Non-BPS solutions and Bions in CPN models

Tatsu MISUMI   Akita U. / Keio U.

Mainly based on TM, M.Nitta, N.Sakai, JHEP05(2016)057 [arXiv:1604.00839]
partly based on 1404.7225, 1409.3444, 1507.00408, 1607.04205

07/18/16@Resurgence2016, Lisbon

◆Resurgence theory in QM and Matrix models

Ecalle, et.al.(61~) Schiappa,Marino,Aniceto,Vaz,Vonk,Russo, et.al.(08~) Argyres,Dunne,Unsal, et.al.(12~)

Series of perturbative series around nontrivial backgrounds : Trans-series

$$\mathcal{P}(g^2) \;=\; P(g^2) \;+\; \sum_{\alpha} C_\alpha e^{-S_\alpha/g^2} P_\alpha(g^2) \qquad\qquad P_\alpha(g^2) = \sum_{n=0} a_n^\alpha g^{2n}$$

➡ Non-perturbative result? $0 = \mathrm{Im}(\mathbb{B}_{[0,0]} + \mathbb{B}_{[1,1]}[\mathcal{I}\bar{\mathcal{I}}] + \mathbb{B}_{[2,2]}[\mathcal{I}\mathcal{I}\bar{\mathcal{I}}\bar{\mathcal{I}}] + \ldots)$

- Resurgence trans-series may give a consistent definition of QT
- We may relate perturbative and nonperturbative contributions

cf.) Resurgence theory in ODE

$$\varphi_\pm(z;C) = \mathcal{S}_\pm\varphi_0(z) \;+\; \sum_{l=1}^{\infty} C^l e^{-lAz}\mathcal{S}_\pm\varphi_l(z)$$ Solution as trans-series

$$\varphi_+(C) = \varphi_-(C+\mathfrak{s})$$ Stokes phenomena & Bridge eq.

$$\mathcal{S}_+\varphi_0(z) - \mathcal{S}_-\varphi_0(z) \approx \mathfrak{s}e^{-Az}\mathcal{S}\varphi_1(z)$$ Relation between different sectors

Resurgence and Bion configurations in QM

- In Resurgent trans-series in Quantum mechanics with degenerate vacua, Instanton-Antiinstanton (Bion) configurations play vital roles.

  Zinn-Justin(81) Jentschura, Zinn-Justin(04) Dunne, Unsal(13) TM, Sakai, Nitta(15)

- In complexified QM with fermions, bions exist as exact complex solutions.

  Behtash, Dunne, Schafer, Sulejmanpasic,Unsal(15) Fujimori, Kamata,TM, Nitta, Sakai (16)

- One-loop and Lefschetz thimble integrals around complex and real solutions are calculated, which are consistent with exact nonperturbative results.

  Fujimori, Kamata,TM, Nitta, Sakai (16)

- At QM levels, the relevance of Bion solutions(configurations) is clarified.

How about field theory? How about bion molecules?

Bions in 2D CPN models with R1 x S1

Dunne, Unsal (12)

- Bion configuration contains two types of quasi modulus (separation and phase), whose moduli integrals give bion contributions.

  TM, Nitta, Sakai (14)(15)

- 2D CPN sigma model is reduced to CPN quantum mechanics.

  TM, Nitta, Sakai (15)

- Complex and real bion solutions in complexified CPN QM are found.

  Fujimori, Kamata, TM, Nitta, Sakai (16)

- One-loop and Lefschetz thimble integrals of complex and real bions are in precise agreement with exact results for SUSY and near-SUSY cases.

  Fujimori, Kamata, TM, Nitta, Sakai (16)

Do bion-molecule saddle points exist? ⇒ Non-BPS solutions

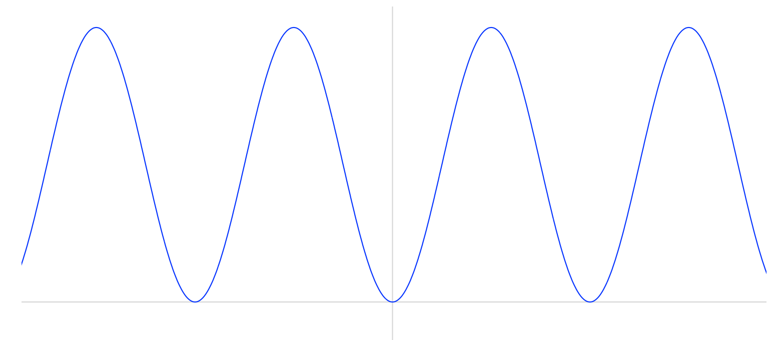
Review of Sine-Gordon QM

Detailed talk by Prof. Sakai on Friday 7/22

◆ Imaginary ambiguity in SG Quantum Mechanics

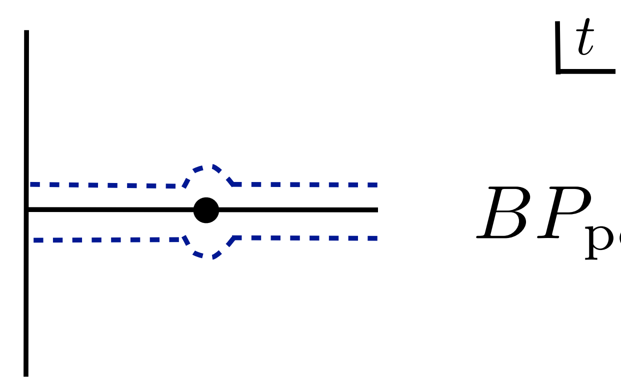
- sine-Gordon QM

$$-\frac{1}{2}\frac{d^2}{dx^2}\psi(x) + \frac{1}{8g^2}\sin^2(2gx)\,\psi(x) = E\,\psi(x)$$



$$\Rightarrow \quad E_{pert} = \sum_{k=0} a_k\, g^{2k}$$

$$a_k = -\frac{2}{\pi}k! \qquad \text{(for large } k\text{)}$$



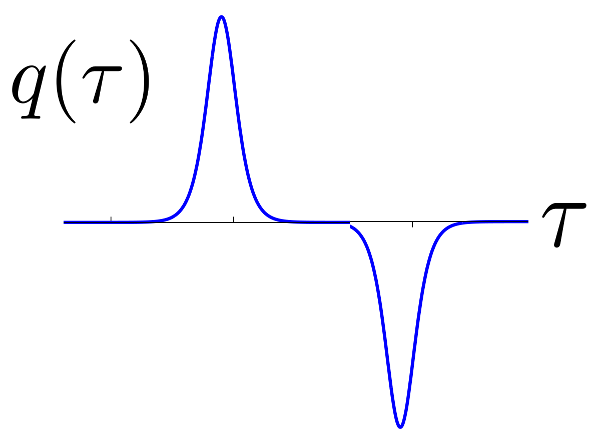
$$BP_{\rm pert}(t) = \frac{C}{1-t}$$

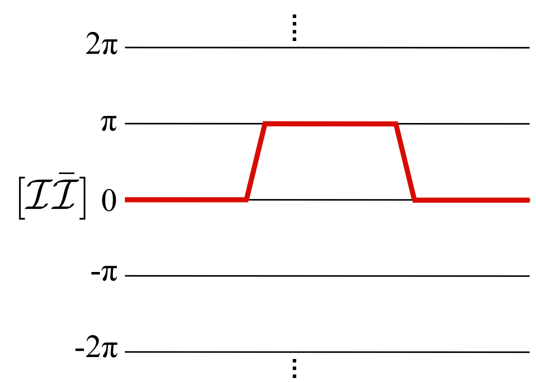
$$\Rightarrow \quad \mathrm{Im}\left[\mathbb{B}_{\rm pert}(g^2)\right] = \mathrm{Im}\left[\int_0^\infty \frac{dt}{g^2} e^{-t/g^2} \frac{C}{1-t}\right] = \boxed{\mp\frac{2\,e^{-1/g^2}}{g^2}} \qquad \boxed{S_I = \frac{1}{2g^2}}$$

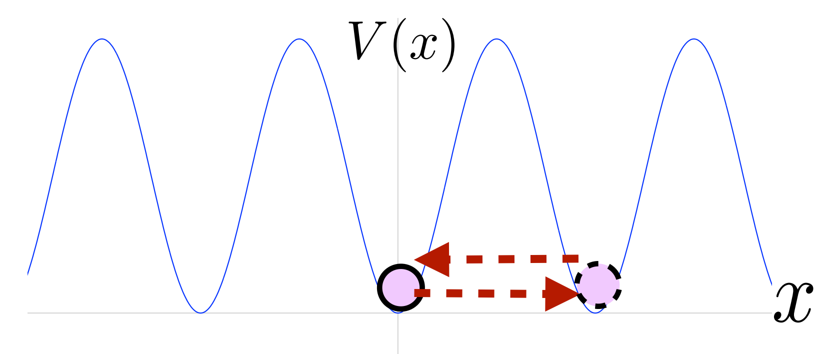
What does the imaginary ambiguity indicate?

◆Instanton + anti-instanton (Bion) in SG

$$x_{\mathcal{I}\bar{\mathcal{I}}}(\tau) \;=\; \frac{1}{g}\,\mathrm{arctan}\, e^{\tau-\tau_{\mathcal{I}}} + \frac{1}{g}\,\mathrm{arctan}\, e^{-\tau+\tau_{\bar{\mathcal{I}}}} + n\pi/(2g)$$

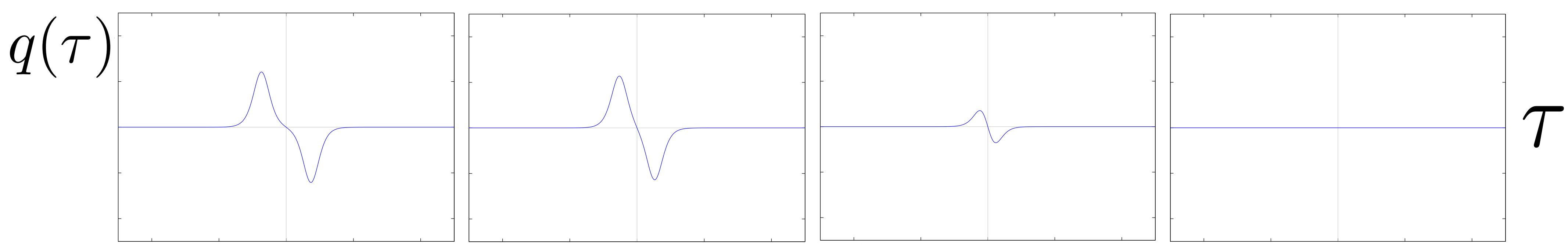






- Not a solution, but a possible configuration
- Effective attractive force for large separation

$$V_{\mathcal{I}\bar{\mathcal{I}}}(R) \;=\; -\frac{2}{g^2}\,\mathrm{exp}[-R]$$



Quasi-moduli integral

$$[\mathcal{I}\bar{\mathcal{I}}]\xi^{-2} = \int_{-\infty}^{\infty} dR\, \exp\left(-\frac{2}{-g^2}e^{-R} - \epsilon R\right)$$

$$\xi \equiv e^{-S_I}/\sqrt{\pi g^2}$$

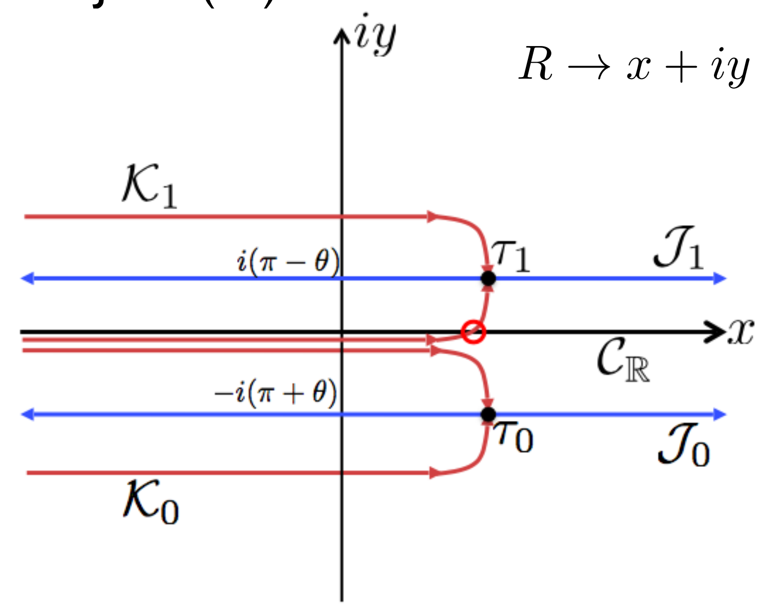
◆Contribution from bion configuration

$$[\mathcal{I}\bar{\mathcal{I}}]\xi^{-2} = \int_{-\infty}^{\infty} dR \, \exp\left(-\frac{2}{-g^2}e^{-R} - \epsilon R\right)$$

- The integral is ill-defined due to attractive force
- Semiclassical dilute-gas description is broken down

・ Bogomolny--Zinn-Justin prescription   Bogomolny(80) Zinn-Justin(81)

1. regards $-g^2$ as positive,
2. perform quasi-moduli integral
3. analytically continue to $-g^2 = e^{\mp i\pi} g^2$



$n_1 = \langle \mathcal{C}_{\mathbb{R}}, \mathcal{K}_1 \rangle = 1$

understood in terms of Lefschetz thimble integral

Behtash,Poppitz,Sulejmanpasic,Unsal(15)   Fujimori, Kamata, TM, Nitta, Sakai (16)

$$\triangle E^{(1,1)} = \xi^2 \left[2\left(\gamma + \log\frac{2}{g^2}\right) \pm 2i\pi\right] \Rightarrow \pm\frac{2e^{-1/g^2}}{g^2}$$

Imaginary ambiguities cancel out !

