Fermion Mass Matrices from Exceptional Field Theory

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Iberian Strings

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MC, O.Varela – arXiv: 2012.05249

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Overview

- KK spectroscopy for fermions from ExFT.
- Spectrum of AdS_4, $\mathcal{N} = 1$ solutions preserving at least SU(3) from two gaugings of $D = 4$, $\mathcal{N} = 8$ vacua:

  G_2 in SO(8) gauging, which uplifts to 11-D M-theory.

  G_2 and SU(3) in ISO(7), which uplifts to type IIA string theory.

- Spectrum of non supersymmetric solutions of the same kind.

  $SO(7)_v$, $SO(7)_c$ and $SU(4)_c$ in SO(8).

  SO(7), SO(6) and G_2 in ISO(7).

- Conclusion and Outlook.

Kaluza-Klein Spectrum: what do we mean?

Example: One-Dimensional reduction:

$$ds^2 = ds^2_{\text{ext}}(x) + L^2 d\theta^2 \,.$$

Consider i.e. a massless scalar field and perform a Fourier expansion on the internal manifold

$$\phi(x,\theta) = \sum_{n=-\infty}^{\infty} \phi_n(x)\mathcal{Y}_n(\theta) = \sum_{n=-\infty}^{\infty} \phi_n(x) e^{\frac{i\pi n\theta}{L}} \,.$$

Then

$$\Box_D \phi(x,\theta) = 0 \quad \longrightarrow \quad \left(\Box_{D-1} + \partial_\theta^2\right)\phi(x,\theta) = \sum_{n=-\infty}^{\infty}\left(\Box_{D-1}\phi_n(x) - \frac{\pi^2 n^2}{L^2}\phi_n(x)\right) e^{\frac{i\pi n\theta}{L}} = 0$$

$E_{7(7)}$ Exceptional Field Theory

A manifest $E_{7(7)}$ duality covariant formulation of supergravity

[O. Hohm, H.Samtleben,'13]

$$E_{7(7)} \; : \; \{\mathcal{A}_\mu^M, g_{\mu\nu}, \mathcal{M}_{MN}, \mathcal{B}_{\mu\nu\,\alpha}, \mathcal{B}_{\mu\nu\,M}\}$$

$$SU(8) \; : \; \{\psi^i{}_\mu, \chi^{[ijk]}\}$$

[H.,M. Godazgar, O.Hohm, H.Nicolai, H.Samtleben, '14]

with $\mu, \nu = 0, ..., 3$ external indices; $\alpha = 1, ..., 133$ and $M, N = 1, ..., 56$ in the adjoint and fundamental of $E_{7(7)}$; $i, j = 1, ..., 8$ in the fundamental of SU(8).

All fields depend on the external x^μ and internal Y^M coordinates, modulo section constraints

Mass Matrices: the computation

Starting point: [H.,M. Godazgar, O.Hohm, H.Nicolai, H.Samtleben, '14]

$$
\begin{aligned}
\mathcal{L}_{\text{ExFT fermi}} \quad = \quad & -i\epsilon^{\mu\nu\rho\sigma}\bar{\psi}^i_\mu\gamma_\nu\mathcal{D}_\rho\psi_{\sigma i} - \tfrac{1}{6}\, e\,\bar{\chi}^{ijk}\gamma^\mu\mathcal{D}_\mu\chi_{ijk} \\
& -4i\,\epsilon^{\mu\nu\rho\sigma}\left(\mathcal{V}^{-1}\right)_{ij}{}^{M}\,\bar{\psi}^i_\mu\gamma_\nu\nabla_M\left(\gamma_\rho\psi^j_\sigma\right) - 4\sqrt{2}\,e\left(\mathcal{V}^{-1}\right)^{ij\,M}\,\bar{\psi}^k_\mu\nabla_M\left(\gamma^\mu\chi_{ijk}\right) \\
& +\tfrac{1}{9}\,e\,\epsilon_{ijklmnpq}\left(\mathcal{V}^{-1}\right)^{ij\,M}\bar{\chi}^{klm}\nabla_M\chi^{npq} + \text{c.c.}
\end{aligned}
$$

