The Swampland -
From Conjectures to Theorems

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The Swampland Program

Effective field theory is the way to describe physics below a cutoff

- in condensed matter physics
- in fluid dynamics
- in statistical physics
- in particle physics
- ...
- in gravitational theories and cosmology

Gravity as a perturbative QFT is non-renormalisable, and hence an EFT with a cutoff

Breakdown of the EFT expected at least at M_{Pl} - what replaces it?

More modest question:
Can we give criteria when an EFT arises at low energies from a consistent theory of quantum gravity?

The Swampland Program

Swampland [Vafa'05]
EFTs consistent as QFT, but inconsistent in presence of gravity

$\Longleftrightarrow$

Landscape
Low-energy approximations to consistent Quantum Gravity theory

Swampland Conjectures:

Proposals for criteria to distinguish both sides

Two possible approaches:

1. Make concrete assumptions about what Quantum Gravity is and deduce consequences for resulting EFTs.
2. Try to find general principles which every hypothetical QG should encompass.

Swampland program pursues both directions.

The Swampland Program

1. Try to find general principles which every hypothetical QG should encompass.

   $\rightsquigarrow$ Necessarily speculative/conjectural:
   Arguments are often heuristic and subject of intense debate.

   $\rightsquigarrow$ Growing web of conjectures with high degree of internal logic and consistency.

2. Make concrete assumptions about what Quantum Gravity is and deduce consequences for resulting EFTs.

   Within String Theory as concrete computational framework:

   $\rightsquigarrow$ Can falsify/prove some of general conjectures and reformulate
   $\rightsquigarrow$ Uncover mathematical structure behind the conjectures
   $\rightsquigarrow$ Inspiration for model building

I) A Web of QG Conjectures in $d \geq 4$

Disclaimer:

- necessarily incomplete list here
- see Reviews: [Brennan,Carta,Vafa'17] [Palti'19] and references citing these for complete picture

Warning:

- Quantum Gravity in $d = 2, 3$ very different from $d \geq 4$
- Following conjectures understood for $d \geq 4$ in general

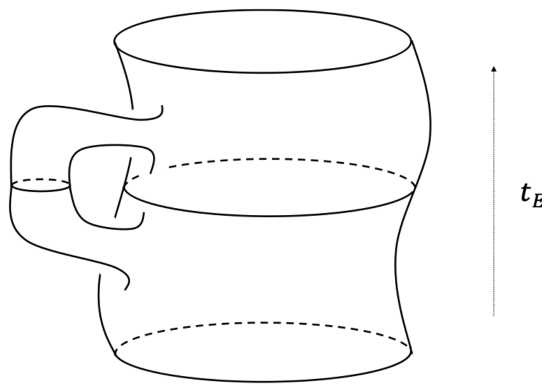
Some QG Conjectures ($d \geq 4$)

1. No free dimensionless parameters

   All couplings in the effective action of a QG should be the VEV of dynamical moduli.

   - reflects background independence
   - violated e.g. in $d = 2$ worldsheet theories, but believed to hold for $d > 3$ (maybe $d \geq 3$?)
   - satisfied in all effective actions of string compactifications

Implies 'Baby-Universe Hypothesis':
'Baby-Universe' Hilbert space must be 1-dimensional [McNamara,Vafa'20]
cf.[Hebecker,Mikhail,Soler'18]



Pic: McNamara,Vafa'20

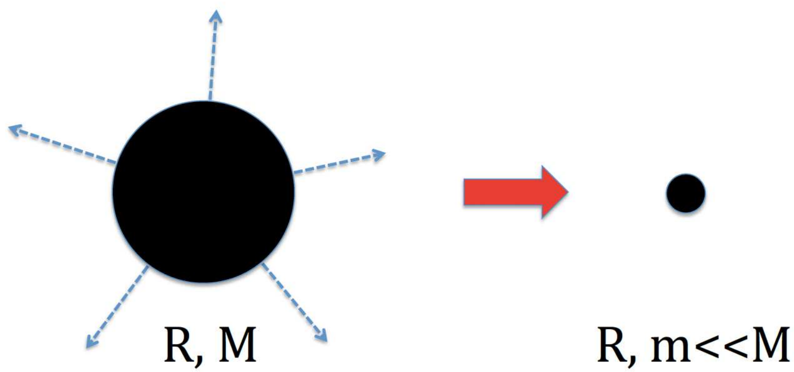
Some QG Conjectures ($d \geq 4$)

2. No Global Symmetries: [Banks,Dixon'88], [Banks,Seiberg'11], [Harlow,Ooguri'18]

There exist no exact global symmetries in presence of gravity.