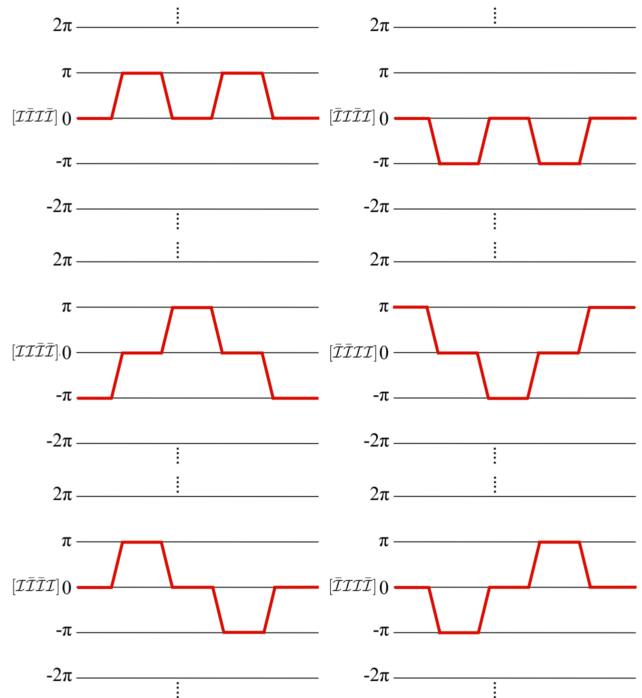
Zinn-Justin(81)

$$\mathrm{Im}[\mathbb{B}_{pert} + [\mathcal{I}\bar{\mathcal{I}}]] = 0$$

up to e^{-4S_I}

◆ Bion molecules in SG QM

Dunne, Unsal (14) TM, Nitta, Sakai (15)



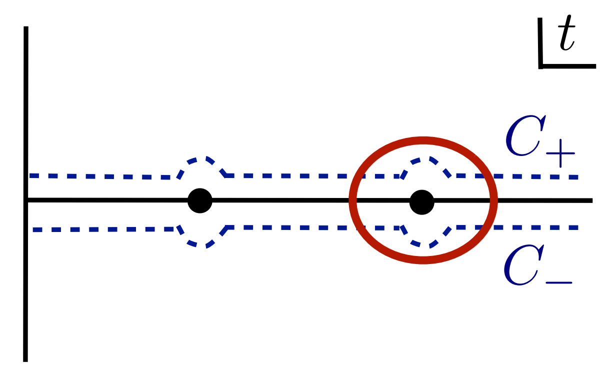
$$[\mathcal{I}\mathcal{I}\cdots\bar{\mathcal{I}}\bar{\mathcal{I}}]_{\rm all} = \left(\frac{e^{-S_{\mathcal{I}}}}{\sqrt{\pi g^2}}\right)^{n+m} e^{i(n-m)\theta} \int dR_1 dR_2 ... dR_{n+m-1}\, e^{-V[R_1]-V[R_2]-\cdots V[R_{n+m-1}]}$$

$$\Rightarrow ([\mathcal{I}\bar{\mathcal{I}}\mathcal{I}\bar{\mathcal{I}}] + [\bar{\mathcal{I}}\mathcal{I}\bar{\mathcal{I}}\mathcal{I}] + [\mathcal{I}\mathcal{I}\bar{\mathcal{I}}\bar{\mathcal{I}}] + [\bar{\mathcal{I}}\bar{\mathcal{I}}\mathcal{I}\mathcal{I}] + [\mathcal{I}\bar{\mathcal{I}}\bar{\mathcal{I}}\mathcal{I}] + [\bar{\mathcal{I}}\mathcal{I}\mathcal{I}\bar{\mathcal{I}}])\xi^{-4}$$

$$= -16\left(\gamma + \log\frac{2}{g^2}\right)^3 + 22\pi^2\left(\gamma + \log\frac{2}{g^2}\right) + \psi^{(2)}(1) \mp i\pi\left[32\left(\gamma + \log\frac{2}{g^2}\right)^2 - \frac{16\pi^2}{3}\right]$$

cancels the imaginary ambiguity at higher orders

$$a_k = \left(\frac{1}{2}\right)^{k+2} (k+1)!\,\left[C + O(\log k/k)\right]$$



$$e^{-4S_I}$$

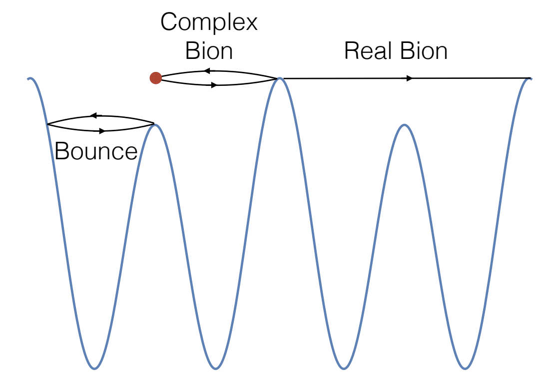
Uniform WKB by Prof. Sakai, Friday 7/22

◆Complexified SG quantum mechanics

Behtash, Dunne, Schafer, Sulejmanpasic, Unsal(15)

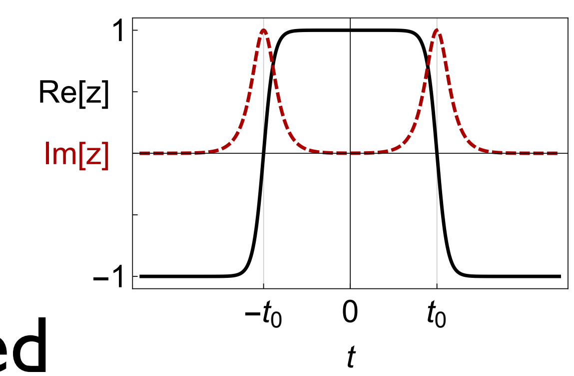
- Bions as solutions in complexified Sine-Gordon QM

$$Z = \int_\Gamma Dz\, e^{-\frac{1}{\hbar}S[z(t)]}, \quad S[z(t)] = \int dt\, \left(\tfrac{1}{2}\dot{z}^2 + V(z)\right)$$



$$\frac{d^2x}{dt^2} = +\frac{\partial V_r}{\partial x} \quad , \quad \frac{d^2y}{dt^2} = -\frac{\partial V_r}{\partial y} \quad \Rightarrow \quad z_{cb}(t) = 2\pi \pm 4\left(\arctan e^{-\omega_{cb}(t-t_0)} + \arctan e^{\omega_{cb}(t+t_0)}\right)$$

Non-perturbative effects is expected to be described by contribution from complexified solutions.



⇒ Exact results for SUSY case should be reproduced

$$-\lim_{\beta\to\infty}\frac{Z_1}{Z_0} = \frac{m}{2\pi}(1+e^{\pm 2\pi i\epsilon})\,\Gamma(2\epsilon)\exp\left[-\frac{2m}{g^2} + (2\epsilon-1)\log\frac{g^2}{4m}\right]$$

One-loop and thimble integrals justify this argument.

Fujimori, Kamata, TM, Nitta, Sakai (16)

See arXiv:1607.04205

Bions in 2D CP^N-1 model

◆Notation in CP^N-1 model

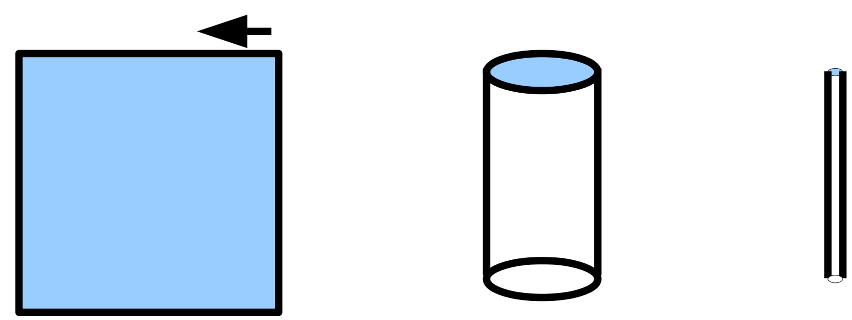
$$S = \frac{1}{g^2}\int d^2x (D_\mu n)^\dagger (D_\mu n)\,, \qquad n(x) \equiv \omega(x)/|\omega(x)|$$

$$Q = \int d^2x\; i\epsilon_{\mu\nu}(D_\mu n)^\dagger (D_\nu n) \qquad A_\mu(x) \equiv -in^\dagger \partial_\mu n$$

- Spatial compactification S^1

$$-\infty \leq x_1 \leq \infty$$

$$0 \leq x_2 \leq L$$

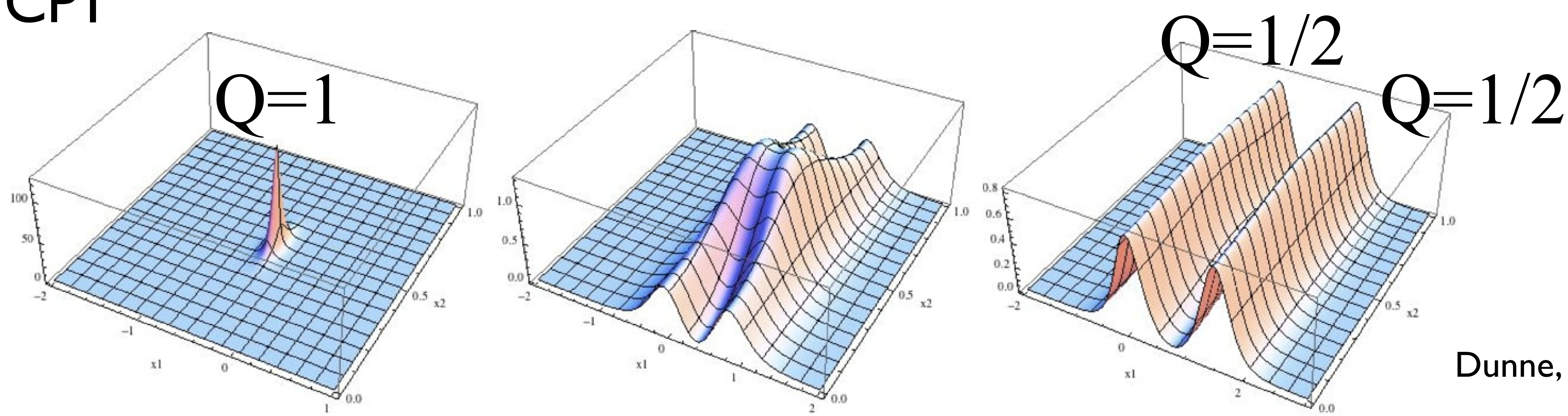


- Z_N twisted b.c. in S^1 direction (favored vacuum for the case with fermions)

$$\omega(x_1, x_2 + L) = \Omega\, \omega(x_1, x_2)\,, \qquad \Omega = \mathrm{diag.}\left[1, e^{2\pi i/N}, e^{4\pi i/N}, \cdots, e^{2(N-1)\pi i/N}\right]$$

◆ Fractional instantons in CP^N-1 on R1 × S1

cf.) CP1

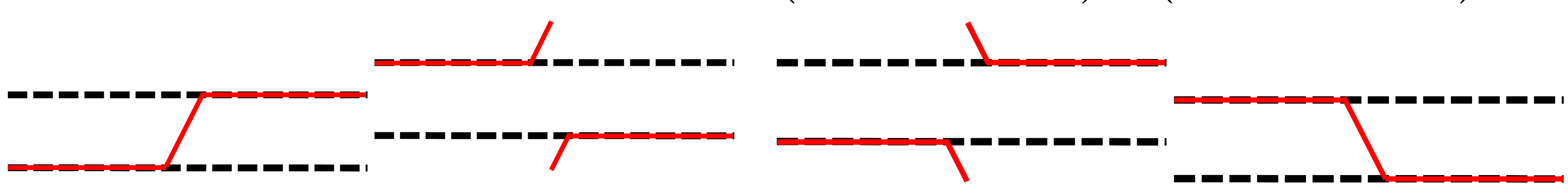


Dunne, Unsal(12)

BPS instanton : $\omega = (1, \lambda_1\, e^{\pi z/L} + \lambda_2\, e^{-\pi z/L}) \qquad z = x_1 + i x_2$

separation : $\tau = -\dfrac{L}{\pi} \log \lambda_1 \lambda_2$

Fractional instanton : $\omega \ = \ (1, \lambda\, e^{\pm \pi z/L}) \quad (1, \lambda\, e^{\pm \pi \bar{z}/L})$



Talk by Prof. Nitta, Wednesday 7/20

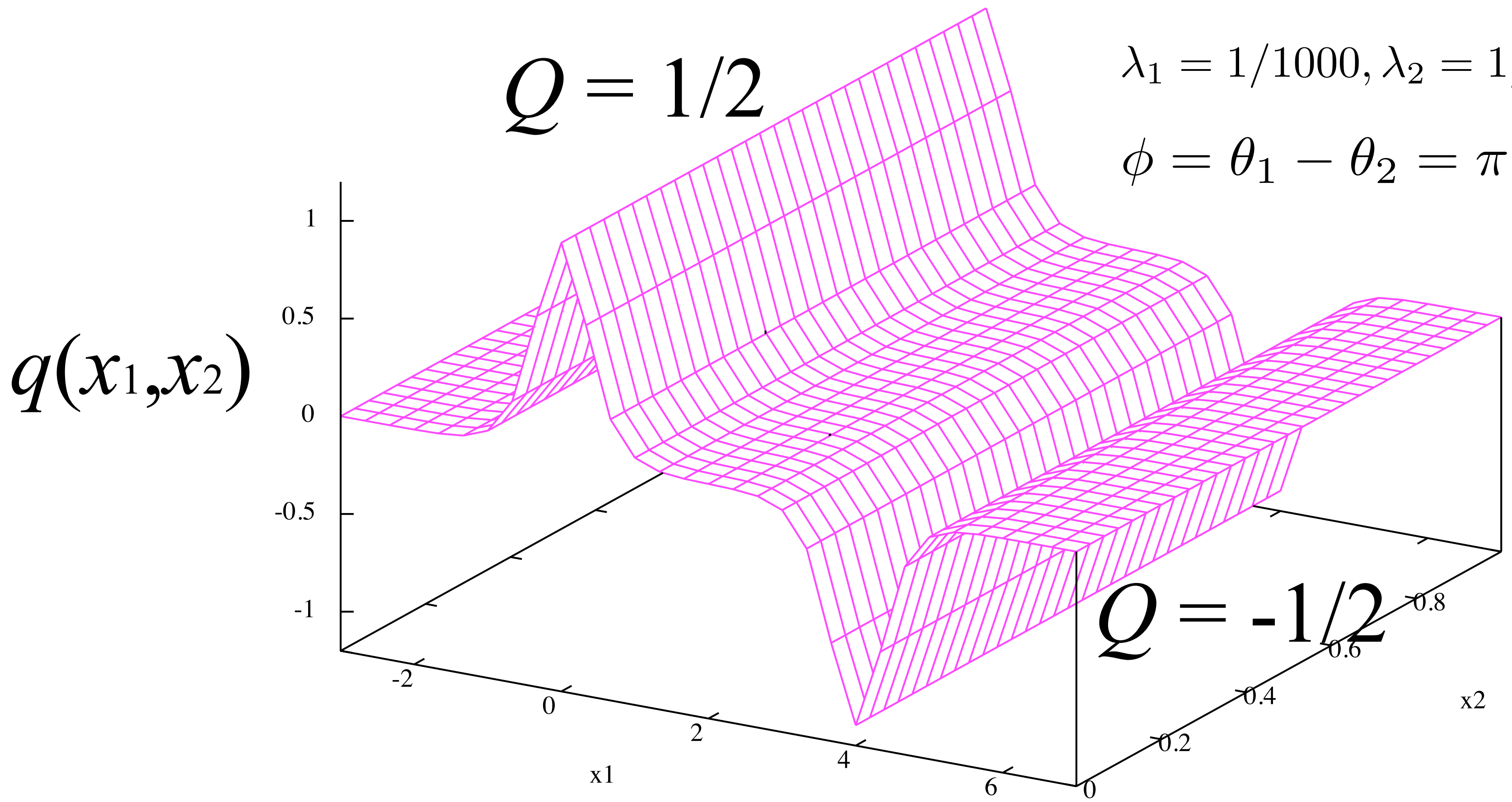
◆Neutral bions in CP^N-1 model on R1 × S1

