# Quantum Corrections for 4d N=1 Emergent Strings and the Weak Gravity Conjecture

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based on:
D. Kläwer, S.-J. Lee, T. Weigand, MW [2011.00024]

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# Introduction

**Swampland Program:** [Vafa '05]

(see review talk by Timo Weigand)

| **Landscape** | **Swampland** |
|---|---|
| EFT + quantum gravity = ✔ | EFT + quantum gravity = ✗ |

**Consistency conditions** imposed by Quantum Gravity

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## Swampland Conjectures:

- No Global Symmtery [Banks, Dixon '88]
- Completeness Hypothesis [Polchinski '03]
- Weak Gravity Conjecture [Arkani-Hamed, Motl, Nicolis, Vafa '06]
- Swampland Distance Conjecture [Ooguri, Vafa '06]
- dS/AdS conjecture [Palti, Shiu, Ooguri, Vafa '18, Lüst, Palti, Vafa'19]
- ...

[review: Brennan, Carta, Vafa '17; Palti '19]

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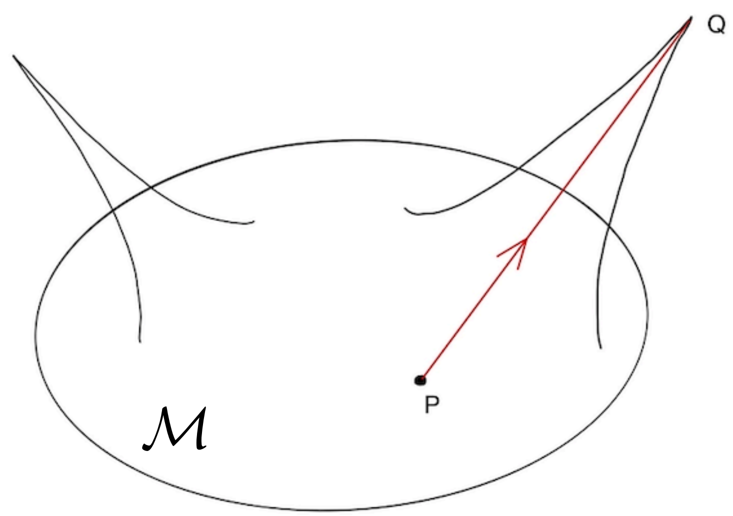
Focus of this talk

[review: Brennan, Carta, Vafa '17; Palti '19]

# Introduction

## Swampland Distance Conjecture [Ooguri, Vafa '06]

At infinite distance in moduli space an infinite tower of states becomes exponentially massless in Planck units.




$$\frac{M(Q)}{M_{pl}} \sim e^{-Ad(P,Q)}$$

## Weak Gravity Conjecture [Arkani-Hamed, Motl, Nicolis, Vafa '06]

Decay of extremal black holes $\Rightarrow$ Self-repulsive state:

$$|F_{\text{grav.}}| \sim \frac{m^2}{M_P^2} \lesssim (gq)^2 \sim |F_{\text{gauge}}| \,.$$

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TEST SDC AND WGC IN 4D $\mathcal{N} = 1$.

- Setup: F-theory on CY 4-folds.

- Look at **infinite distance** limits in *classical* Kähler moduli space:
  "$t \sim \lambda \to \infty$" [cf. Lee, Lerche, Weigand '19]

- if $\exists$ gauge theory with $g \sim e^{-a\lambda}$: [Heidenreich, Reece, Rudelius '15-'18; Kläwer, Palti '16]
  $\Rightarrow$ WGC requires states with $m_n \lesssim q_n e^{-b\lambda} M_{pl}$

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  $\Rightarrow$ WGC requires states with $m_n \lesssim q_n e^{-b\lambda} M_{pl}$
- What are these states?

**KK-tower**

$m_n^2 \sim n^2 M_{KK}^2$

(partial) **decompactification!**

**String states**

$m_n^2 \sim n M_S^2$

not necessarily decompactification:

[Lee, Lerche, Weigand '19]

**Emergent String Proposal:**

*Theory reduces to weakly coupled ST and massless states are excitations of weakly-coupled* critical *string.*

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*Theory reduces to weakly coupled ST and massless states are excitations of weakly-coupled* critical *string.*

# Three Core Messages

[Kläwer, Lee, Weigand, MW '20]

1. **Uniqueness:** For equi-dimensional infinite distance points the emergent tensionless and weakly-coupled string is *unique*.