Heuristic argument against continuous global symmetry:

- Form a black hole of arbitrarily high representation R of G (continuous)
- Hawking radiation does not lead to net discharge since G is not gauged



- BH reaches point where either

(a) the BH entropy $S = \frac{\text{Area}}{4G}$ cannot account for high representation,

(b) or where it becomes a charged remnant.

Rigorous argument for special case of AdS spacetime via holography in [Harlow,Ooguri'18]

Some QG Conjectures ($d \geq 4$)

2. No Global Symmetries: applies to generalised p-form symmetries

   continuous p-form symmetry: [Gaiotto,Kapustin,Seiberg,Willett'14]
   conserved Noether current is $(p+1)$ form J_{p+1} with $d * J_{p+1} = 0$

   Example: [Cordova,Freed,Lam,Seiberg'19/20]
   A coupling constant would imply a (-1) form symmetry

   - $S = S_0 + \lambda \int \mathcal{L}_\lambda$ with $d\mathcal{L}_\lambda = 0$ (top form)
   - 0-form Noether current $J_0 := *\mathcal{L}_\lambda$: $\quad d * J_0 = 0$

   No global (-1)-form symmetries $\Rightarrow$ No free parameters [McNamara,Vafa'20]
   (-1)-form symmetry must be gauged, i.e. $S = S_0 + \int \phi \mathcal{L}_\lambda$

   Swampland-Applications to higher p-symmetries:
   [McNamara,Vafa] [Montero,Vafa'20]

   [Heidenreich,McNamara,Montero,Reece,Rudelius,Valenzuela'20]

Some QG Conjectures ($d \geq 4$)

3. Completeness Conjecture: [Polchinski'03]

   The full charge/weight lattice is populated by states in the physical Hilbert space.

   - In particular: Generators of charge/weight lattice realised by physical states, but of a priori arbitrary mass.
   - Very different from QFT: Operators in general representation need not correspond to a physical state.

   Example:

   Pure Einstein-Maxwell theory (no charged matter at all) is in the swampland!

Some QG Conjectures ($d \geq 4$)

4. Weak Gravity Conjecture: [Arkani-Hamed,Motl,Nicolis,Vafa'06]

Electric Version:

In a $U(1)$ gauge theory coupled to gravity, there must exist some 'super-extremal' charged state with

$$\frac{\mathfrak{q}^2 g_{\rm YM}^2}{m^2}\Big|_{\rm state} \overset{!}{\geq} \frac{Q^2 g_{\rm YM}^2}{M^2}\Big|_{\rm B.H.}$$

Heuristic Argument:

'Charged black holes should be able to decay, at least for $g_{\rm YM}$ small'.
[AMNV'06]

Fact: Existence of super-extremal state is necessary condition for BH to decay.

Warning:
Not fully clear if or why charged BH really has to decay in gauge theory.

If non-decaying charged BHs lead to charged remnants, then for $g_{\rm YM} \to 0$ would get infinitely many such remnants below energy M_0 since $g_{\rm YM} Q \leq M_0$ for BH of charge M_0.

Some QG Conjectures ($d \geq 4$)

4. Weak Gravity Conjecture: [Arkani-Hamed,Motl,Nicolis,Vafa'06]

   ∃! 'super-extremal' state w.r.t. charged extremal black hole

$$\frac{q_k^2 g_{\rm YM}^2}{M_k^2}\Big|_{\rm state} \overset{!}{\geq} \frac{Q^2 g_{\rm YM}^2}{M^2}\Big|_{\rm B.H.} = \frac{\#}{M_{\rm Pl}^{d-2}}$$

   Alternative viewpoint:

