AMPLITUDES POSITIVITY AND IR-UV CONNECTIONS IN GRAVITON EXCHANGE

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SISSA & IFPU & INFN

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BASED ON 2011.11652
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The final theory

The physicist


Swamp
Basic Land — Swamp
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XLN • EN Raoul Vitale
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Effective Field Theories

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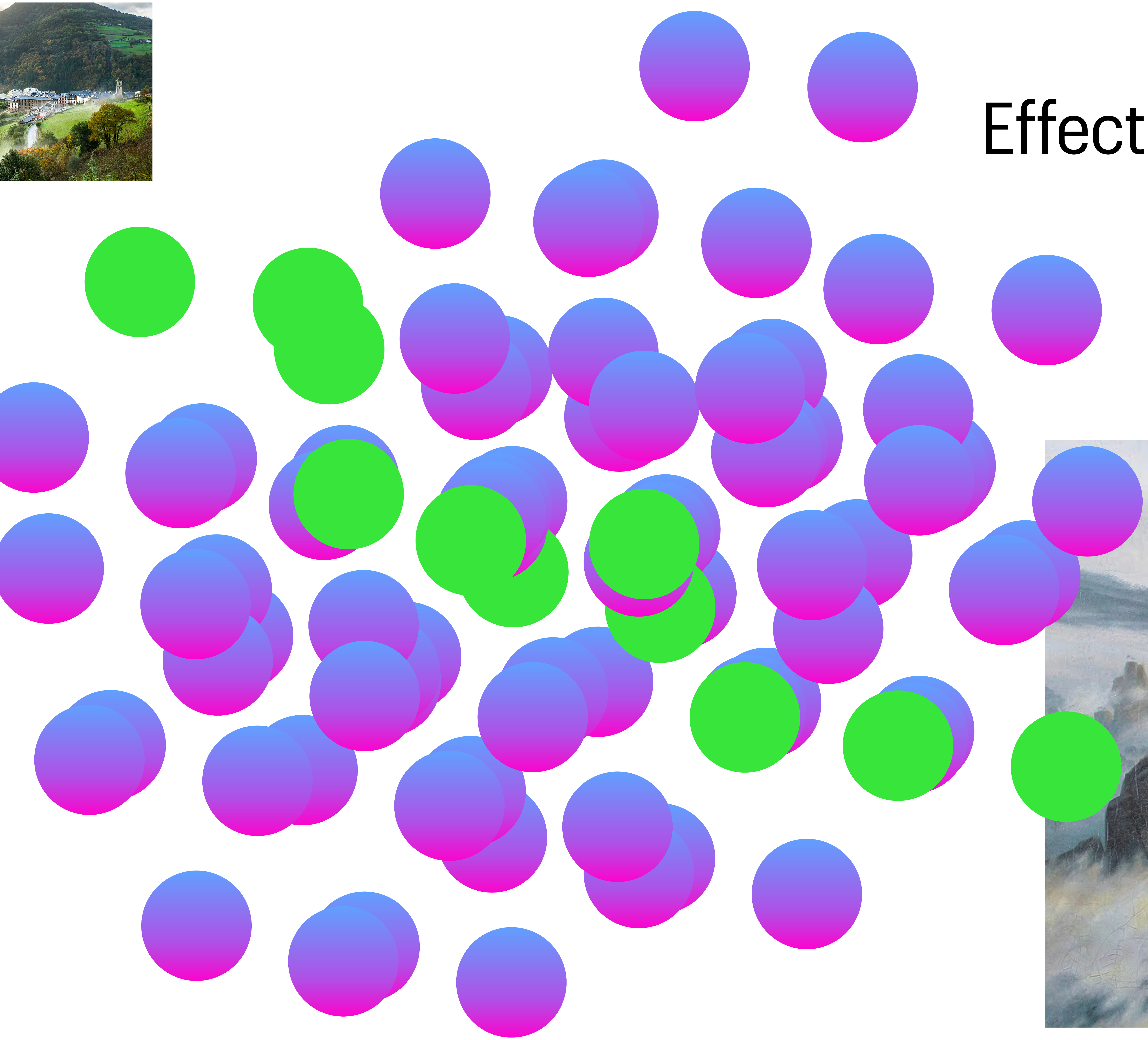

What is an Effective Field Theory (EFT)?

In most applications in physics we do not use fundamental theories

- We don't know the fundamental theory

- Physical phenomena are ordered by energy scales

  For example soft-collinear effective field theory (SCEFT)

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For example soft-collinear effective field theory (SCEFT)

EFT's are characterised by a derivative expansion with infinite terms compatible with the symmetries of the system

$$\mathscr{L} \sim (\partial\phi)^2 + \frac{c_1}{\Lambda^4}(\partial\phi)^4 + \frac{c_2}{\Lambda^8}(\partial\phi)^6 + \ldots \Rightarrow$$

Taylor Series of the complete theory

What is an Effective Field Theory (EFT)?

$$\mathscr{L} \sim (\partial\phi)^2 + \frac{c_1}{\Lambda^4}(\partial\phi)^4 + \frac{c_2}{\Lambda^8}(\partial\phi)^6 + \dots \quad \Rightarrow \quad \text{Taylor Series of the complete theory}$$

In an ideal world we obtain the EFT from the UV complete theory

Taylor series from the full function

What is an Effective Field Theory (EFT)?

$$\mathscr{L} \sim (\partial\phi)^2 + \frac{c_1}{\Lambda^4}(\partial\phi)^4 + \frac{c_2}{\Lambda^8}(\partial\phi)^6 + \ldots$$ ➡ Taylor Series of the complete theory

In an ideal world we obtain the EFT from the UV complete theory

Taylor series from the full function

However, in many cases we do not know the full theory, we just know some terms in the EFT

General Relativity

$$\mathscr{L} = -\sqrt{|g|}\; M_P^2\, R \sim \partial h^2 + \frac{1}{M_P}h(\partial h)^2 + \frac{1}{M_P^2}h^2(\partial h)^2 + \ldots$$

NOT EVERY POLYNOMIAL IS A TAYLOR SERIES OF A FUNCTION WITH SOME DEFINITE PROPERTIES

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

NOT EVERY EFT IS A LOW ENERGY EXPANSION OF A THEORY WITH SOME DEFINITE PROPERTIES

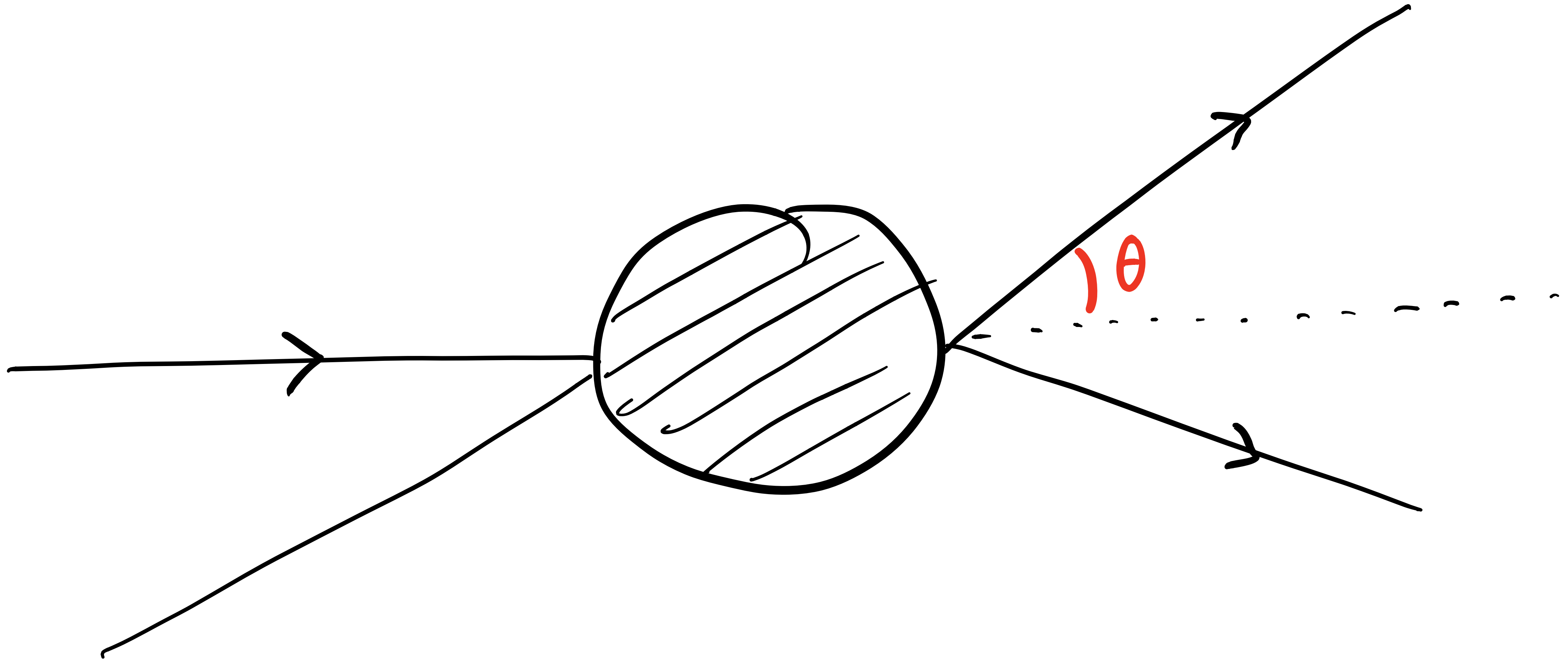
$$\mathscr{L} \sim (\partial\phi)^2 + \frac{c_1}{\Lambda^4}(\partial\phi)^4 + \frac{c_2}{\Lambda^8}(\partial\phi)^6 + \dots$$