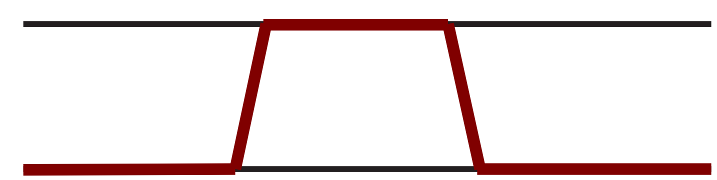
Dunne,Unsal (12)
TM, Nitta, Sakai (14)

cf.) CP1 $$\omega = \left(\lambda_1 e^{i\theta_1} e^{-\pi z} + \lambda_2 e^{i\theta_2} e^{\pi \bar{z}},\, 1\right)^T \qquad z = x_1 + i x_2$$

$$\lambda_1 = 1/1000, \lambda_2 = 1/1000$$

$$\phi = \theta_1 - \theta_2 = \pi/4$$





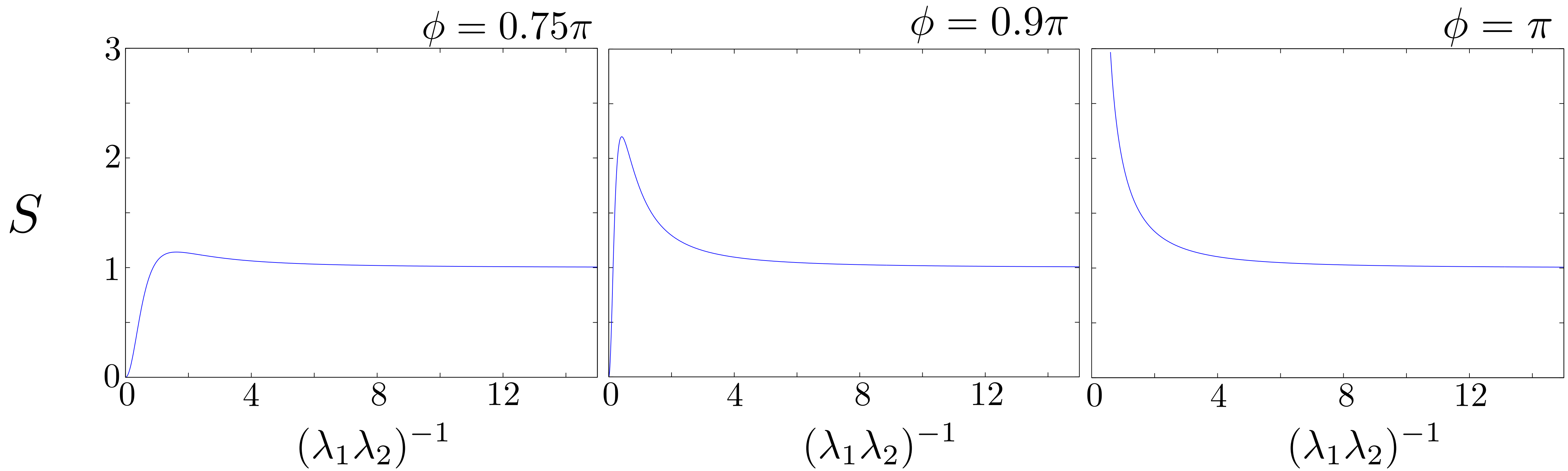
separation : $$\tau = -\frac{L}{\pi} \log \lambda_1 \lambda_2$$

It is notable that it has no x2-dependence

◆Phase moduli of bions in CP^N-1 model

TM, Nitta, Sakai (14)

cf.) CP1 $\omega = \left(\lambda_1 e^{i\theta_1} e^{-\pi z} + \lambda_2 e^{i\theta_2} e^{\pi \bar{z}},\, 1\right)^T$ $\phi \equiv \theta_1 - \theta_2$



$$V[R] = -\frac{4\kappa L}{g^2}\cos\phi\, e^{-\kappa R} \qquad \kappa = \frac{2\pi}{LN}$$

Effective interaction potential depends on relative phase

◆Quasi-moduli integral of CP^N-1 bion

TM, Nitta, Sakai (15)

$$V[R] = -\frac{4\kappa L}{g^2}\cos\phi\, e^{-\kappa R} \qquad \kappa = \frac{2\pi}{LN} \qquad 0 \leq \phi < 2\pi$$

- Quasi moduli integral over separation & relative phase

$$\begin{aligned}
&\int_0^{2\pi} d\phi\, I(v^2\cos\phi) \\
&\quad = 2\left[\int_0^{\pi/2} d\phi \left\{-\left(\gamma + \log\left(\frac{8\pi v^2}{N}\cos\phi\right)\right) \mp i\pi\right\} + \int_{\pi/2}^{\pi} d\phi \left\{-\left(\gamma + \log\left(-\frac{8\pi v^2}{N}\cos\phi\right)\right)\right\}\right] \\
&\quad = -2\pi\left(\gamma + \log\left(\frac{4\pi v^2}{N}\right)\right) \mp i\pi^2.
\end{aligned}$$

$$\Rightarrow \quad [\mathcal{B}_{ii}] = Ce^{-\frac{2S_I}{N}}\left[-2\pi\left(\gamma + \log\left(\frac{4\pi v^2}{N}\right)\right) \mp i\pi^2\right]$$

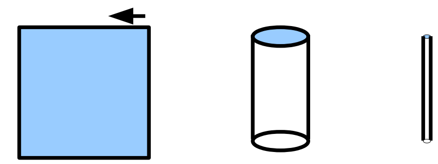
This is what we want to compare with perturbative Borel resummation in CPN sigma model.

Bion solutions in complexified CPN model

See arXiv:1607.04205

◆Complexified CPN quantum mechanics

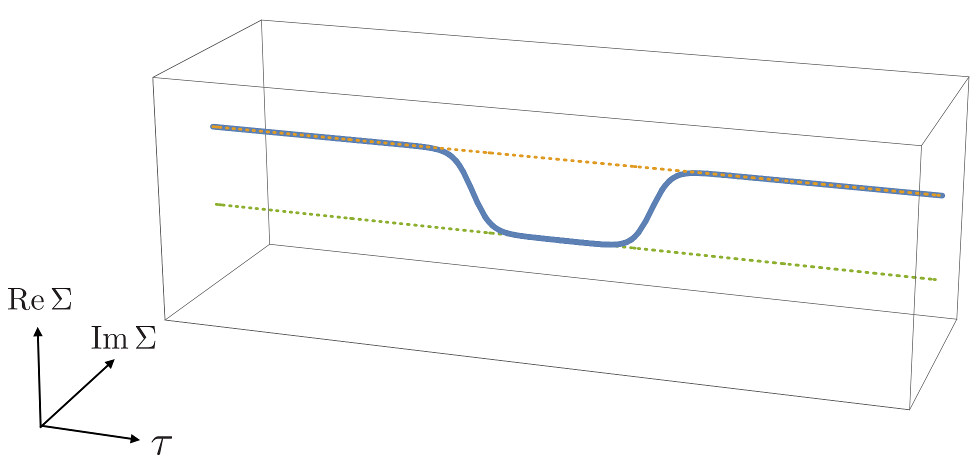
Fujimori, Kamata, TM, Nitta, Sakai (16)

- Reduced quantum mechanics 

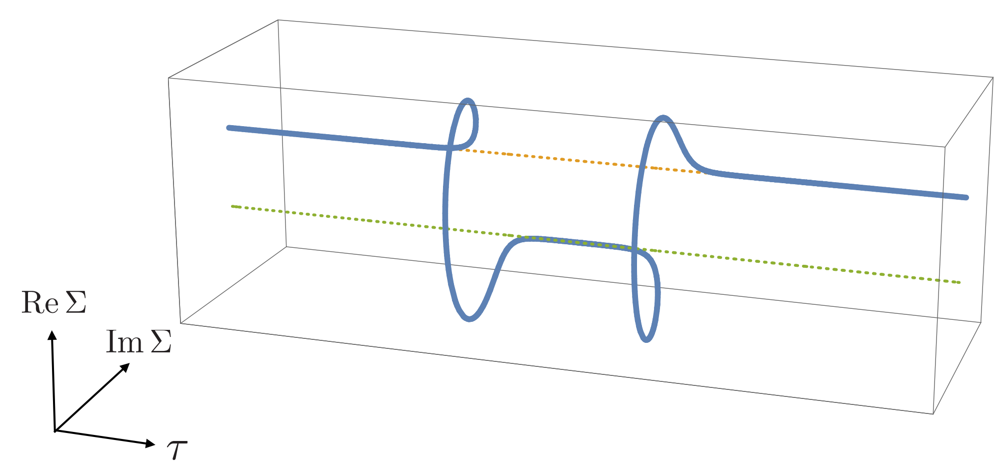
$$L = \frac{1}{g^2}\frac{\partial_t\varphi\partial_t\bar{\varphi}}{(1+\varphi\bar{\varphi})^2} - V(\varphi\bar{\varphi}), \qquad V(\varphi\bar{\varphi}) \equiv \frac{1}{g^2}\frac{m^2\varphi\bar{\varphi}}{(1+\varphi\bar{\varphi})^2} - \epsilon m\frac{1-\varphi\bar{\varphi}}{1+\varphi\bar{\varphi}},$$

Real bion solution : $\varphi = e^{i\phi_0}\sqrt{\frac{\omega^2}{\omega^2-m^2}}\frac{1}{i\sinh\omega(\tau-\tau_0)}$

Complex bion solution : $\varphi = e^{i\phi_0}\sqrt{\frac{\omega^2}{\omega^2-m^2}}\frac{1}{\cosh\omega(\tau-\tau_0)}, \qquad \tilde{\varphi} = -e^{-i\phi_0}\sqrt{\frac{\omega^2}{\omega^2-m^2}}\frac{1}{\cosh\omega(\tau-\tau_0)}$



Real bion



Complex bion

◆ One-loop determinant and Lefschetz thimbles

Fujimori, Kamata, TM, Nitta, Sakai (16)

-STEP1

1-loop det :
$$\frac{\int \mathcal{D}\xi \exp\left(-\frac{1}{2}\int d\tau\, \xi^{\mathrm{T}} \Delta\, \xi\right)}{\int \mathcal{D}\xi \exp\left(-\frac{1}{2}\int d\tau\, \xi^{\mathrm{T}} \Delta_0 \xi\right)} = \int d\tau_0 d\phi_0 \sqrt{\det\left(\frac{1}{\pi} M K_+^\dagger K_-\right)} = \beta \frac{16i\, e^{\omega(\bar\tau_0-\tau_0)}\omega^4}{g^2(\omega^2-m^2)}$$

$$\Rightarrow \quad -\lim_{\beta\to\infty}\frac{1}{\beta}\frac{Z_1}{Z_0} = \pm i(1-e^{2\pi\epsilon i})\frac{16\omega^4}{g^2(\omega^2-m^2)}\exp\left(-\frac{2\omega}{g^2} - 2\epsilon\log\frac{\omega+m}{\omega-m}\right)$$

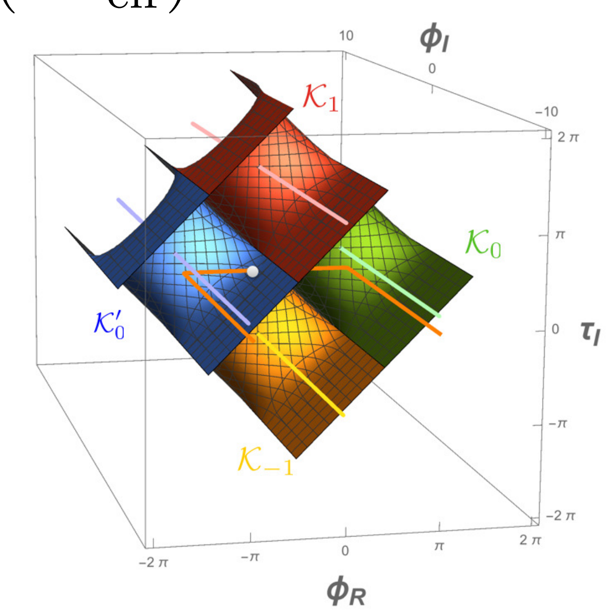
1-loop contribution from complex and real bions are consistent with SUSY results, although nearly flat direction cannot be incorporated.

-STEP2

Thimble integral :
$$-\lim_{\beta\to\infty}\frac{1}{\beta}\frac{Z_1}{Z_0} \approx -\frac{8m^4}{\pi g^4}\int d\tau_r d\phi_r \exp\left(-V_{\text{eff}}\right)$$

$$\Rightarrow \quad -\frac{2m}{\pi}\left(\frac{g^2}{2m}\right)^{2(\epsilon-1)} e^{\pm\epsilon\pi i}\sin\epsilon\pi\,\Gamma(\epsilon)^2 e^{-\frac{2m}{g^2}}$$

Thimble integral from complex and real bions reproduce both SUSY and near-SUSY exact results.



Non-BPS solutions in 2D CPN model

◆Non-BPS solutions in CP^N-1 models

Dabrowski, Dunne(13)

- Non-BPS solution = solution of 2nd-order EOM

BPS EOM $\cancel{D_\mu n = \pm i\epsilon_{\mu\nu} D_\nu n}$

Full EOM $D_\mu D_\mu n - (n^\dagger \cdot D_\mu D_\mu n)\, n = 0$

- Systematic method to obtain Non-BPS solution

Din-Zakrzewski projection $Z_+ :\ \omega \ \to\ Z_+\omega \equiv \partial_z \omega - \frac{(\partial_z\omega)\omega^\dagger}{\omega\omega^\dagger}\omega$

enables us to construct non-BPS solutions from BPS solutions

BPS solution $\omega_{\mathcal{II}} = \left(l_1 e^{i\theta_1} e^{-\frac{4\pi}{3}z},\, l_2 e^{i\theta_2} e^{-\frac{2\pi}{3}z},\, 1\right)$

Non-BPS solution

$$\omega_{\rm nbps} = \left(e^{i\theta_1}\left(\frac{2l_1}{l_2}e^{-\frac{2\pi}{3}z} + l_1 l_2 e^{-\frac{2\pi}{3}(2z+\bar z)}\right),\ e^{i\theta_2}\left(1 - l_1^2 e^{-\frac{4\pi}{3}(z+\bar z)}\right),\quad l_2 e^{-\frac{2\pi}{3}\bar z} + \frac{2l_1^2}{l_2}e^{-\frac{2\pi}{3}(z+2\bar z)}\right)$$

◆Non-BPS solutions in CP^N-1 models

Dabrowski, Dunne(13)

- Non-BPS solution = solution of 2nd-order EOM

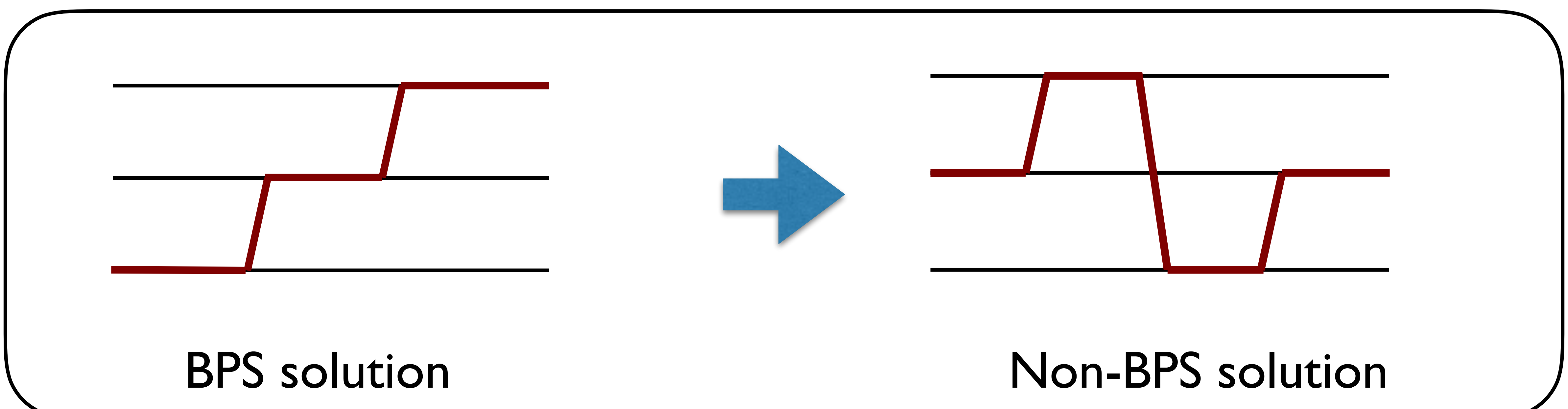
BPS EOM $\cancel{D_\mu n = \pm i\epsilon_{\mu\nu} D_\nu n}$