   State with highest charge-to-mass ratio must satisfy

$$\begin{aligned} |F_{\rm Coulomb}| \quad &\geq \quad |F_{\rm Grav}| \\ \frac{g_{\rm YM}^2 q_k^2}{M_k^2} \quad &\overset{!}{\geq} \quad \frac{1}{M_{\rm Pl}^{d-2}} \frac{d-3}{d-2} \end{aligned}$$

   $\Longrightarrow$ No stable charged remnants

Some QG Conjectures ($d \geq 4$)

4. Weak Gravity Conjecture: [Arkani-Hamed,Motl,Nicolis,Vafa'06]

   $\exists!$ 'super-extremal' state w.r.t. charged extremal black hole

$$\frac{q_k^2 g_{\rm YM}^2}{M_k^2}\Big|_{\rm state} \overset{!}{\geq} \frac{Q^2 g_{\rm YM}^2}{M^2}\Big|_{\rm B.H.} = \frac{\#}{M_{\rm Pl}^{d-2}}$$

In presence of massless scalars: [Palti'17]

[Lee,Lerche,TW'18][Heidenreich,Reece,Rudelius'19]

State with highest charge-to-mass ratio must satisfy

$$\begin{array}{ccc} |F_{\rm Coulomb}| & \geq & |F_{\rm Grav}| + \qquad |F_{\rm Yukawa}| \\ \dfrac{g_{\rm YM}^2 q_k^2}{M_k^2} & \overset{!}{\geq} & \dfrac{1}{M_{\rm Pl}^{d-2}}\left(\dfrac{d-3}{d-2} + \dfrac{1}{4}\dfrac{M_{\rm Pl}^4}{M^4} g^{rs}\partial_r \dfrac{M^2}{M_{\rm Pl}^2}\partial_s \dfrac{M^2}{M_{\rm Pl}^2}\right) \end{array}$$

Some QG Conjectures ($d \geq 4$)

4. Weak Gravity Conjecture

Which particles must satisfy the WGC bound?

- **Tower WGC** [Andriolo,Junghans,Noumi,Shiu'18]

  Tower of infinitely many states of increasing mass

  Sufficient condition to ensure that the WGC is stable under S^1 compactification

- **Sublattice WGC**: Sublattice of charge lattice

  [Heidenreich,Reece,Rudelius'15'16] [Montero,Shiu,Soler'16]

  Observed in all known string compactifications (see later), but might be too strong in general

Important consequence:

New physics at tower scale $m^2 \sim g_{\rm YM}^2 M_{\rm Pl}^{d-2}$

cf. **Species scale** $\Lambda \sim \frac{1}{\sqrt{N}} M_{\rm Pl}$ (for $d = 4$) [Dvali,Gabadadze,Kolanovic,Nitti '01] [Grimm,Palti,Venezuela'18]

Some QG Conjectures ($d \geq 4$)

4. Weak Gravity Conjecture - Mild Form

[AMNV'06] [Kats,Motl,Padi'06] . . . [Cheung,Liu,Remmen'18] [Hamada,Noumi,Shiu'18] [Loges,Noumi,Shiu'19] [Bellzzini,Lewandowski,Serra'19]

WGC states can be BHs of lower mass for suitable signs of Wilson coefficients in effective action

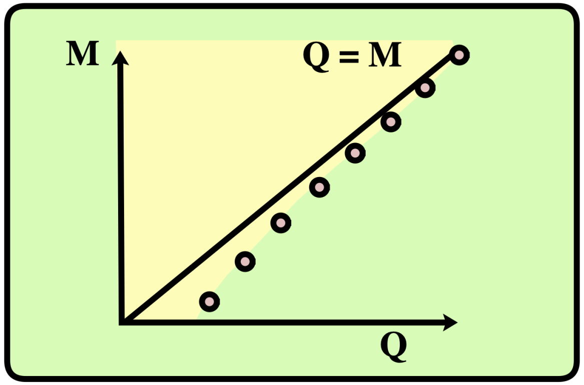


Image: [AMNV'06]

Generalisation to higher p-form gauge potentials
[AMNV'06],[Heidenreich,Reece,Rudelius'15'16]

$$\frac{q_p^2 g_p^2}{T_p^2} \geq \frac{1}{M_{\rm Pl}^{d-2}} \left(\frac{d-2}{p(d-p-2)} + \dots \right) \qquad (p \neq 0)$$

Special case: $p = 0 \implies$ axionic WGC [AMNV'06]

$$\frac{1/f}{S_{\rm inst}} \geq \frac{1}{M_{\rm Pl}} \qquad (\text{in} \quad d = 4)$$

Some QG Conjectures ($d \geq 4$)

5. Swampland Distance Conjecture: [Ooguri,Vafa'06]

In a QG with a moduli space parametrised by massless scalar fields, there exists a geodesic path of infinite geodesic distance.