Unitarity, Lorentz invariance, symmetries… impose conditions

See talks by Aalsma and Weigan

WE CAN LOOK AT THE 2—>2 SCATTERING AMPLITUDE AROUND FLAT SPACE-TIME

$$\mathscr{A}(s,t)$$



$s \equiv$ center of mass energy squared

$$t = -\frac{s}{2}(1-\cos\theta)$$

$$\sigma \sim \int d(\text{phase space})\ |\mathscr{A}|^2$$

WE CAN LOOK AT THE 2—>2 SCATTERING AMPLITUDE

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- Lorentz invariance implies that it only depends on s and t.
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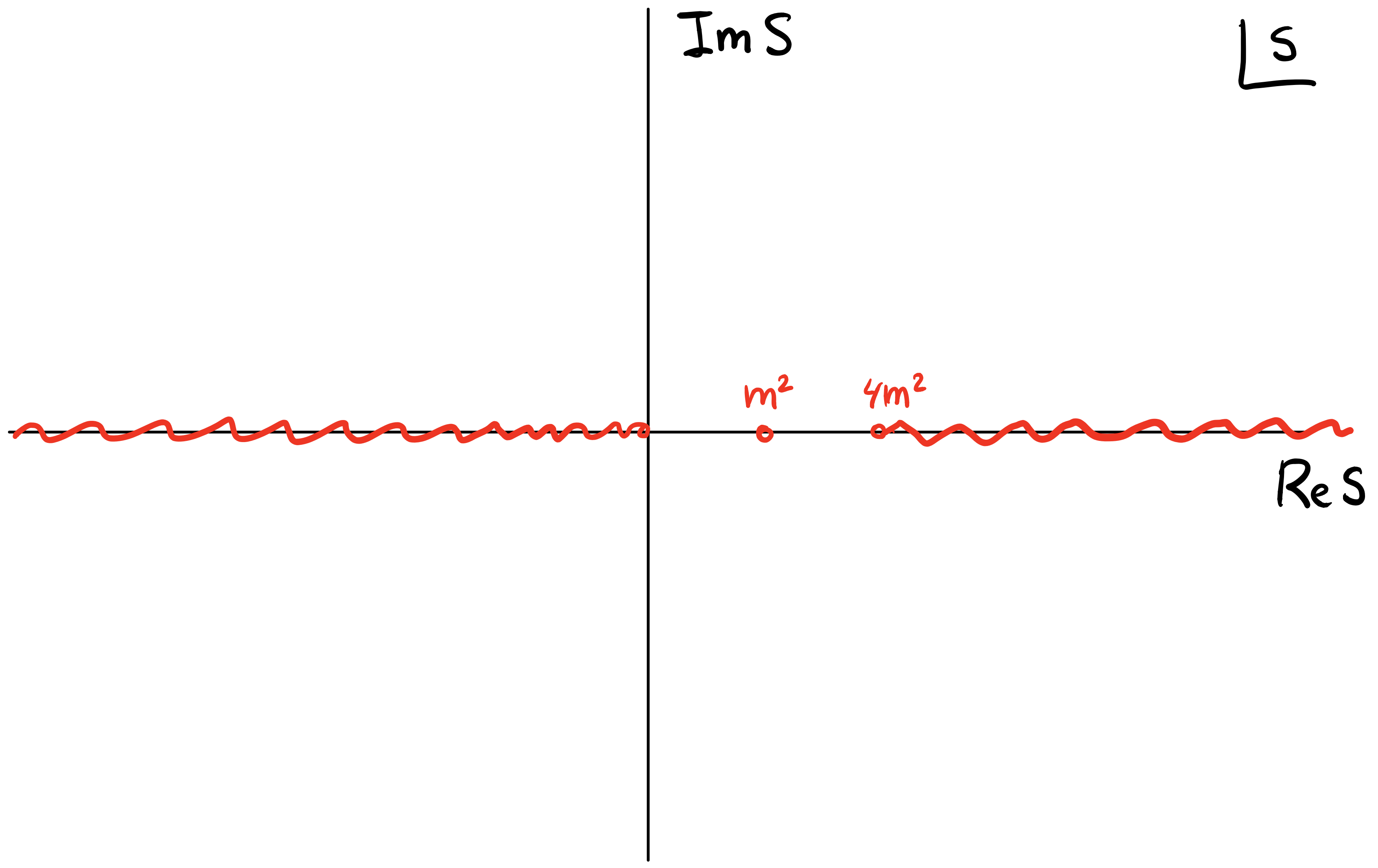
- Spin statistics is imposed by using crossing symmetry.

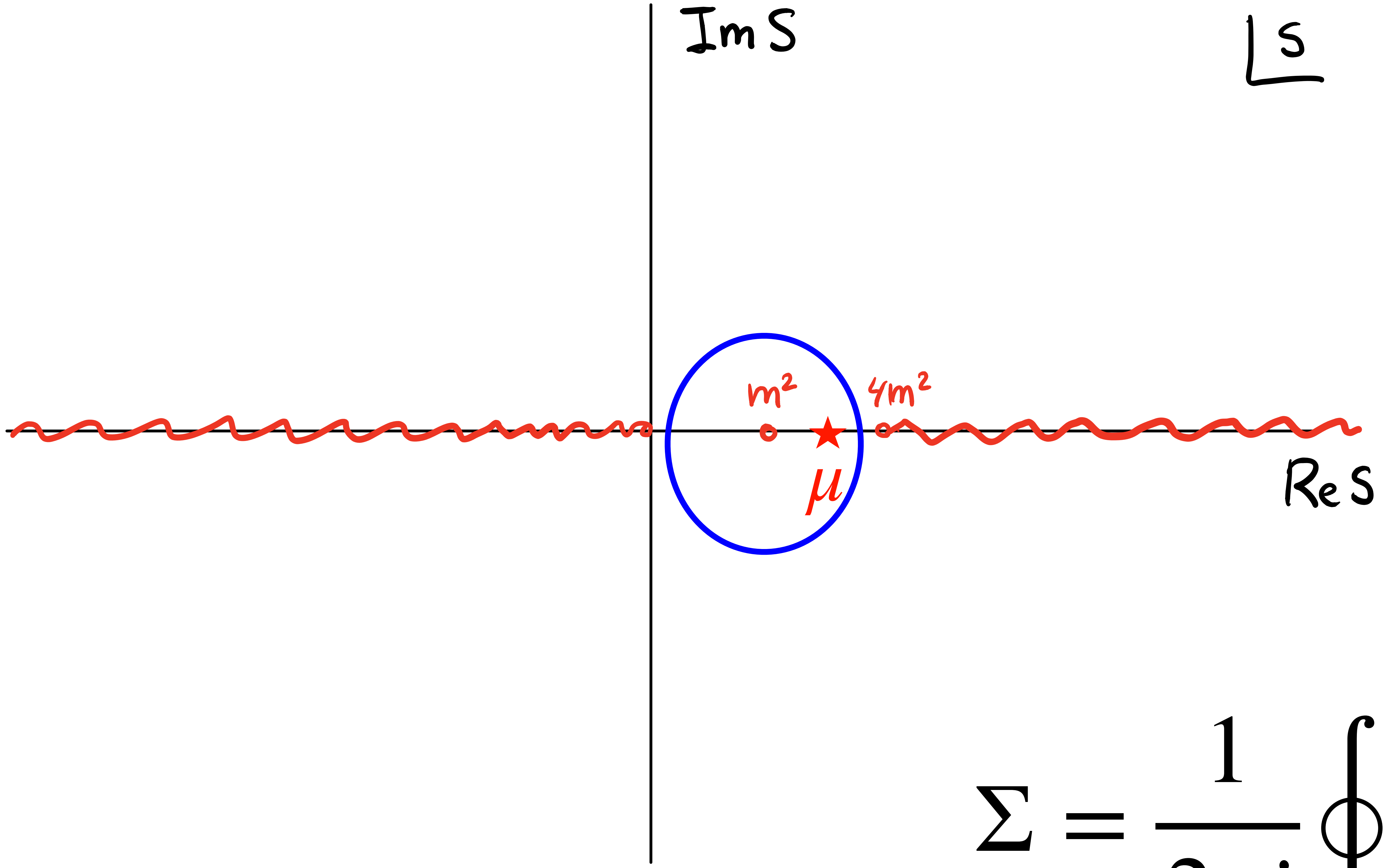
- In the forward limit (theta=0) t=0:



