

# Quantum BTZ black hole

Roberto Emparan  
ICREA & UBarcelona

**Iberian Strings 2021**  
IST Lisboa, 22 Jan 2021

Based on [2007.15999](#) [hep-th]



Antonia Frassino



Benson Way

# What's a Quantum Black Hole?

A large- $N$  matrix, with  
fastly scrambling entries?

# quantum black hole

Simpler goal:

Classical geometry of black hole modified  
(possibly a lot) by quantum fields

Much insight gained this way

# Quantum backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

classical Einstein tensor & metric

quantum matter renorm stress tensor  
(many fields)

*Coupled system: metric +  $\langle$ QFT $\rangle$*

Very hard to solve simultaneously

Perturbative backreaction: limited insight

# Quantum backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

Exact backreaction:

2D models: CGHS/RST, JT+CFT

*cf Thorlacius's talk @ IbSt*

**Holographic reformulation**

*RE+Fabri+Kaloper 2002*

*Tanaka 2002*

# Quantum backreaction — What for?

Radiating/Evaporating Black hole

Quantum (generalized) entropy  $S_{gen} = \frac{A}{4G} + S_{out}$

Page curve turns when  $S_{out} \sim \frac{A}{4G}$

*Penington 2019*  
*Almheiri+al 2019*

# Holographic approach

$\langle T_{\mu\nu} \rangle$ : CFT on boundary geometry of bulk dual  
classical bulk  $\Leftrightarrow$  planar CFT ( $N \rightarrow \infty$ )

