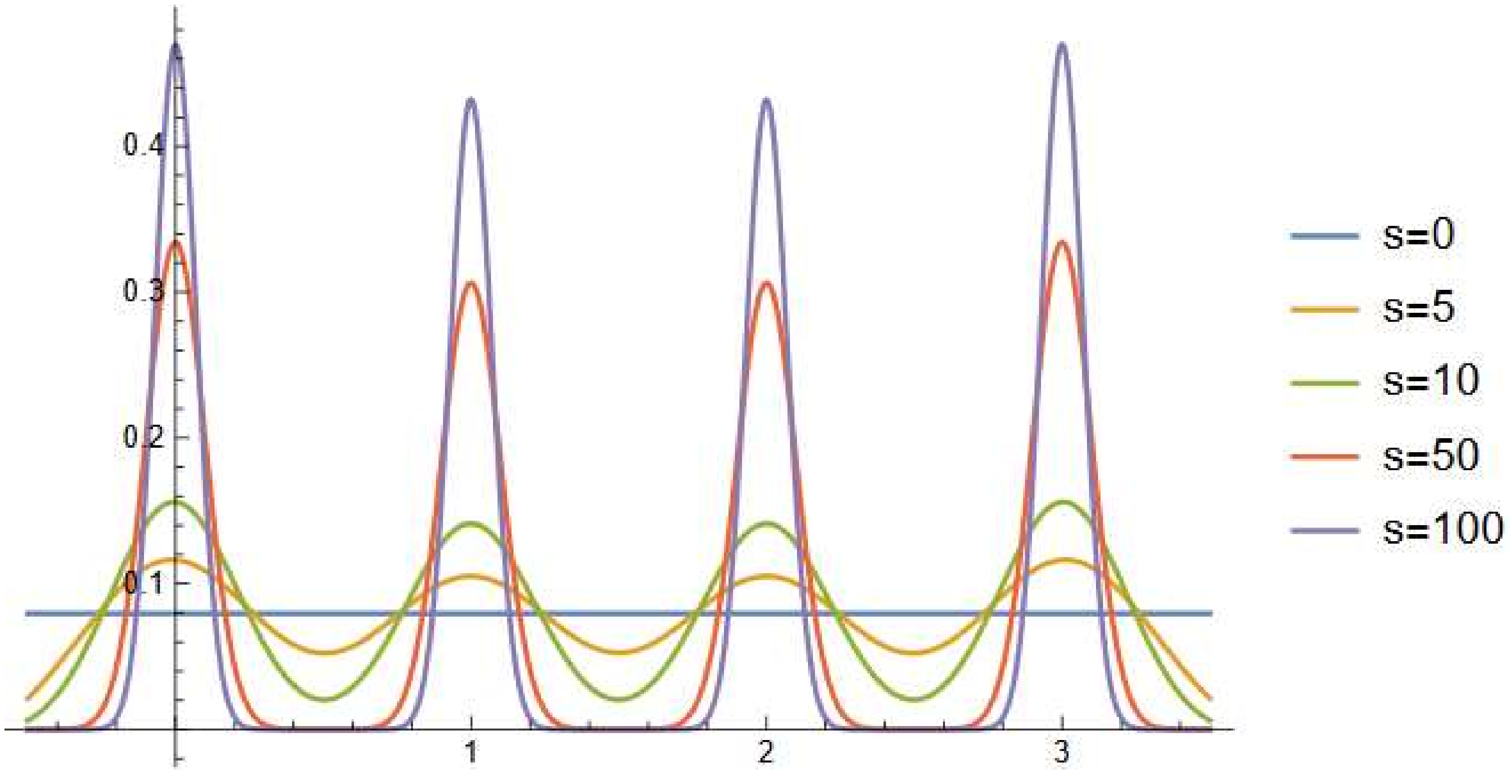
Generalized Coherent State Transform

The deformations defined previously are transformations of the Kähler structure, which describes the classical system.

