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Massive supermembrane in ten non compact dimensions

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Based on

M.P García del Moral, P. Leon, A. Restuccia, Massive supermembrane on a knot. arXiv:2101.04018[hep-th]

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Outline

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   - Massive Supermembrane on $M_9 \times LCD$
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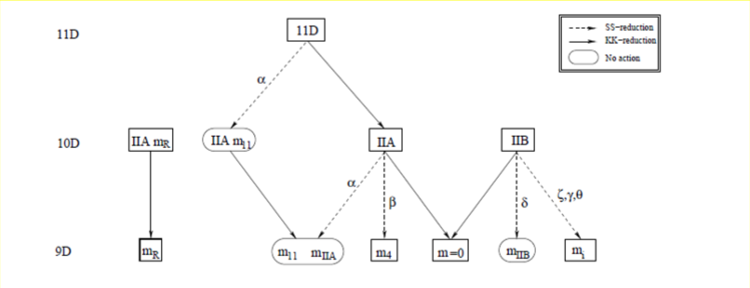
Motivation

- The 11D Supergravity, which is the lower energy limit of the Supermembrane Theory (M-theory), is unique.
- The maximal and gauge supergravities in lower dimensions can be obtained from the 11D supergravity through KK and SS reductions, among other possibilities.
- In ten dimensions there is a massive deformations of maximal type IIA supergravity knwon by the name of Romans Supergravity whose origin in M-theory is unknown. **L.J. Romans. PLB (1986)**

Bergshoeff, E., de Wit, T., Gran, U., Linares, R., Roest, D. NPB (2003).



- There exist an uplift of the 10D Romans Supergravity to 11D, in which a cosmological constant is present. However, the resulting action in 11D is non covariant.

$$
\begin{aligned}
S_{11D} &= \frac{1}{k}\int d^{11}x\left[\sqrt{|\hat{g}|}\left\{\hat{R}-\frac{1}{48}(\hat{G}^{(4)})^2+\frac{1}{2}m^2|\hat{k}|^4\right\}\right.\\
&+ \left.\frac{\epsilon^{\mu_1\ldots\mu_{11}}}{(144)^2}\left\{2^4\partial\hat{C}\partial\hat{C}\hat{C}+18\partial\hat{C}\hat{C}(i_{\hat{k}}\hat{C})^2+\frac{3^3}{5}m^2\hat{C}(i_{\hat{k}}\hat{C})^4\right\}_{\mu_1\ldots\mu_{11}}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
k &\equiv 16\pi G_N^{(11)}, \quad |\hat{k}|=\sqrt{-\hat{k}^\mu\hat{k}^\nu\hat{g}_{\mu\nu}}, \quad \mathcal{L}_{\hat{k}}\hat{g}_{\mu\nu}=\mathcal{L}_{\hat{k}}\hat{C}_{\mu\nu\rho}=0,\\
\hat{G}^{(4)}_{\mu\nu\rho\sigma} &\equiv 4\partial_{[\mu}\hat{C}_{\nu\rho\sigma]}+3m(i_{\hat{k}}\hat{C})_{[\mu\nu}(i_{\hat{k}}\hat{C})_{\rho\sigma]},\\
i_{\hat{k}}T^{(r)}_{\mu_1\ldots\mu_{r-1}} &\equiv \hat{k}^\mu T^{(r)}_{\mu_1\ldots\mu_{r-1}\mu}
\end{aligned}
$$

E. Bergshoeff,Y. Lozano, T. Ortin. NPB (1998).

- It was conjectured that the M-theory origin of Romans supergravity could be obtained by performing a T duality transformation and a non trivial uplift to ten non compact dimensions of the M-theory formulated on a torus bundle with parabolic monodromy.

C. Hull. JHEP (1998).

- Nevertheless, the concrete realization of Hull's conjecture was a long term open problem.
- We obtain a formulation of the supermembrane that we believe correspond to a concrete realization of Hull's conjecture.

