

Quantum BTZ black hole

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Based on [2007.15999](#) [hep-th]



Antonia Frassino



Benson Way

What's a Quantum Black Hole?

A large- N matrix, with
fastly scrambling entries?

quantum black hole

Simpler goal:

Classical geometry of black hole modified
(possibly a lot) by quantum fields

Much insight gained this way

Quantum backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

classical Einstein tensor & metric

quantum matter renorm stress tensor
(many fields)

Coupled system: metric + \langle QFT \rangle

Very hard to solve simultaneously

Perturbative backreaction: limited insight

Quantum backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

Exact backreaction:

2D models: CGHS/RST, JT+CFT

cf Thorlacius's talk @ IbSt

Holographic reformulation

RE+Fabrizio+Kaloper 2002

Tanaka 2002

Quantum backreaction — What for?

Radiating/Evaporating Black hole

Quantum (generalized) entropy $S_{gen} = \frac{A}{4G} + S_{out}$

Page curve turns when $S_{out} \sim \frac{A}{4G}$

Penington 2019
Almheiri+al 2019

Holographic approach

$\langle T_{\mu\nu} \rangle$: CFT on boundary geometry of bulk dual
classical bulk \Leftrightarrow planar CFT ($N \rightarrow \infty$)

Conventional AdS/CFT has *fixed boundary* geometry

Make boundary geometry dynamical

Braneworld gravity

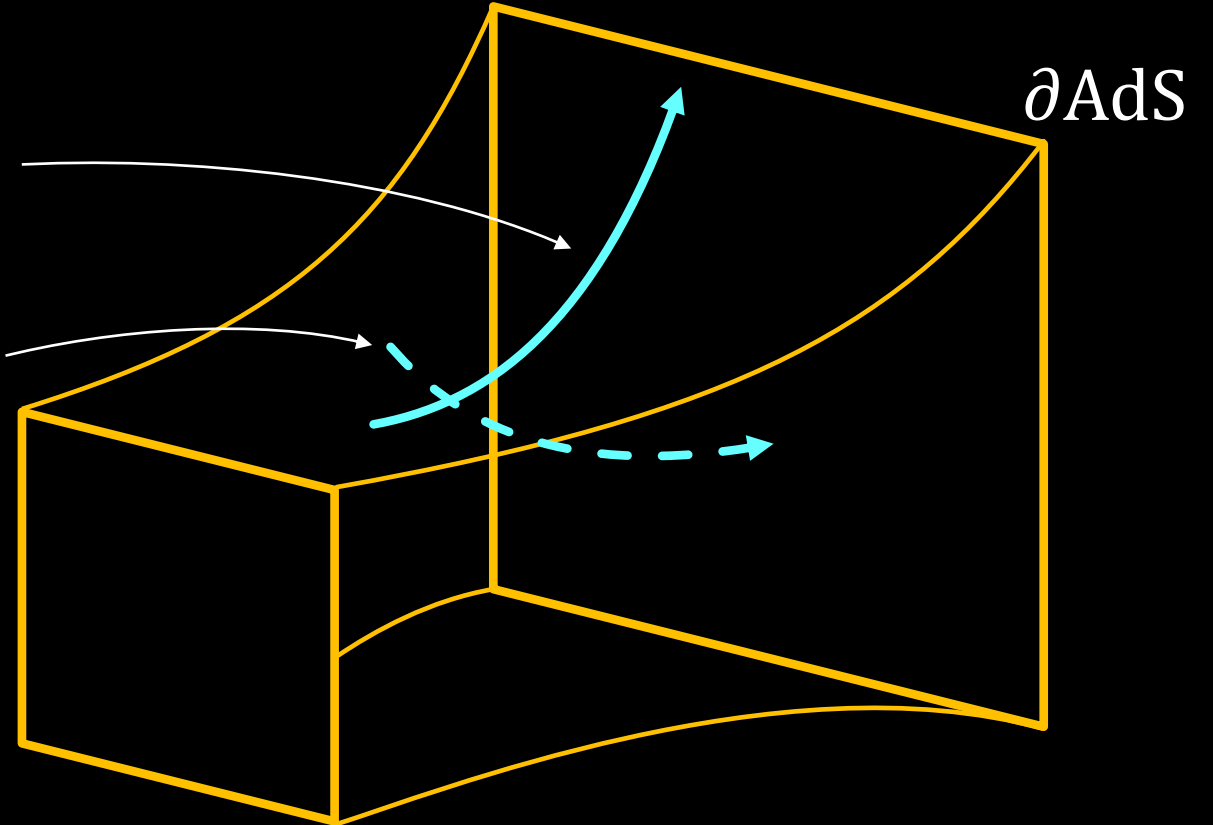
Randall+Sundrum 1999

AdS bulk

Gravitational fluctuations

non-normalizable

normalizable



Braneworld gravity

Randall+Sundrum 1999

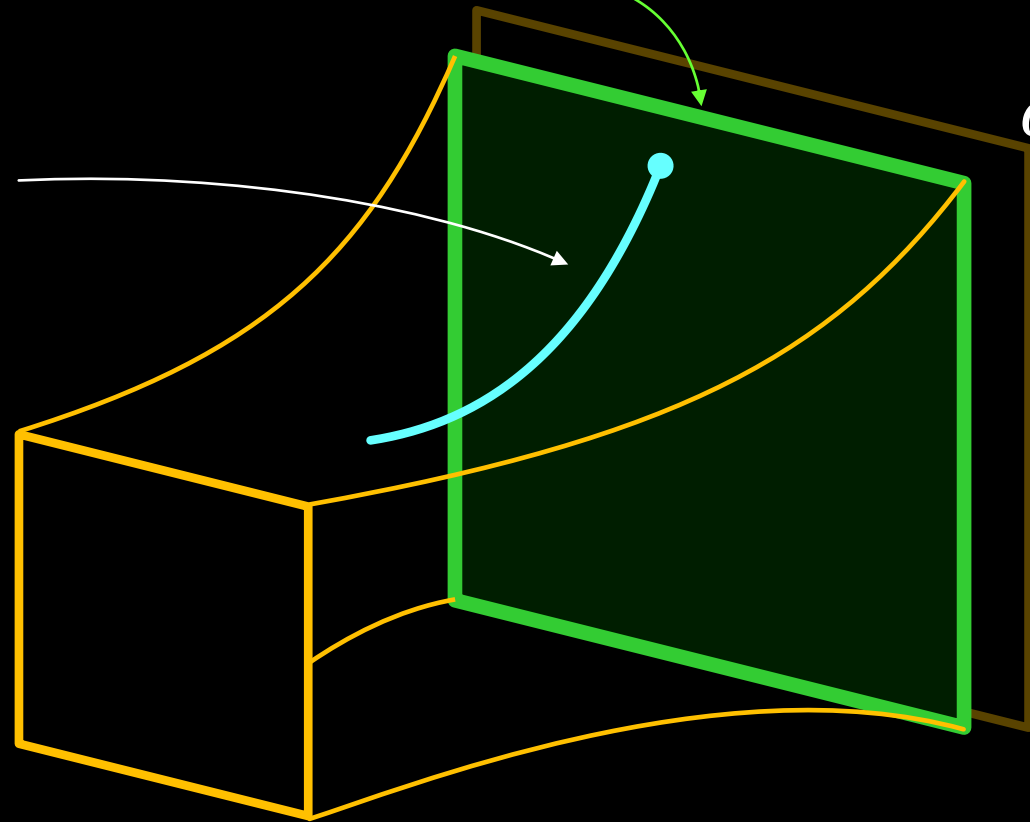
Gravitational fluctuations

non-normalizable

AdS bulk

Planck brane

∂AdS



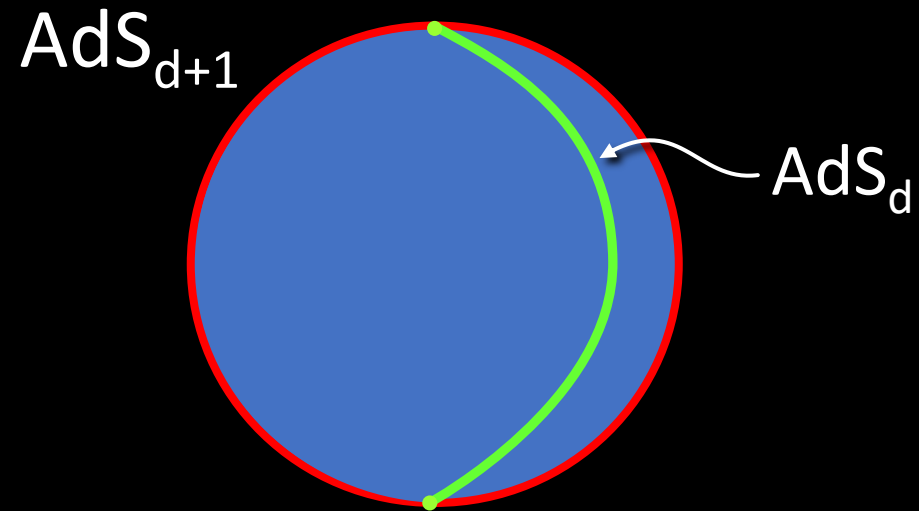
Braneworld dynamics

Graviton on the brane

Remaining grav bulk dynamics
is dual to CFT (cutoff & planar)