where $i,j = 1,\ldots,8$, $\mathcal{V}_M{}^{\underline{A}} = \left(\mathcal{V}_M{}^{[ij]}, \mathcal{V}_{M\,[ij]}\right)$ and

$$
\nabla_M\xi_i = \partial_M\xi_i - \tfrac{1}{4}\omega_M{}^{\alpha\beta}\gamma_{\alpha\beta}\,\xi_i + \tfrac{1}{2}\mathcal{Q}_{M\,i}{}^{j}\xi_j - \tfrac{2}{3}\,\lambda\,\Gamma_{KM}{}^{K}\xi_i\ .
$$

Mass Matrices: the computation

For solutions coming from consistent truncation of a 10 or 11 dimensional theory compactified on a topological sphere we can employ a generalised Scherk-Schwarz ansatz for the vielbeine: [O.Hohm, H.Samtleben, '15]

$$\boldsymbol{e}_\mu{}^\alpha(x,Y) = \rho(Y)^{-1}\, e_\mu{}^\alpha(x)\,, \qquad \boldsymbol{\mathcal{V}}_M{}^{\underline{A}}(x,Y) = U_M{}^{\underline{M}}(Y)\, \mathcal{V}_{\underline{M}}{}^{\underline{A}}(x)\,,$$

where the (Y-dependent) twist matrix $(U^{-1})^M{}_{\underline{N}}$ and the real parameter ρ satisfy the conditions:

$$\mathcal{L}_{\mathcal{U}_{\underline{M}}}\mathcal{U}_{\underline{N}} - F_{\underline{MN}} = X_{\underline{MN}}{}^{\underline{K}}\mathcal{U}_{\underline{K}}\,, \qquad \mathcal{U}_{\underline{M}} \equiv \rho^{-1}(U^{-1})_{\underline{M}}$$

Furthermore

$$\omega_N^{\alpha\beta}(x,Y) = 0\,, \quad \mathcal{Q}^{ik}{}_k{}^j = -\frac{1}{4}\rho(Y)A_1^{ij}(x)\,, \quad \mathcal{Q}^{[ij}{}_k{}^{l]} = -\frac{1}{12}\rho(Y)A_{2\,k}{}^{ijl}(x)\,.$$

Mass Matrices: the computation

For the same kind of solutions, one can show that all KK fluctuations can be expanded in terms of a complete basis of scalar functions on the internal manifold [E.Malek, H.Samtleben, '20]

$$\boldsymbol{\psi}^{i}_{\mu}\,(x,Y) = \rho(Y)^{-\frac{1}{2}}\,\psi^{i\Lambda}_{\mu}(x)\,\mathcal{Y}_{\Lambda}(Y)\,, \qquad \boldsymbol{\chi}^{ijk}\,(x,Y) = \rho(Y)^{\frac{1}{2}}\,\chi^{ijk\Lambda}(x)\,\mathcal{Y}_{\Lambda}(Y)$$

The $\mathcal{Y}_{\Lambda}(Y)$ can be chosen to be the scalar harmonics of the largest symmetry group, i.e. the maximally symmetric point of the gauged SUGRA. For SO(8) and ISO(7) gaugings, they live in the $\oplus_{n=0}^{\infty}[n,0,0,0]$ of SO(8) or the $\oplus_{n=0}^{\infty}[n,0,0]$ of SO(7) , and satisfy

$$\rho^{-1}\,(U^{-1})_{\underline{N}}{}^{M}\partial_{M}\,\mathcal{Y}_{\Lambda} = -(\mathcal{T}_{\underline{N}})_{\Lambda}{}^{\Sigma}\,\mathcal{Y}_{\Sigma}\;,$$

$$[\mathcal{T}_{\underline{M}},\mathcal{T}_{\underline{N}}] = -X_{\underline{MN}}{}^{\underline{P}}\,\mathcal{T}_{\underline{P}}$$

$$\Updownarrow$$

$$[X_{\underline{M}},X_{\underline{N}}] = -X_{\underline{MN}}{}^{\underline{P}}X_{\underline{P}}$$