Conventional AdS/CFT has *fixed boundary* geometry

Make boundary geometry dynamical



# Braneworld gravity

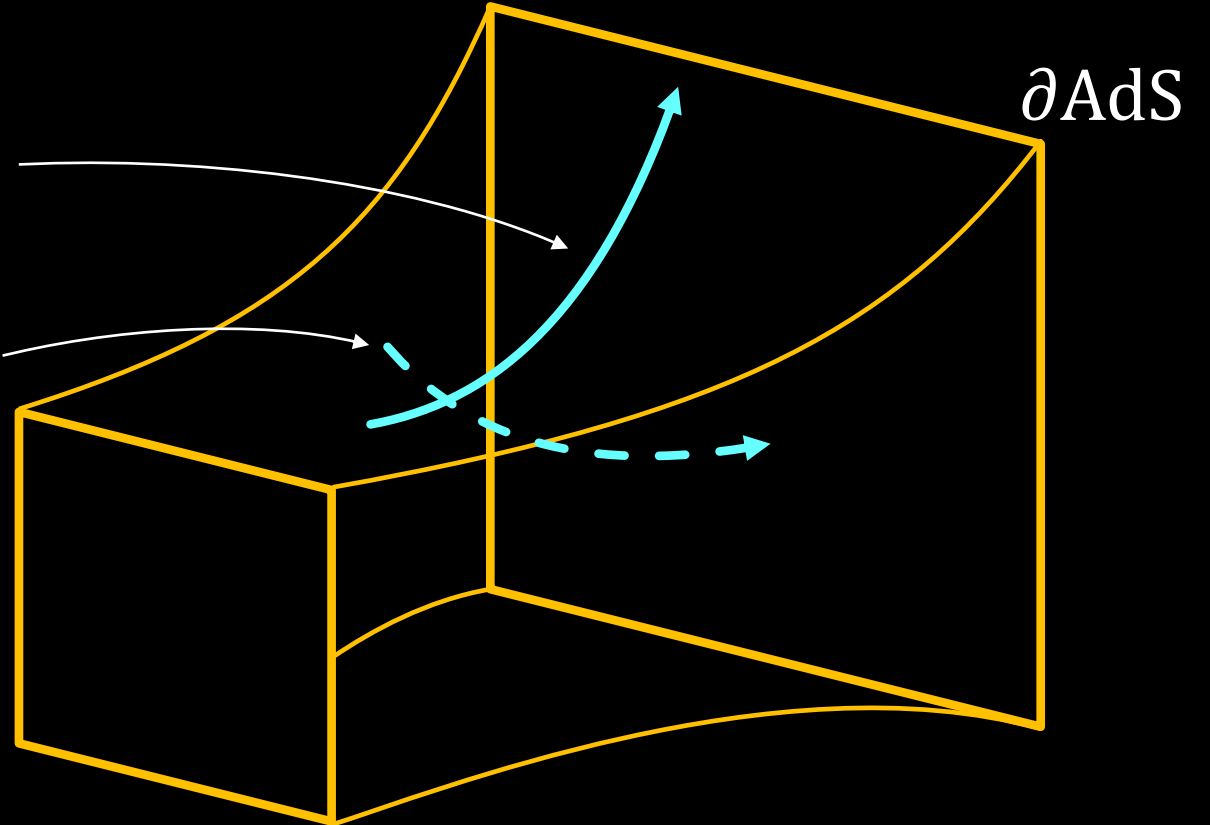
*Randall+Sundrum 1999*

AdS bulk

Gravitational fluctuations

non-normalizable

normalizable



# Braneworld gravity

*Randall+Sundrum 1999*

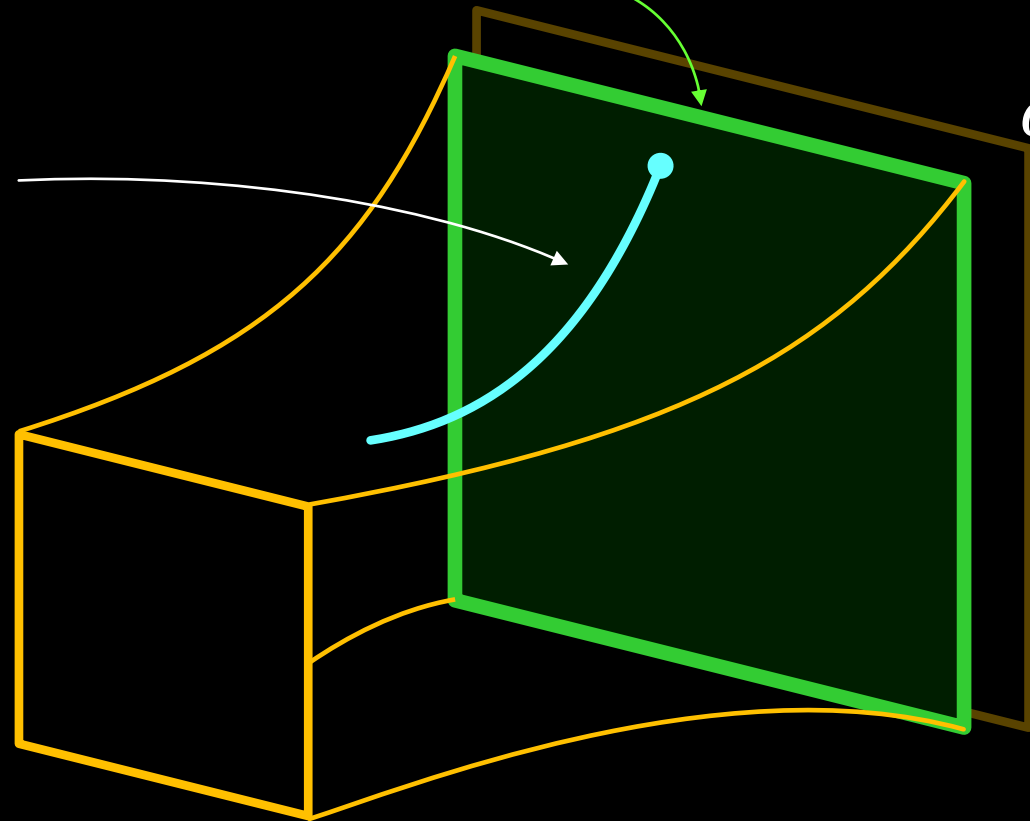
Gravitational fluctuations

non-normalizable

AdS bulk

Planck brane

$\partial\text{AdS}$



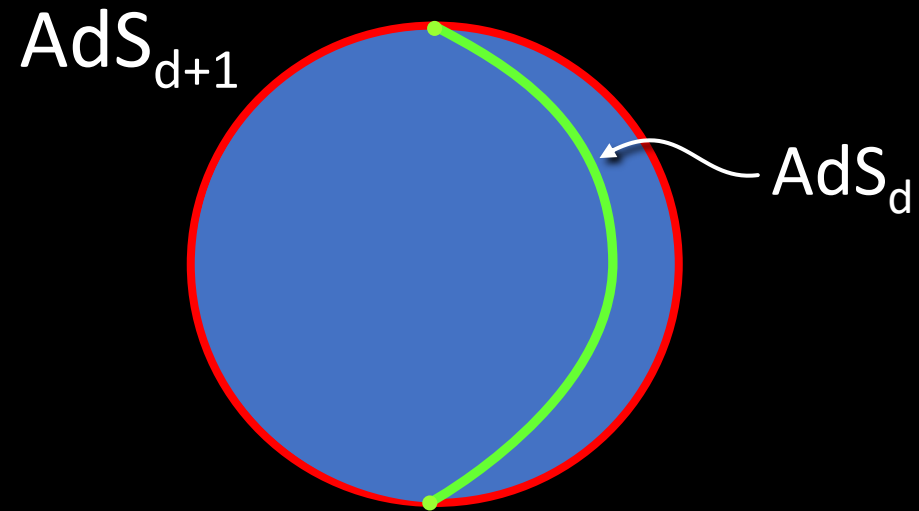
# Braneworld dynamics

Graviton on the brane

Remaining grav bulk dynamics  
is dual to CFT (cutoff & planar)