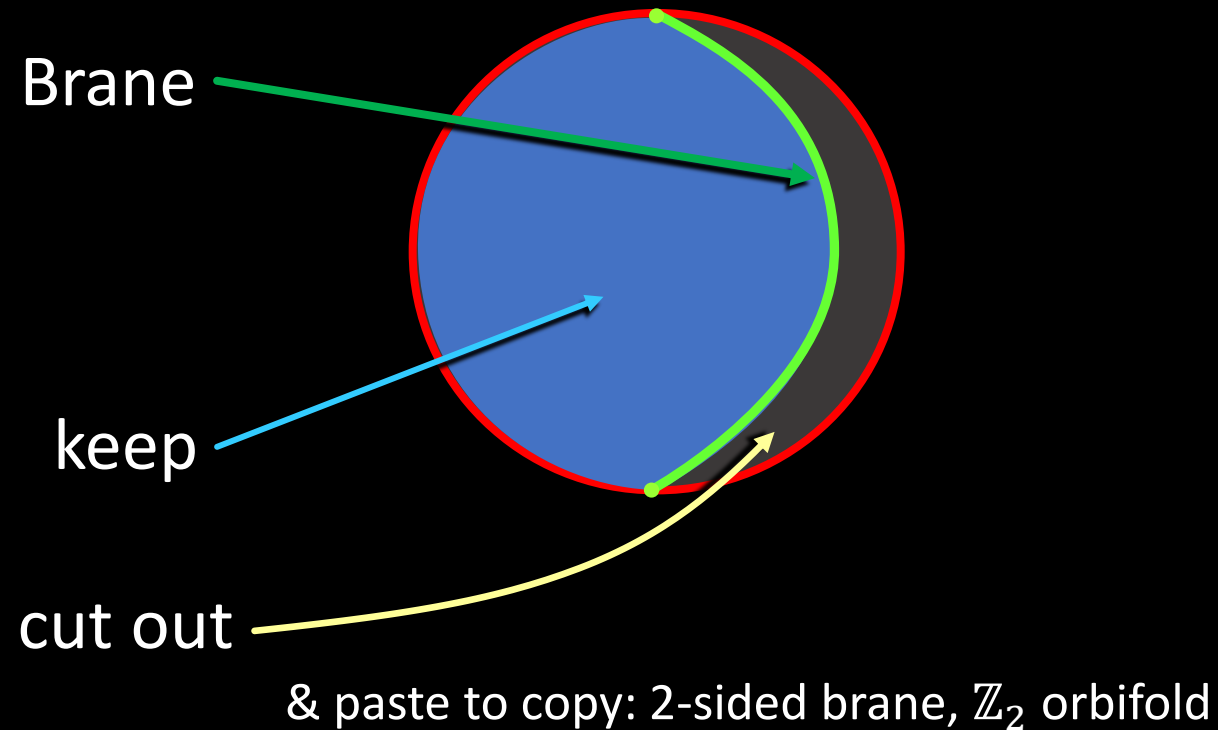
Slicing AdS

AdS slicing



Braneworlds

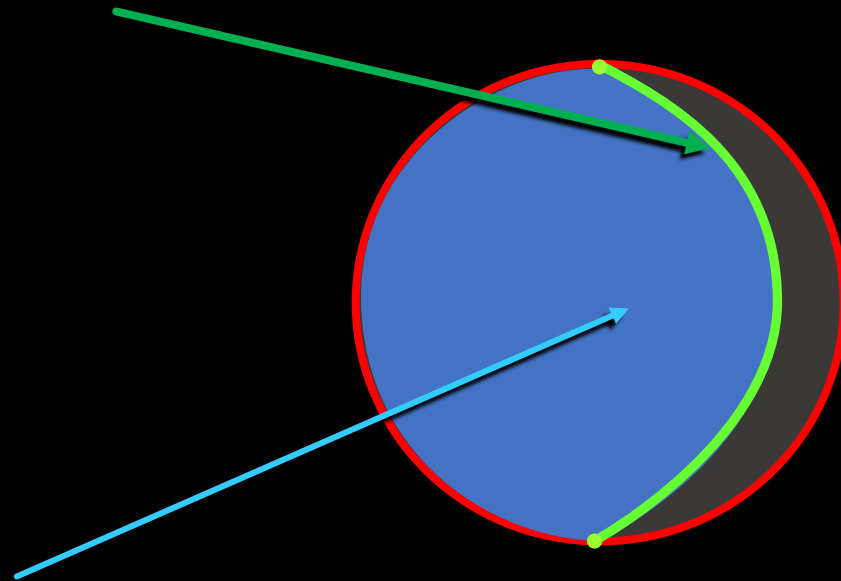
Karch-Randall: AdS branes



Randall+Sundrum 1999
Karch+Randall 2000

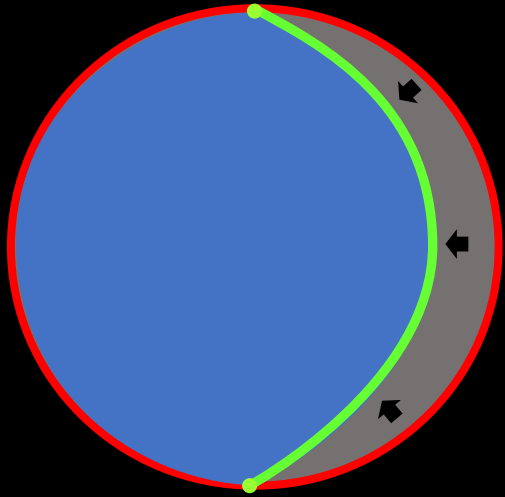
Braneworld holographies

Gravity effectively induced on brane



Holographic CFT describes bulk

Effective gravity + CFT

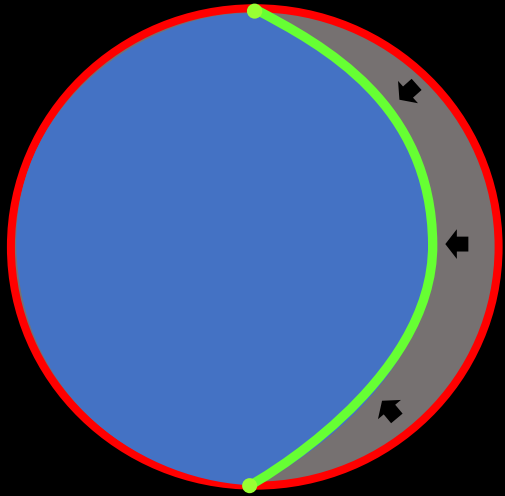


Integrate from boundary to brane
à la Fefferman-Graham
(holographic renormalization)

i.e. solve bulk Einstein eqs with prescribed
boundary metric, perturbatively away
from the bdry

deHaro+Skenderis+Solodukhin 2000

Effective gravity + CFT



Do not introduce counterterms!
Keep finite cutoff: brane position

This integrates the CFT UV degrees of freedom and generates the effective action

deHaro+Skenderis+Solodukhin 2000

Braneworld holography

$$\mathcal{G}_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle_{\text{planar}}$$