- Poles in the masses intermediate particles s=m^2

- A branch cut above production threshold s=4m^2

WE CAN LOOK AT THE 2—>2 SCATTERING AMPLITUDE



$$\Sigma = \frac{1}{2\pi i}\oint ds \frac{\mathcal{A}(s,0)}{(s-\mu)^3}$$

ADAMS, ARKANI-HAMED, DUBOVSKY, NICOLIS, RATTAZZI, TRINCHERINI, 0912.4258 &0602178

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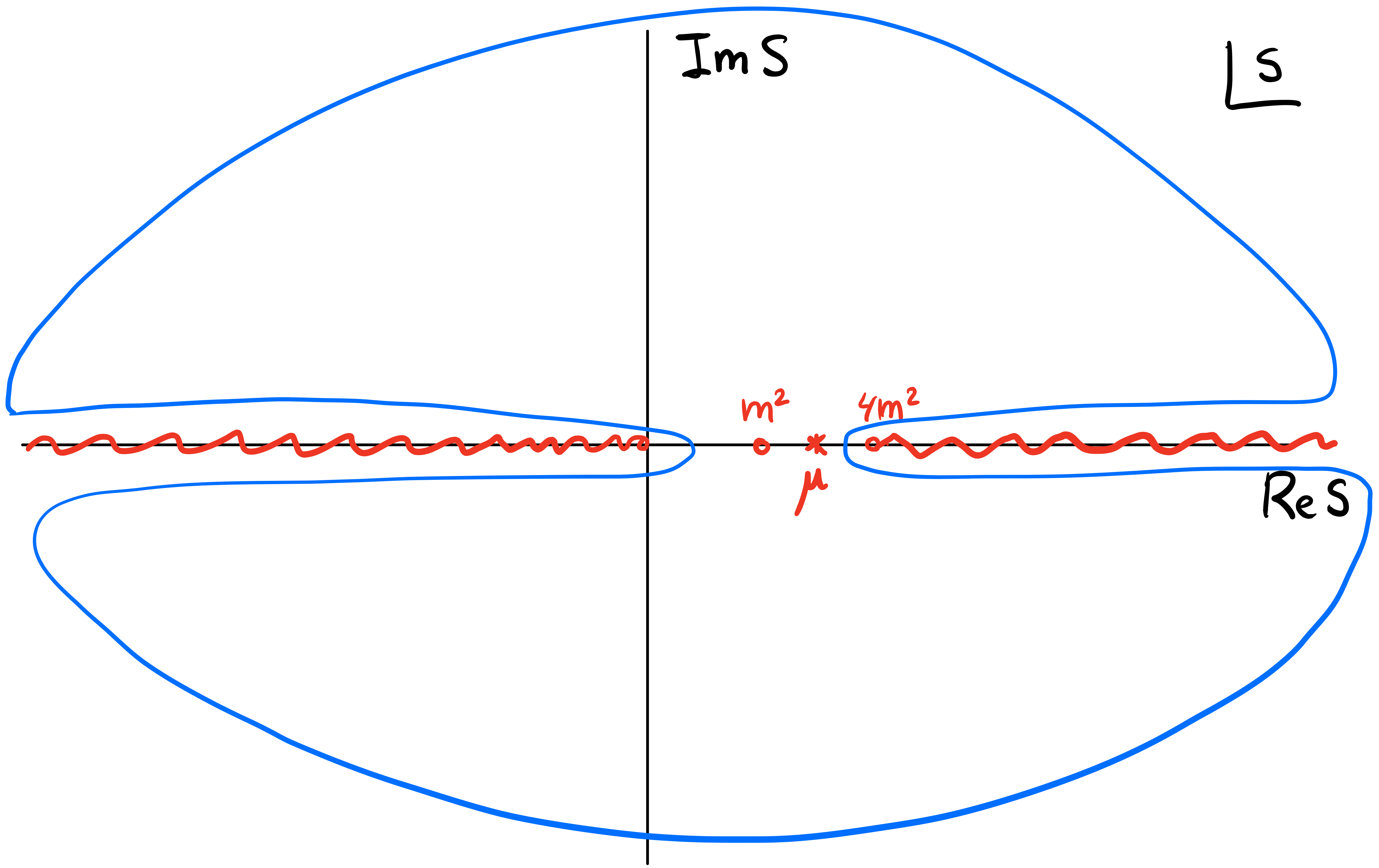
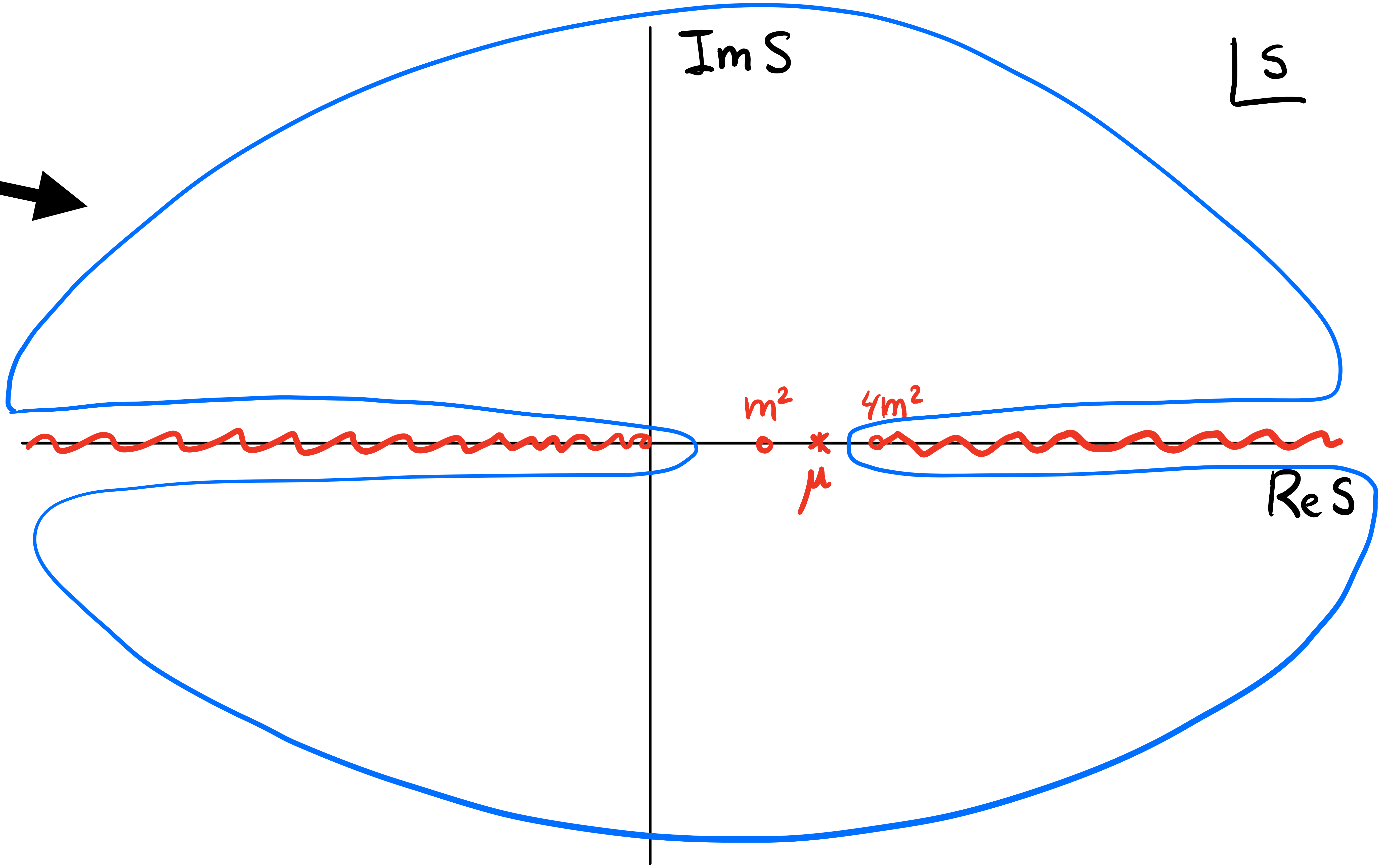


WE CAN LOOK AT THE 2—>2 SCATTERING AMPLITUDE

$$\lim_{s\to\infty} \frac{|\mathscr{A}|}{s^2} = 0$$



MARTIN, *NUOVO CIM.A* 42 (1965) 930-953

WHERE TO LOOK?