Full EOM $D_\mu D_\mu n - (n^\dagger \cdot D_\mu D_\mu n)\, n = 0$

- Systematic method to obtain Non-BPS solution

Din-Zakrzewski projection $Z_+ : \omega \to Z_+ \omega \equiv \partial_z \omega - \frac{(\partial_z \omega)\omega^\dagger}{\omega\omega^\dagger}\omega$

enables us to construct non-BPS solutions from BPS solutions

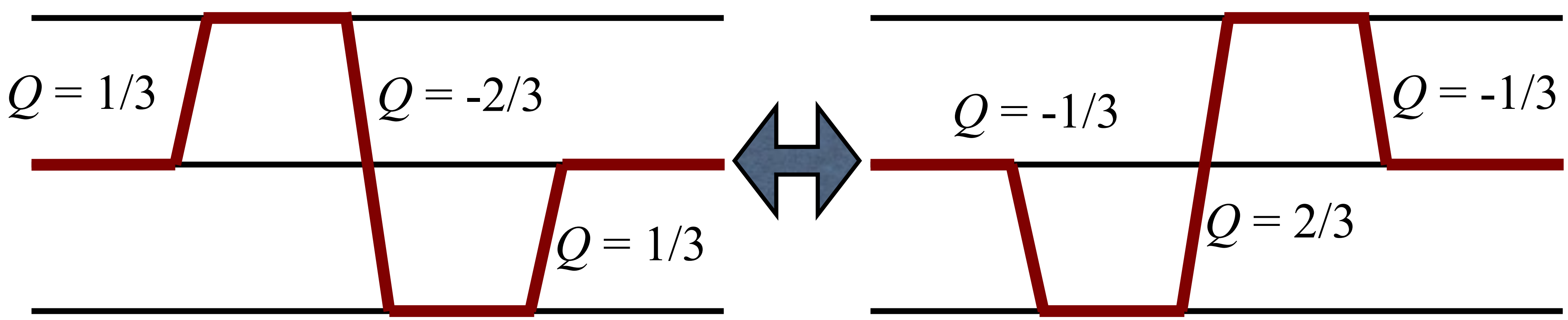


◆Generic properties of Non-BPS solution

TM, Nitta, Sakai (16)

ex.) CP2 model

$$\omega = \left(e^{i\theta_1} \left(l_1 l_2^2 e^{-\frac{2\pi}{3}(2z+\bar{z})} + 2l_1 e^{-\frac{2\pi}{3}z} \right) \quad e^{i\theta_2} \left(-l_1^2 l_2 e^{-\frac{4\pi}{3}(z+\bar{z})} + l_2 \right) \quad \cdot 2l_1^2 e^{-\frac{2\pi}{3}(z+2\bar{z})} + l_2^2 e^{-\frac{2\pi}{3}\bar{z}} \right)$$

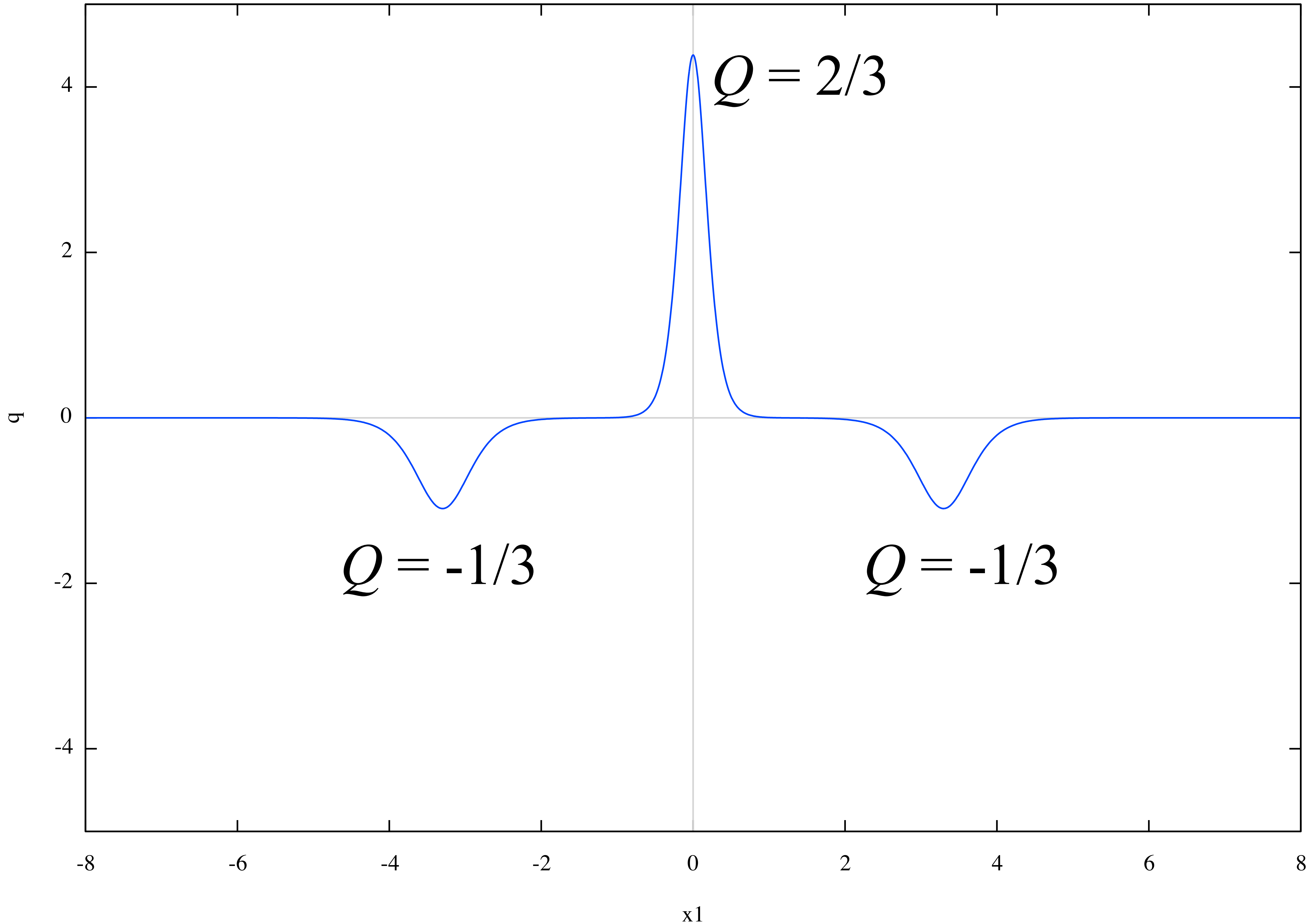


$2l_1 \gg l_2^2$ $\qquad$ $2l_1 \ll l_2^2$

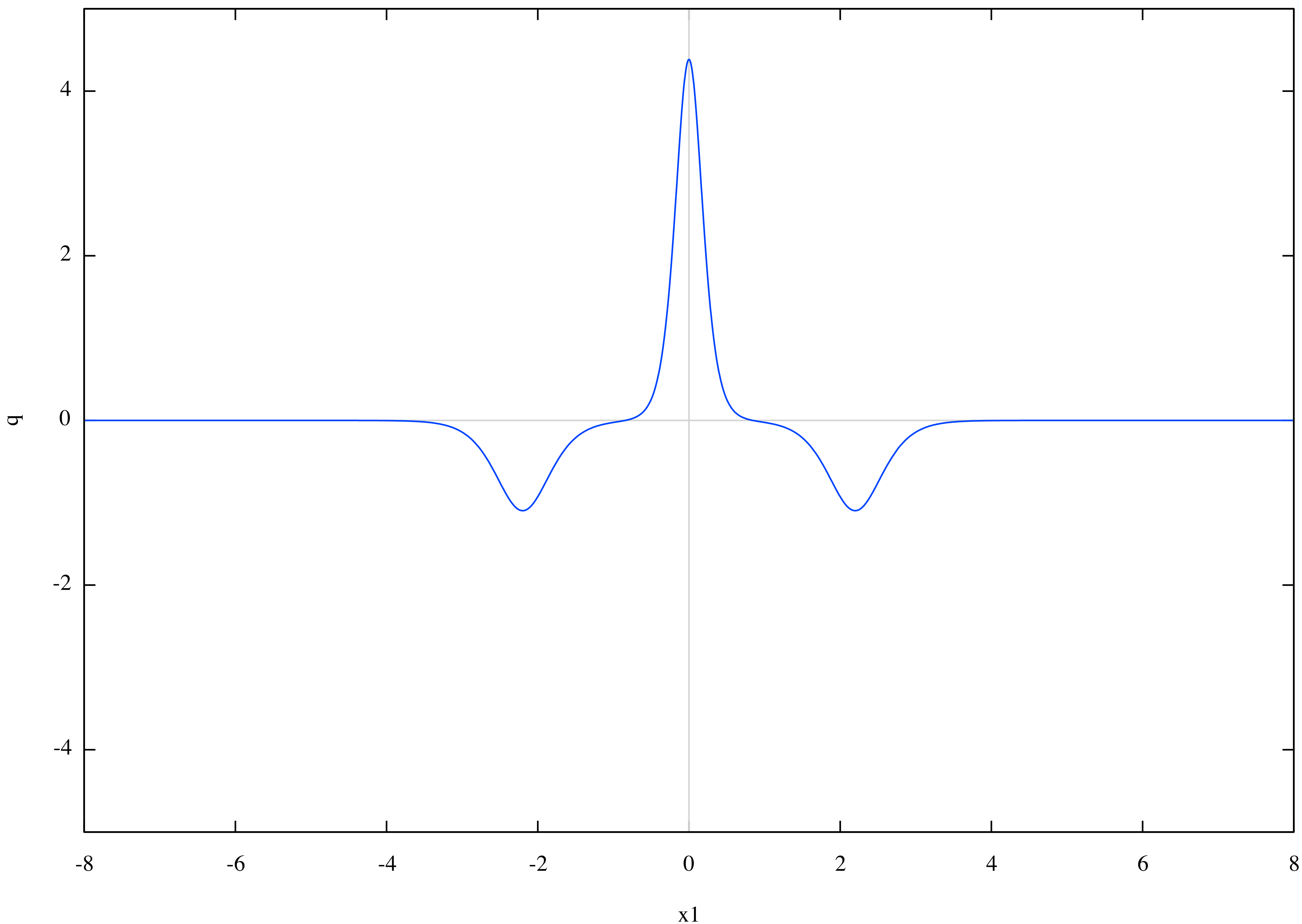
Transition between 2 distinct configurations occurs as the moduli parameters are varied.

Flipping partners

Topological charge density

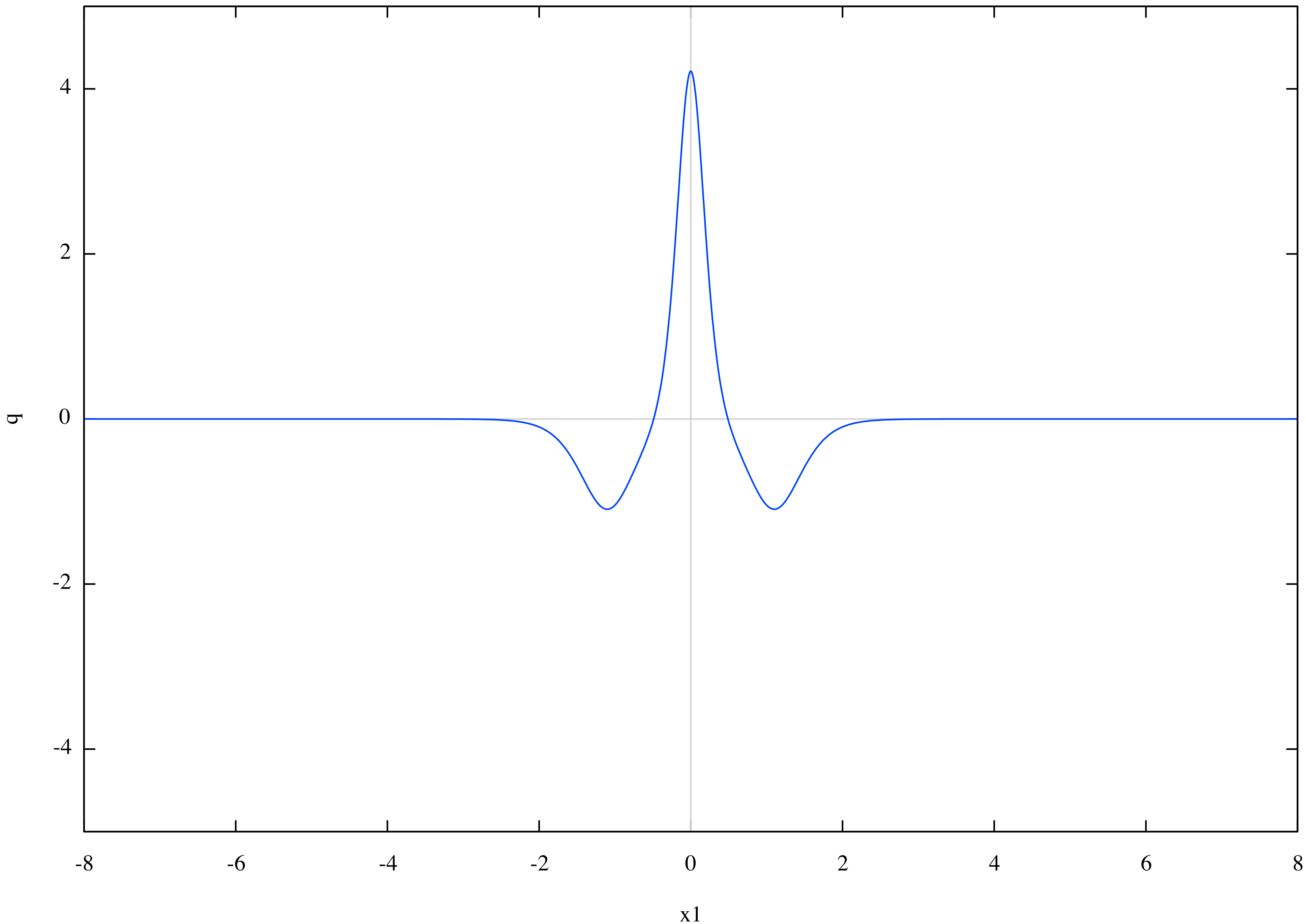


Topological charge density of the non-BPS solution
(l_1=1, l_2=1000)