Massive M2-brane in ten non compact dimensions

- There is a formulation of the M2-brane on torus bundle with monodromies in $Sl(2, Z)$. Its lower energy limit correspond to gauge supergravities in 9D.

  M. P. García del Moral, J. M. Peña, A. Restuccia. JHEP (2012).

- This formulation correspond to a M2-brane toroidally compactified with C_- fluxes. It has discrete supersymmetric spectrum and hence describe a sector of the microscopical degrees of freedom of M-theory.

  M. P. García del Moral, C. Las Heras,P. Leon, J. M. Peña, A. Restuccia PLB (2019) and JHEP (2020).

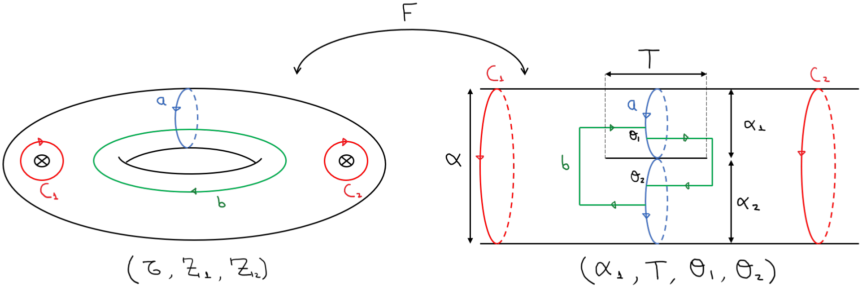
- It was proposed that the 10D massive type IIA supergravity could be obtained as the uplift of the M2-brane on a torus bundle with fluxes and parabolic monodromies. It correspond to a M2-brane on a punctured torus.

  M.P García del Moral, A. Restuccia. FP (2018).

- Then, with this in mind, we propose a formulation of the M2-brane in a background characterized by $M_9 \times LCD$, where LCD is the one loop light cone closed string interaction diagram.

- This model is based in the known relation that exist between the light cone closed string interaction diagrams and the Riemann surfaces with punctures.

The Torus with two punctures



$$F(z) = \alpha \ln \left[\frac{\Theta_1(z - Z_1|\tau)}{\Theta_1(z - Z_2|\tau)} \right] - 2\pi i \frac{Im(Z_1 - Z_2)}{Im\tau}(z - z_0)$$

$$2\pi i(Z_1 - Z_2) = (\theta_1 + \theta_2)\alpha_1 - \theta_2 - 2\pi i T$$

$$\Theta_1(z) = i \sum_{n=-\infty}^{\infty} (-1)^n q^{(n-1/2)^2} e^{i\pi(2n-1)z}, \quad q = e^{i\pi\tau}$$

S.B Giddings, S.A. Wolpert (1987).

- The Mandelstam map has the following properties
  1. The 1-form dF is an Abelian differential of third kind

$$dF \to \frac{(-1)^{i+1}dz}{z-Z_i} \quad \text{if} \quad z \to Z_i, \quad dF \to D(P_i)(z-P_i)dz \quad \text{if} \quad z \to P_i$$

     where

$$D(P_i) = \sum_i (-1)^{i+1}\left[\frac{\partial_z^2\Theta_1(P_i-Z_i,\tau)}{\Theta_1(P_i-Z_i,\tau)} - \left(\frac{\partial_z\Theta_1(P_i-Z_i,\tau)}{\Theta_1(P_i-Z_i,\tau)}\right)^2\right].$$

  2. The integrals over the curves a, b, C_u are given by

$$\int_a dF = 2\pi i\alpha_1, \quad \int_b dF = \frac{i}{2\pi}(\alpha_1\theta_1 - \alpha_2\theta_2), \quad \int_{C_u} dF = (-1)^u 2\pi i\alpha$$

  3. Decomposing the Mandelstam map as $F = G + iH$
     - $G \equiv ReF$ is single-valued, but dG is harmonic since is exact with poles.