2. **Quantum Consistency:** Leading perturbative corrections at $\mathcal{O}(\alpha'^2)$ ensure *consistency* of emergent string limits.

3. **WGC:** Classical super-extremality bound generically satisfied.
But: quantum effects modify super-extremality bound

$\longrightarrow$ deduce mass renormalisation of string states from quantum corrections.

# Setup

- F-theory on elliptic CY 4-fold: $E_\tau \to X_4 \to B_3$.

- Kähler moduli space spanned by $h^{1,1}(B_3)$ moduli $v^\alpha$.

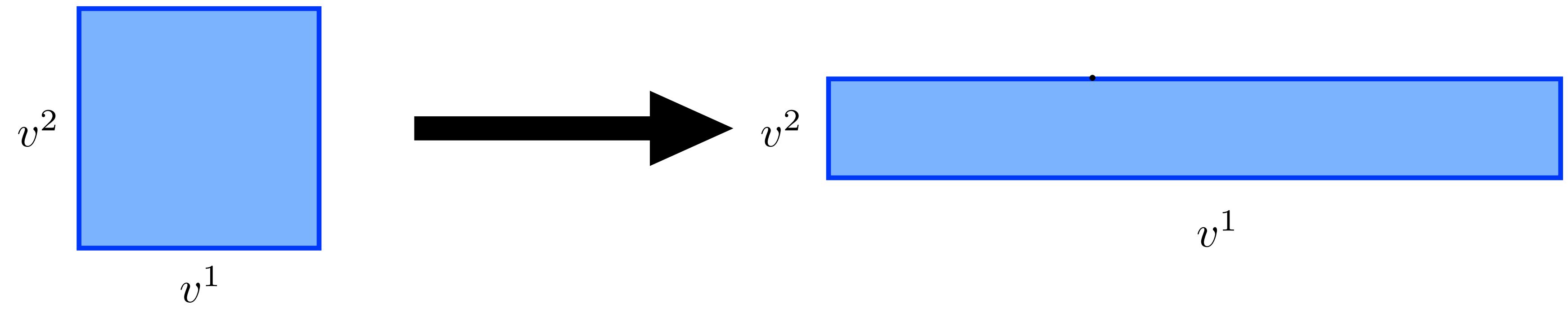
$$J = v^\alpha J_\alpha \qquad \Rightarrow \qquad \mathcal{V}_{B_3} = \frac{1}{6}\int_{B_3} J^3 = \frac{1}{6} k_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma \,.$$

- To avoid decompactification limit, cannot just take

$$v^\alpha \sim \mu \to \infty \qquad \forall \alpha \,.$$

- Have to take a non-trivial limit:

Intuition:




[Lee, Lerche, Weigand '19]

- possible if $J_0^3 = 0$ for some generator $J_0$.

# Setup

[Lee, Lerche, Weigand '19]

Two possibilities

$J_0^3 = 0$ → $J_0^2 = 0$ J-class B

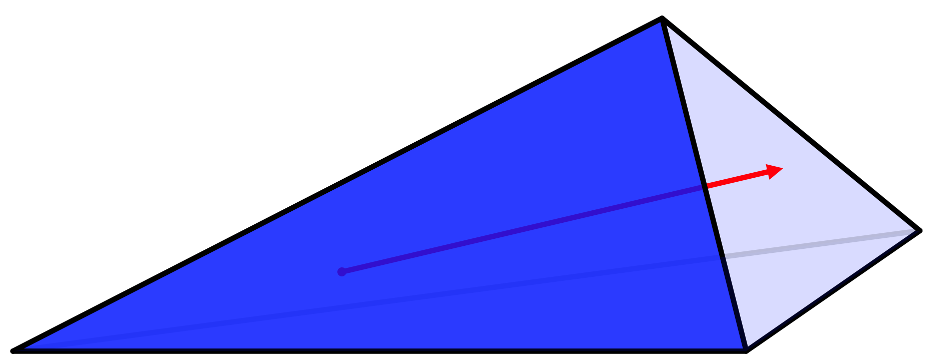
$J_0^3 = 0$ → $J_0^2 \neq 0$ J-class A

Focus here

$$J = \lambda J_0 + \sum_{\mu \in \mathcal{I}_1} \frac{1}{\lambda^2} J_\mu + \sum_{r \in \mathcal{I}_3} v^r J_r \,,$$