At infinite geodesic distance

$$\Delta\phi = \int_{\tau_i}^{\tau_f} \sqrt{g_{ij} \frac{d\phi^i}{d\tau} \frac{d\phi^j}{d\tau}} d\tau$$

in moduli space, an infinite tower of states becomes massless as

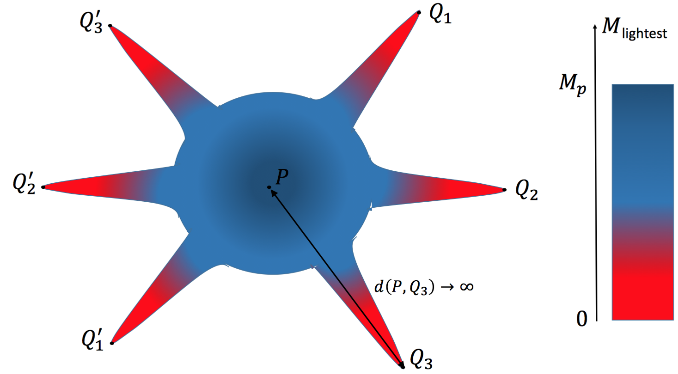


Image: Palti,1903.06239

$$m(\phi) = m_0 \, e^{-c \frac{\Delta\phi}{M_{\text{Pl}}}} \quad \text{for } \Delta\phi \to \infty \qquad c \geq 0$$

Main Motivation: String theory experience (see later)

Refined SDC: $c = \mathcal{O}(1)$ [Klaewer,Palti'16] [Baume,Palti'16]

Conjecture: $c \geq \frac{2}{d\sqrt{(d-2)(d-1)}}$ [Bedroya,Vafa'19]

Some QG Conjectures ($d \geq 4$)

6. Emergent String Conjecture: [Lee,Lerche,TW'19]

   If a quantum gravity theory (in Minkowski space) admits an infinite distance limit, then

   - *either it reduces to a weakly coupled string theory*
     $\Rightarrow$ *infinite tower of string excitations*
   - *or it decompactifies*
     $\Rightarrow$ *infinite tower of Kaluza-Klein excitations*

Note:

- In presence of several towers at different scale, the physics is determined by the lowest lying tower
- In Minkowski space expect string tower
  (1) at KK tower $\rightarrow$ weakly coupled string theory or
  (2) above KK tower $\rightarrow$ decompactification limit

SDC as RG flow with string defects: [Lanza,Marchesano,Martucci,Valenzuela'20]

Some QG Conjectures ($d \geq 4$)

7. CFT Distance Conjecture

[Perlmutter,Rastelli,Vafa,Valenzuela] [Baume,Calderon Infante]'20

All infinite distance points on the conformal manifold of a unitary CFT in $d \geq 3$ exhibit higher spin symmetry and hence an infinite tower of higher spin J operators with dimension $\Delta_J = d - 2 + J$.

Motivated holographically via Swampland Distance Conjecture

Note:

Higher Spin points describe free CFTs, hence the infinite distance points are weak coupling points

Quantum Gravity Conjectures

8. AdS Distance Conjecture [Lüst,Palti,Vafa '19]; cf. [Alday,Perlmutter'19]

For QG on AdS spacetime in $d \geq 4$ with cosmological constant Λ, there exists an infinite towers of states with mass

$$m \sim |\Lambda|^{\alpha} \qquad \text{for} \quad |\Lambda| \to 0 \qquad \alpha = \mathcal{O}(1), \quad \alpha > 0$$

Argument: By application of SDC to variation of spacetime metric
under certain technical assumptions

Strong version: *For supersymmetric AdS vacua:* $\alpha = \frac{1}{2}$

- Would imply no separation of scales for AdS-vacua

  cf. [Tsimpis'12] [Gautason, Schillo,Van Riet, Williams'15] [Gautason,Van Hemelryck,Van Riet'18] [Blumenhagen,Brinkmann,Makridou'19] [Font,Herraez,Ibanez'19][Apruzzi,DeLua,Gnecchi,LoMonaco,Tomasiello'19]

- Violated by Type IIA AdS construction of DeWolfe,Giryavets,Kachru,Taylor'05

  cf.[Junghans'20] [Marchesano,Palti,Quirant,Tomasiello'20] . . .