$$dG \to \frac{(-1)^{i+1}dz}{|z-Z_i|} \quad \text{if} \quad z \to Z_i.$$

     - $H \equiv ImF$ is multi-valued and therefore dH is harmonic.

$$dH \to (-1)^{i+1}d\varphi \quad \text{if} \quad z \to Z_i.$$

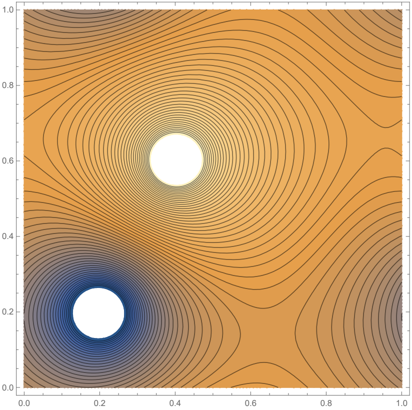


Figura: Curves $G = Const$ with $\tau = i$, $Z_1 = 0.2(1+i)$ and $Z_2 = 0.4 + 0.6i$.

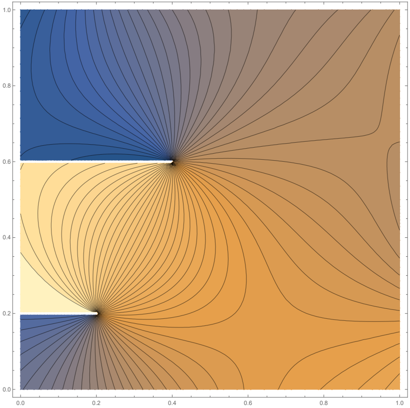


Figura: Curves $H = Const$ with $\tau = i$, $Z_1 = 0.2(1+i)$ and $Z_2 = 0.4 + 0.6i$.

Massive Supermembrane on $M_9 \times LCD$

- We propose the following metric for the target space

$$ds^2 = 2dX^+dX^- + \delta_{mn}dx^m dx^n + dK^2 + dH^2, \quad K \equiv \tanh(G)$$

- Now, defining the non trivial maps of the supermembrane as

$$dX^k = dK + dA^k, \quad dX^H = dH + dA^H.$$

- We define the determinant of the world-volume metric as

$$\sqrt{W} = \epsilon^{ab}\partial_a K \partial_b H$$

- This definition of $\sqrt{W}$ ensures that the Lie Bracket of the variables K and H is well defined and

$$\{K, H\} = 1, \quad \{A, B\} = \frac{\epsilon^{ab}}{\sqrt{W}}\partial_a A \partial_b B.$$

- The Hamiltonian of the M2 is defined, in general, as the integration of the Hamiltonian density over over the world-volume Σ of the membrane.

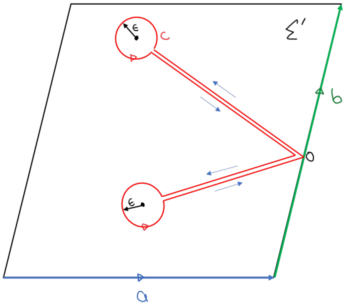
$$H = \int_{\Sigma} \mathcal{H}.$$

- Now, we obtain that if $S \equiv (P_1, P_2)$ then

$$H = \int_{\Sigma} \mathcal{H} = \int_{\Sigma/S} \mathcal{H}.$$

Then we only have to worry about the punctures.
- In order to deal with the punctures we define

$$H = \int_{\Sigma} \mathcal{H} = \lim_{\epsilon \to 0} \int_{\Sigma'} \mathcal{H}$$