$$J_0^2 . J_\mu \neq 0 \qquad \forall \mu \in \mathcal{I}_1 \,,$$
$$J_0^2 . J_r = J_0 . J_s . J_r = 0 \qquad \forall r, s \in \mathcal{I}_3$$

→ $\mathcal{V}_{B_3}$ =const. for $\lambda \to \infty$



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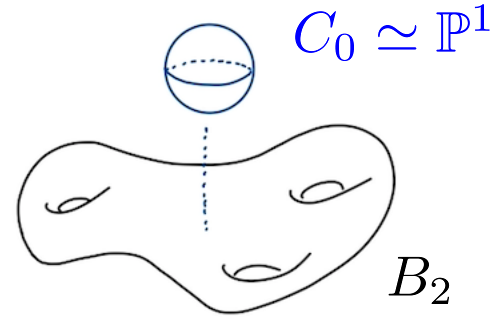
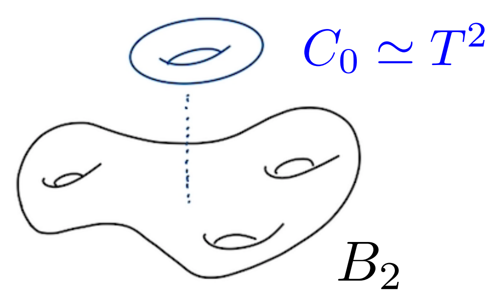
- The curve $C_0 = J_0.J_0$ shrinks in the limit: $\mathcal{V}_{C_0} \sim \frac{1}{\lambda^2}$
- $B_3$ is fibration $C_0 \to B_2$:

1




$$\bar{K}_{B_3}.C_0 = 2$$

2

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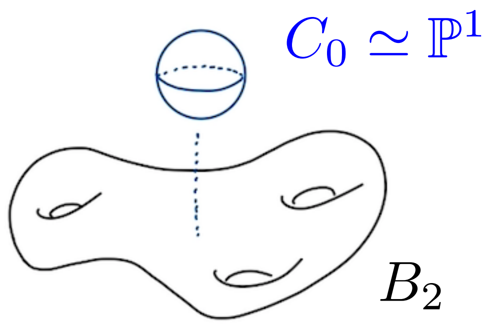
# Emergent Strings

[Lee, Lerche, Weigand '19]

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- D3-brane wrapped on $C_0$ gives rise to *emergent heterotic string*:

$$\frac{M_{\rm het}^2}{M_S^2} \sim \lambda^{-2}$$

- Gauge theory on divisor $\mathcal{S}$ with $\mathcal{S}.C_0 \neq 0$ becomes weakly coupled:

$$\frac{1}{g_{YM}^2} = \mathcal{V}_{\mathcal{S}} \sim \lambda^2$$

- String excitation modes satisfy WGC!

$$M_k^2 \sim M_{\rm het}^2(n_k - 1)$$
$$\exists q_k\colon\ q_k^2 \geq 4mn_k$$

# Emergent Strings

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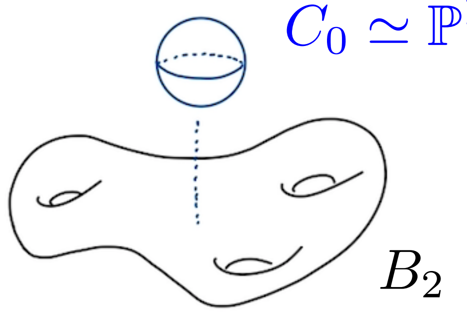


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**So everythings fine?**

# Open Problems

1 Can there be multiple strings that become tensionless at the same rate?

$$B_3 = \underbrace{\mathbb{P}^1}_{\sim\lambda^{-1/2}} \times \underbrace{\mathbb{P}^1}_{\sim\lambda^{-1/2}} \times \underbrace{\mathbb{P}^1}_{\sim\lambda^{1}}$$

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**2** Analysis so far was purely classical!