Mass Matrices: the computation

Final result: [MC, O.Varela, '20]

$$
\begin{aligned}
\mathcal{L}_{\text{KK fermi}} \quad = \quad & -i\epsilon^{\mu\nu\rho\sigma}\,\bar{\psi}_\mu^{i\Lambda}\,\gamma_\nu\,\mathcal{D}_\rho\psi_{\sigma i\Lambda} - \tfrac{1}{6}\,e\,\bar{\chi}^{ijk\Lambda}\,\gamma^\mu\,\mathcal{D}_\mu\chi_{ijk\Lambda} \\
& + e\,A_{1\,i\Lambda,j\Sigma}\;\bar{\psi}_\mu^{i\Lambda}\,\gamma^{\mu\nu}\,\psi_\nu^{j\Sigma} + \frac{\sqrt{2}}{6}\,e\,A_{2\,i\Lambda}{}^{jkl\Sigma}\;\bar{\psi}_\mu^{i\Lambda}\,\gamma^\mu\,\chi_{jkl\Sigma} \\
& + \sqrt{2}\,e\,A_3^{ijk\Lambda,lmn\Sigma}\;\bar{\chi}_{ijk\Lambda}\,\chi_{lmn\Sigma} \;+\; \text{c.c.}\,,
\end{aligned}
$$

with

$$
\begin{aligned}
& A_{1\,i\Lambda,j\Sigma} \equiv A_{1ij}\,\delta_{\Lambda\Sigma} - 8\,(\mathcal{V}^{-1})_{ij}{}^{\underline{M}}(\mathcal{T}_{\underline{M}})_{\Lambda\Sigma}\;, \\
& A_{2\,i\Lambda}{}^{jkl\Sigma} \equiv A_{2i}{}^{jkl}\,\delta_\Lambda^\Sigma - 24\,\delta_i^{[j}\,(\mathcal{V}^{-1})^{kl]\underline{N}}(\mathcal{T}_{\underline{N}})_\Lambda{}^\Sigma\;, \\
& A_3{}^{ijk\Lambda,lmn\Sigma} \equiv A_3{}^{ijk,lmn}\,\delta^{\Lambda\Sigma} + \frac{\sqrt{2}}{18}\,\epsilon^{ijklmnpq}\,(\mathcal{V}^{-1})_{pq}{}^{\underline{N}}(\mathcal{T}_{\underline{N}})^{\Lambda\Sigma}\;.
\end{aligned}
$$

Mass Matrices: an $\mathcal{N}=8$, $D=4$ comparison

For the D=4, $\mathcal{N}=8$ gauged supergravities the fermion mass matrices are specific components of the so called *T-tensor*

$$T_{\underline{AB}}{}^{\underline{C}} = (\mathcal{V}^{-1})_{\underline{A}}{}^{\underline{M}} (\mathcal{V}^{-1})_{\underline{B}}{}^{\underline{N}} X_{\underline{MN}}{}^{\underline{P}} \mathcal{V}_{\underline{P}}{}^{\underline{C}} \, .$$

$$A_1^{ij} = \tfrac{4}{21} T^{ikjl}{}_{kl} \quad , \quad A_{2h}{}^{ijk} = 2\, T_{mh}{}^{mijk} \, , \quad A_{3\, ijk,lmn} = \frac{\sqrt{2}}{108} \epsilon_{ijkpq[rlm} A_{2\, n]}{}^{pqr} \, ,$$

They satisfy a number of properties

$$A_1^{ij} = A_1^{(ij)} \, , \qquad A_{2h}{}^{ijk} = A_{2h}{}^{[ijk]} \, , \quad A_{2k}{}^{ijk} = 0 \, ,$$
$$A_{3\, ijk,lmn} = A_{3\, lmn,ijk}$$

and

$$\delta\psi_\mu^i = 2\, A_1{}^{ij} \gamma_\mu \epsilon_j + \ldots \, , \qquad \delta\chi^{ijk} = -2\sqrt{2}\, A_{2\, h}{}^{ijk} \epsilon^h + \ldots$$