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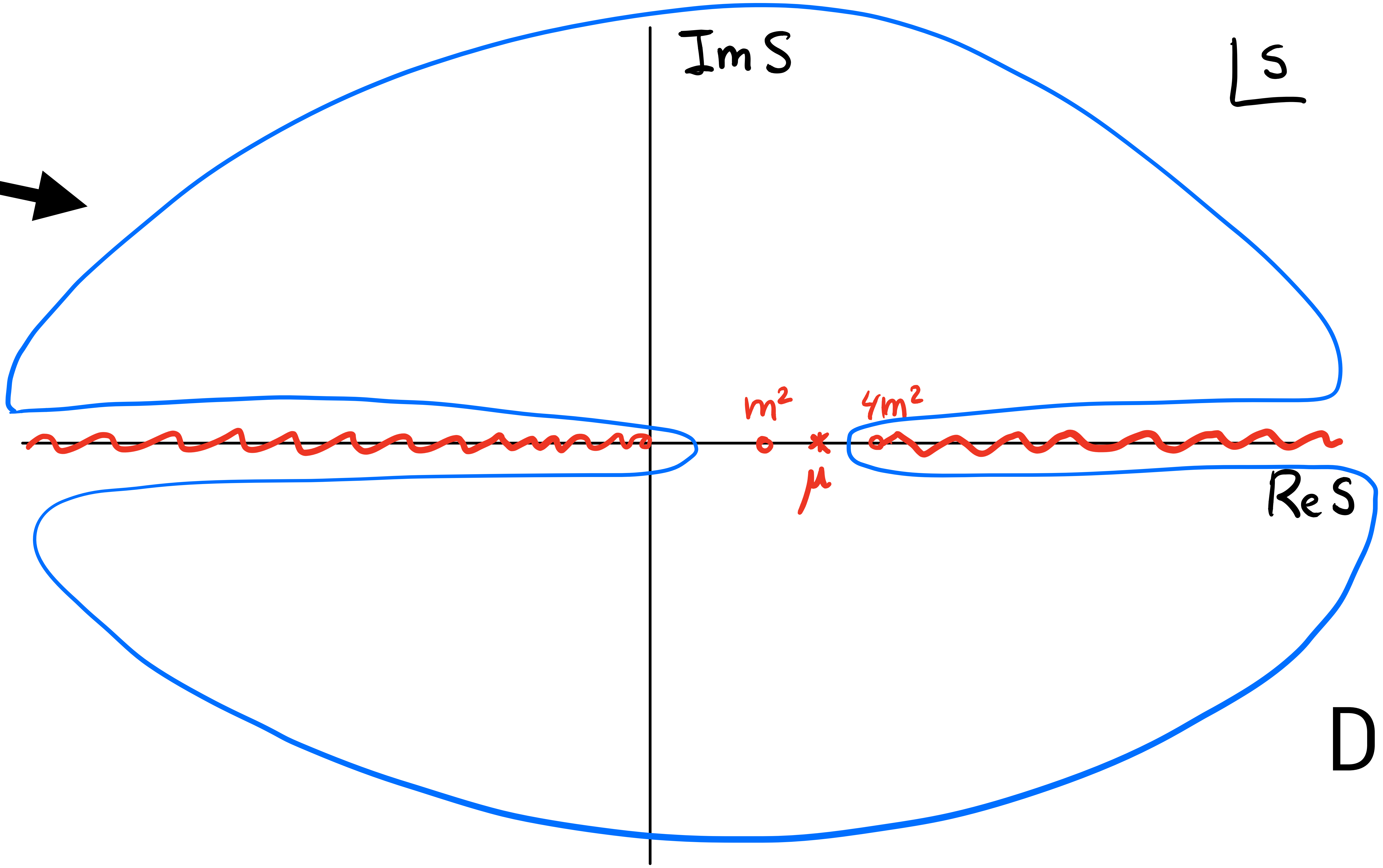

$$\mathrm{Disc}\mathscr{A} = 2\mathrm{Im}\mathscr{A}$$

WE CAN LOOK AT THE 2—>2 SCATTERING AMPLITUDE

$$\Sigma = \frac{1}{2\pi i}\oint ds \frac{\mathscr{A}(s,0)}{(s-\mu)^3} = \int_{4m^2}^{\infty} \frac{ds}{\pi}\left(\frac{\mathrm{Im}\mathscr{A}}{(s-\mu)^3} + \frac{\mathrm{Im}\mathscr{A}^{\times}}{(s-4m^2+\mu)^3}\right)$$

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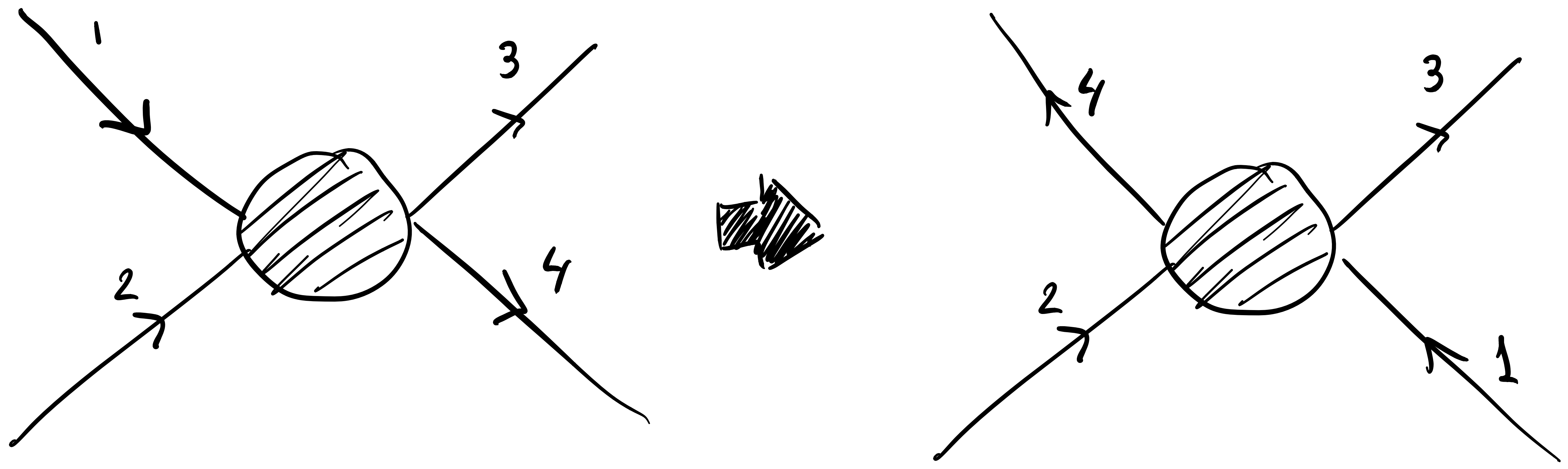

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$$Im\mathscr{A}(s,0) = s\sqrt{1-\frac{4m^2}{s}}\,\sigma(s) > 0 \quad \text{for} \quad s > 4m^2$$