- $\mathcal{N} = 1$ moduli space is **not** classically exact.
- Quantum Corrections can obstruct classical infinite distance limits!

Already observed in $\mathcal{N} = 2$ setups:

1. Vector multiplet moduli space of IIA on $CY_3$. [Lee, Lerche, Weigand '19]
2. Hypermultiplet moduli space of IIB on $CY_3$. [Marchesano, MW '19]

- Shrinking of fibral curve can be problematic:

$$\mathcal{V}_{C_0} = \frac{M_{\text{het}}^2}{M_S^2} \precsim \frac{M_{KK}^2}{M_S^2}\,.$$

$\Rightarrow$ Critical 4D string theory?!

In type IIB hypermultiplet moduli space the quantum corrections precisely obstruct limits with classically intrinsically 4D strings! [Baume, Marchesano, MW '19]

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3 Does the classical WGC relation also receive quantum corrections?

# Summary of Results

[Kläwer, Lee, Weigand, MW '20]

1 **Uniqueness:** For any J-class A or J-class B limit one of the following holds:

1. $B_3$ has a *unique* $\mathbb{P}^1$-fibration and the fiber shrinks at the fastest rate in the limit (unique emergent heterotic string).
2. $B_3$ has a *unique* $T^2$-fibration and the fiber shrinks at the fastest rate in the limit. (unique emergent type II string)
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2 For $T^2$-fibrations:

- $\alpha'$-corrections do **not** ostruct infinite distance limits with *finite* volume: $\to$ Classical analysis goes through!
- Reason: Winding modes along the torus fiber give the leading KK tower $\Rightarrow M^2_{IIB}/M^2_{KK} \sim \mathcal{O}(1)$: Emergent string limit!

# Summary of Results

[Kläwer, Lee, Weigand, MW '20]

2 For $\mathbb{P}^1$-fibrations:

- $\alpha'$-corrections obstruct infinite distance limits with *finite* volume:
  $\rightarrow$ Perturbative control lost at finite distance in Kähler moduli space
  $\Rightarrow$ No emergent string limit!

- Limit: $J = \tilde{\lambda} J_0 + \frac{1}{\tilde{\lambda}} J_{\mathcal{I}_1} + b J_{\mathcal{I}_3}$ *marginally* allowed with $M_{\rm het}/M_{\rm KK} \sim \mathcal{O}(1)$.
  $\Rightarrow$ Emergent String Limit survives quantum corrections!

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**3** The WGC relation (only applies for the $\mathbb{P}^1$ case):

- Classically satisfied by excitations of heterotic string.

- Corrections to F-theory geometry yield correction to the superextremality bound:

$$\frac{g_{\text{YM}}^2 q_k^2 M_{\text{Pl}}^2}{M_k^2} \geq \frac{g_{\text{YM}}^2 Q_{\text{BH}}^2 M_{\text{Pl}}^2}{M_{\text{BH}}^2} = 1 - \frac{1}{2}(\Delta_0 + \Delta_1)\,.$$

- For repulsive force conjecture to be satisfied: find mass renormalisation of string excitation states.

# Quantum Corrections

[Grimm, Keitel, Mayer, Pugh, Savelli, Weissenbacher '13-'19]

- The classical 4D effective theory has $h^{(1,1)}(B_3)$ chiral multiplets with saxionic component

$$\mathrm{Re}\, T^0_\alpha = \frac{1}{2}\int_{D_\alpha} J\wedge J = \frac{1}{2}k_{\alpha\beta\gamma}v^\beta v^\gamma\,,$$

with Kähler potential and dual linear multiplets

$$K = -2\log \mathcal{V}^0_{B_3}\,,\qquad L_0^\alpha = -\frac{\partial K}{\partial \mathrm{Re}\, T_\alpha} = \frac{v^\alpha}{\mathcal{V}^0_{B_3}}\,.$$