Mass Matrices: an $\mathcal{N}=8$, $D=4$ comparison

Analogously, for the Kaluza-Klein fluctuations

$$A_{1\,i\Lambda,j\Sigma} = A_{1\,j\Sigma,i\Lambda}\ , \qquad A_{2\,h\Lambda}{}^{ijk\Sigma} = A_{2\,h\Lambda}{}^{[ijk]\Sigma}\ , \qquad A_{2\,k\Lambda}{}^{ijk\Lambda} = 0\ ,$$

$$A_{3\,ijk\Lambda,lmn\Sigma} = A_{3\,lmn\Sigma,ijk\Lambda}\ , \qquad A_{3\,ijk\Lambda,lmn\Sigma} = \frac{\sqrt{2}}{108}\,\epsilon_{ijkpq[rlm}\,A_{2\,n]\Sigma}{}^{pqr\Omega}\,\delta_{\Omega\Lambda}\ ,$$

and

$$\delta\psi_\mu^{i\Lambda} = 2\,A_1{}^{i\Lambda,j\Sigma}\,\gamma_\mu\epsilon_{j\Sigma} + \ldots\ , \qquad \delta\chi^{ijk\Lambda} = -2\sqrt{2}\,A_{2\,h\Sigma}{}^{ijk\Lambda}\,\epsilon^{h\Sigma} + \ldots$$

What have we done?

From ExFT we have built the Fermion Mass matrices for the full KK towers.

- Reminder I : possible for AdS_4 solutions of $D = 11$ and type IIA supergravities that uplift consistently on topologically S^7, S^6 from gauged supergravities in lower dimensions.
- Reminder II : For concreteness, we shall focus on $\mathcal{N} = 1$, at least SU(3) preserving solutions in the SO(8) and ISO(7) gauging: G_2 in SO(8) , G_2 and SU(3) in ISO(7).

What we are going to do

- We can use the mass matrices in order to compute the spectra of gravitini and spin 1/2 fermions for the solutions specified above.
- Combining the fermionic results with the computed spectra for fields of other spins the complete spectra of these solutions is provided.
- They come organised in $\mathrm{OSp}(4|1) \times G$ supermultiplets, where G is the specific internal symmetry group.

Applications: $\mathcal{N}=1$, G_2 solution in SO(8) gauging

$n=0$

$[0,0]$ MGRAV $\left(\frac{5}{2}\right)$ CHIRAL $\left(1+\sqrt{6}\right)$	$[0,1]$ MVEC $\left(\frac{3}{2}\right)$
$[1,0]$ GINO $\left(1+\frac{\sqrt{6}}{2}\right)$	
$[2,0]$ CHIRAL $\left(1+\frac{\sqrt{6}}{6}\right)$	

$n=1$

$[0,0]$ GRAV $\left(1+\frac{\sqrt{106}}{4}\right)$ CHIRAL $\left(1+\frac{\sqrt{166}}{4}\right)$	$[0,1]$ GINO $\left(1+\frac{3}{4}\sqrt{6}\right)$ VEC $\left(1+\frac{\sqrt{74}}{4}\right)$
$[1,0]$ GRAV $\left(1+\frac{\sqrt{66}}{4}\right)$ VEC $\left(1+\frac{\sqrt{114}}{4}\right)$ CHIRAL $\left(1+\frac{3}{2}\sqrt{\frac{7}{2}}\right)$ GINO $\left(1+\frac{\sqrt{94}}{4}\right)$	$[1,1]$ VEC $\left(1+\frac{\sqrt{14}}{4}\right)$
$[2,0]$ GINO $\left(1+\frac{1}{2}\sqrt{\frac{61}{6}}\right)$ VEC $\left(1+\sqrt{\frac{91}{24}}\right)$ CHIRAL $\left(1+\frac{\sqrt{654}}{12}\right)$	
$[3,0]$ CHIRAL $\left(1+\sqrt{\frac{3}{8}}\right)$	