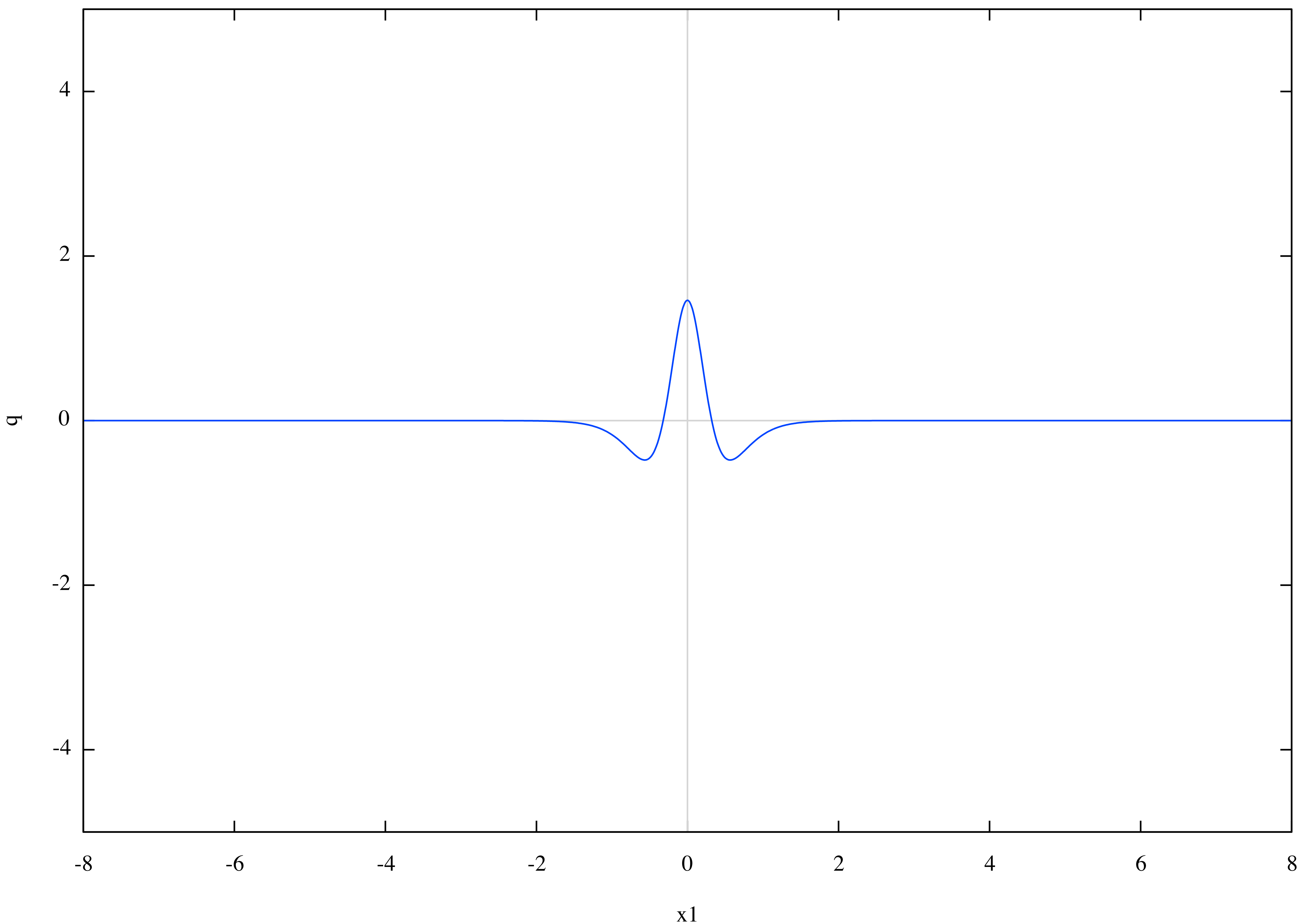


Topological charge density of the non-BPS solution

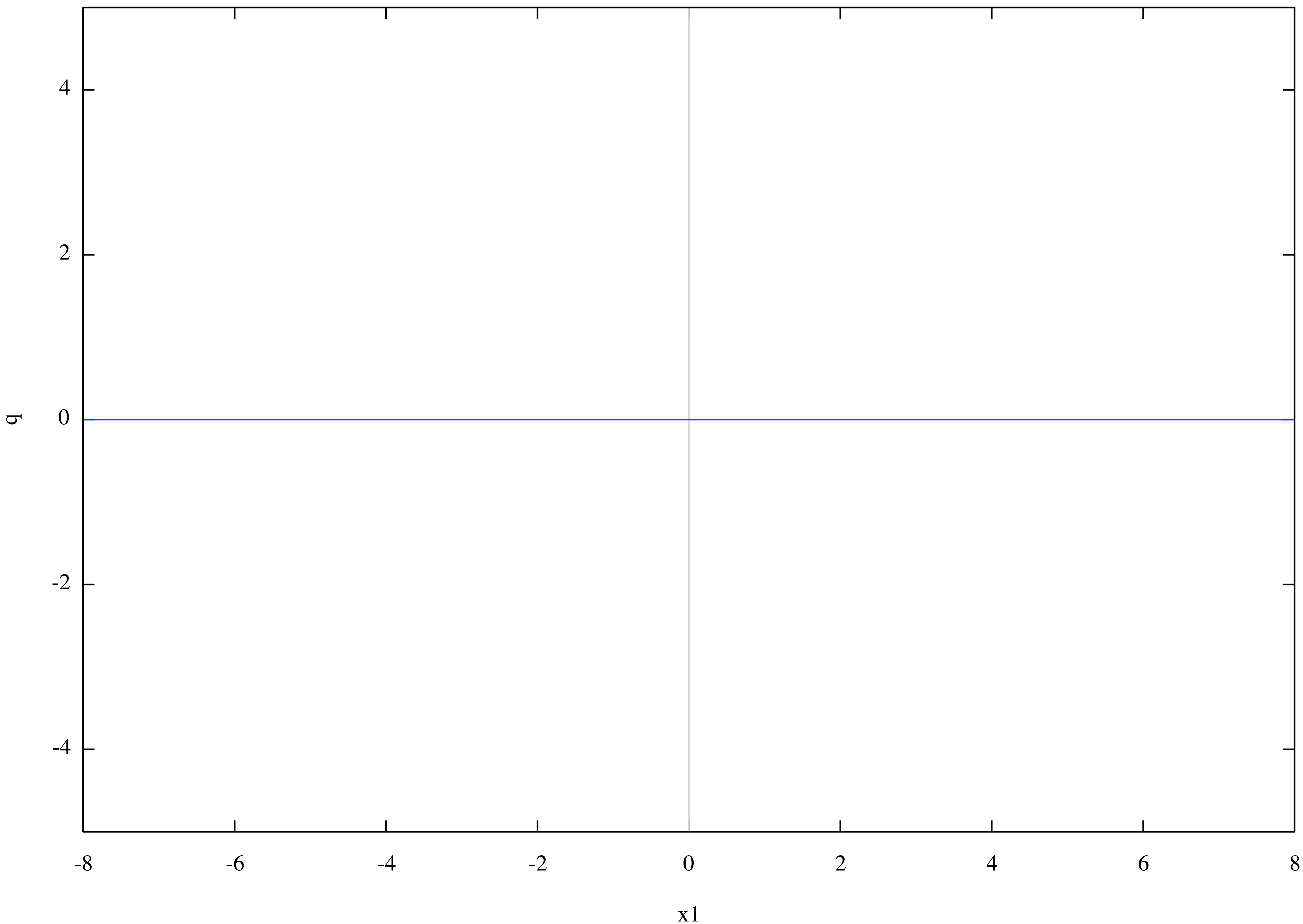
($l_1=1$, $l_2=100$)



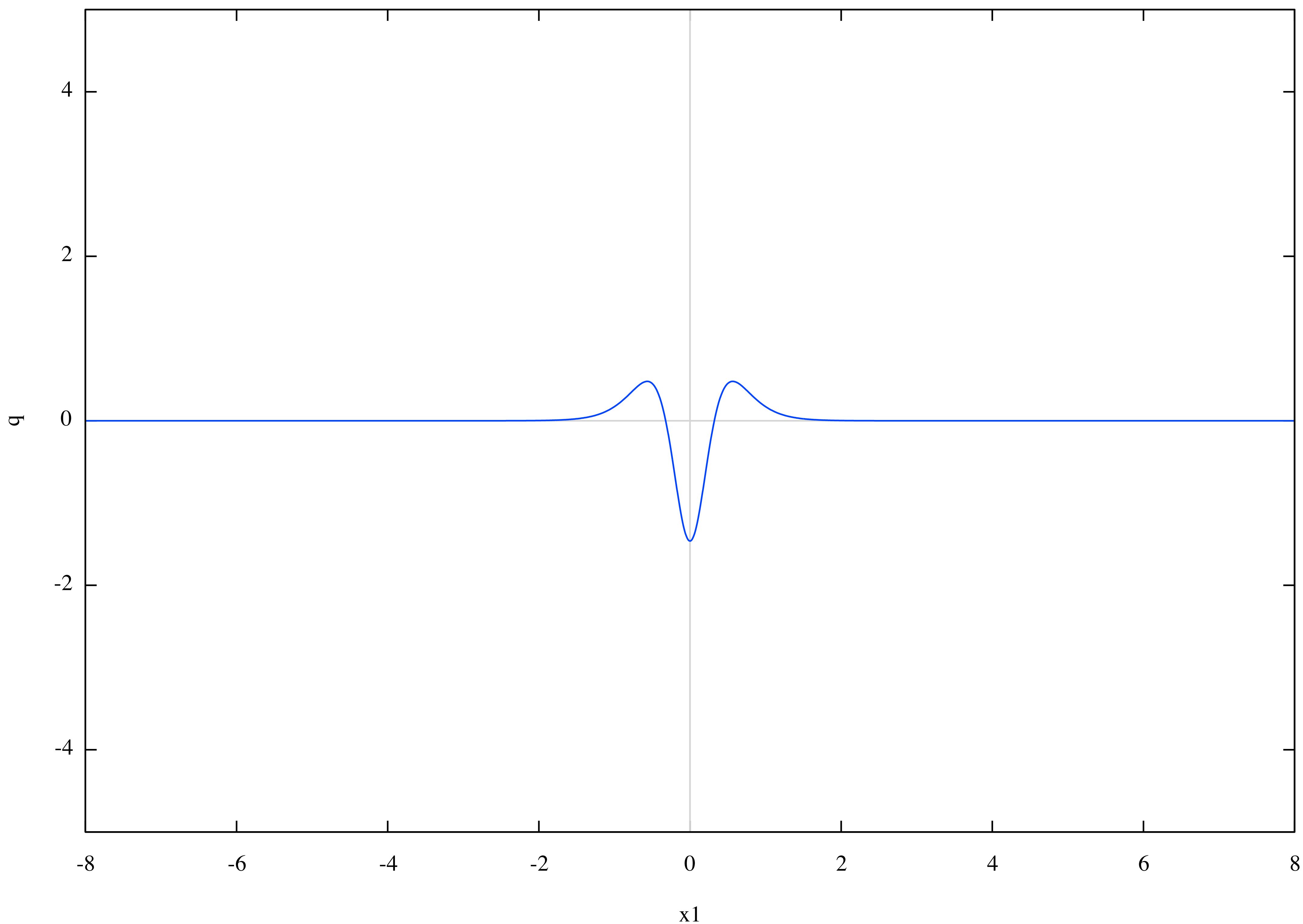
Topological charge density of the non-BPS solution
$(l_1=1, l_2=10)$



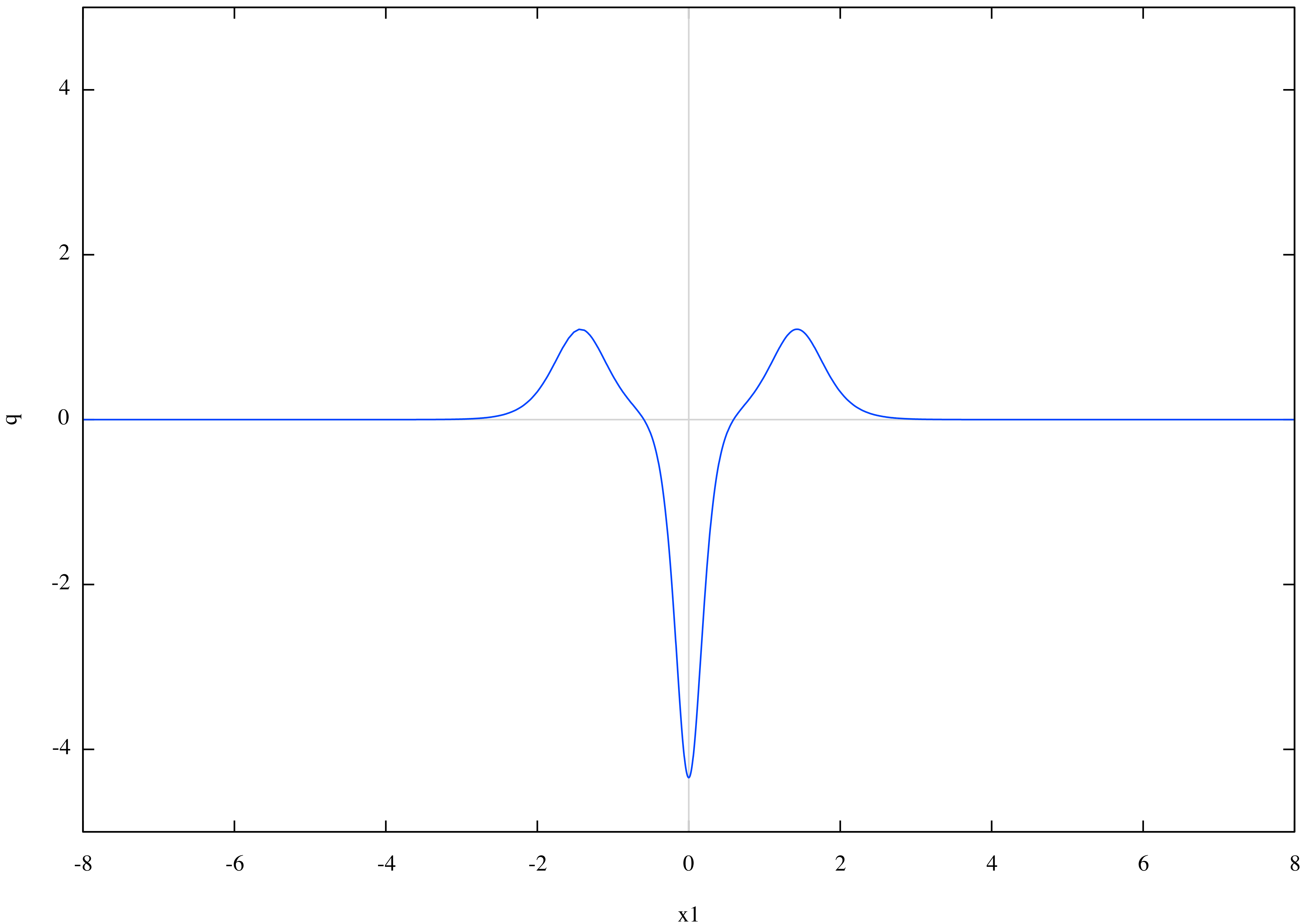
Topological charge density of the non-BPS solution
(l_1=1, l_2=2)