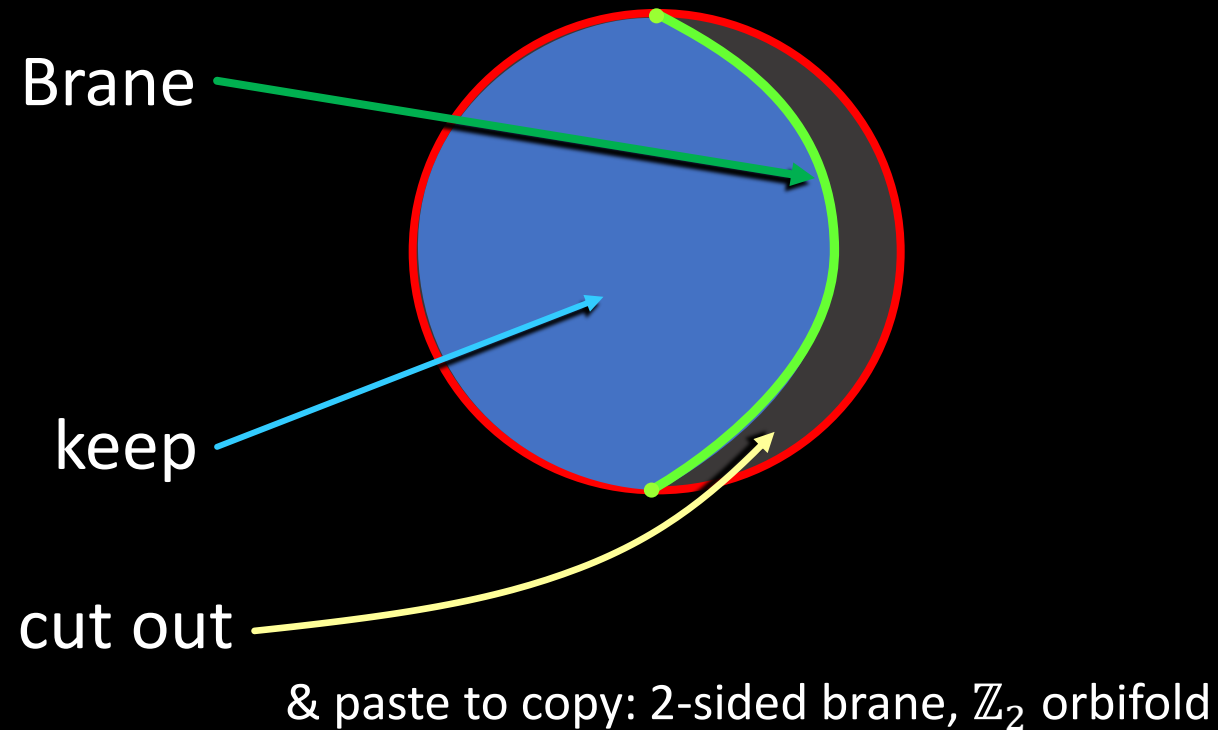
# Slicing AdS

AdS slicing



# Braneworlds

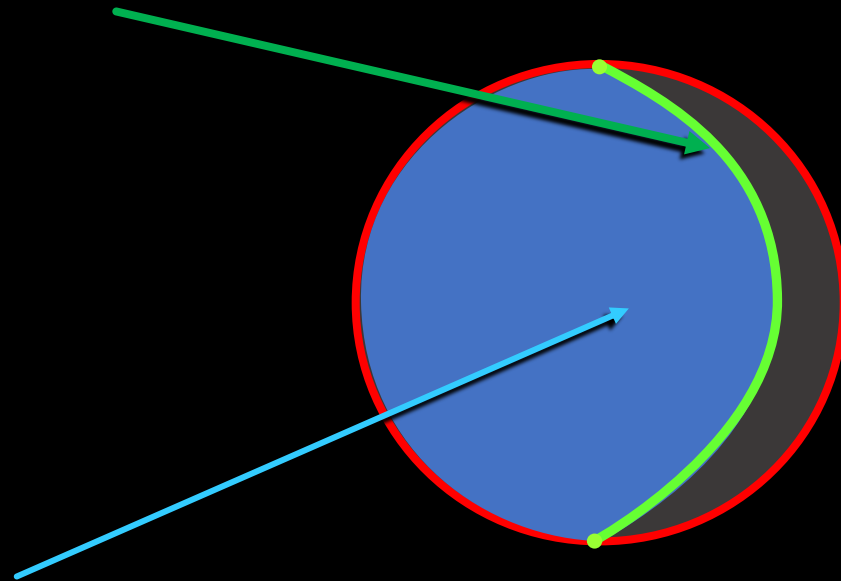
Karch-Randall: AdS branes



*Randall+Sundrum 1999*  
*Karch+Randall 2000*

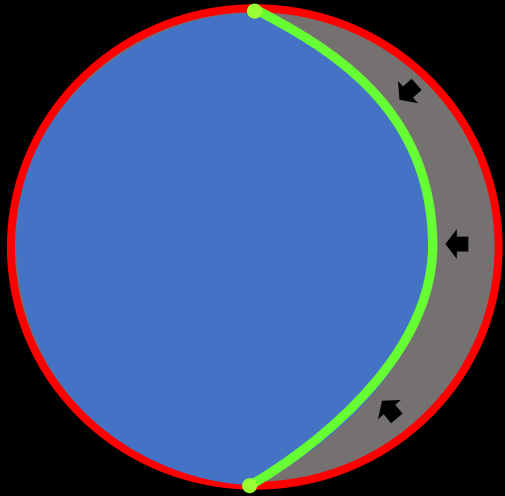
# Braneworld holographies

Gravity effectively induced on brane



Holographic CFT describes bulk

# Effective gravity + CFT

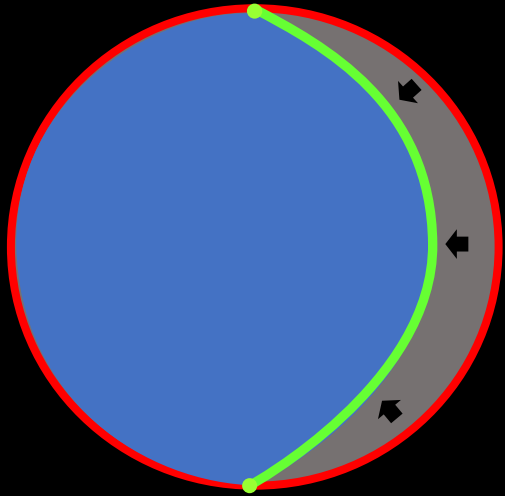


Integrate from boundary to brane  
*à la* Fefferman-Graham  
(holographic renormalization)

i.e. solve bulk Einstein eqs with prescribed  
boundary metric, perturbatively away  
from the bdry

*deHaro+Skenderis+Solodukhin 2000*

# Effective gravity + CFT



*Do not introduce counterterms!*  
Keep finite cutoff: brane position

This integrates the CFT UV degrees of freedom and generates the effective action

*deHaro+Skenderis+Solodukhin 2000*



# Braneworld holography

$$\mathcal{G}_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle_{\text{planar}}$$