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$$\boxed{\Sigma > 0}$$

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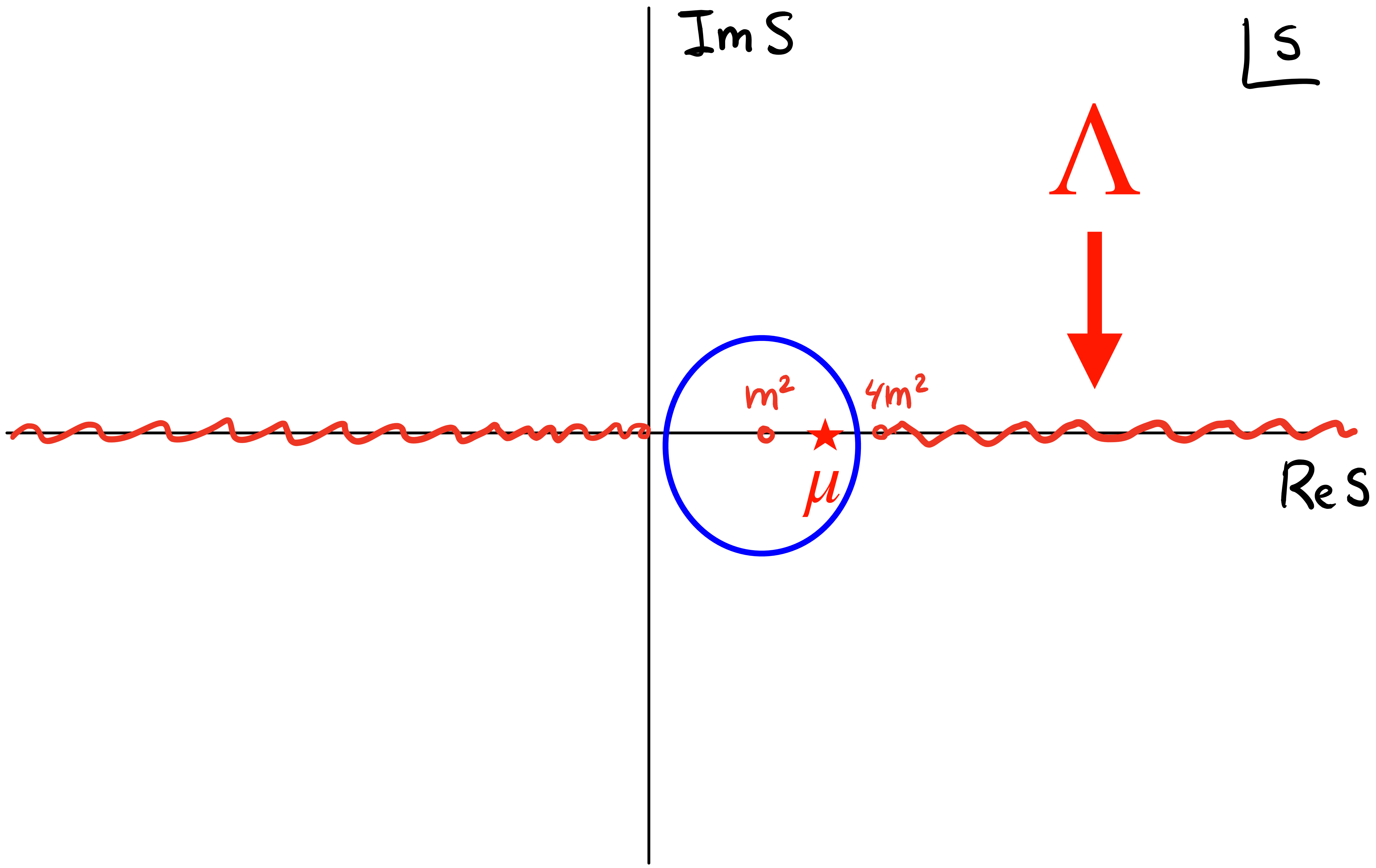
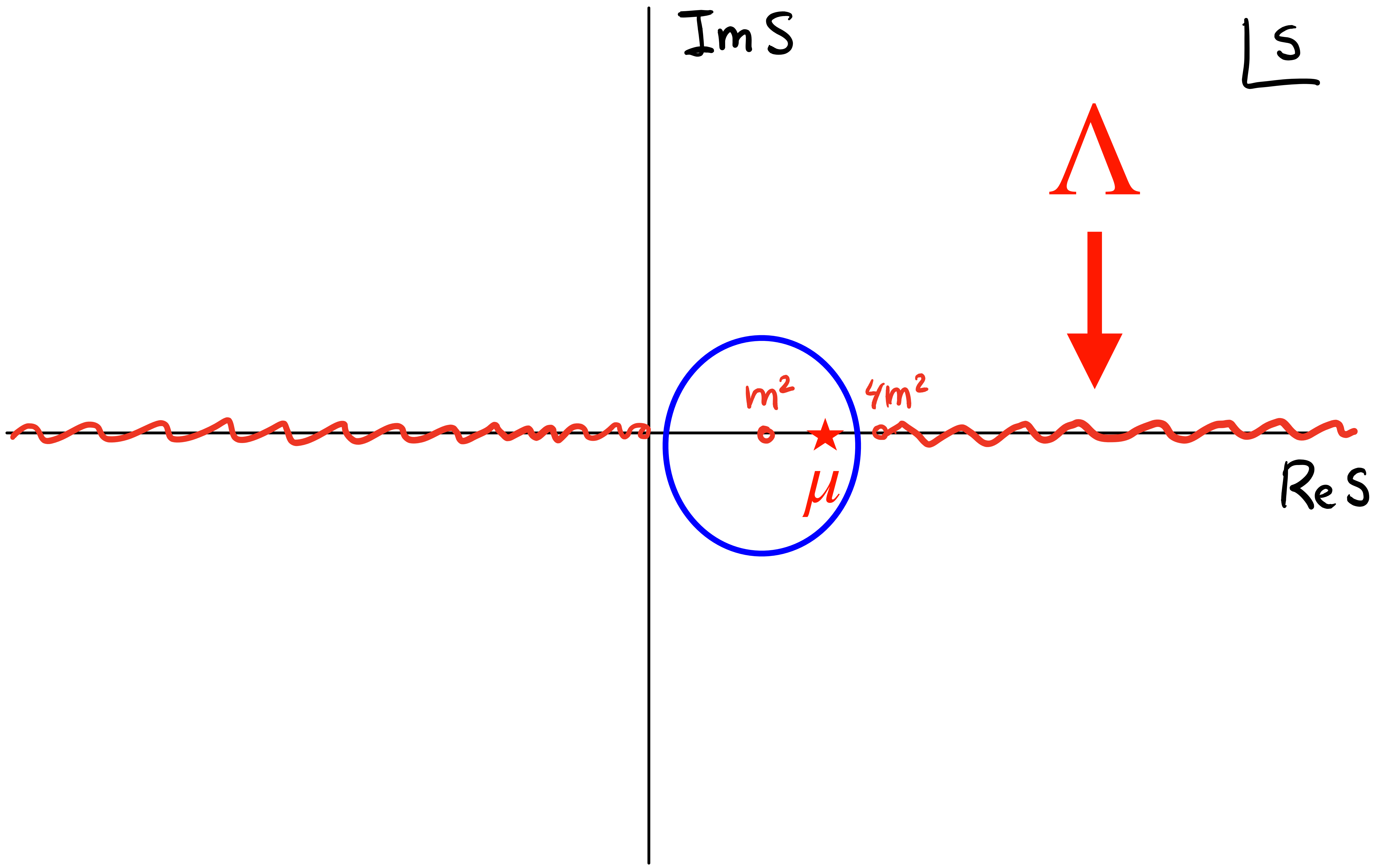