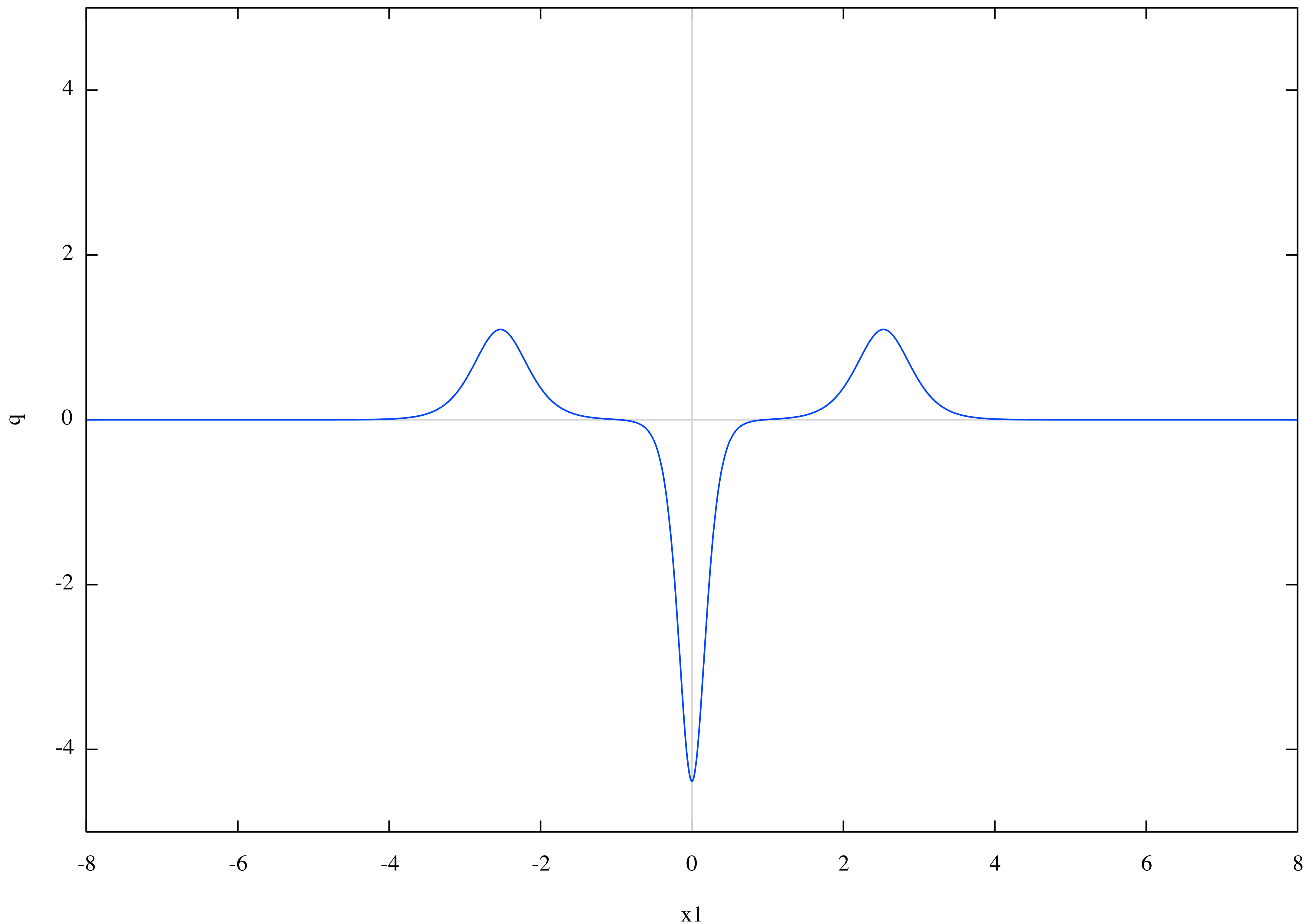
Topological charge density of the non-BPS solution
($l_1=1, l_2=\sqrt{2}$)



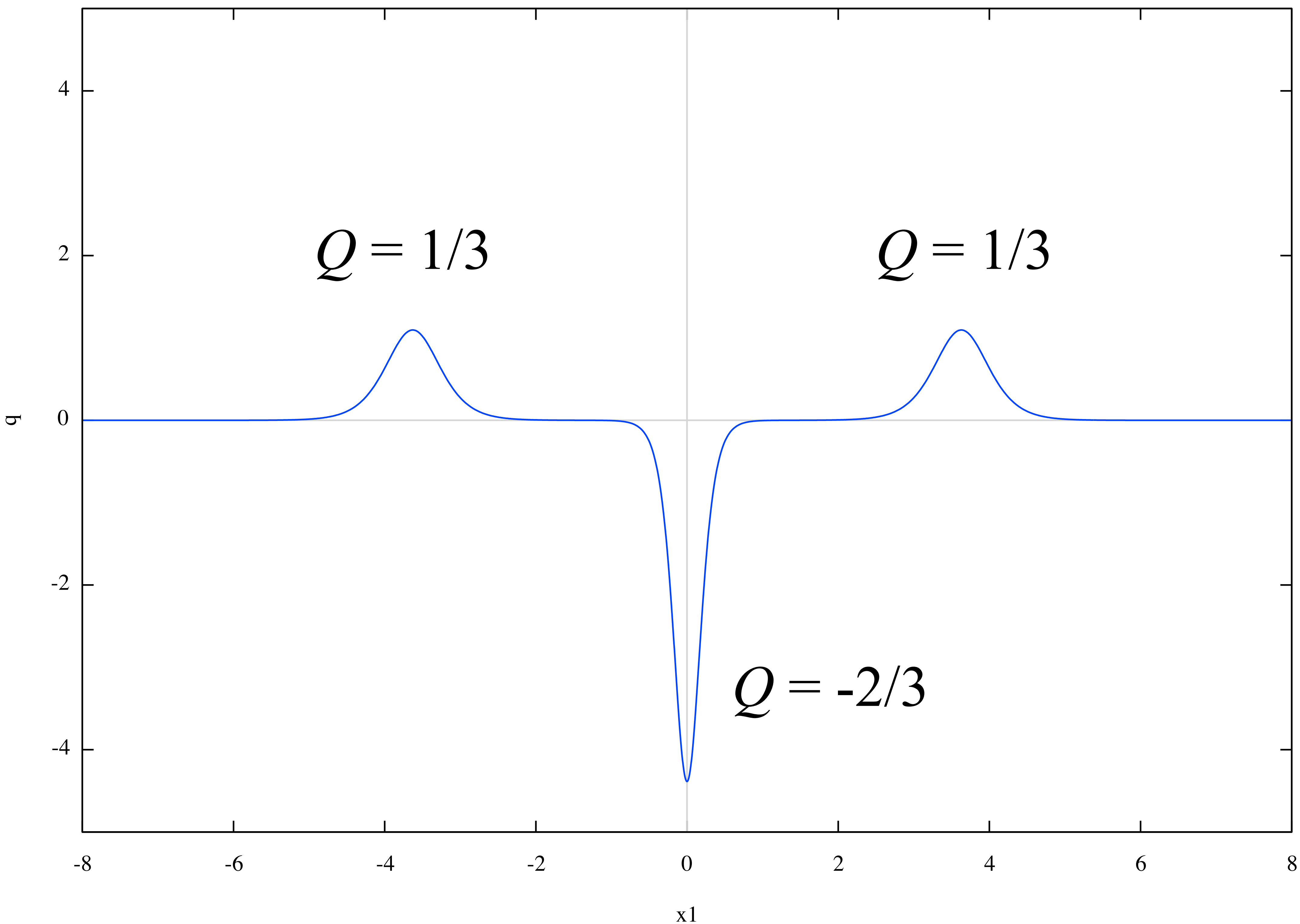
Topological charge density of the non-BPS solution
(l_1=1, l_2=1)



Topological charge density of the non-BPS solution
(l_1=1, l_2=1/10)



Topological charge density of the non-BPS solution
($l_1=1$, $l_2=1/100$)



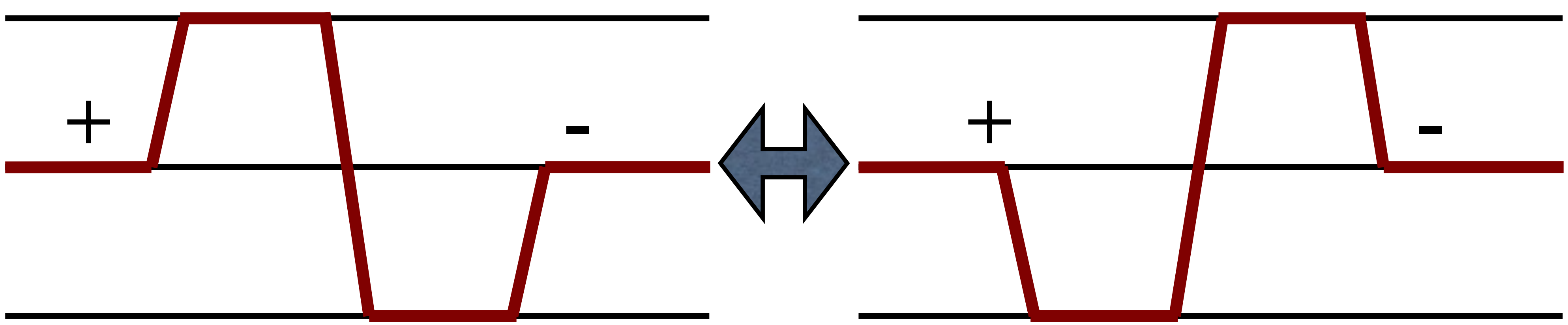
Topological charge density of the non-BPS solution
(l_1=1, l_2=1/1000)

◆ Generic properties of Non-BPS solution

TM, Nitta, Sakai (16)

ex.) CP2 model

$$\omega = \left(e^{i\theta_1} \left(l_1 l_2^2 e^{-\frac{2\pi}{3}(2z+\bar{z})} + 2 l_1 e^{-\frac{2\pi}{3} z} \right) \quad e^{i\theta_2} \boxed{\left(-l_1^2 l_2 e^{-\frac{4\pi}{3}(z+\bar{z})} + l_2 \right)} \quad 2 l_1^2 e^{-\frac{2\pi}{3}(z+2\bar{z})} + l_2^2 e^{-\frac{2\pi}{3}\bar{z}} \right)$$



Instanton and anti-instanton constituents at the both sides has a opposite sign, leading to absence of attractive force.

Relative sign (phase)

We again note that relative phase is essential in CPN model

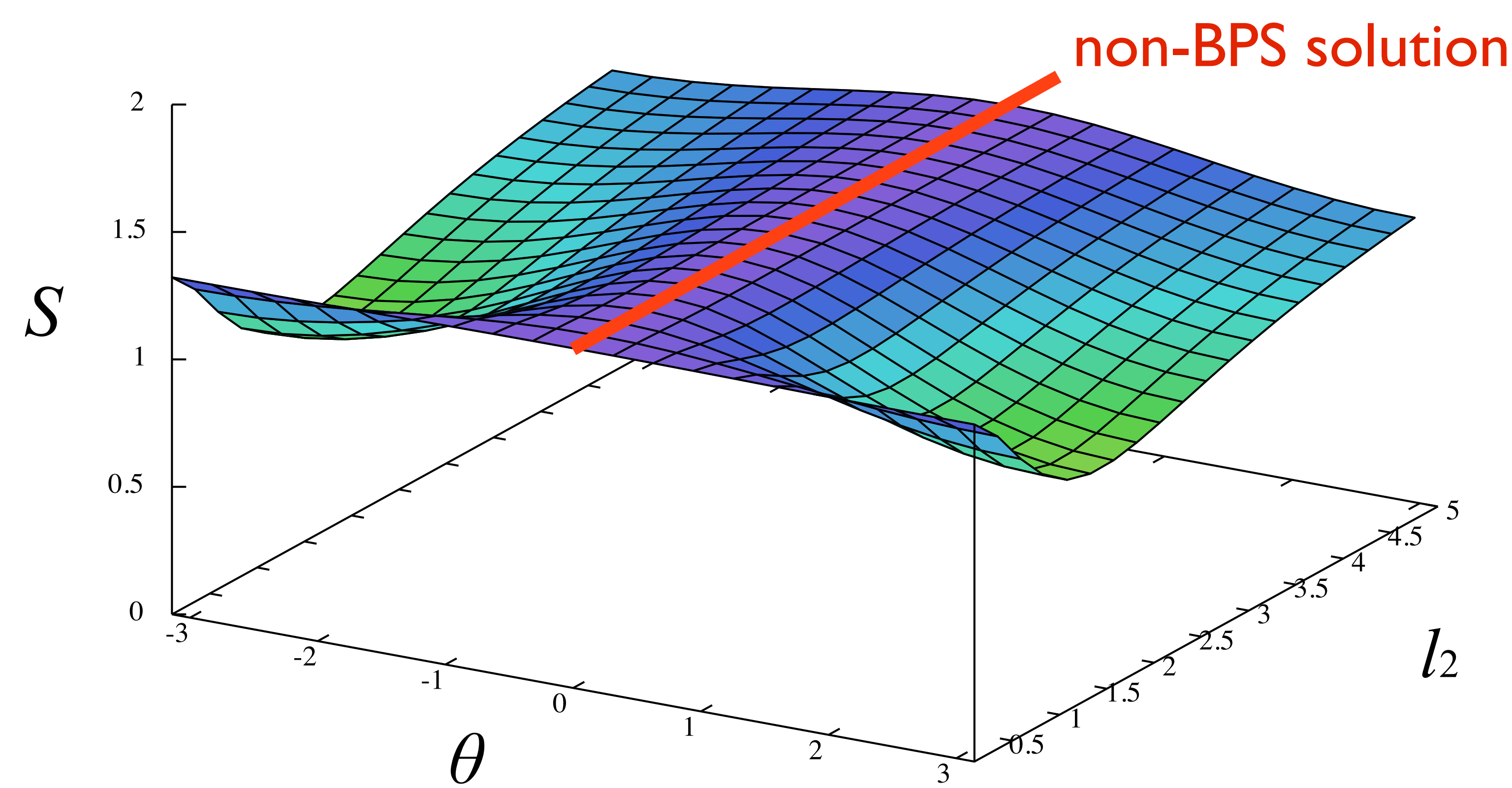
◆ Generic properties of Non-BPS solution

TM, Nitta, Sakai (16)

ex.) CP2 model

$e^{i(\pi+\theta)}$

$$\omega = \left(e^{i\theta_1}\left(l_1 l_2^2 e^{-\frac{2\pi}{3}(2z+\bar{z})} + 2l_1 e^{-\frac{2\pi}{3}z}\right) \quad e^{i\theta_2}\left(-l_1^2 l_2 e^{-\frac{4\pi}{3}(z+\bar{z})} + l_2\right) \quad 2l_1^2 e^{-\frac{2\pi}{3}(z+2\bar{z})} + l_2^2 e^{-\frac{2\pi}{3}\bar{z}} \right)$$



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◆Generic properties of Non-BPS solution

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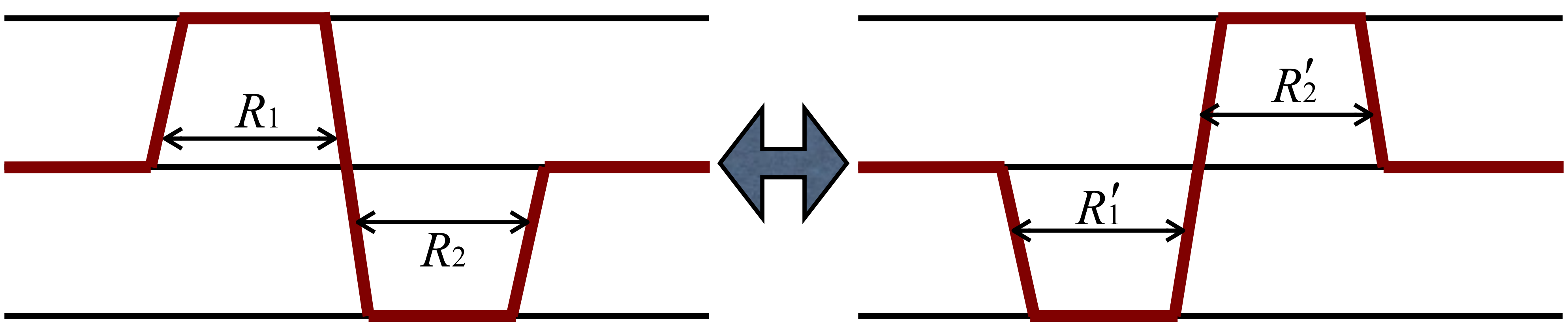


$$R_1 = R_2 = \frac{3}{4\pi} \log(4 l_1 / l_2^2) \qquad R'_1 = R'_2 = \frac{3}{4\pi} \log(l_2^2 / l_1)$$