Quantum Gravity Conjectures

9. No dS Conjecture [Dvali,Gomez'13/14] [Obied,Ooguri,Spodyneiko,Vafa],[Andriot]'18 [Dvali,Gomez,(Zell)][Garg,Krishnan][Palti,Shiu,Ooguri,Vafa]'18

In QG, the potential satisfies one of the following two conditions

$$(1) \quad |\nabla V| \geq c \frac{V}{M_{\rm Pl}} \qquad \text{or} \quad (2) \qquad \min(\nabla_i \nabla_j V) \leq -c' \frac{V}{M_{\rm Pl}^2}$$

Implies: For $V > 0$ every critical point cannot be a minimum

Arguments:

- Inconsistency of deSitter spacetimes via mechanism of 'quantum breakup' in quantum corpuscular approach $\Rightarrow c = c(V)$ [Dvali,Gomez'13/14] [Dvali,Gomez,Zell'18] [Blumenhagen,Kneissl,Makridou'20]
- (1) matches behaviour of known <u>perturbative</u> string vacua [Obied,Ooguri,Spodyneiko,Vafa'18]

  (2) needed to evade various potential counter-examples [Denef,Hebecker,Wrase'18][Conlon'18][Murayam,Yamazaki,Yanagida'18]...

Quantum Gravity Conjectures

9. No dS Conjecture

$$(1) \quad |\nabla V| \geq c\frac{V}{M_{\rm Pl}} \qquad \text{or} \qquad (2) \qquad \min(\nabla_i \nabla_j V) \leq -c' \frac{V}{M_{\rm Pl}^2}$$

Arguments (continued):

- On basis of SDC in strict perturbative regime towards boundary of moduli space: [Palti,Shiu,Ooguri,Vafa]'18

(a) For $V > 0$: if $|\nabla V| \leq \sqrt{2}\frac{V}{M_{\rm Pl}}$ and $\min(\nabla_i \nabla_j V) \geq -c' \frac{V}{M_{\rm Pl}^2}$
$\Longrightarrow$ horizon with entropy $S_{GH} \sim R^2 \sim 1/\Lambda \sim \log(\dim(\mathcal{H}))$ ($M_{\rm Pl} = 1$)

(b) In strict perturbative regime (infinity distance weak coupling limit $\phi \to \infty$):
$N(\phi) \sim e^{b\phi}$ light states by SDC
entropy $S_{\rm tower} \sim N^\gamma R^\delta \overset{!}{\leq} R^2 \sim 1/V(\phi)$ (for Bousso-bound) $\Rightarrow (1)$

Latter argument does not preclude dS vacua in strong coupling regime (e.g. KKLT)
inspired considerable investigations into explicit constructions

Quantum Gravity Conjectures

10. Transplanckian Censorship Conjecture (TCC) [Bedroya,Vafa'19]

The expansion of the Universe should not allow early transplanckian modes to beome classical, i.e. $\frac{a_f}{a_i}\ell_{\rm Pl} < \frac{1}{H_f}$

Motivation:
Avoids Transplanckian Problem of inflation - though not required.

Result-oriented approach

Constrains potential via $V = \Lambda = \frac{(d-2)(d-1)}{2} H_\Lambda^2$

- Asymptotic regime: $\frac{|\nabla V|}{V} > c_\infty = \frac{2}{(d-2)(d-1)}$
- Small ϕ: Meta-stable dS vacua allowed with $T < \frac{1}{H_\Lambda}\ln(\frac{M_{\rm Pl}}{H_\Lambda})$

Also constrains constant $c \geq \frac{2}{d\sqrt{(d-2)(d-1)}}$ in SDC $m(\phi) = m_0\, e^{-c\frac{\Delta\phi}{M_{\rm Pl}}}$

A Web of Conjectures

The conjectures are heuristic, but logically connected:

Example: What happens if take $g_{\rm YM} \to 0$ at $M_{\rm Pl}$ finite?