Einstein+higher curvature

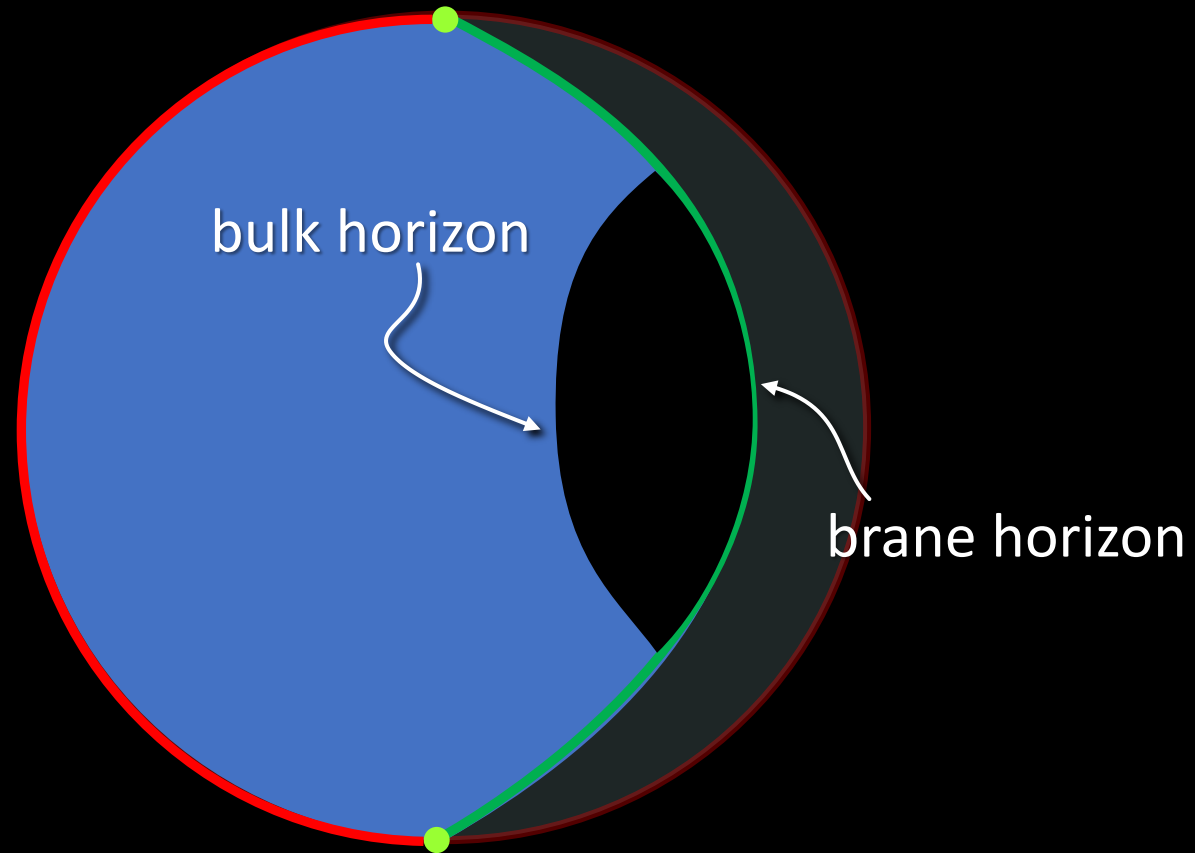
large-N CFT stress tensor

Effective theory w/ cutoff (brane position)

Einstein+higher-curvature terms: induced by CFT above cutoff

$\langle T_{\mu\nu} \rangle$ : holographic CFT below cutoff

# Black hole on brane

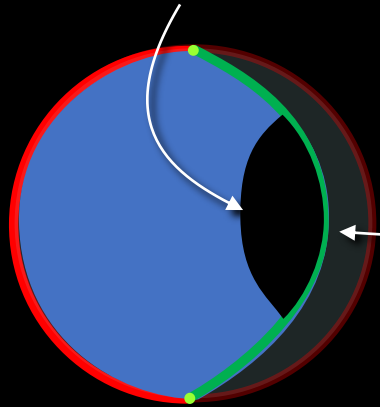


# Quantum BH Entropy in bw holography

$$S_{gen} = S_{Wald} + S_{out}$$

Bulk bh entropy

$$S_{gen} = \frac{A_{d+1}}{4G_{d+1}}$$



Brane bh entropy

$$S_{Wald} = \frac{A_d}{4G_d} + \dots$$

CFT entanglement entropy

$$S_{out} = S_{gen} - S_{Wald}$$

(CFT below cutoff)

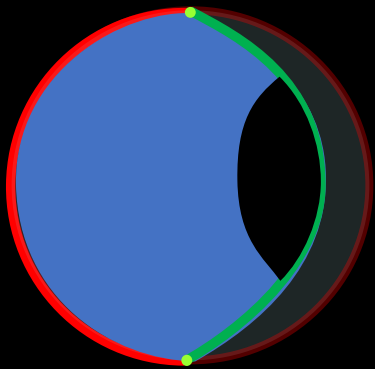
induced by entanglement of CFT above cutoff

RE 2006

# Quantum BH Entropy

If the holographic interpretation of braneworlds is consistent, then

$$T dS_{gen} = dM - \Omega dJ$$

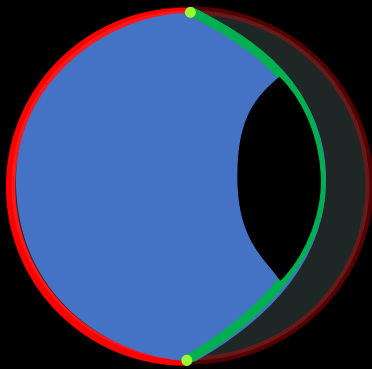


$$\Delta S_{gen} \geq 0$$

$S_{Wald}$  should not satisfy these

# Quantum BH Entropy: Second law

$$\Delta S_{gen} \geq 0 \quad \text{bulk 2nd law}$$



$S_{Wald}$  can decrease

for perturbative backreaction (non-holo): [Wall 2011](#)

# Quantum BH Entropy: First law

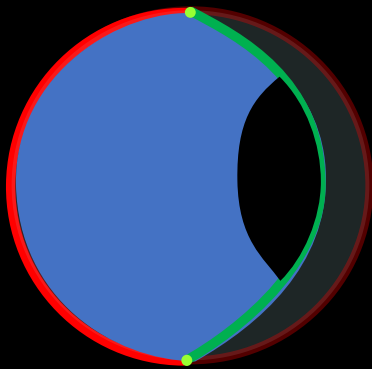
$$TdS_{gen} = dM - \Omega dJ$$

Not trivial!

*Bulk 1st law?*

In  $d + 1$  dim bulk

On  $d$  dim brane  
w/ higher curvature gravity

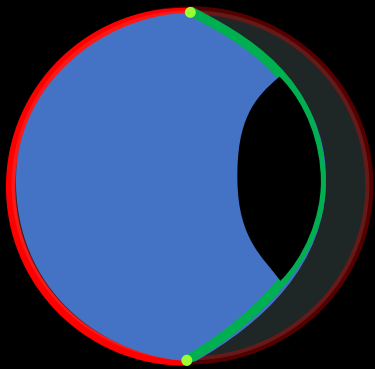


$S_{Wald}$ : no 1st law

# Quantum BH Entropy: First law

$$T dS_{gen} = dM - \Omega dJ$$

We will test this in explicit  
exact solutions



# Black hole on the brane: AdS C-metric

*Plebański + Demiański 1976*

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2} \left( -H(r)dt^2 + \frac{dr^2}{H(r)} + r^2 \left( \frac{dx^2}{G(x)} + G(x)d\phi^2 \right) \right)$$

