Symbolic Formulation for Principal Component Analysis of Interval-Valued Data

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supervised by
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Overview

Analyse grades of students.

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Conventional approach:

- average
- variance
- minimum and maximum
- ...

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Single valued variables summarise object of interest.

Analyse grades of students.

Symbolic approach:

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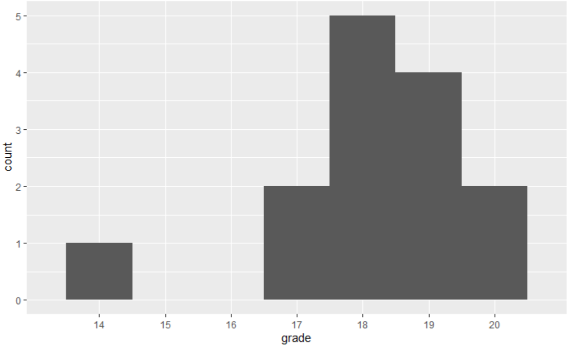
Symbolic approach:

- histogram



Analyse grades of students.

Symbolic approach:

- histogram
- interval: $[14, 20]$
- ...

Analyse grades of students.

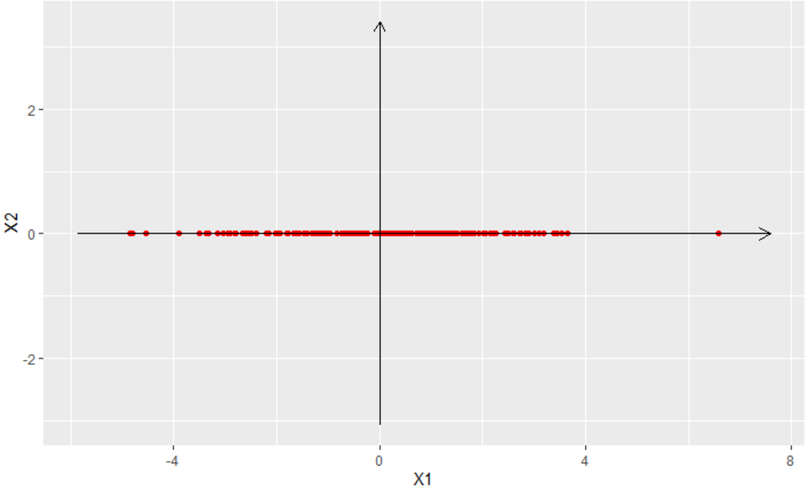
Symbolic approach:

- histogram
- interval
- ...

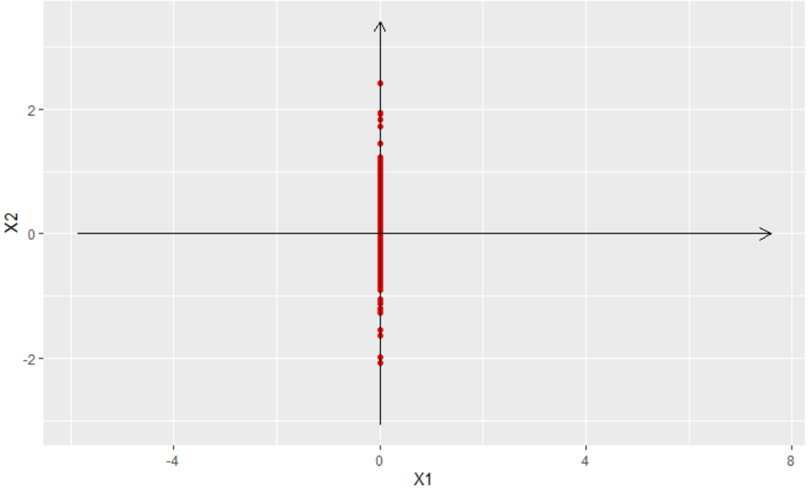
Data account for intrinsic variability.

Let us study some 2d data.

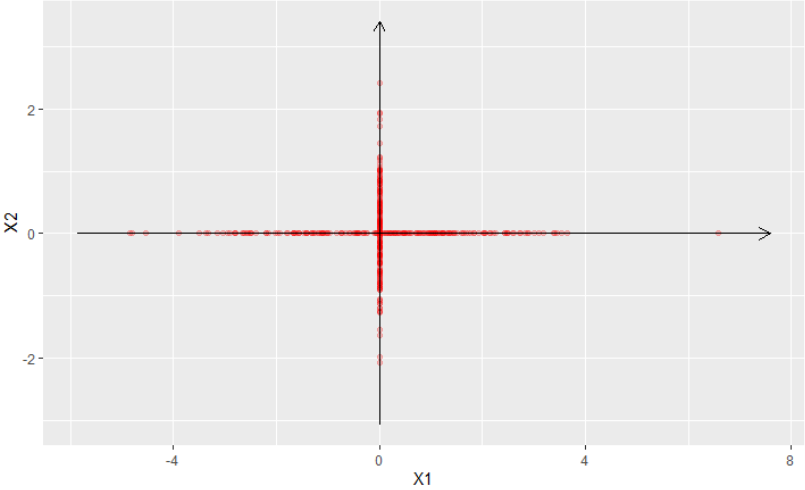
Let us study some 2d data... along the x axis.