$$\text{(M)GRAV} \;:\; E_0 = 1+\sqrt{\frac{9}{4}+\frac{5}{8}n(n+6)-\frac{5}{4}\,\mathcal{C}_2(p,q)}\,,$$

$$\text{GINO} \;:\; E_0 = 1+\sqrt{4+\frac{5}{8}n(n+6)-\frac{5}{4}\,\mathcal{C}_2(p,q)}\;,$$

$$\text{(M)VEC} \;:\; E_0 = 1+\sqrt{\frac{21}{4}+\frac{5}{8}n(n+6)-\frac{5}{4}\,\mathcal{C}_2(p,q)}\;,$$

$$\text{CHIRAL} \;:\; E_0 = 1+\sqrt{6+\frac{5}{8}n(n+6)-\frac{5}{4}\,\mathcal{C}_2(p,q)}\;.$$

$$\mathcal{C}_2(p,q) \equiv \frac{1}{3}\,p(p+5)+q(q+3)+\,pq$$

Applications: $\mathcal{N}=1$, G_2 solution in ISO(7) gauging

$n=0$

$[0,0]$ MGRAV $\left(\frac{5}{2}\right)$ CHIRAL $\left(1+\sqrt{6}\right)$	$[0,1]$ MVEC $\left(\frac{3}{2}\right)$
$[1,0]$ GINO $\left(1+\frac{\sqrt{6}}{2}\right)$	
$[2,0]$ CHIRAL $\left(1+\frac{\sqrt{6}}{6}\right)$	

$n=1$

$[0,0]$ GINO (4)	$[0,1]$ GINO (3) CHIRAL $\left(1+\sqrt{6}\right)$
$[1,0]$ GRAV $\left(1+\frac{\sqrt{19}}{2}\right)$ VEC $\left(1+\frac{\sqrt{31}}{2}\right)$ CHIRAL $\left(1+\frac{\sqrt{34}}{2}\right)$	$[1,1]$ VEC $\left(1+\sqrt{\frac{3}{2}}\right)$
$[2,0]$ GINO $\left(1+\sqrt{\frac{19}{6}}\right)$ VEC $\left(1+\frac{\sqrt{159}}{6}\right)$	
$[3,0]$ CHIRAL (2)	

$$\text{(M)GRAV} \quad : \quad E_0 = 1+\sqrt{\tfrac{9}{4}+\tfrac{5}{12}n(n+5)}\,,$$

$$\text{GINO} \quad : \quad E_0 = 1+\sqrt{4+\tfrac{5}{6}n(n+5)-\tfrac{5}{4}\,\mathcal{C}_2(p,q)}\,,$$

$$\text{(M)VEC} \quad : \quad E_0 = 1+\sqrt{\tfrac{21}{4}+\tfrac{5}{6}n(n+5)-\tfrac{5}{4}\,\mathcal{C}_2(p,q)}\,,$$

$$\text{CHIRAL} \quad : \quad E_0 = 1+\sqrt{6+\tfrac{5}{6}n(n+5)-\tfrac{5}{4}\,\mathcal{C}_2(p,q)}\,.$$

$$\mathcal{C}_2(p,q) \equiv \tfrac{1}{3}\,p(p+5)+q(q+3)+\,pq$$

Applications:
$\mathcal{N}=1$, SU(3) solution in ISO(7) gauging

$n=0$

[0, 0] MGRAV $\left(\frac{5}{2}\right)$ GINO (3) CHIRAL $2\times\left(1+\sqrt{6}\right)$	[0, 1] conj. to [1, 0]	[0, 2] conj. to [2, 0]
[1, 0] GINO $\left(\frac{7}{3}\right)$ VEC $\left(1+\frac{\sqrt{109}}{6}\right)$	[1, 1] MVEC $\left(\frac{3}{2}\right)$ CHIRAL (2)	
[2, 0] CHIRAL $\left(\frac{5}{3}\right)$		