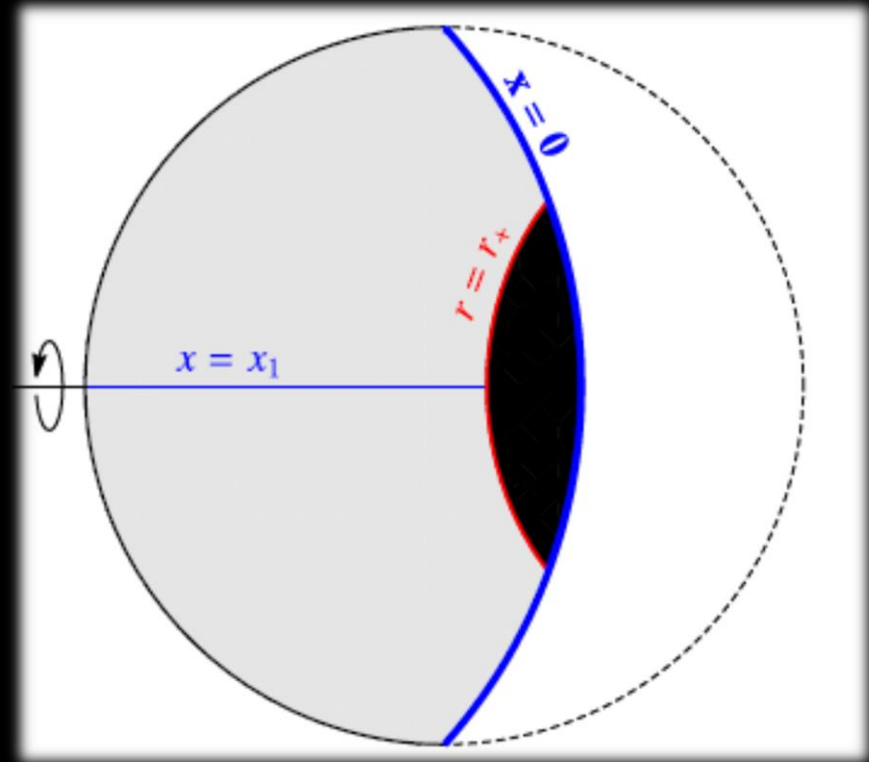
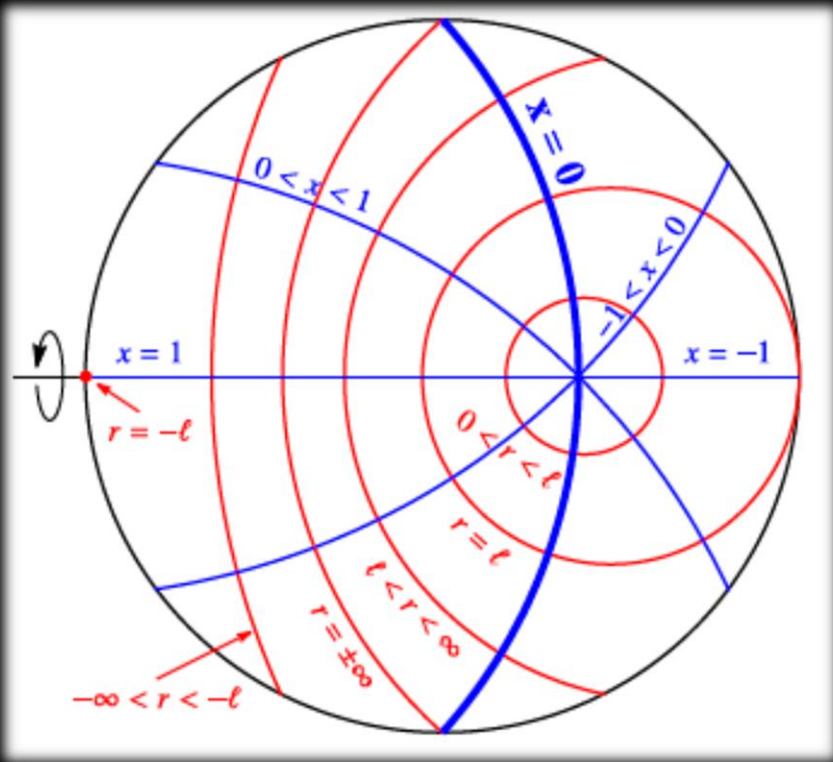
$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}$$

$$G(x) = 1 - \kappa x^2 - \mu x^3$$



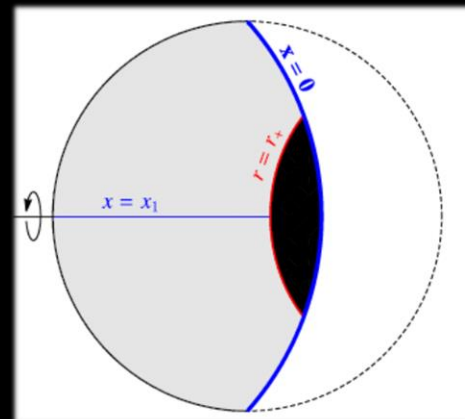
# Adapted coordinates

Brane at  $\mathbf{x} = \mathbf{0}$ :  $K_{ab} = -\frac{1}{\ell} h_{ab}$



# quBTZ metric

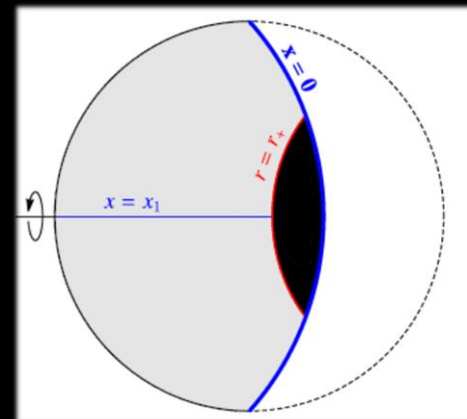
3D metric induced on brane at  $x = 0$



$$ds^2 = - \left( \frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

# quBTZ metric

3D metric induced on brane at  $x = 0$

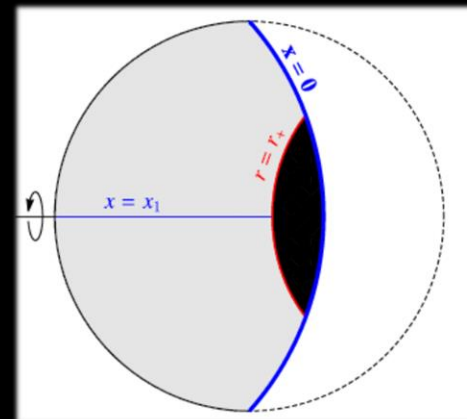


$$ds^2 = - \left( \frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

$\ell = 0$  : BTZ black hole

# quBTZ metric

3D metric induced on brane at  $x = 0$



$$ds^2 = - \left( \frac{r^2}{\ell_3^2} - 8G_3M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