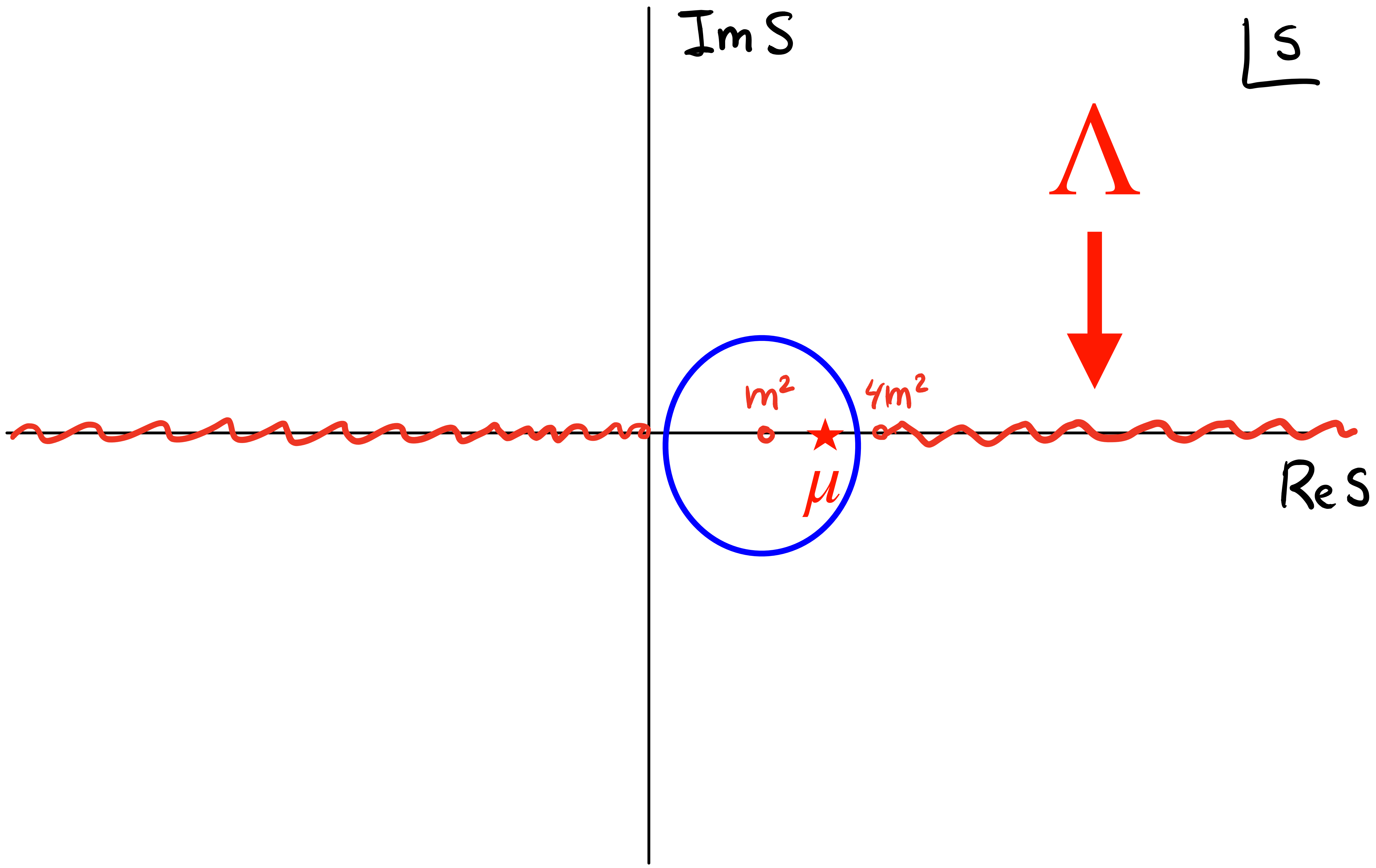
BUT WHAT IF WE ONLY KNOW AN EFT VALID UP TO AN ENERGY LAMBDA?

WE CAN LOOK AT THE 2—>2 SCATTERING AMPLITUDE



Computable in the EFT!! $\Sigma = \frac{1}{2\pi i}\oint ds \frac{\mathcal{A}(s,0)}{(s-\mu)^3} > 0$

WE CAN LOOK AT THE 2—>2 SCATTERING AMPLITUDE



$$\Sigma = \frac{1}{2\pi i}\oint ds \frac{\mathscr{A}(s,0)}{(s-\mu)^3} = \int_{4m^2}^{\Lambda^2} + \int_{\Lambda^2}^{\infty} \longrightarrow \bar{\Sigma} = \frac{1}{2\pi i}\oint ds \frac{\mathscr{A}(s,0)}{(s-\mu)^3} - \int_{4m^2}^{\Lambda^2} = \int_{\Lambda^2}^{\infty} > 0$$

WHERE TO LOOK?

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$$\Sigma = c_1 > 0$$

APPLICATIONS TO A MYRIAD OF TOPICS: MASSIVE GRAVITY, THE A-THEOREM, HIGGS PHYSICS, COSMOLOGY...

BUT THERE IS AN IMPORTANT PIECE MISSING: GRAVITY

THE FORWARD LIMIT IS ILL-DEFINED

$$\mathcal{A}(s,t) = \frac{1}{M_P^2}\left(\frac{s^2}{t} + \frac{t^2}{4m^2 - s - t} + \frac{(4m^2 - s - t)^2}{s}\right) + \frac{1}{M_P^4}\left(s^2 \log(t) + t^2 \log(4m^2 - s - t) + (4m^2 - s - t)^2 \log(s)\right)$$

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SO AS MOST WE CAN GO TO THE ASYMPTOTIC FORWARD LIMIT

$$\mathscr{A}(s, t \to 0) = \frac{a_0}{t} + a_l \, log(t) + a(s)$$

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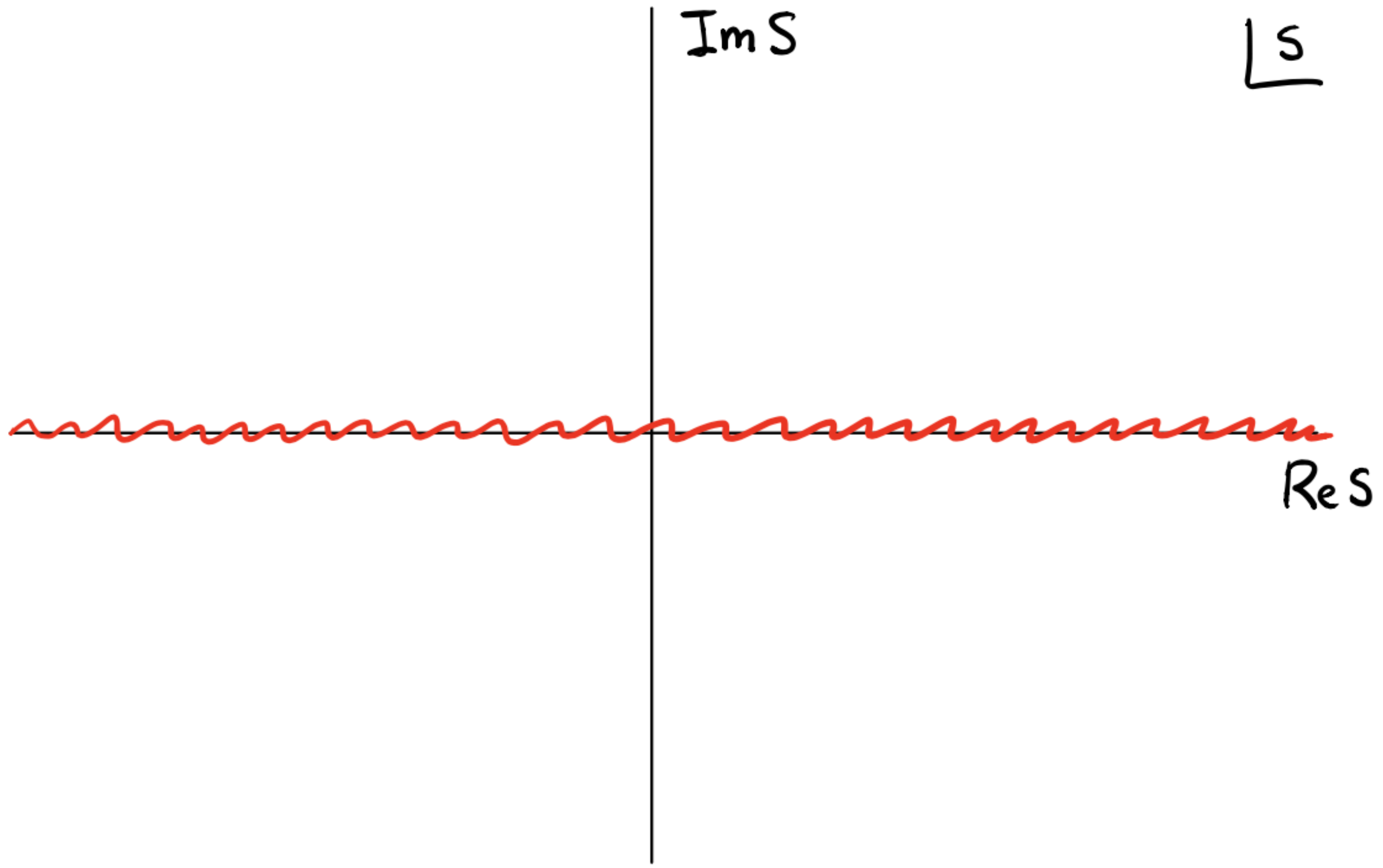