No-global-symmetry Conjecture

In presence of gravity, no global symmetries are possible. [Banks,Dixon'88]

suggests:

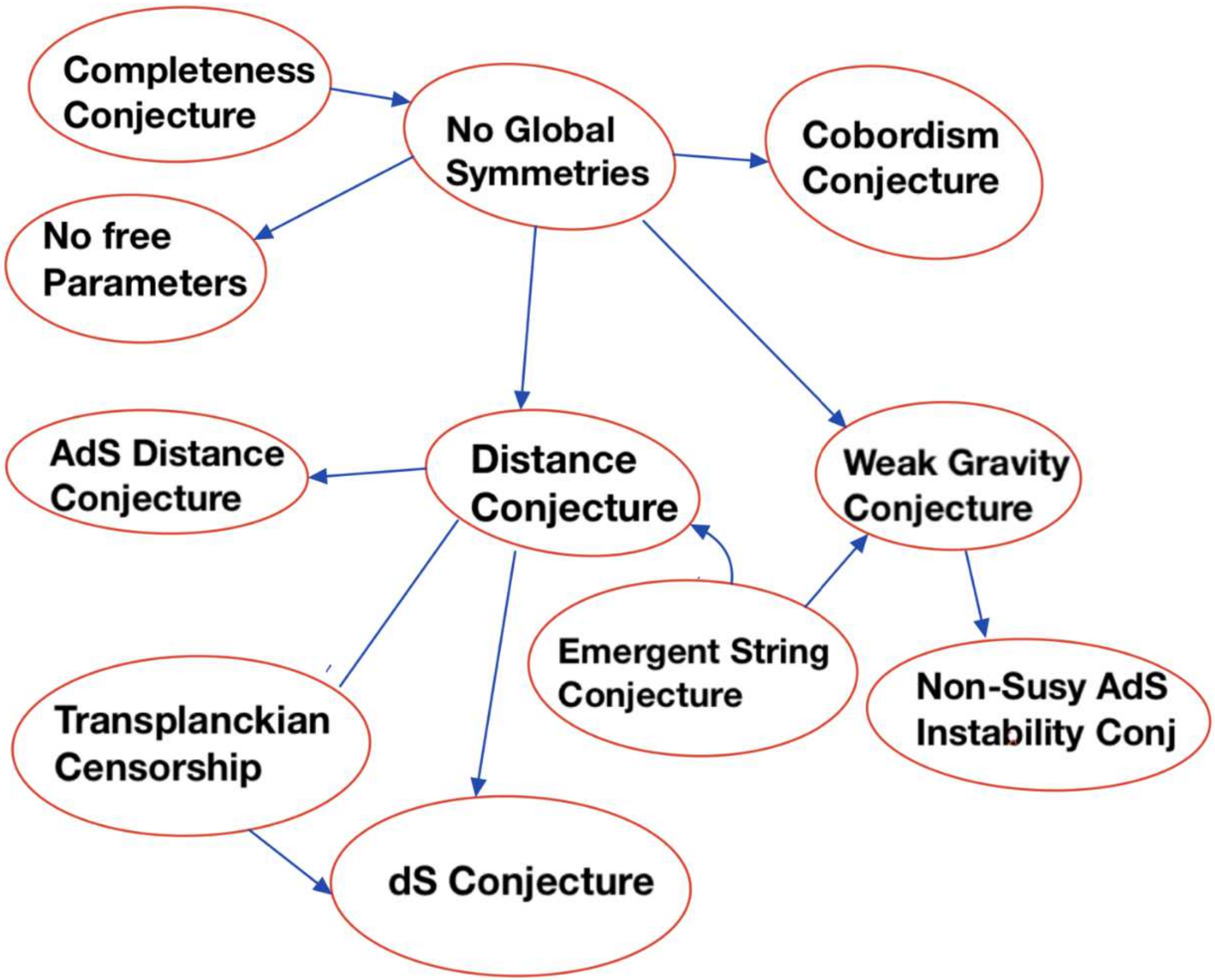
- Offensive limit should be at infinite distance (beyond reach).
- Effective theory must break down (quantum gravity censorship).