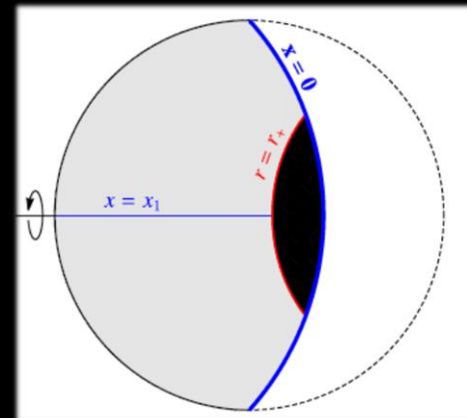
$\ell = 0$  : BTZ black hole

$\ell > 0$  : quantum-corrected BTZ  $\neq \text{AdS}_3/\Gamma$

# quBTZ metric

quantum correction:  $\ell = 2 \hbar c G_3 + \dots$

CFT central charge



$$ds^2 = - \left( \frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

$F(M)$  determined by bulk regularity

# Rotating quBTZ

Can have  $M < J/\ell_3$

$\langle T_{ab} \rangle_{CFT}$  smooth at inner Cauchy horizon

but see *RE+Tomašević 2020 “Strong Cosmic Censorship in BTZ”*

# Quantum BH Entropy: First law

$M$   $J$  measured in 3D eff theory

w/ rotation added

$T$   $\Omega$

(lengthy expressions)

$$S_{gen} = \frac{A_{bulk}}{4G_4}$$

$$TdS_{gen} = dM - \Omega dJ$$

So

Fully quantum-backreacted BTZ and CFT stress tensor  
can be described *exactly* and *in detail*

Quantum entropy from braneworld holography: *consistent*

Efficient use of holography to *solve a hard quantum problem*



# Thank you



Antonia Frassino



Benson Way

# Effective action gravity + CFT

Integrate 4D bulk action

*RE+Johnson+Myers 1999*

→ 3D eff action

3D  $h_{ab}, R_{ab}$

$\frac{1}{L_3^2}$  : 4D  $\frac{1}{\ell_4^2}$  and brane tension  $\frac{1}{\ell}$

$\ell$ : brane position  $\sim$  tension $^{-1} \sim$  cutoff $^{-1}$

$$I = \frac{1}{16\pi G_3} \int d^3x \sqrt{-h} \left[ \frac{2}{L_3^2} + R + \ell^2 \left( \frac{3}{8} R^2 - R_{ab} R^{ab} \right) + \dots \right]$$

$$G_3 = \frac{G_4}{2\ell_4}$$

+  $I_{CFT}$

holographic CFT

same as "New 3D massive gravity"

*Bergshoeff+Hohm+Townsend 2009*

# Effective action gravity + CFT

$$I = \frac{1}{16\pi G_3} \int d^3x \sqrt{-h} \left[ \frac{2}{L_3^2} + R + \ell^2 \left( \frac{3}{8} R^2 - R_{ab} R^{ab} \right) + \dots \right] + I_{CFT}$$

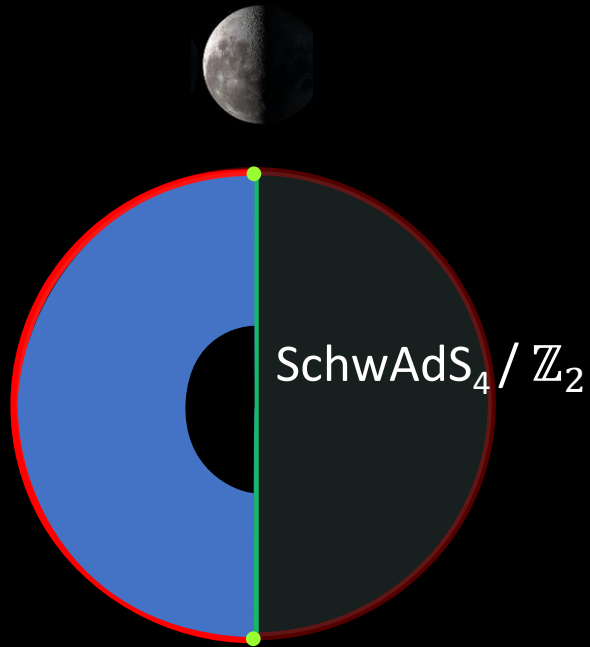
An exact 4D bulk solution is a 3D effective theory solution

exact CFT stress tensor

exact backreaction of the CFT (planar)

exact in *all* higher curvature corrections

# Strength of backreaction: $\ell$



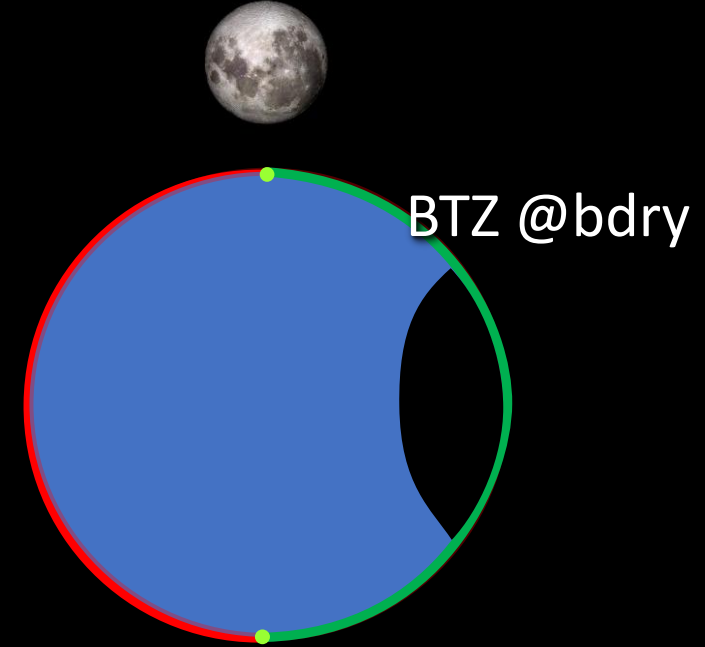
$$\ell \rightarrow \infty$$

zero tension brane  
"maximal backreaction"  
no 3D gravity



$$0 < \ell < \infty$$

brane tension  $\propto 1/\ell$   
backreaction  $\propto \ell$   
cutoff energy  $\propto 1/\ell$



$$\ell \rightarrow 0$$

brane tension  $\rightarrow \infty$   
backreaction  $\rightarrow 0$

$$ds^2 = - \left( \frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r} \right) dt^2 + \frac{dr^2}{\frac{r^2}{\ell_3^2} - 8G_3 M - \ell \frac{F(M)}{r}} + r^2 d\phi^2$$