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- Higer derivative contributions to the 11D M-theory supergravity effective action give $\alpha'$ corrections to these quantities:

$$\mathrm{Re}\, T_\alpha = \mathrm{Re}\, T^0_\alpha + \alpha^2\left((\kappa_3+\kappa_5)\frac{\mathrm{Re}\, T^0_\alpha \mathcal{Z}}{\mathcal{V}^0_{B_3}} + \kappa_5 \frac{\mathrm{Re}\, T^0_\alpha \mathcal{T}}{\mathcal{V}^0_{B_3}} + \kappa_4 \mathcal{Z}_\alpha \log \mathcal{V}^0_{B_3} + \kappa_6 \mathcal{T}_\alpha + \kappa_7 \mathcal{Z}_\alpha\right)\,,$$

$$e^{K/2} = \mathcal{V}^0_{B_3} + \alpha^2\left((\tilde\kappa_1+\tilde\kappa_2)\mathcal{Z} + \tilde\kappa_2 \mathcal{T}\right)\,.$$

$$\mathcal{Z}_\alpha = \int_{Y_4} c_3(Y_4)\wedge \pi^*(J_\alpha)\,,$$

$$\mathcal{T}_\alpha = -18(1+\alpha_2)\frac{1}{\mathrm{Re}\, T^{\mathrm{cl.}}_\alpha}\int_{D_\alpha} c_1(B_3)\wedge J \int_{D_\alpha} J_\alpha \wedge J\,,$$

$$\mathcal{T} = \mathcal{T}_\alpha v^\alpha\,, \qquad \mathcal{Z} = \mathcal{Z}_\alpha v^\alpha\,,$$

# Quantum Obstructions

[Kläwer, Lee, Weigand, MW '20]

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$B_3 : T^2 \to B_2$:

- In generic models: $\mathcal{Z}_0 = 0$ independent of chosen model.
- $\bar{K}.C_0 = 0 \Rightarrow \mathcal{T}_0 = 0$ independently of chosen model.

$\Rightarrow$ Classical limit stays in perturbative F-theory regime!

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$B_3 : \mathbb{P}^1 \to B_2$:

- In generic models: $\mathcal{Z}_0 \neq 0$ but precise value model dependent.
- $\bar{K}.C_0 = 2 \Rightarrow \mathcal{T}_0 = -36(1 + \alpha_2)$ independently of chosen model.

$\Rightarrow$ Classical limit leads out of perturbative F-theory regime!