$\Longleftrightarrow$ **WGC particle tower:**

$$\mathfrak{q}^2 g_{\rm YM}^2 \overset{!}{\geq} \# \frac{m^2}{M_{\rm Pl}^{d-2}} \qquad \Longrightarrow \text{tower of light states for } g_{\rm YM} \to 0 \qquad \checkmark$$

$\Longleftrightarrow$ **Swampland Distance Conjecture** $\checkmark$

A Web of Conjectures



II) QG Conjectures in String Theory

QG Conjectures in String Theory

1) Quantitative test of the conjectures in computational framework, especially in situations with minimal supersymmetry

2) Understand mathematical foundations of conjectures

3) Refine and unify conjectures

SDC for 4d N=2 theories

Type IIB on CY_3 at infinite distance limits in $\mathcal{M}_{\rm cplx\,str.} = \mathcal{M}^{\rm IIB}_{\rm VM}$

[Grimm,Palti,Valenzuela'18] [Grimm,Li,Palti'18] [Joshi,Klemm'19] [Gendler,Valenzuela'20]

[Grimm'20] [Font,Herraez,Ibanez'19]...

(mirror dual: [Corvilain,Grimm,Valenzuela'18] [Lee,Lerche,TW'19])

- Degenerations of Hodge structure $\leftrightarrow$ degenerating 3-cycles
- 3 types of degenerations at infinite distance
  $\Longleftrightarrow$ rate of vanishing of volume
- Towers of asymptotically massless BPS states:
  D3-branes on degenerating special Lagrangians

$$M_n = n M_0 \quad \text{with} \quad \frac{M_n}{M_{\rm Pl}} \sim e^{-\sqrt{d}\frac{\Delta\Phi}{M_{\rm Pl}}} \quad d = 1, 2, 3$$

$\Longrightarrow$ particle spectrum from SDC ✓

Note: $M_n^2 = n^2 M_0^2$ indistinguishable from KK spectrum

Is there a denser spectrum in addition?

Emergent Strings for 5d N=1

M-theory on CY_3 at infinite distance limit in $\mathcal{M}_{\mathrm{Kahler}}$ [Lee,Lerche,TW'19]

3 types of infinite distance limits

1. $\mathcal{V}_{\mathrm{CY}_3} \to \infty \Longrightarrow$ decompactification limit (KK tower)
2. $\mathcal{V}_{\mathrm{CY}_3}$ finite, but $\mathcal{V}_D \to \infty$ $\quad D$: 2 or 4-cycle
   $\Rightarrow \exists$ unique fastest shrinking non-contract. cycle Σ with $\mathcal{V}_\Sigma \to 0$

$$\left\{ \begin{array}{c} T^2 \\ \mathrm{K3}/T^4 \end{array} \right\} \text{Limit} \iff \Sigma \text{ is } \left\{ \begin{array}{c} T^2 \\ \mathrm{K3}/T^4 \end{array} \right\} \text{fiber}$$

T^2 Limit: Tower of M2-branes on T^2 $(M_n = nM_0)$:
Decompactification limit (6d F-theory limit)

$\mathrm{K3}/T^4$ Limit: i) M5 brane on $K3/T^4 \Rightarrow$ heterotic/Type II string
String excitations $M_n = \sqrt{n} M_0$
ii) Wrapped M2-branes $M_n = nM_0$ (KK modes)
$\Longrightarrow$ **emergent string limit**: $M_{\mathrm{str}} \sim M_{\mathrm{KK}}$

Emergent String Conjecture

Quantitative tests survive quantum corrections:

In flat space emergent string limits string tension always parametrically coupled to M_{KK}

- 4d N=2 hypermultiplets (Type IIB on CY_3)

  [Marchesano,Wiesner'19] [Baume,Marchesano,Wiesner'19]

- 4d N=1 F-theory [Kläwer,Lee,Wiesner,TW'20]
- 4d N=1 G2 manifold [Xu'20]