$G_3$  "renormalized" by higher curvatures  
 quantum correction:  $\ell = 2 \hbar c G_3 + \dots$   
 CFT central charge

$F(M)$  determined by bulk regularity outside horizon

$$\langle T^a_b \rangle = \frac{\ell}{16\pi G_3} \frac{F(M)}{r^3} \text{diag}\{1, 1, -2\} + \dots$$

not thermal plasma  $\text{diag}\{-2, 1, 1\}$

# $\langle T^a_b \rangle$ for free fields

*Steif, Shiraishi+Maki, Lifschytz+Ortiz 1993*

*Casals+Fabbri+Martínez+Zanelli 2016,2019*

## Free conformal scalar in BTZ

Method of images:  $\text{BTZ} = \text{AdS}_3/\Gamma$

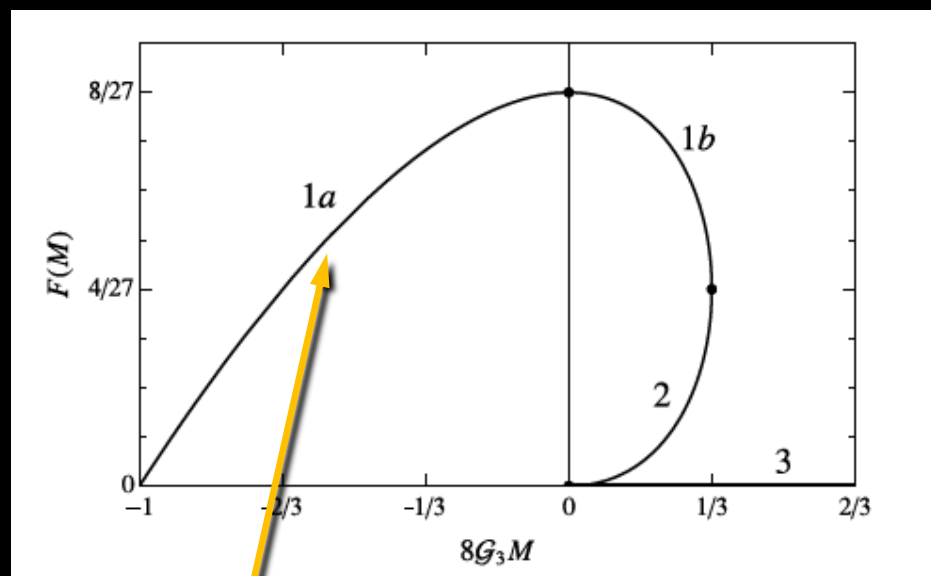
Satisfy KMS, Hartle-Hawking at horizon

Same structure:  $\langle T^a_b \rangle = \frac{c}{8\pi} \frac{F(M)}{r^3} \text{diag}\{1, 1, -2\}$

different  $F(M) = \sum_{n=1}^{\infty} F_n(M)$  (images)