$n=1$

[0, 0] GRAV $\left(1+\frac{\sqrt{29}}{2}\right)$ GINO $2\times(4)$ VEC $2\times\left(1+\frac{\sqrt{41}}{2}\right)$ CHIRAL $2\times\left(1+\sqrt{11}\right)$	[0, 1] conj. to [1, 0]	[0, 2] conj. to [2, 0]	[0, 3] conj. to [3, 0]
[1, 0] GRAV $\left(1+\frac{\sqrt{181}}{6}\right)$ GINO $2\times\left(1+\frac{\sqrt{61}}{3}\right)$ VEC $3\times\left(\frac{23}{6}\right)$ CHIRAL $3\times\left(1+\frac{\sqrt{79}}{3}\right)$	[1, 1] GINO $2\times(3)$ VEC $3\times\left(1+\frac{\sqrt{21}}{2}\right)$ CHIRAL $2\times\left(1+\sqrt{6}\right)$	[1, 2] conj. to [2, 1]	
[2, 0] GINO $\left(1+\frac{\sqrt{31}}{3}\right)$ VEC $2\times\left(\frac{19}{6}\right)$ CHIRAL $\left(\frac{10}{3}\right)$	[2, 1] VEC $\left(\frac{13}{6}\right)$ CHIRAL $\left(1+\frac{\sqrt{19}}{3}\right)$		
[3, 0] CHIRAL (2)			

$$\begin{aligned}
\text{(M)GRAV} \quad &: \quad E_0 = 1+\sqrt{\tfrac{9}{4}+\tfrac{5}{6}n(n+5)-\tfrac{5}{3}\,\mathcal{C}_2(p,q)}\;, \\
\text{GINO} \quad &: \quad E_0 = 1+\sqrt{4+\tfrac{5}{6}n(n+5)-\tfrac{5}{3}\,\mathcal{C}_2(p,q)}\;, \\
\text{(M)VEC} \quad &: \quad E_0 = 1+\sqrt{\tfrac{21}{4}+\tfrac{5}{6}n(n+5)-\tfrac{5}{3}\,\mathcal{C}_2(p,q)}\;, \\
\text{CHIRAL} \quad &: \quad E_0 = 1+\sqrt{6+\tfrac{5}{6}n(n+5)-\tfrac{5}{3}\,\mathcal{C}_2(p,q)}\;.
\end{aligned}$$

$$\mathcal{C}_2(p,q) \equiv \tfrac{1}{3}\left[p(p+3)+q(q+3)+pq\right]$$

Applications:
A general formula for $\mathrm{OSp}(4|\mathcal{N}) \times G$ supermultiplets

A more general formula can be derived for the conformal dimensions of the supermultiplets [MC,O.Varela, '20]

$$E_0 = s_0^{(2)} - \tfrac{1}{2} + \sqrt{\tfrac{9}{4} + s_0^{(2)}(s_0^{(2)} + 1) - s_0(s_0 + 1) + \alpha\, n(n + d - 1) + \mathcal{Q}_2}\ ,$$

where

$$s_0^{(2)} = \begin{cases} \tfrac{1}{2}\,(4 - \mathcal{N}) & , \quad \text{if } 1 \leq \mathcal{N} \leq 4 \\ 0 & , \quad \text{if } 4 \leq \mathcal{N} \leq 8 \end{cases}$$

s_0 is the spin of the superconformal primary of the multiplet,

$\mathcal{Q}_2$ is a homogeneous quadratic polynomial in the integer Dynkin labels of G .

Applications: A general formula for $\mathrm{OSp}(4|\mathcal{N}) \times G$ supermultiplets

$$E_0 = s_0^{(2)} - \tfrac{1}{2} + \sqrt{\tfrac{9}{4} + s_0^{(2)}(s_0^{(2)}+1) - s_0(s_0+1) + \alpha\, n(n+d-1) + \mathcal{Q}_2}$$

$$\mathcal{N}=1: \quad E_0 = 1 + \sqrt{6 - s_0(s_0+1) + \alpha\, n(n+d-1) - \beta\, \mathcal{C}_2(p,q)}$$

$$\mathcal{N}=2\,,\mathrm{SU}(3)\times\mathrm{U}(1): \quad E_0 = \tfrac{1}{2} + \sqrt{\tfrac{17}{4} - s_0(s_0+1) + \alpha\, n(n+d-1) - \tfrac{4}{3}\,\mathcal{C}_2(p,q) + \tfrac{1}{2}\, y_0^2}$$