Einstein+higher curvature

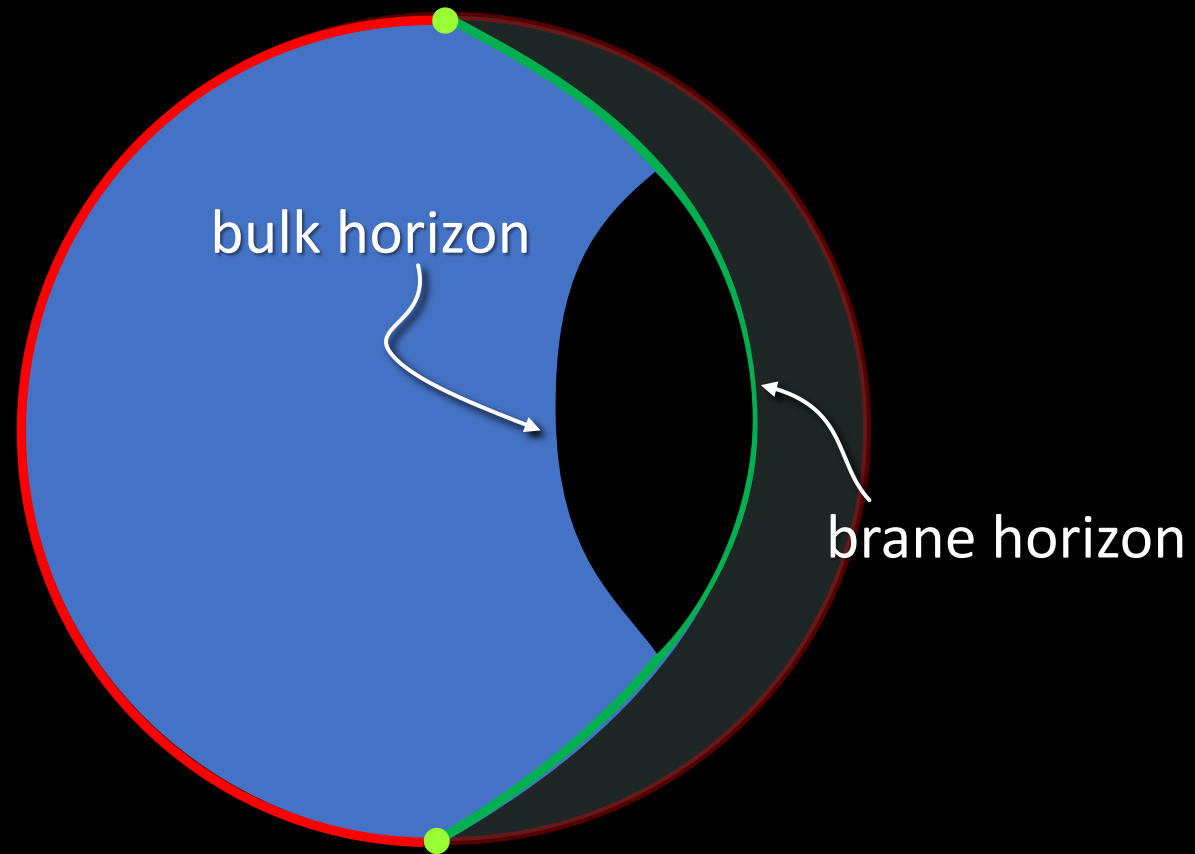
large-N CFT stress tensor

Effective theory w/ cutoff (brane position)

Einstein+higher-curvature terms: induced by CFT above cutoff

$\langle T_{\mu\nu} \rangle$: holographic CFT below cutoff

Black hole on brane

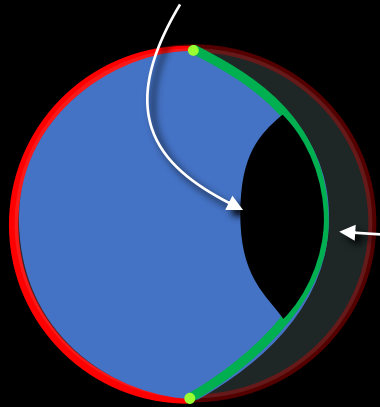


Quantum BH Entropy in bw holography

$$S_{gen} = S_{Wald} + S_{out}$$

Bulk bh entropy

$$S_{gen} = \frac{A_{d+1}}{4G_{d+1}}$$



Brane bh entropy

$$S_{Wald} = \frac{A_d}{4G_d} + \dots$$

CFT entanglement entropy

$$S_{out} = S_{gen} - S_{Wald}$$

(CFT below cutoff)

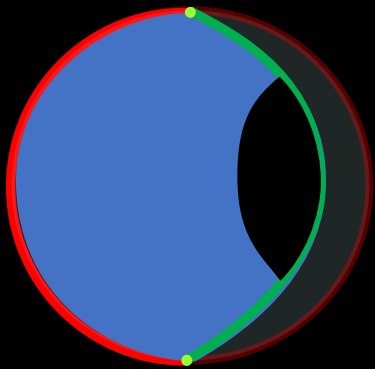
induced by entanglement of CFT above cutoff

RE 2006

Quantum BH Entropy

If the holographic interpretation of braneworlds is consistent, then

$$TdS_{gen} = dM - \Omega dJ$$

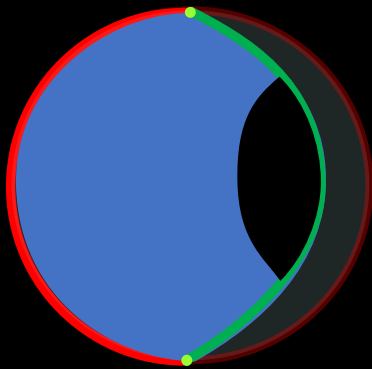


$$\Delta S_{gen} \geq 0$$

S_{Wald} should not satisfy these

Quantum BH Entropy: Second law

$$\Delta S_{gen} \geq 0 \quad \text{bulk 2nd law}$$



S_{Wald} can decrease

for perturbative backreaction (non-holo): [Wall 2011](#)

Quantum BH Entropy: First law

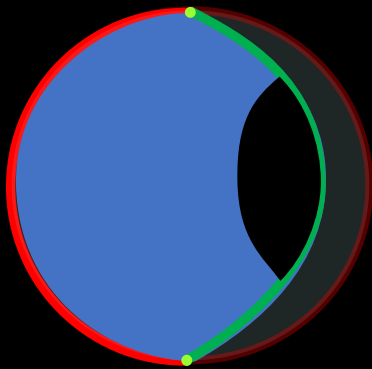
$$TdS_{gen} = dM - \Omega dJ$$

Not trivial!

Bulk 1st law?