# Holographic quantum black holes: small $\text{AdS}_3$ bhs

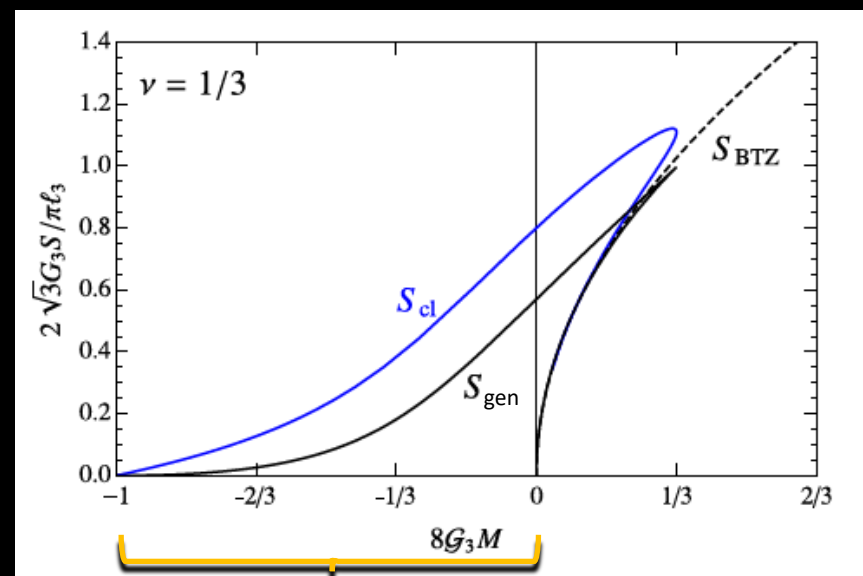
Stress tensor



Casimir energy on a cone  
Conical singularities dressed by quantum horizon

*Quantum Cosmic Censorship*

Entropy

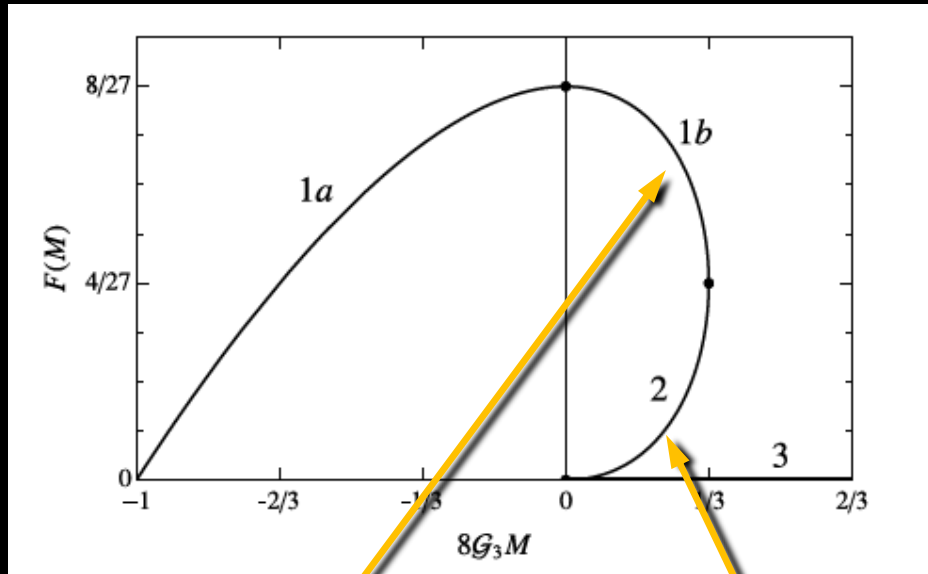


Small  $\text{AdS}_3$  black holes  $-\frac{1}{8G_3} < M < 0$

*RE+Fabri+Kaloper 2002*

# Holographic quantum black holes: quBTZ

Stress tensor

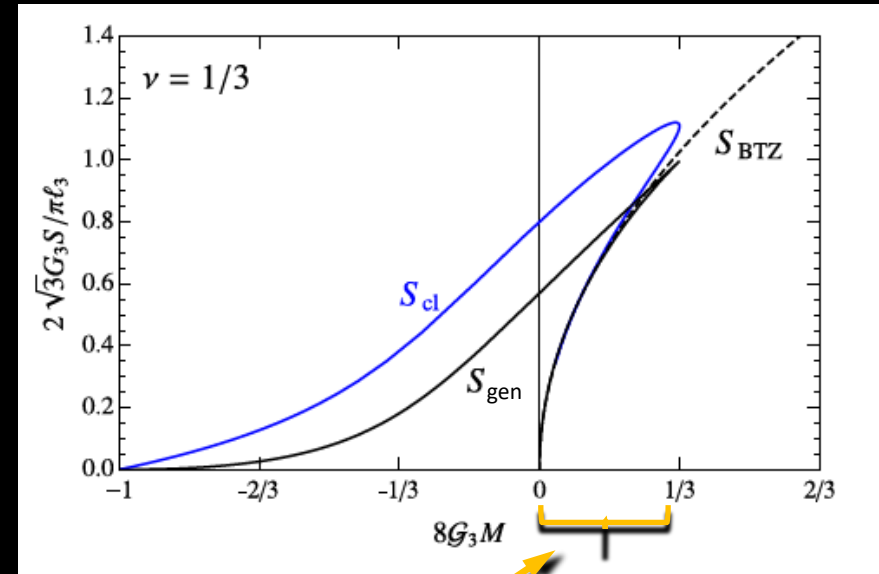


CFT in BTZ

Casimir-dominated

Thermal-dominated

Entropy



Quantum-corrected BTZ:  $0 < M < \frac{1}{24G_3}$



# Quantum BH Entropy: First law

$$M = \frac{1}{2G_3} \frac{z^2(1 - \nu z^3)(1 + \nu z)}{(1 + 3z^2 + 2\nu z^3)^2}$$

(measured in 3D eff theory)

$$T = \frac{1}{2\pi\ell_3} \frac{z(2 + 3\nu z + \nu z^3)}{1 + 3z^2 + 2\nu z^3}$$

$\nu$ : backreaction parameter  $\ell/\ell_3$

$z$ : mass parameter

$$S_{gen} = \frac{A_{bulk}}{4G_4} = \frac{\pi\ell_3}{G_3} \frac{z}{1 + 3z^2 + 2\nu z^3}$$

$$T \partial_z S_{gen} = \partial_z M$$

$$\Rightarrow T dS_{gen} = dM$$

# Rotating quBTZ

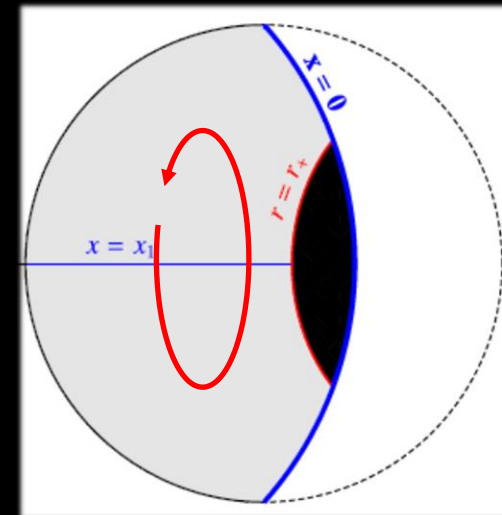
Rotating AdS C-metric

Bulk structure similar to Kerr(-AdS<sub>4</sub>)

inner&outer horizons, ring singularity

$\ell = 0$  : rot BTZ black hole

$\ell > 0$  : quantum-corrected rot BTZ  $\neq \text{AdS}_3/\Gamma$



# Rotating quBTZ metric

$$ds^2 = - \left( \frac{r^2}{\ell_3^2} - 8G_3M - \ell \frac{F_1(M, J)}{r} \right) dt^2 + \left( r^2 + \ell \frac{F_2(M, J)}{r} \right) d\phi^2$$
$$- 8G_3J \left( 1 + \ell \frac{F_3(M, J)}{r} \right) dt d\phi + \left( \frac{r^2}{\ell_3^2} - 8G_3M + \frac{(4G_3J)^2}{r^2} + \ell \frac{F_4(M, J)}{r} \right)^{-1} dr^2$$

$\ell = 0$  : rot BTZ black hole

$\ell > 0$  : quantum-corrected rot BTZ  $\neq \text{AdS}_3/\Gamma$

# Going further

General holographic proof of  $TdS_{gen} = dM - \Omega dJ$

More classical proofs of quantum theorems?

Holographic duals of massive gravities – 3D and higher d

Extensions: charge, higher-d?

Exact entanglement islands

Dual Hawking evaporation? *RE+Kaloper+Tanaka+al 2002-20??*