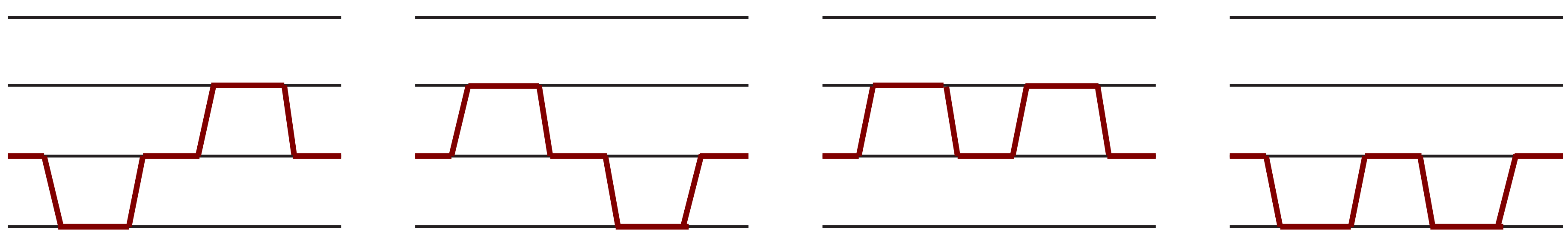
Symmetric separations

◆Bion configurations incorporate Non-BPS solution

TM, Nitta, Sakai (16)

- Two-bion configuration

$$\omega = \left(\cdots,\ a e^{-\frac{2\pi}{N} z} + c e^{-\frac{2\pi}{N}(2z+\bar{z})},\ 1 + b e^{-\frac{2\pi}{N}(z+\bar{z})} + d e^{-\frac{4\pi}{N}(z+\bar{z})},\ f e^{-\frac{2\pi}{N}\bar{z}} + g e^{-\frac{2\pi}{N}(z+2\bar{z})}, \cdots \right)$$



As a special choice of parameter set, it results in a non-BPS solution.

$$a = 2l_1/l_2,\ c = l_1 l_2,\ b = 0,\ d = -l_1^2,\ f = \quad l_2,\ g = \quad 2l_1^2/l_2$$

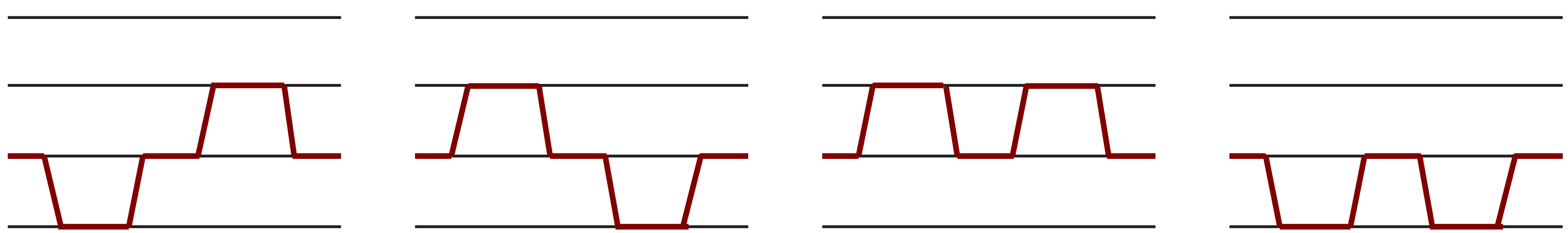
It indicates Non-BPS solutions are relevant in resurgence trans-series.

◆Bion configurations incorporate Non-BPS solution

TM, Nitta, Sakai (16)

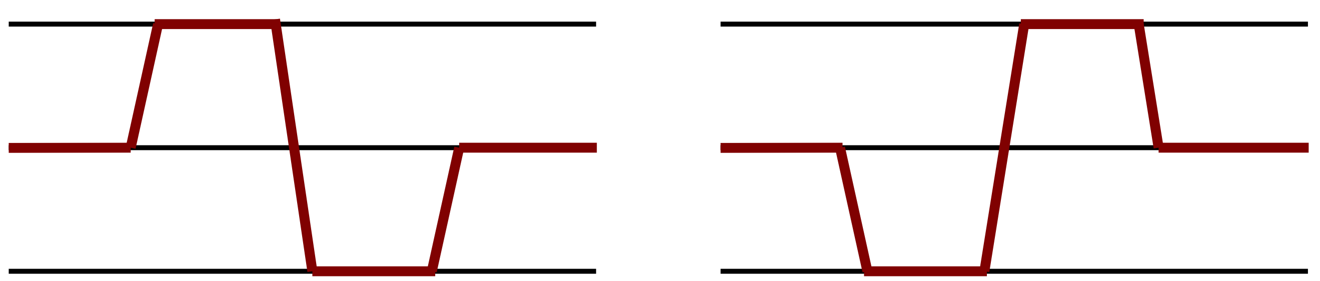
- Two-bion configuration

$$\omega = \left(\cdots,\ a e^{-\frac{2\pi}{N} z} + c e^{-\frac{2\pi}{N}(2z+\bar{z})},\ 1 + b e^{-\frac{2\pi}{N}(z+\bar{z})} + d e^{-\frac{4\pi}{N}(z+\bar{z})},\ f e^{-\frac{2\pi}{N}\bar{z}} + g e^{-\frac{2\pi}{N}(z+2\bar{z})}, \cdots \right)$$



As a special choice of parameter set, it results in a non-BPS solution.

$$\omega = \left(e^{i\theta_1}\left(l_1 l_2^2 e^{-\frac{2\pi}{3}(2z+\bar{z})} + 2 l_1 e^{-\frac{2\pi}{3} z} \right) \quad e^{i\theta_2}\left(-l_1^2 l_2 e^{-\frac{4\pi}{3}(z+\bar{z})} + l_2 \right) \quad 2 l_1^2 e^{-\frac{2\pi}{3}(z+2\bar{z})} + l_2^2 e^{-\frac{2\pi}{3}\bar{z}} \right)$$



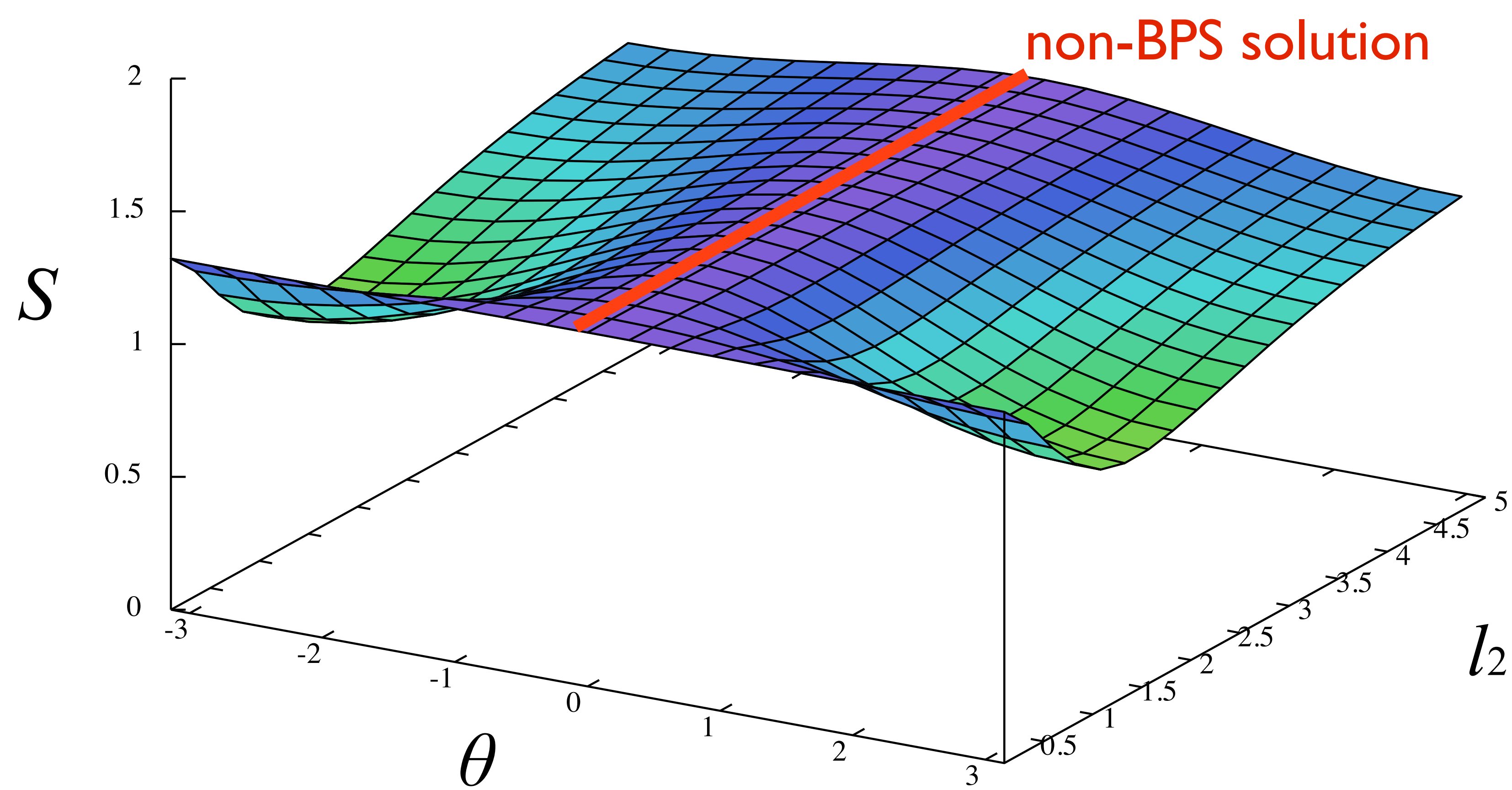
It indicates Non-BPS solutions are relevant in resurgence trans-series.

◆Negative modes of Non-BPS solution

TM, Nitta, Sakai (16)

Non-BPS solutions are unstable and have negative modes.

$$\omega = \left(l_1 l_2^2 e^{-\frac{2\pi}{3}(2z+\bar{z})} + 2l_1 e^{-\frac{2\pi}{3}z} \quad -\gamma l_1^2 l_2 e^{-\frac{4\pi}{3}(z+\bar{z})} + \gamma' e^{i\theta} l_2 \quad -2l_1^2 e^{-\frac{2\pi}{3}(z+2\bar{z})} - l_2^2 e^{-\frac{2\pi}{3}\bar{z}} \right)$$



Shift of the relative phase is a negative mode

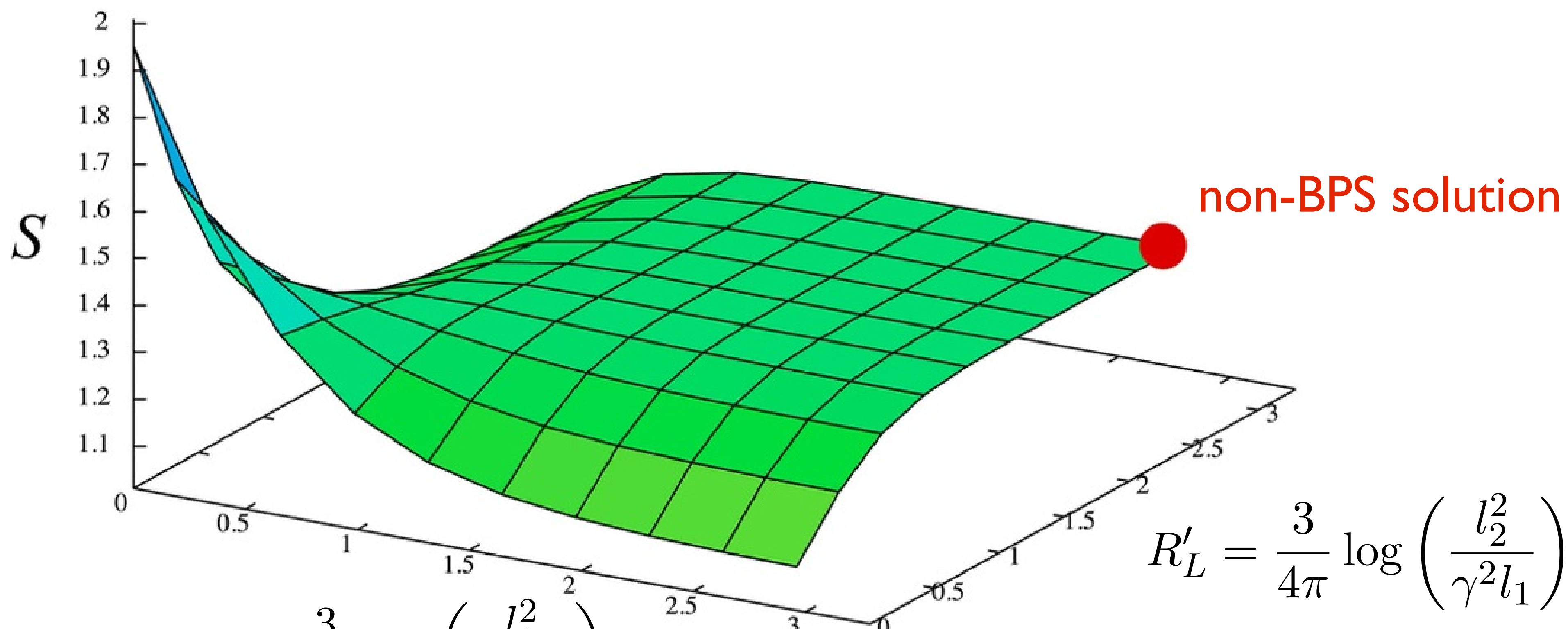
$$\frac{\partial^2 S}{\partial (R'_R, R'_L, \theta)^2} \approx \begin{pmatrix} -0.0606 & 0.0852 & O(10^{-6}) \\ 0.0852 & -0.0606 & O(10^{-6}) \\ O(10^{-6}) & O(10^{-6}) & -0.0204 \end{pmatrix}$$

◆Negative modes of Non-BPS solution

TM, Nitta, Sakai (16)

Non-BPS solutions are unstable and have negative modes.