- After all this consideration, the Hamiltonian can be written as

$$\begin{aligned} H &= \lim_{\epsilon\to 0}\int_{\Sigma'} d^2\sigma \frac{1}{2}\sqrt{W}\left[\left(\frac{P_m}{\sqrt{W}}\right)^2+\left(\frac{P_K}{\sqrt{W}}\right)^2+\left(\frac{P_H}{\sqrt{W}}\right)^2\right. \\ &+ \left.\frac{1}{2}\{X^m,X^n\}^2+\{X^m,X^K\}^2+\{X^m,X^H\}^2+\{X^K,X^H\}^2\right] \end{aligned}$$

- Now, we can impose the gauge

$$\{K,A^k\}+\{H,A^H\}=0$$

- Using the definition

$$P_M=\sqrt{W}P_{0M}+\Pi_M$$

The massive supermembrane Hamiltonian

$$
\begin{aligned}
H &= 2\pi\alpha + 2\pi\alpha\left[P_{0m}^2 + P_{0K}^2 + P_{0H}^2\right] + 2\pi\alpha\left[A^K(Z_2) - A^K(Z_1)\right] \\
&+ \lim_{\epsilon\to 0}\int_{\Sigma'} dK\wedge dH \frac{1}{2}\Big[\Pi_m^2 + \Pi_K^2 + \Pi_H^2 + \frac{1}{2}\{X^m, X^n\}^2 \\
&+ (\partial_H X^m + \{A^K, X^m\})^2 + (\partial_K X^m - \{A^H, X^m\})^2 \\
&+ (\partial_H A^H + \{A^K, A^H\})^2 + (\partial_K A^K + \{A^K, A^H\})^2 \\
&+ (\partial_K A^H)^2 + (\partial_H A^K)^2 - \{A^K, A^H\}^2 \\
&+ 2\bar{\Psi}\Gamma^-\Gamma_m\{X^m, \Psi\} + 2\bar{\Psi}\Gamma^-\Gamma_K\{A^K, \Psi\} + 2\bar{\Psi}\Gamma^-\Gamma_H\{A^H, \Psi\} \\
&+ 2\bar{\Psi}\Gamma^-\Gamma_K\partial_H\Psi - 2\bar{\Psi}\Gamma^-\Gamma_H\partial_K\Psi\Big],
\end{aligned}
$$

- where we have obtained massive terms

$$
\begin{aligned}
(\partial_K X^m)^2 + (\partial_H X^m)^2 &\neq 0 \quad ,\\
(\partial_K A^K)^2 + (\partial_H A^K)^2 &\neq 0 \quad ,\\
(\partial_K A^H)^2 + (\partial_H A^H)^2 &\neq 0 \quad .
\end{aligned}
$$

- Reminiscent of the flux condition

$$\int_\Sigma dX^K \wedge dX^H = \int_\Sigma dK \wedge dH = 2\pi\alpha$$

- The Hamiltonian is subject to the following constrains

$$d\chi = 0, \quad \int_a \chi = 0, \quad \int_b \chi = 0, \quad \int_{C_1} \chi = 0 \tag{1}$$

where

$$\chi = P_K dX^K + P_H dX^H + P_m dX^m + \bar{\Psi}\Gamma^- d\Psi \tag{2}$$

Conclusions

- We have obtained the description of a massive $D = 11$ supermembrane formulated on a target space $M_9 \times LCD$, so that the theory contains ten non-compact dimensions.
- The nontriviality of the Mandelstam maps induce the presence of mass terms different from zero for all the dynamical fields. If there exists a matrix regularization of the Hamiltonian should have a discrete spectrum.
- The theory is globally and locally invariant under area preserving diffeomorphisms that leave fixed the punctures. This give a close relation with the (1,1) knots theory.
- This formulation represent a new sector of M-theory with well-defined quantum properties.
- We believe that this formulation of the M2-brane represents a concrete realization of Hull's conjecture.
- In a paper to appear we will show that a non trivial double dimensional reduction generates a massive type IIA superstring in 10D.

M.P Garcia del Moral, P. Leon, A. Restuccia arXiv:2101.xxxx[hep-th]

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