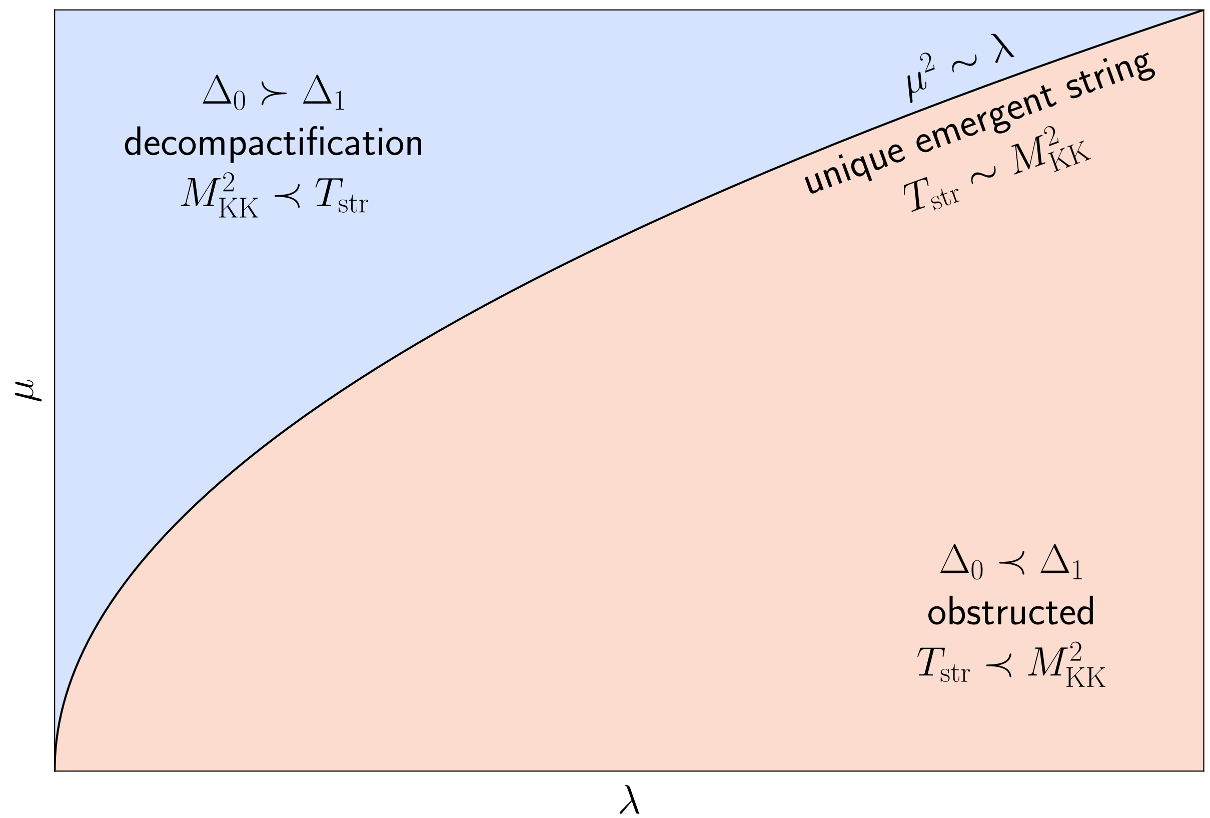
# Quantum Obstructions — Consequences

[Kläwer, Lee, Weigand, MW '20]

To keep weakly-coupled, tensionless heterotic string limit have to rescale:

$$J \to J' = \mu J$$

- For $\mu = \lambda^{1/2}$ have $M_{\text{het}}/M_{KK} \sim \mathcal{O}(1)$ which is *marginally* allowed by the quantum corrections (only numerical suppression).
- No further obstruction due to non-perturbative superpotential.




**Question:** Is the WGC relation affected by the quantum corrections?

# Weak Gravity Conjecture

[Kläwer, Lee, Weigand, MW '20]

From now on, assume that we have U(1) gauge theory realised on divisor **S**. In general, the WGC requires

$$\frac{g_{\rm YM}^2 q_k^2}{M_k^2} \overset{!}{\geq} \frac{g_{\rm YM}^2 Q_{\rm BH}^2}{M_{\rm BH}^2} \quad \overset{g_{\rm YM}\to 0}{=} \quad \frac{1}{M_{\rm Pl}^2}$$

Here the mass $M_k$ of the states is

$$M_k^2 = 8\pi M_{\rm het}^2 (n_k - 1)$$

Proof of this in the asymptotic regime has two components:

- Combinatorial Part: Establish the existence of excitation states satisfying $q_k^2 \geq 4m\, n_k$.
- Geometric Part: Establish a geometric relation between curve volumes

$$g_{\rm YM}^2 = \frac{2\pi}{\mathcal{V}_{\mathbf{S}}}\,, \qquad M_{\rm het}^2 = 2\pi \mathcal{V}_{C_0}\,, \qquad M_{\rm Pl}^2 = (4\pi)\mathcal{V}_{B_3}\,.$$

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  - Excitation spectrum captured by elliptic genus of heterotic string.
  - Use general properties of 4D elliptic genus to infer existence of such states. [2 x (Lee, Lerche, Lockhart, Weigand '20)]
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  - Classically, in the strict asymptotic limit

$$\frac{\mathcal{V}_{\mathbf{S}}^{(0)} \mathcal{V}_{C_0}^{(0)}}{2m \mathcal{V}_{B_3}^{(0)}} = 1$$

[see also: Lee, Lerche, Weigand '19]

  - But what about corrections?