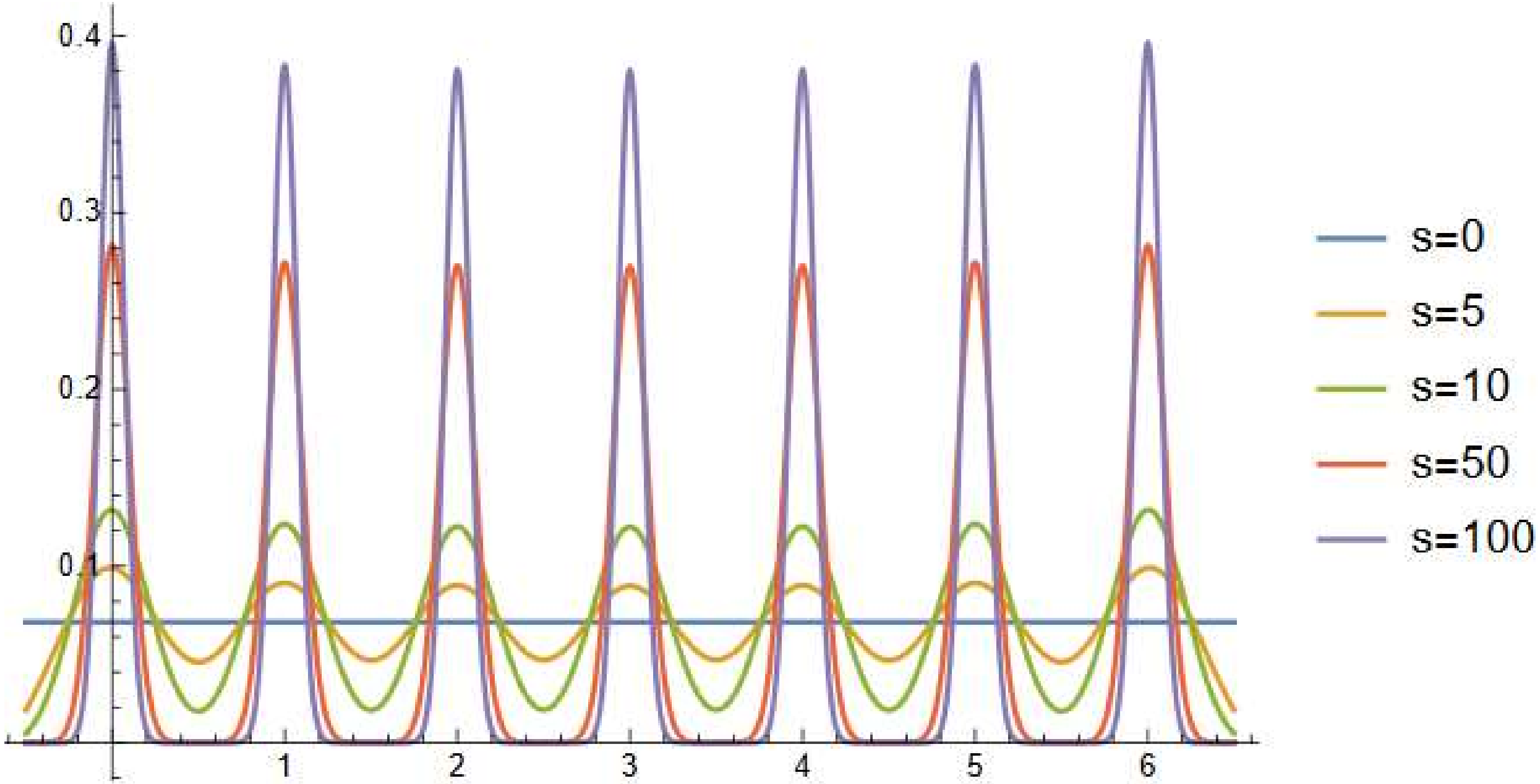
How do we lift these transformations to the Hilbert space in order to deform the quantum states?

In this thesis, we provided evidence that supports a transformation of quantum states known as Generalized Coherent State transform, which differs from other (perhaps more intuitive) transformations.

Density profiles



Density profiles



Thank you!