In $d + 1$ dim bulk

On d dim brane
w/ higher curvature gravity

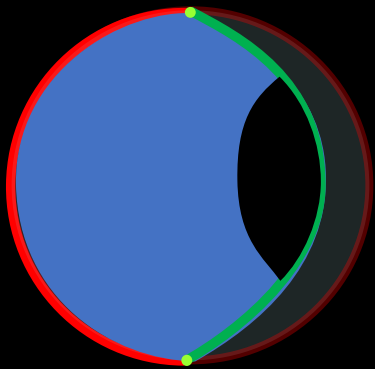


S_{Wald} : no 1st law

Quantum BH Entropy: First law

$$T dS_{gen} = dM - \Omega dJ$$

We will test this in explicit
exact solutions



Black hole on the brane: AdS C-metric

Plebański + Demiański 1976

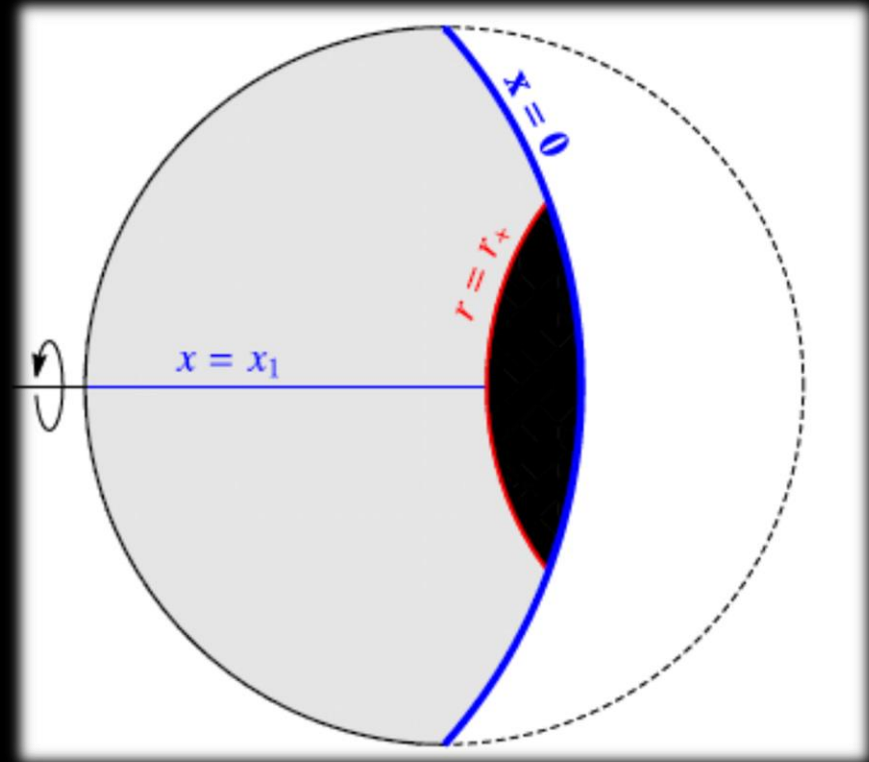
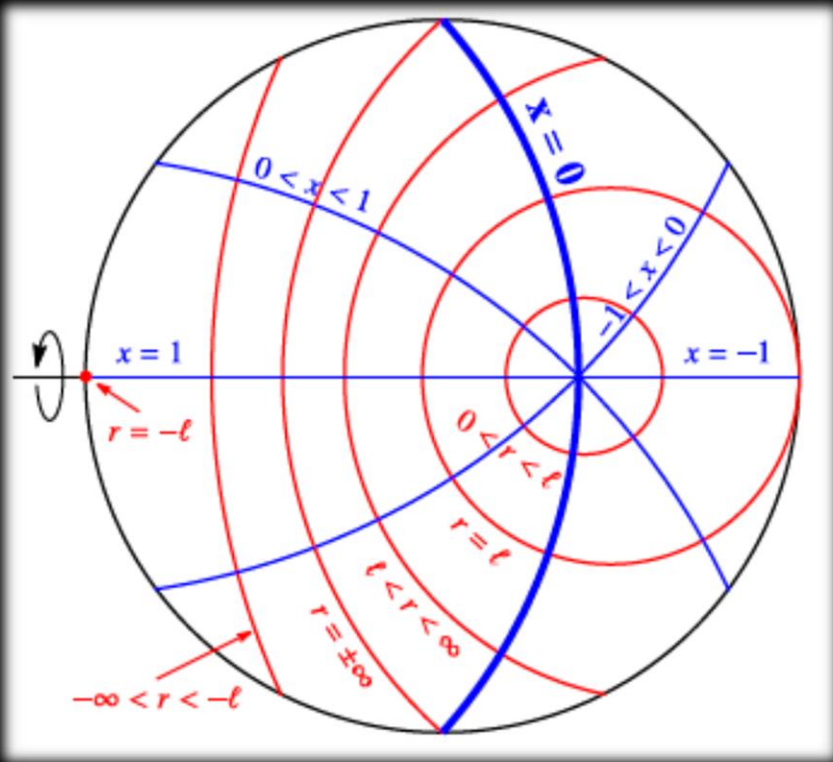
$$ds^2 = \frac{\ell^2}{(\ell + xr)^2} \left(-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2 \left(\frac{dx^2}{G(x)} + G(x)d\phi^2 \right) \right)$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}$$

$$G(x) = 1 - \kappa x^2 - \mu x^3$$

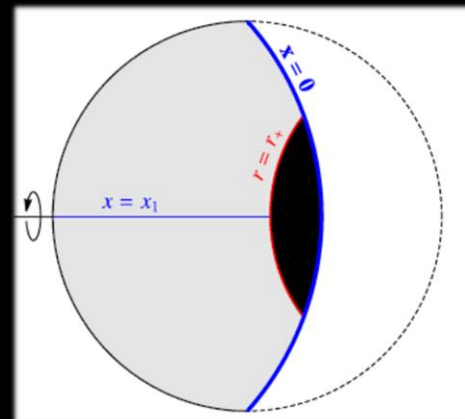
Adapted coordinates

Brane at $\mathbf{x} = \mathbf{0}$: $K_{ab} = -\frac{1}{\ell} h_{ab}$



quBTZ metric

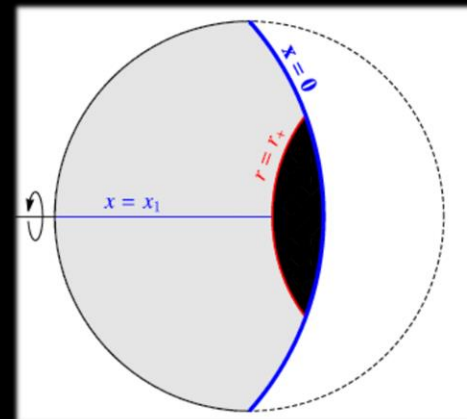
3D metric induced on brane at $x = 0$



$$ds^2 = - \left(\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

quBTZ metric

3D metric induced on brane at $x = 0$

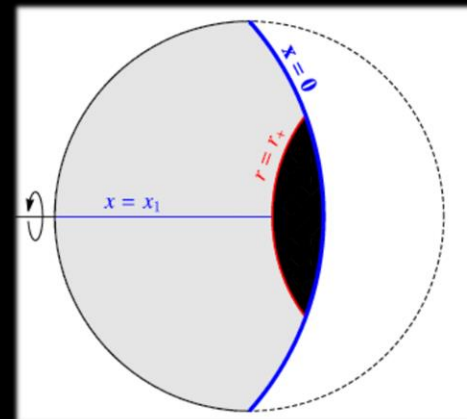


$$ds^2 = - \left(\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

$\ell = 0$: BTZ black hole

quBTZ metric

3D metric induced on brane at $x = 0$



$$ds^2 = - \left(\frac{r^2}{\ell_3^2} - 8G_3M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