$$\omega = \left(l_1 l_2^2 e^{-\frac{2\pi}{3}(2z+\bar{z})} + 2 l_1 e^{-\frac{2\pi}{3}z} \quad \underline{-\gamma} l_1^2 l_2 e^{-\frac{4\pi}{3}(z+\bar{z})} \underline{+\gamma'} \underline{e^{i\theta}} l_2 \quad -2 l_1^2 e^{-\frac{2\pi}{3}(z+2\bar{z})} - l_2^2 e^{-\frac{2\pi}{3}\bar{z}} \right)$$



$$R'_R = \frac{3}{4\pi} \log\left(\frac{l_2^2}{\gamma'^2 l_1}\right)$$

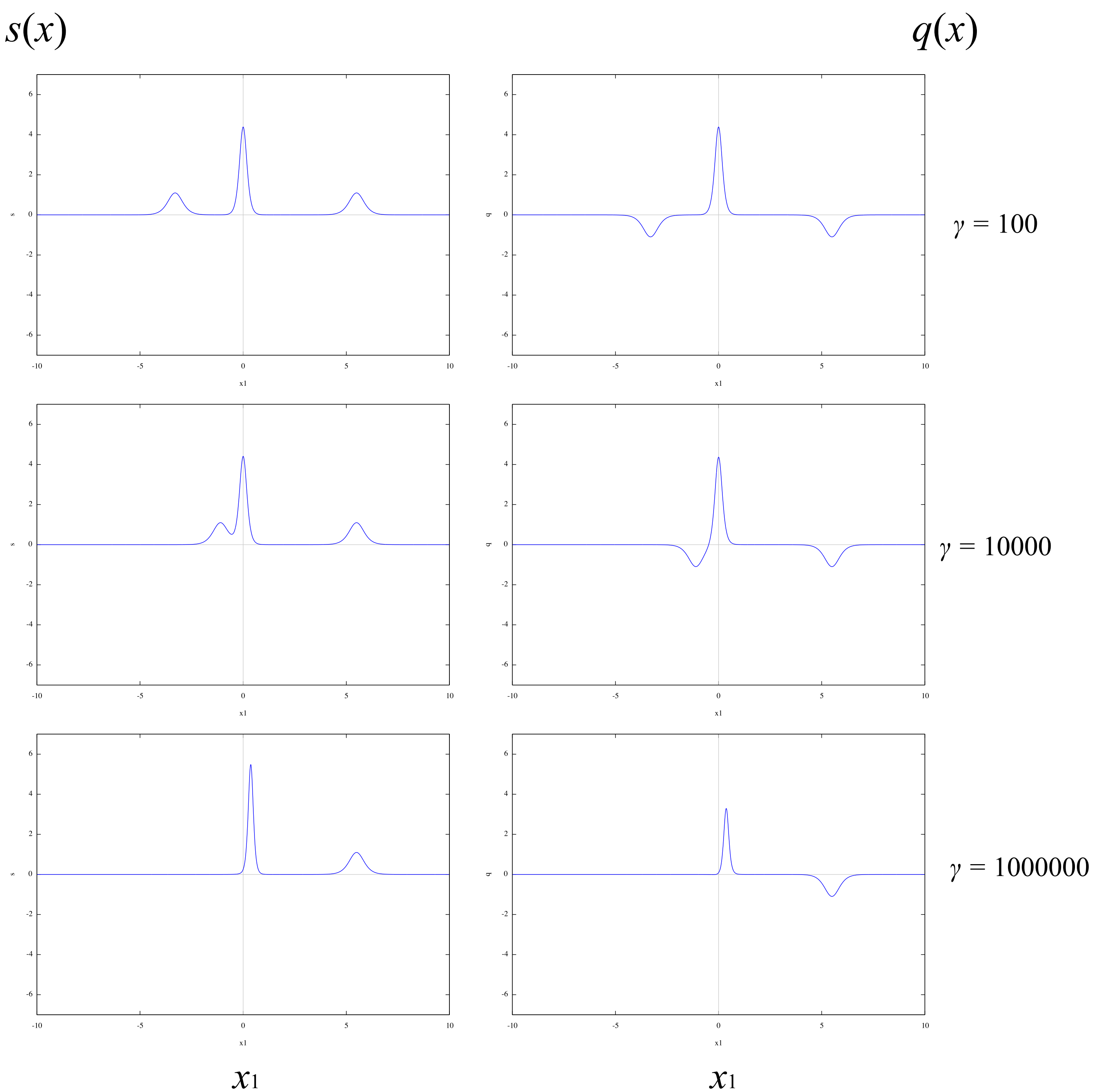
$$R'_L = \frac{3}{4\pi} \log\left(\frac{l_2^2}{\gamma^2 l_1}\right)$$

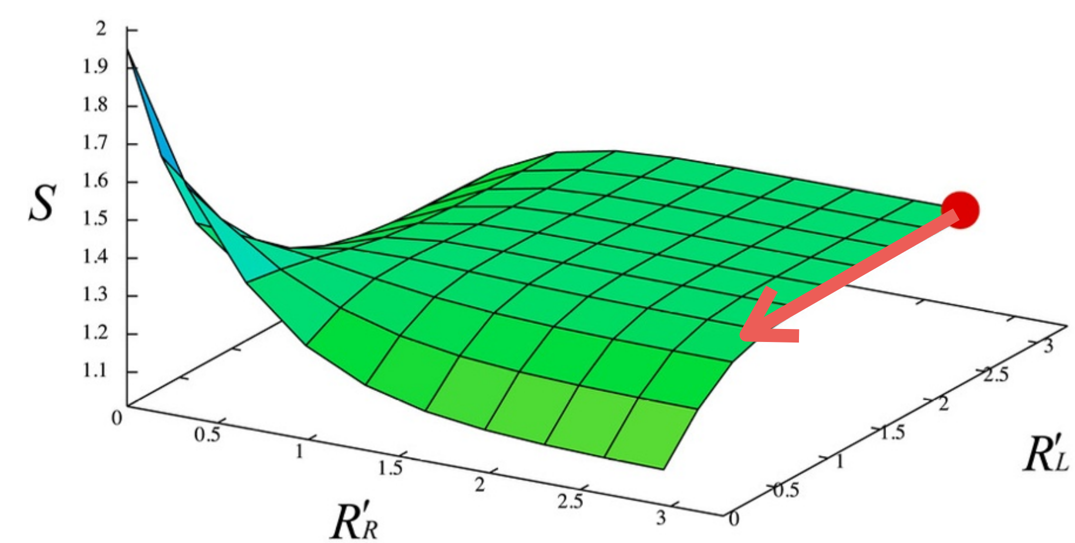
Breaking the symmetric separation leads to a negative mode

◆Negative modes of Non-BPS solution

TM, Nitta, Sakai (16)

Non-BPS solutions are unstable and have negative modes.



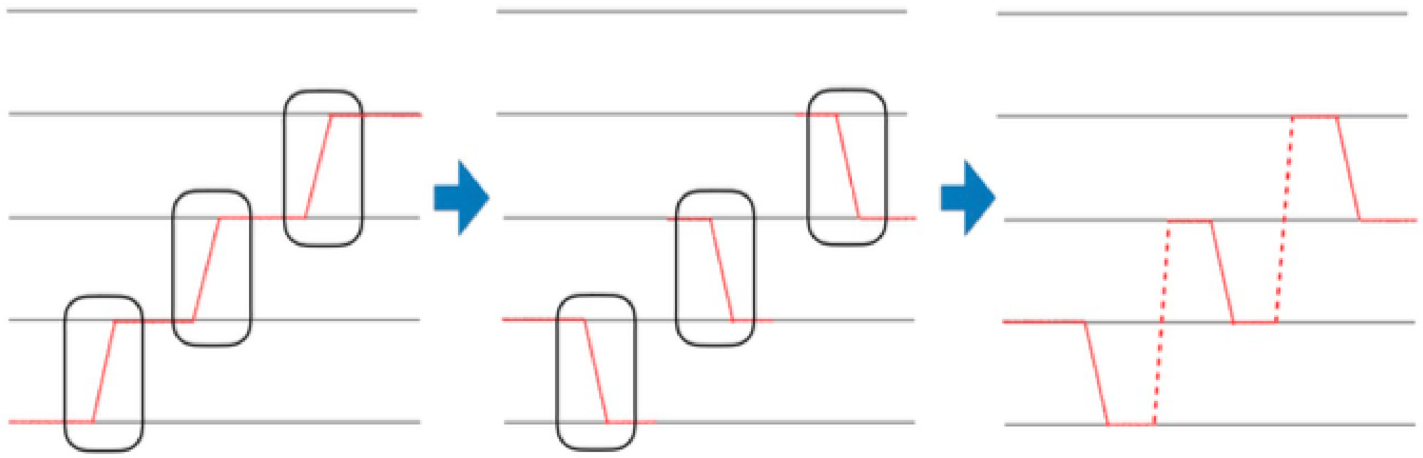


Breaking the separation symmetry leads to a negative mode

◆Generic construction of Non-BPS solutions

TM, Nitta, Sakai (16)

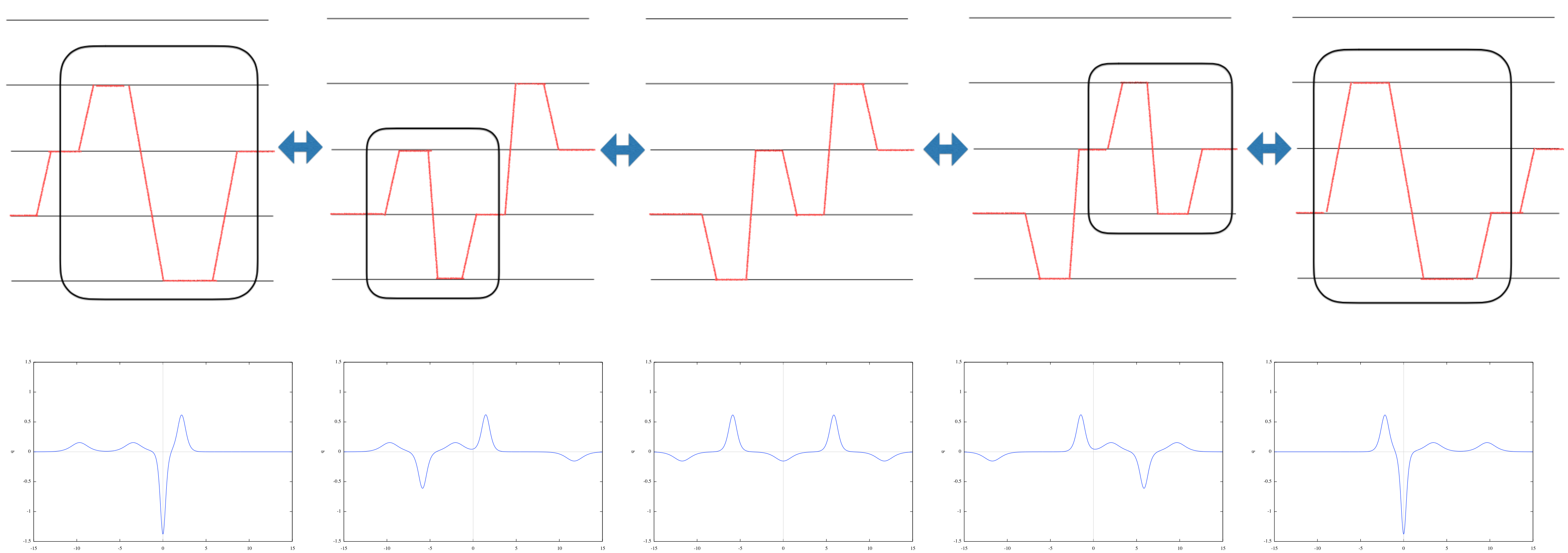
Diagrammatic construction of non-BPS solutions



Original BPS solution | Turn elements upside down | Connecting the elements

1. Turn elements upside down
2. Connect the elements
3. Find flipping partners

Flipping partners appear depending on parameters



Non-BPS solutions in 2D CPN models

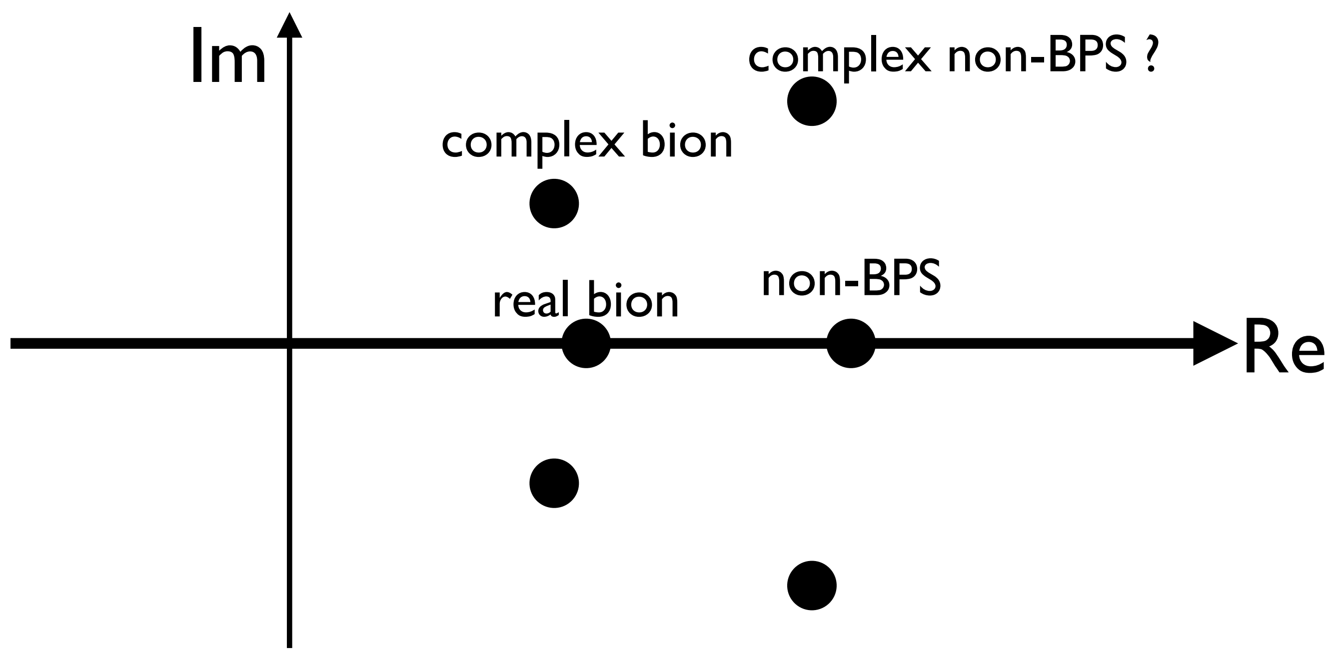
- Non-BPS solutions is realized based on the very subtle balance: (i)flipping partners, (ii)relative sign, (iii)symmetric separation.
- Non-BPS solutions have negative modes, so the solutions are unstable saddle-point solutions.
- Non-BPS solutions are seen as special cases of multi-bion configurations, thus they are also essential in the resurgent trans-series.
- We find a generic way how to construct non-BPS solutions graphically.

How about non-BPS solutions in complexified theory?

◆Non-BPS solutions in Complexified theory

$$\omega = \left(e^{i\theta_1}\left(l_1 l_2^2 e^{-\frac{2\pi}{3}(2z+\bar{z})} + 2l_1 e^{-\frac{2\pi}{3}z}\right) \quad e^{i\theta_2}\left(-l_1^2 l_2 e^{-\frac{4\pi}{3}(z+\bar{z})} + l_2\right) \quad -2l_1^2 e^{-\frac{2\pi}{3}(z+2\bar{z})} - l_2^2 e^{-\frac{2\pi}{3}\bar{z}}\right)$$

- These non-BPS solutions are also solutions in complexified theory
- The complex versions of the solutions have larger modulus
- It means they will be nontrivial saddle points to which thimbles attach



Conjectural configuration space?

It indicates CPN model has other saddle points rather than real and complex bions, contributing to the resurgent expansion

What have been done

- Resurgence structure in QM is clarified.
- Relevance of non-BPS solutions in CPN model
- Relation of Bion and Non-BPS solutions