Let us study some 2d data... along the y axis.

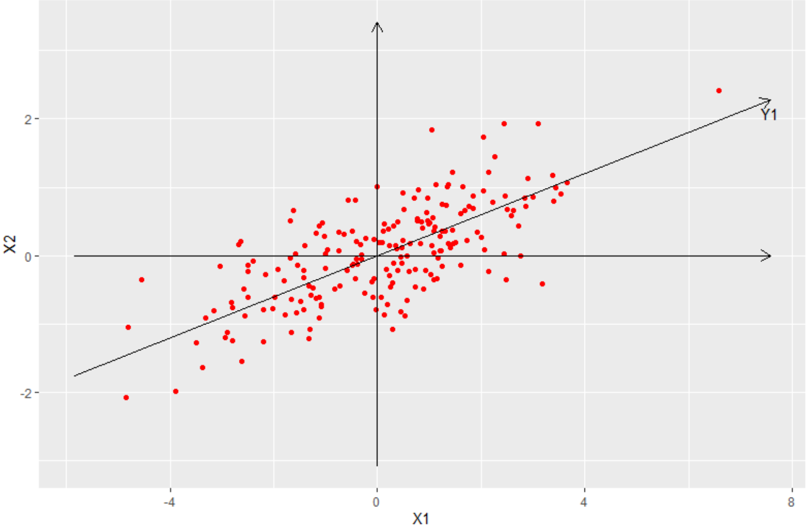


Let us study some 2d data... which one is best?



Is that the best we can do?

Is that the best we can do? – No



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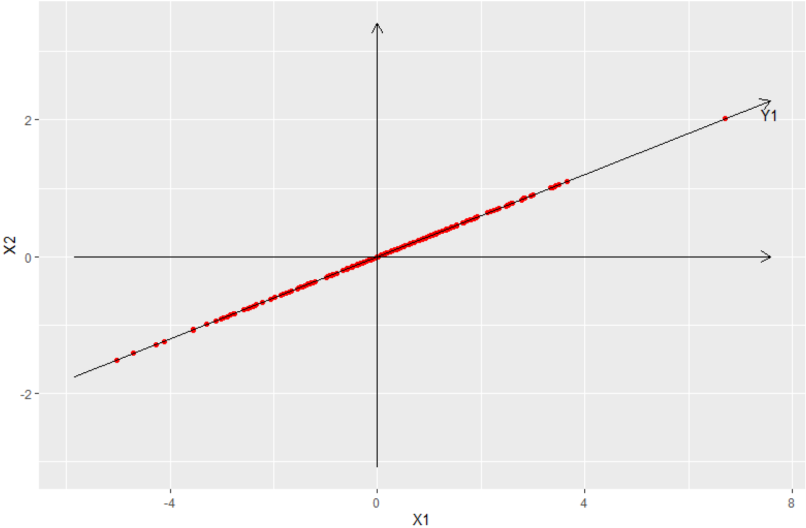

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$$\gamma_1 = \arg\max_{\gamma : ||\gamma|| = 1} \mathrm{Var}(\gamma^T \boldsymbol{X})$$

Random vector $\boldsymbol{X} = (X_1, \cdots, X_p)^T$

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$$\gamma_i = \begin{cases} \arg\max_{\gamma : ||\gamma||=1} \mathrm{Var}(\gamma^T \boldsymbol{X}) \\ \mathrm{Cov}(\gamma^T \boldsymbol{X}, \gamma_j^T \boldsymbol{X}) = 0,\ j = 1, \cdots, i-1, \quad \text{if} \quad i > 1 \end{cases}$$

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$$\gamma_i = \begin{cases} \arg\max_{\gamma:||\gamma||=1} \text{Var}_k(\gamma^T \boldsymbol{X}) \\ \gamma^T \gamma_j = 0, \ j = 1, \cdots, i-1, \quad \text{if} \quad i > 1 \end{cases}$$

Some other tasks:

- Implement the method;
- Create a robust variant;
- Explore real data;

Thank you.