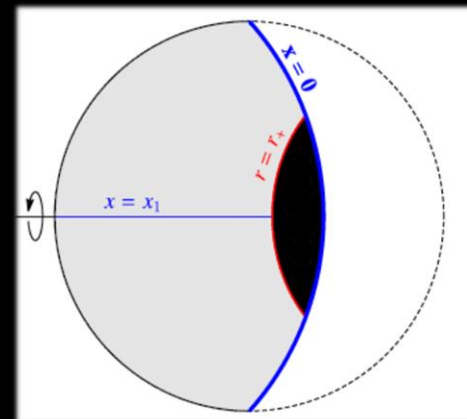
$\ell = 0$: BTZ black hole

$\ell > 0$: quantum-corrected BTZ $\neq \text{AdS}_3/\Gamma$

quBTZ metric

quantum correction: $\ell = 2 \hbar c G_3 + \dots$

CFT central charge



$$ds^2 = - \left(\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

$F(M)$ determined by bulk regularity

Rotating quBTZ

Can have $M < J/\ell_3$

$\langle T_{ab} \rangle_{CFT}$ smooth at inner Cauchy horizon

but see *RE+Tomašević 2020 “Strong Cosmic Censorship in BTZ”*

Quantum BH Entropy: First law

M J measured in 3D eff theory w/ rotation added

T Ω

(lengthy expressions)

$$S_{gen} = \frac{A_{bulk}}{4G_4}$$

$$TdS_{gen} = dM - \Omega dJ$$

So

Fully quantum-backreacted BTZ and CFT stress tensor
can be described *exactly* and *in detail*

Quantum entropy from braneworld holography: *consistent*

Efficient use of holography to *solve a hard quantum problem*

Thank you



Antonia Frassino



Benson Way

Effective action gravity + CFT

Integrate 4D bulk action

RE+Johnson+Myers 1999

→ 3D eff action

3D h_{ab}, R_{ab}

$\frac{1}{L_3^2}$: 4D $\frac{1}{\ell_4^2}$ and brane tension $\frac{1}{\ell}$

ℓ : brane position \sim tension $^{-1} \sim$ cutoff $^{-1}$

$$I = \frac{1}{16\pi G_3} \int d^3x \sqrt{-h} \left[\frac{2}{L_3^2} + R + \ell^2 \left(\frac{3}{8} R^2 - R_{ab} R^{ab} \right) + \dots \right]$$

$$G_3 = \frac{G_4}{2\ell_4}$$

+ I_{CFT}

holographic CFT

same as "New 3D massive gravity"

Bergshoeff+Hohm+Townsend 2009

Effective action gravity + CFT

$$I = \frac{1}{16\pi G_3} \int d^3x \sqrt{-h} \left[\frac{2}{L_3^2} + R + \ell^2 \left(\frac{3}{8} R^2 - R_{ab} R^{ab} \right) + \dots \right] + I_{CFT}$$

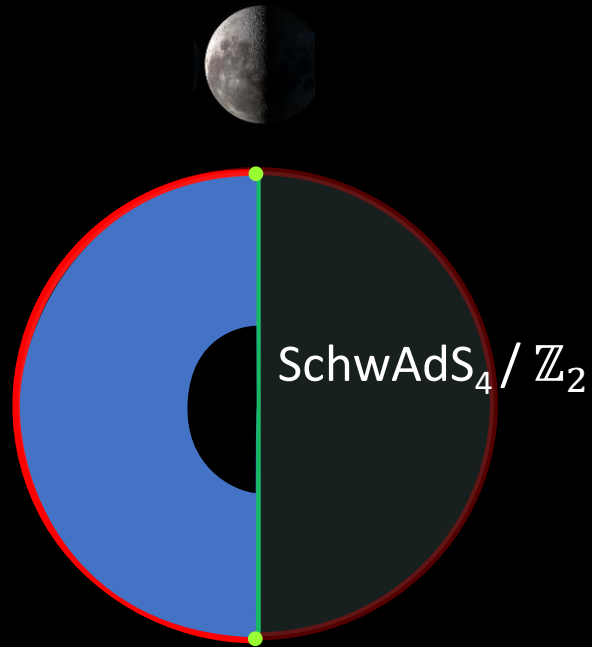
An exact 4D bulk solution is a 3D effective theory solution

exact CFT stress tensor

exact backreaction of the CFT (planar)

exact in *all* higher curvature corrections

Strength of backreaction: ℓ



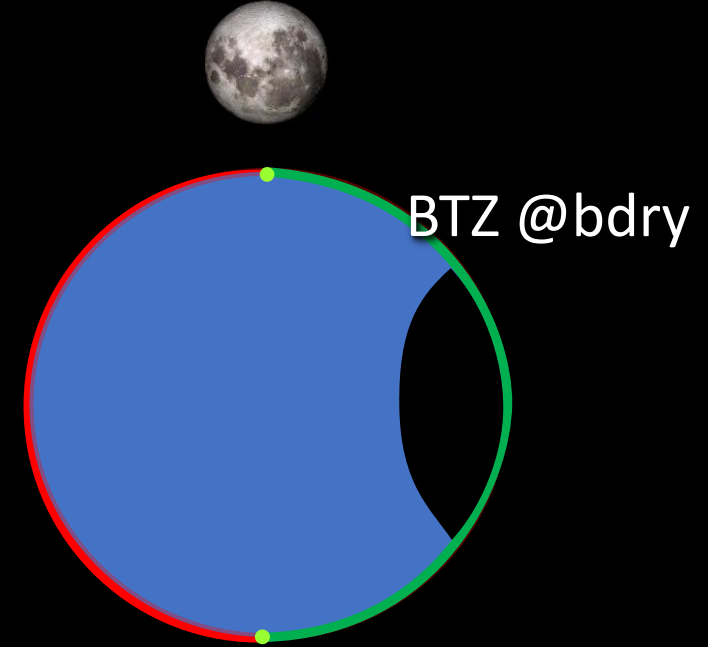
$$\ell \rightarrow \infty$$

zero tension brane
"maximal backreaction"
no 3D gravity



$$0 < \ell < \infty$$

brane tension $\propto 1/\ell$
backreaction $\propto \ell$
cutoff energy $\propto 1/\ell$



$$\ell \rightarrow 0$$

brane tension $\rightarrow \infty$
backreaction $\rightarrow 0$

$$ds^2 = - \left(\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

G_3 "renormalized" by higher curvatures
 quantum correction: $\ell = 2 \hbar c G_3 + \dots$
 CFT central charge