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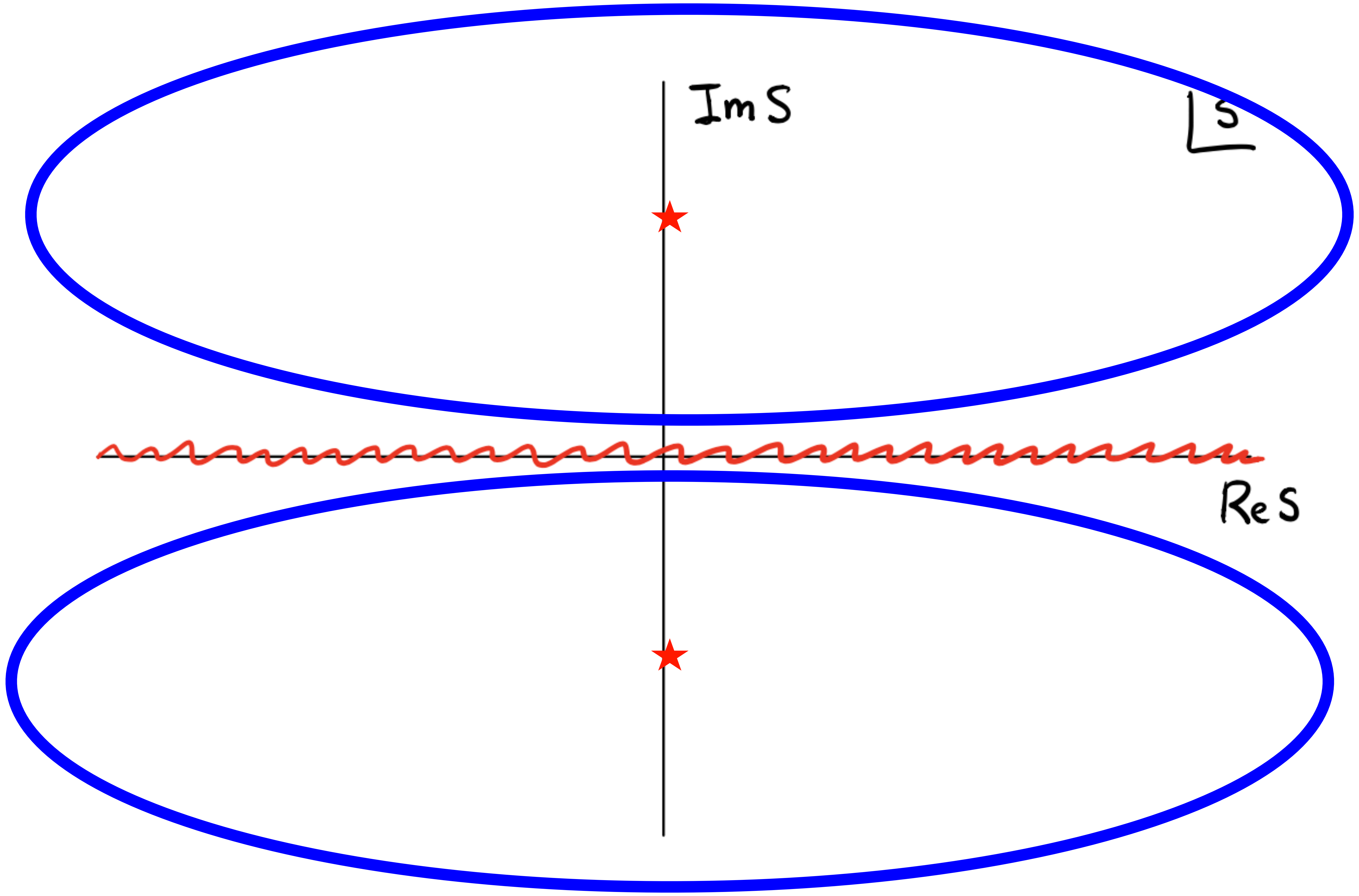

INSTEAD WE DEFINE

$$\mathcal{B}(s,0^-) = \mathrm{Re}\mathcal{A}(s+i\kappa,0^-) - \frac{(s-\mu)^n}{2\pi}\int_0^{4m^2} dz\ \frac{\mathrm{Im}\mathcal{A}(z+i\epsilon,0^-)}{(z-s)(z-\mu)^n}.$$

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$$\Sigma > 0$$

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BUT WE HAVE $\Sigma \sim \frac{a_0}{t}, \quad \text{with} \quad a_0 > 0$

$$\Sigma = \frac{1}{2\pi i}\oint ds \frac{s^3\mathscr{B}(s,0)}{(s^2+\delta^2)^3} = \int_{4m^2}^{\infty}\frac{ds}{\pi}\left(\frac{s^3\,\mathrm{Im}\mathscr{A}}{(s^2+\delta^2)^3} + \frac{(s-4m^2)^3\,\mathrm{Im}\mathscr{A}^{\times}}{((s-4m^2)^2+\delta^2)^3}\right)$$

This has divergences 1/t and log(t)

This must also have divergences 1/t and log(t)

$$\Sigma = \underbrace{\frac{1}{2\pi i}\oint ds \frac{s^3\mathscr{B}(s,0)}{(s^2+\delta^2)^3}}_{\text{This has divergences } 1/t \text{ and } \log(t)} = \int_{4m^2}^{\infty}\frac{ds}{\pi}\underbrace{\left(\frac{s^3\mathrm{Im}\mathscr{A}}{(s^2+\delta^2)^3} + \frac{(s-4m^2)^3\mathrm{Im}\mathscr{A}^{\times}}{((s-4m^2)^2+\delta^2)^3}\right)}_{\text{This must also have divergences } 1/t \text{ and } \log(t)}$$

IR **Energy** $\longrightarrow$ **UV**

$\underbrace{\hspace{4cm}}_{\text{Well described by the EFT}}$ $\underbrace{\hspace{4cm}}_{\text{Quantum Gravity?}}$

WHY GRAVITY WORKS?

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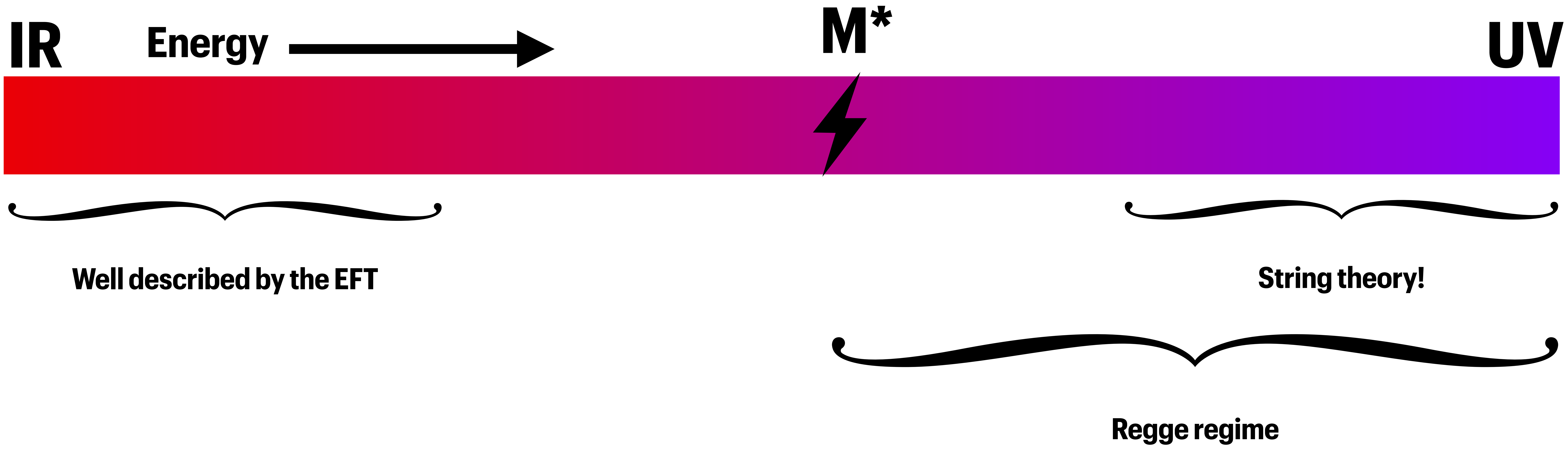
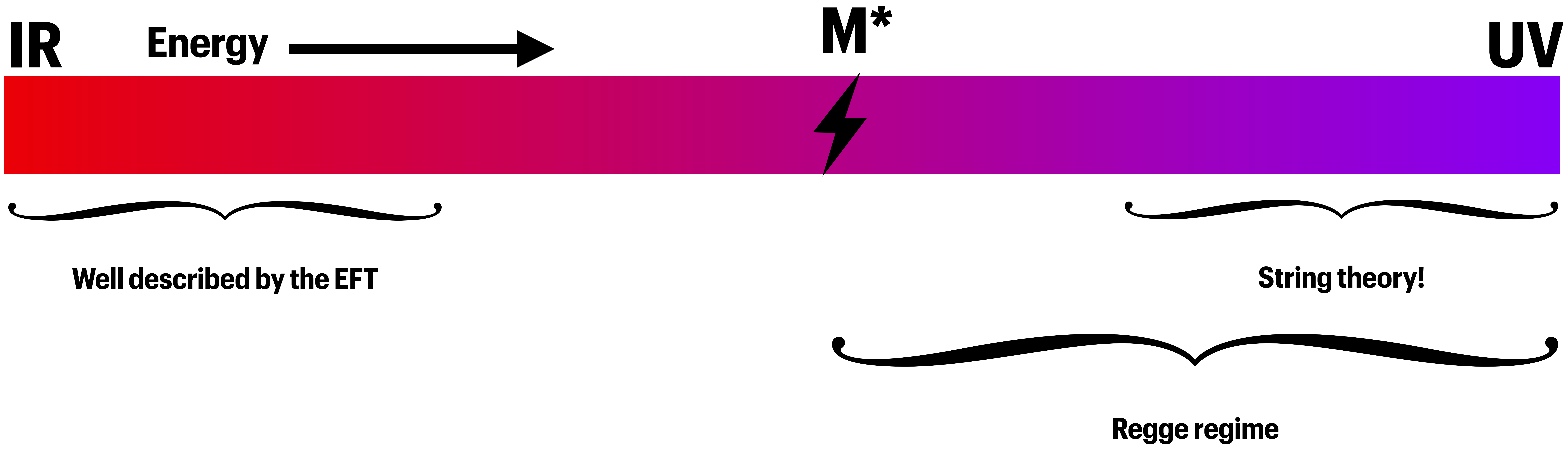