# Corrections in F-theory/Heterotic Duality

[Kläwer, Lee, Weigand, MW '20]

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$$\frac{2\pi}{g_{YM}^2}\bigg|_{M_{\text{het}}} = (2m)\text{Re}\, S_{\text{het}} = \frac{1}{2}\frac{M_{pl}^2}{M_{\text{het}}^2}$$

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- $\mathcal{O}(S_{\text{het}}^0)$ holomorphic and non-holomorphic corrections:

$$\mathcal{O}(S_{\text{het}}^0) = \tilde{\Delta} = \underbrace{2\pi\,\text{Re} f^{(1)}(M)}_{=\tilde{\Delta}_0} + \underbrace{\frac{c}{8\pi}K^H(M,\bar{M})}_{=\tilde{\Delta}_1} + \ldots$$

- Geometric classical relation in the weak coupling limit $\frac{1}{\mathcal{V}_{\mathbf{S}}} = \frac{2m\mathcal{V}_{B_3}}{\mathcal{V}_{C_0}}$

- F-theory corrections split into two parts

$$\frac{\mathcal{V}_{\mathbf{S}}\mathcal{V}_{C_0}}{2m\mathcal{V}_{B_3}} = \frac{\mathcal{V}_{\mathbf{S}}^{(0)}\mathcal{V}_{C_0}^{(0)}}{2m\mathcal{V}_{B_3}^{(0)}}(1+\Delta_0+\Delta_1+\ldots)$$

$\Delta_0$: classical corrections to the weak coupling limit from four-fold geometry
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Can now match the corrections at the heterotic string scale

$$\frac{2\pi}{g^2_{\rm YM}}\bigg|_{M_{\rm het.}} = 2m\mathrm{Re}\, S_{\rm het}\,(1+\Delta_0+\Delta_1) - \underbrace{\frac{b}{2\pi}\log\left(2\pi\mathcal{V}_{C_0}\right)}_{\text{Running from } M_{\rm IIB}\to M_{\rm het.}}$$

$\longrightarrow$

$$\tilde{\Delta}_0 = 2m\mathrm{Re}\, S_{\rm het}\,\Delta_0$$

$$\tilde{\Delta}_1 = 2m\mathrm{Re}\, S_{\rm het}\,\Delta_1 - \frac{b}{8\pi}\log(2\pi\mathcal{V}_{C_0})\,.$$

(Identification allows to fix certain coefficients of the F-theory corrections)

# WGC — Perturbative Corrections

[Kläwer, Lee, Weigand, MW '20]

- Including corrections we get the WGC relation

$$\frac{g_{\rm YM}^2 q_k^2}{M_k^2} \geq \frac{g_{\rm YM}^2}{4\pi}\frac{2m}{M_{\rm het}^2} = \frac{2m\mathcal{V}_{B_3}}{\mathcal{V}_{\mathbf{S}}\mathcal{V}_{C_0}}\frac{1}{M_{\rm Pl}^2} = (1-\Delta_0-\Delta_1+\ldots)\frac{1}{M_{\rm Pl}^2}$$

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- So WGC not fulfilled except in the strict limit?

- So far only considered corrections to the *gauge coupling*.
- Mass of string excitations also get renormalised through stringy effects, i.e. corrections to

$$M_k^2 = 8\pi M_{\rm het}^2(n_k-1)$$

- In general: very difficult to calculate these effects.
- **Idea:** Repulsive force conjecture (RFC) should still be satisfied away from strict weak coupling limit:

$$|F_{\rm Coulomb}| \overset{!}{\geq} |F_{\rm Grav}| + |F_{\rm Yuk}|$$

# WGC — Perturbative Corrections

[Kläwer, Lee, Weigand, MW '20]

- Use this to *predict* the form of the mass renormalisation at one loop!

$$\frac{M_k^2}{M_{\rm Pl}^2} =: 8\pi(n_k - 1)\frac{M_{\rm het}^2}{M_{\rm Pl}^2}(1+\delta)$$

$$\overset{\rm RFC}{\Longrightarrow} \qquad \delta = -\frac{1}{2}(\Delta_0 + \Delta_1)\ .$$

- We thus get the 1-loop WGC relation

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Note: This is not to be confused with the higher derivative corrections to charge-to-mass ratio of extremal black holes!