$F(M)$ determined by bulk regularity outside horizon

$$\langle T^a_b \rangle = \frac{\ell}{16\pi G_3} \frac{F(M)}{r^3} \text{diag}\{1, 1, -2\} + \dots$$

not thermal plasma $\text{diag}\{-2, 1, 1\}$

$\langle T^a_b \rangle$ for free fields

Steif, Shiraishi+Maki, Lifschytz+Ortiz 1993

Casals+Fabbri+Martínez+Zanelli 2016,2019

Free conformal scalar in BTZ

Method of images: $\text{BTZ} = \text{AdS}_3/\Gamma$

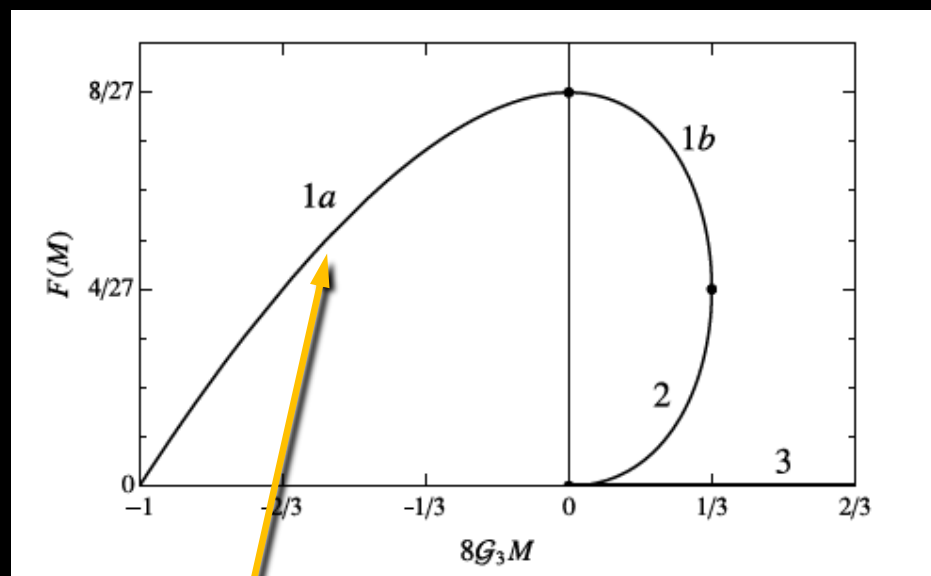
Satisfy KMS, Hartle-Hawking at horizon

Same structure: $\langle T^a_b \rangle = \frac{c}{8\pi} \frac{F(M)}{r^3} \text{diag}\{1, 1, -2\}$

different $F(M) = \sum_{n=1}^{\infty} F_n(M)$ (images)