Weak Gravity Conjecture

Follows from SDC/Emergent String Conjecture

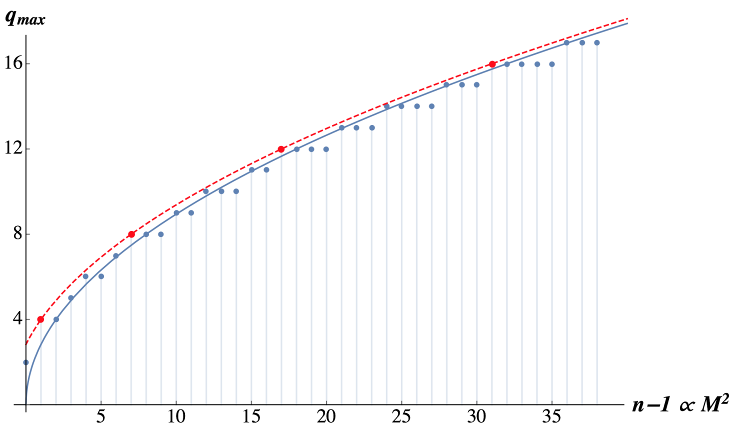
4d $N = 2$: BPS particles of SDC are the WGC states

[Grimm,Palti,Valenzuela'18] [Gendler,Valenzuela'20]

F-theory in 6d $N = 1$/4d $N = 1$:

Explicit proof of sublattice WGC based on modular properties of emergent heterotic string [Lee,Lerche,TW'18'19'20]

[Kläwer,Lee,Wiesner,TW'20] [Cota,Klemm,Schimannek'20]



In limit $g_{\rm YM} \to 0$:

$$F_{\rm Coulomb} \geq F_{\rm grav} + F_{\rm Yuk}$$

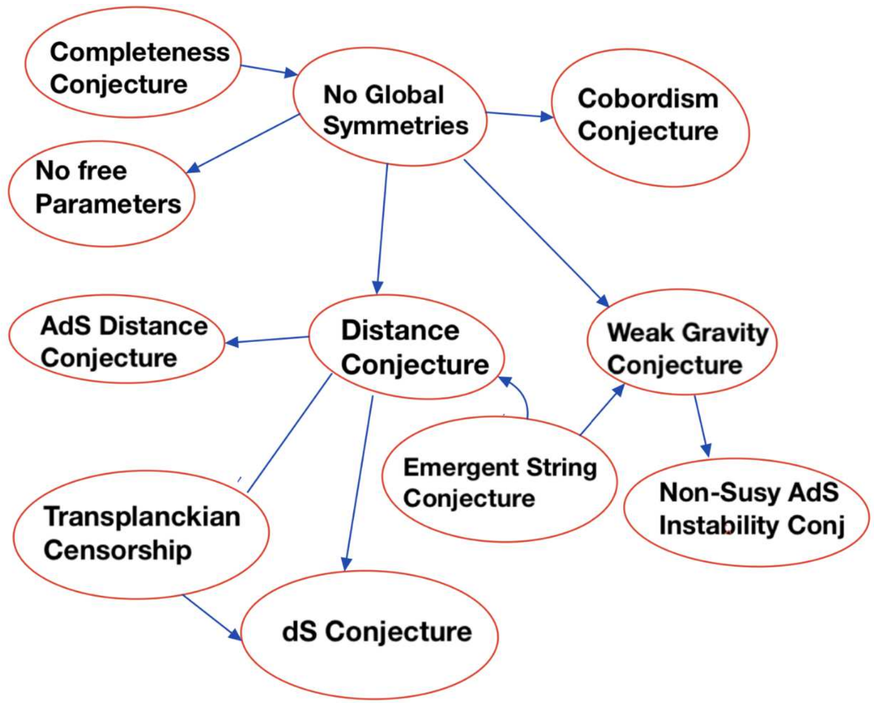
$$q_k^2 g_{\rm YM}^2 \geq \left(\frac{d-3}{d-2} + \frac{1}{d-2}\right) \frac{M_k^2}{M_{\rm Pl}^{d-2}}$$

Quantum corrections in 4d $N = 1$: [Kläwer,Lee,Wiesner,TW'20]

Summary

A web of conjectures

- many in excellent agreement with general predictions of String Theory
- others challenging putative String Theory constructions



Many open questions:

Push predictions further - e.g. within String Theory

Find better general arguments for some conjectures

Is there a Mother Conjecture from which everything follows?