[see e.g. Kats, Motl, Padi '06, Arkani-Hamed, Motl, Nicolis, Vafa '06]

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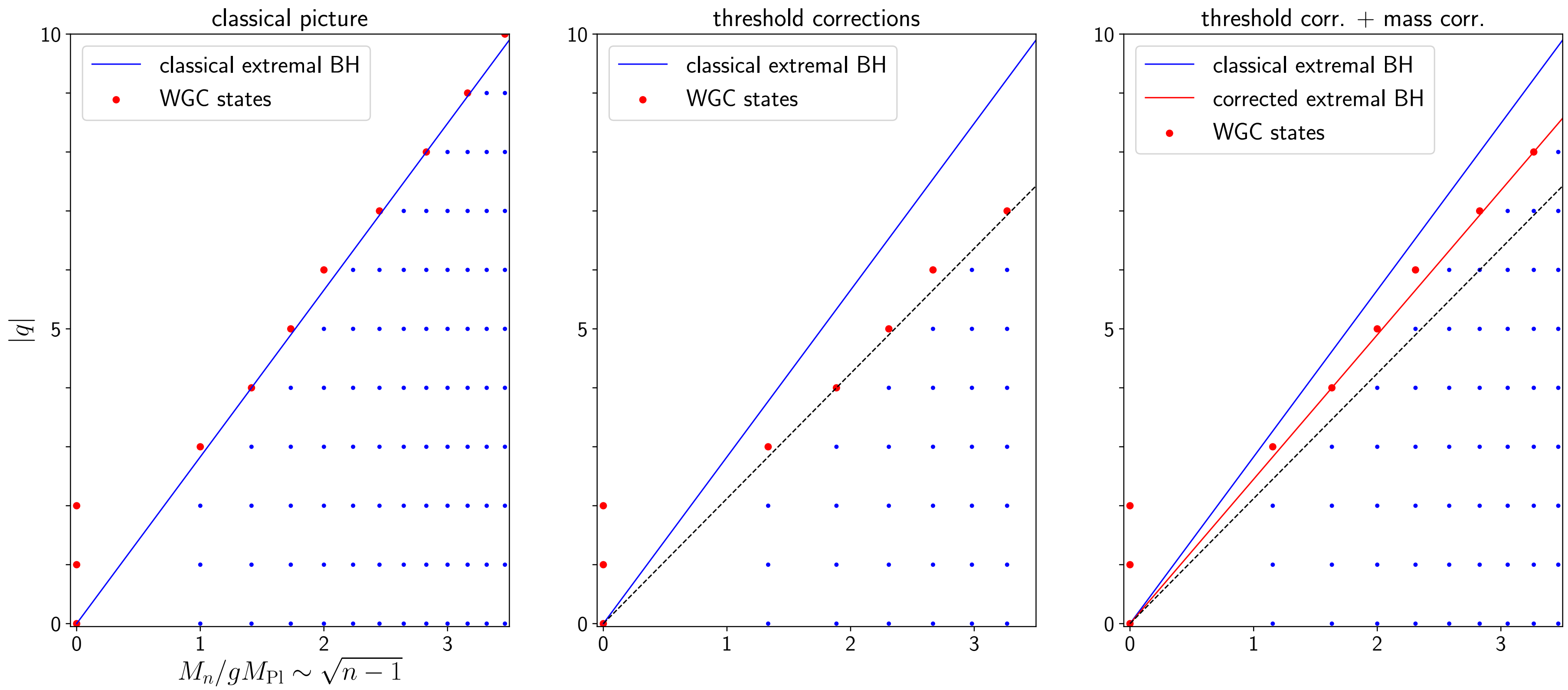

# Summary

- In equi-dimensional infinite distance limits in the Kähler moduli space of F-theory on $CY_4$, there is always a **unique** emergent string.

- F-theory $\alpha'$-corrections ensure that $M_{KK}^2 \precsim M_{\text{string}}^2$ is always fulfilled.

- *finite* volume limits for rationally fibered $B_3$ are out of perturbative control but allowed for elliptically fibered $B_3$.

- Matching F-theory $\alpha'$-corrections to dual heterotic threshold corrections $\rightarrow$ predict mass renormalisation of string excitation modes.

- Gauge threshold corrections and mass renormalisation modify the WGC super-extremality bound at one-loop.

**Thank you!**