TOKUDA, AOKI, HIRANO, 2007.15009

WHY GRAVITY WORKS?



$$\mathrm{Im}\mathscr{A}(s,t) = r(t)(\alpha' s)^{2+l(t)}\left(1+\frac{\zeta}{\log(\alpha' s)}+\mathcal{O}\left(\frac{1}{\alpha' s}\right)\right), \quad \text{for} \quad E > M^*$$

TOKUDA, AOKI, HIRANO, 2007.15009

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$$\int_{(M_*)^2}^{\infty} \sim -\frac{r(0)\alpha'^2}{\pi}\left(-\frac{1}{l'(0)t} + \zeta log(t)\right) + \mathcal{O}(r(0)\alpha'^2)$$

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No definite sign!!!

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$$\boxed{\begin{gathered}\Sigma - \text{divergences} > \mathcal{O}(r(0)\alpha'^2) \\ \\ r(0)\alpha'^2 \sim (M*)^{-2} M_P^{-2}\end{gathered}}$$

$$\boxed{\begin{array}{c} \Sigma - \text{divergences} > \mathcal{O}(r(0)\alpha'^2) \\ \\ r(0)\alpha'^2 \sim (M^*)^{-2} M_P^{-2} \end{array}}$$

SO IF I HAVE A THEORY WITH PHYSICS AT A SCALE $\Lambda < M^*$

THIS IS GREATLY PREDICTIVE

SCALAR FIELD

$$S = \int d^4x \sqrt{|g|}\ \left(-\frac{R}{2\kappa^2} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \right),$$

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$$\mathcal{A}(s, 0^-) = -\frac{\kappa^2 s^2}{t} - \frac{33\kappa^4 s^2}{24\pi^2} \left(\log(s) + \log(-s) \right) \\ - \frac{33\kappa^4 s^2}{24\pi^2} \log(t).$$

$$S = \int d^4x \sqrt{|g|} \, \left(-\frac{R}{2\kappa^2} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \right),$$

$$- \frac{33\kappa^4}{24\pi^2} \left(\frac{3}{2} + \log(\delta^2) \right) > \mathcal{O}(r(0)\alpha'^2),$$

$$\frac{y(j)\kappa^2}{\pi^2 \delta^{4j-4}} > 0.$$

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$$\frac{y(j)\kappa^2}{\pi^2\delta^{4j-4}} > 0.$$

Automatically satisfied

$$S = \int d^4x \sqrt{|g|} \left[-\frac{R}{2\kappa^2} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi \right. \\ \left. + \frac{1}{2}\partial_\mu \psi \partial^\mu \psi - \frac{1}{2}\Lambda^2 \psi^2 - \lambda \Lambda \phi \psi^2 \right]. \tag{36}$$

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Σ – divergences $< 0!!!$

If $\Lambda < M^* < M_P$

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SCALAR QED REQUIRES PHYSICS AT A SCALE BELOW THE PLANCK SCALE

ALBERTE, DE RHAM, JAITLY, TROLLEY, 2012.05798

$$S=\int d^4x\sqrt{|g|}\Big[-\frac{R}{2\kappa^2}+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\frac{1}{2}\partial_\mu\chi\partial^\mu\chi$$
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SCALAR QED REQUIRES PHYSICS AT A SCALE BELOW THE PLANCK SCALE

There are some more recent claims about real QED, but careful…

ALBERTE, DE RHAM, JAITLY, TROLLEY, 2012.05798

In Einstein-Maxwell theory

$$S = \int d^4x \sqrt{|\hat{g}|} \left[\frac{m_{\rm Pl}^2}{2} \hat{R} - \frac{1}{4} \hat{F}^{MN} \hat{F}_{MN} \right.$$

Extremal charged black-holes satisfy

$$\left(\frac{\sqrt{2}|Q|}{M/m_{\rm Pl}} \right)_{\rm extr.} = 1 \,.$$

BELLAZZINI, LEWANDOWSKY, SERRA 1902.03250

In Einstein-Maxwell theory

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Extremal charged black-holes satisfy

$$\left(\frac{\sqrt{2}|Q|}{M/m_{\mathrm{Pl}}} \right)_{\mathrm{extr.}} = 1 \, .$$

However, the Weak Gravity Conjecture states

$$\left(\frac{\sqrt{2}\,|Q|}{M/m_{Pl}} \right)_{\mathrm{extr.}} > 1$$

BELLAZZINI, LEWANDOWSKY, SERRA 1902.03250

In Einstein-Maxwell EFT

$$S=\int d^4x\sqrt{|\hat{g}|}\left[\frac{m_{\rm Pl}^2}{2}\hat{R}-\frac{1}{4}\hat{F}^{MN}\hat{F}_{MN}\right.$$
$$+\frac{\alpha_1}{4m_{\rm Pl}^4}\left(\hat{F}^{MN}\hat{F}_{MN}\right)^2+\frac{\alpha_2}{4m_{\rm Pl}^4}\left(\hat{\tilde{F}}^{MN}\hat{F}_{MN}\right)^2$$
$$\left.+\frac{\alpha_3}{2m_{\rm Pl}^2}\hat{F}_{AB}\hat{F}_{CD}\hat{W}^{ABCD}\right],$$

Extremal charged black-holes satisfy

$$\left(\frac{\sqrt{2}|Q|}{M/m_{\rm Pl}}\right)_{\rm extr.}=1+\frac{4}{5}\frac{(4\pi)^2m_{\rm Pl}^2}{M^2}(2\alpha_1-\alpha_3)$$

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In Einstein-Maxwell theory EFT

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In Einstein-Maxwell theory EFT

$$S = \int d^4x\sqrt{|\hat{g}|}\left[\frac{m_{\rm Pl}^2}{2}\hat{R} - \frac{1}{4}\hat{F}^{MN}\hat{F}_{MN} + \frac{\alpha_1}{4m_{\rm Pl}^4}\left(\hat{F}^{MN}\hat{F}_{MN}\right)^2 + \frac{\alpha_2}{4m_{\rm Pl}^4}\left(\hat{\tilde{F}}^{MN}\hat{F}_{MN}\right)^2 + \frac{\alpha_3}{2m_{\rm Pl}^2}\hat{F}_{AB}\hat{F}_{CD}\hat{W}^{ABCD}\right],$$

Extremal charged black-holes satisfy

$$\left(\frac{\sqrt{2}|Q|}{M/m_{\rm Pl}}\right)_{\rm extr.} = 1 + \frac{4}{5}\frac{(4\pi)^2 m_{\rm Pl}^2}{M^2}(2\alpha_1 - \alpha_3)$$

$$\mathscr{A} \sim s^2(2\alpha_1 - \alpha_3)$$

$$\Sigma \sim (2\alpha_1 - \alpha_3) > 0$$

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CONCLUSIONS

ALWAYS BE POSITIVE :)

- Positivity is a way to bound the zoo of EFTs that we crazily propose in the IR.

- There used to be fundamental obstacles to apply positivity bounds to theories with gravity, but we have overcome them.

- We have the power to bound several modes of gravity+matter.

$$\Sigma = \frac{1}{2\pi i}\oint ds \frac{s^3 \mathscr{B}(s,0)}{(s^2+\delta^2)^3} - \text{divergences} > \mathcal{O}(r(0)\alpha'^2)$$

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