Holographic quantum black holes: small AdS_3 bhs

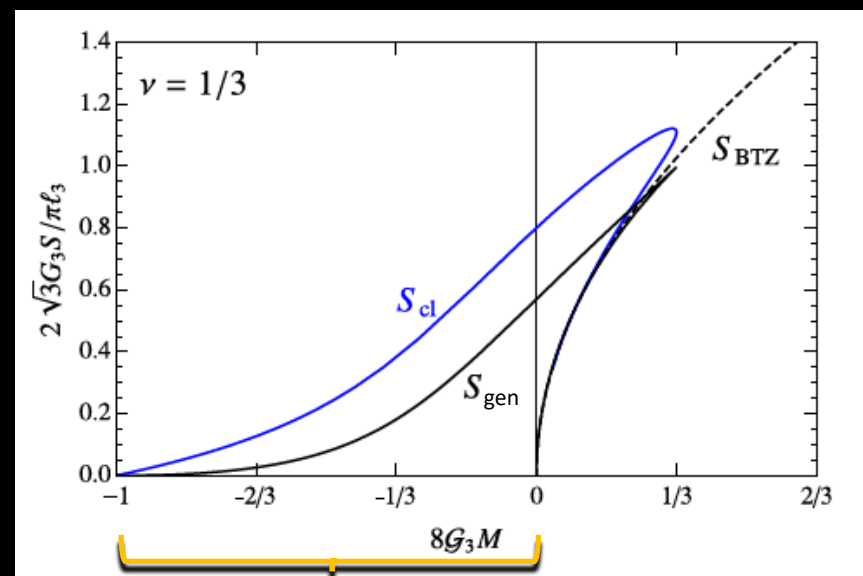
Stress tensor



Casimir energy on a cone
Conical singularities dressed by quantum horizon

Quantum Cosmic Censorship

Entropy

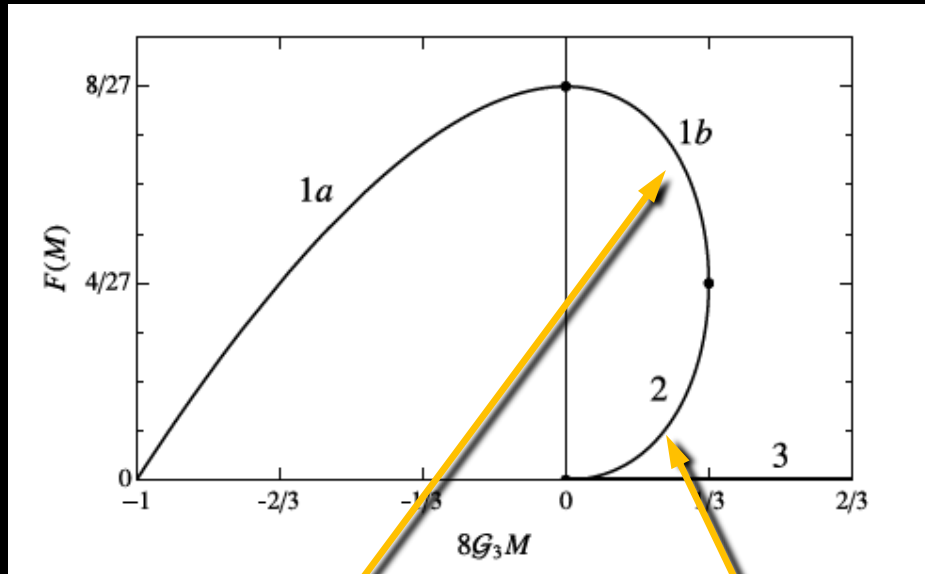


Small AdS_3 black holes $-\frac{1}{8G_3} < M < 0$

RE+Fabri+Kaloper 2002

Holographic quantum black holes: quBTZ

Stress tensor

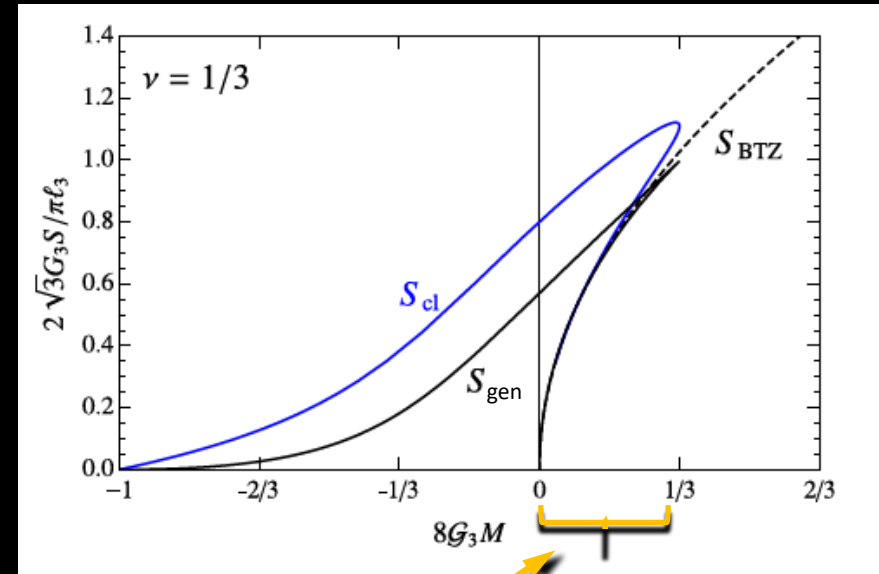


CFT in BTZ

Casimir-dominated

Thermal-dominated

Entropy



Quantum-corrected BTZ: $0 < M < \frac{1}{24G_3}$

Quantum BH Entropy: First law

$$M = \frac{1}{2\mathcal{G}_3} \frac{z^2(1 - \nu z^3)(1 + \nu z)}{(1 + 3z^2 + 2\nu z^3)^2}$$

(measured in 3D eff theory)

$$T = \frac{1}{2\pi\ell_3} \frac{z(2 + 3\nu z + \nu z^3)}{1 + 3z^2 + 2\nu z^3}$$

ν : backreaction parameter ℓ/ℓ_3

z : mass parameter

$$S_{gen} = \frac{A_{bulk}}{4G_4} = \frac{\pi\ell_3}{\mathcal{G}_3} \frac{z}{1 + 3z^2 + 2\nu z^3}$$

$$T \partial_z S_{gen} = \partial_z M$$

$$\Rightarrow T dS_{gen} = dM$$

Rotating quBTZ

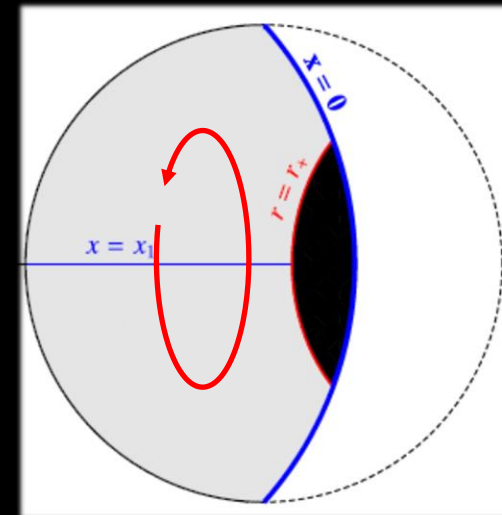
Rotating AdS C-metric

Bulk structure similar to Kerr(-AdS₄)

inner&outer horizons, ring singularity

$\ell = 0$: rot BTZ black hole

$\ell > 0$: quantum-corrected rot BTZ $\neq \text{AdS}_3/\Gamma$



Rotating quBTZ metric

$$ds^2 = - \left(\frac{r^2}{\ell_3^2} - 8G_3M - \ell \frac{F_1(M, J)}{r} \right) dt^2 + \left(r^2 + \ell \frac{F_2(M, J)}{r} \right) d\phi^2$$
$$- 8G_3J \left(1 + \ell \frac{F_3(M, J)}{r} \right) dt d\phi + \left(\frac{r^2}{\ell_3^2} - 8G_3M + \frac{(4G_3J)^2}{r^2} + \ell \frac{F_4(M, J)}{r} \right)^{-1} dr^2$$

$\ell = 0$: rot BTZ black hole

$\ell > 0$: quantum-corrected rot BTZ $\neq \text{AdS}_3/\Gamma$

Going further

General holographic proof of $TdS_{gen} = dM - \Omega dJ$

More classical proofs of quantum theorems?

Holographic duals of massive gravities – 3D and higher d

Extensions: charge, higher-d?

Exact entanglement islands

Dual Hawking evaporation? *RE+Kaloper+Tanaka+al 2002-20??*