[E.Malek, H.Samtleben,'20]
[O.Varela, '20]

$$\mathcal{N}=3\,,\mathrm{SO}(3)_{\mathcal{R}}\times\mathrm{SO}(3)_{\mathcal{F}}: \quad E_0 = \sqrt{3 - s_0(s_0+1) + \tfrac{1}{2}\, n(n+d-1) + \tfrac{1}{2}\, j(j+1) - \tfrac{3}{2}\, h(h+1)}$$

[O.Varela, '20]

$$\mathcal{N}=8: \; E_0 = -\frac{1}{2} + \sqrt{\frac{9}{4} + \frac{1}{4} n(n+d-1)}$$

[F.Englert, H.Nicolai, '83]

Applications:
$\mathcal{N}=0$ solutions in SO(8), ISO(7) gauging

$D=11$ sol.	s	L^2M^2
$\mathcal{N}=0$, SO$(7)_v$	2	$\frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$\frac{3}{2}$	$\frac{9}{2} + \frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	1	$6 + \frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$\frac{1}{2}$	$\frac{15}{2} + \frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$0^{(*)}$	$6 + \frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
$\mathcal{N}=0$, SO$(7)_c$	2	$\frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$\frac{3}{2}$	$\frac{9}{2} + \frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	1	$6 + \frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$\frac{1}{2}$	$\frac{15}{2} + \frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$0^{(*)}$	$6 + \frac{3}{4}n(n+6) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
$\mathcal{N}=0$, SU$(4)_c$	2	$\frac{3}{4}n(n+6) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
	$\frac{3}{2}$	$\frac{9}{2} + \frac{3}{4}n(n+6) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
	1	$6 + \frac{3}{4}n(n+6) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
	$\frac{1}{2}$	$\frac{15}{2} + \frac{3}{4}n(n+6) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
	$0^{(*)}$	$6 + \frac{3}{4}n(n+6) - \frac{3}{2}\mathcal{C}_2(p,q,r)$

IIA sol.	s	L^2M^2
$\mathcal{N}=0$, SO(7)	2	$n(n+5) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$\frac{3}{2}$	$\frac{9}{2} + n(n+5) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	1	$6 + n(n+5) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$\frac{1}{2}$	$\frac{15}{2} + n(n+5) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
	$0^{(*)}$	$6 + n(n+5) - \frac{6}{5}\mathcal{C}_2(p,q,r)$
$\mathcal{N}=0$, SO(6)	2	$n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
	$\frac{3}{2}$	$\frac{9}{2} + n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
	1	$6 + n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
	$\frac{1}{2}$	$\frac{15}{2} + n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
	$0^{(*)}$	$6 + n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q,r)$
$\mathcal{N}=0$, G_2	2	$n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q)$
	$\frac{3}{2}$	$\frac{9}{2} + n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q)$
	1	$6 + n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q)$
	$\frac{1}{2}$	$\frac{15}{2} + n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q)$
	0	$6 + n(n+5) - \frac{3}{2}\mathcal{C}_2(p,q)$

Previous partial results:

[Y.Pang, J.Rong, O.Varela, '18]
[K.Dimmitt, G.Larios, P.Ntokos, O.Varela, '20]
[A.Guarino, E.Malek, H.Samtleben, '20]

Conclusion and future directions

- We have shown how to derive Fermion Mass Matrices from ExFT for solutions of M-theory and type IIA string theory with topologically spherical internal spaces.
- We provided the complete spectrum for $\mathcal{N} = 1$ solutions in the SO(8) and ISO(7) gauging preserving at least SU(3) symmetry.
- We commented on the non-supersymmetric solutions in the same class. Our results allowed us to conjecture the spectra for the relative scalars.

- Is there a more general structure underlying the generalised fermion shifts?
- Do other kind of solutions follow the same pattern?