1/4 BPS AdS_3/CFT_2

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Introduction & Motivation

- There is a large effort in the classification of Type II supergravity or M-theory backgrounds with AdS_{d+1} factors.

- The $\mathcal{N} = (0,4)$ AdS_3 solutions remained however largely unexplored.

  - There are certain AdS_3 solutions, but they involve only NS-NS fields.
    (Maldacena, Oouguri; Maldacena, Oouguri, Son)

  - Canonical example: Near horizon of D1-D5. $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{CY}_2$ geometry realising (4,4) superconformal symmetry.

One avatar where the CFT side is rather more developed than the gravity side is $\mathrm{AdS}_3/\mathrm{CFT}_2$

Introduction & Motivation

- 2d CFTs play a prominent role in string theory and provide the best arena to test the AdS/CFT correspondence.

- The conformal group in 2d is infinite dimensional and this makes two dimensional CFTs much more tractable than their higher dimensional counterparts.

- More general (0,4) 2d CFTs such as the ones described by the quivers constructed in Haghighat, Lockhart, Vafa and collab.; Hanany, Okazaki are however still lacking a holographic description, one of the motivations of this work will be to fill this gap.

We will fill this gap and provide a classification of AdS_3 solutions to massive IIA supergravity with (small) (0,4) SUSY and a concrete proposal to their 2d dual CFTs.

From these classification, through T-duality, we obtain new AdS_2 geometries with $\mathcal{N} = 4$ SUSY in type IIB and propose a dual SCQM.

TALK OUTLINE

- AdS_3 solutions to massive IIA with small (0,4) SUSY
- Our 2d CFTs proposal
  - An illustrative example
- $\mathrm{AdS}_2 \times S^2 \times \mathrm{CY}_2 \times S^1 \times I$ solutions to supergravity IIB with $\mathcal{N} = 4$ SUSY
  - SCQM proposal
- Conclusions

The geometry

Our approach to finding AdS_3 solutions with small N=(0,4) superconformal symmetry is to construct spinors which manifestly realize the bosonic sub-algebra

$$\mathfrak{sl}(2) \oplus \mathfrak{su}(2).$$

We shall seek solutions with metric decomposing as

$$ds^2 = e^{2A} ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{M}_7),$$

$$ds^2(\mathrm{M}_7) = e^{2C} ds^2(\mathrm{S}^2) + ds^2(\mathrm{M}_5).$$

To guarantee small N=(0,4) symmetry we must solve the supersymmetric constraints.

Our strategy

- Construct spinors that ensure consistency with the bosonic sub-algebra of small N=(0,4) superconformal symmetry.
- Exploit an existing N=1 AdS_3 classification to obtain sufficient conditions on the geometry and fluxes for a solution with small N=(0,4) in IIA to exist. (Dibitetto, Lo Monaco, Passias, Petri, Tomasiello)
- Study the classes consistent with our assumptions.

There are 2 classes of solutions

$$\mathrm{M}_4 = \mathrm{CY}_2$$

$$\mathrm{M}_4 = \text{Kähler}$$

The geometry

We focus in the Class I solutions,

$$ds^2 = \frac{u}{\sqrt{h_4 h_8}}\left(ds^2(\text{AdS}_3) + \frac{h_8 h_4}{4h_8 h_4 + (u')^2} ds^2(\text{S}^2)\right) + \sqrt{\frac{h_4}{h_8}} ds^2(\text{CY}_2) + \frac{\sqrt{h_4 h_8}}{u} d\rho^2,$$

$$e^{-\Phi} = \frac{h_8^{\frac{3}{4}}}{2h_4^{\frac{1}{4}}\sqrt{u}}\sqrt{4h_8 h_4 + (u')^2}, \qquad B_2 = \frac{1}{2}\left(-\rho + 2\pi k + \frac{uu'}{4h_4 h_8 + (u')^2}\right)\text{vol}(\text{S}^2),$$ NS5

$$\hat{F}_0 = h_8',$$ D8

$$\hat{F}_2 = -\frac{1}{2}\left(h_8 - h_8'(\rho - 2\pi k)\right)\text{vol}(\text{S}^2),$$ D6

$$\hat{F}_4 = h_4' \text{vol}(\text{CY}_2),$$ D4

$$\hat{F}_6 = \left(h_4 - h_4'(\rho - 2\pi k)\right)\text{vol}(\text{CY}_2) \wedge \text{vol}(\text{S}_2),$$ D2

The background is a SUSY solution of the Massive IIA equations if the functions satisfy,

$$u'' = 0, \quad h_8'' = 0, \quad h_4'' = 0.$$

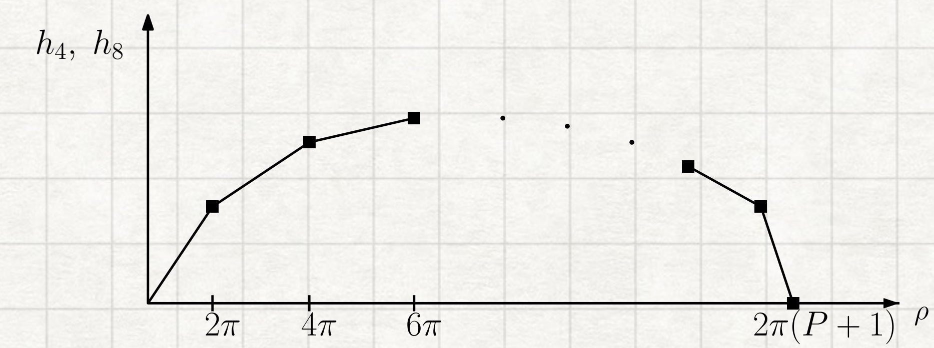
The geometry

The three functions are thus linear and piecewise continuous,

$$h_4(\rho) = \begin{cases} \frac{\beta_0}{2\pi}\rho & 0 \le \rho \le 2\pi \\ \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k) & 2\pi k \le \rho \le 2\pi(k+1) \\ \alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2\pi P) & 2\pi P \le \rho \le 2\pi(P+1)\,. \end{cases} , \qquad h_8(\rho) = \begin{cases} \frac{\nu_0}{2\pi}\rho & 0 \le \rho \le 2\pi \\ \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k) & 2\pi k \le \rho \le 2\pi(k+1) \\ \mu_P - \frac{\mu_P}{2\pi}(\rho - 2\pi P) & 2\pi P \le \rho \le 2\pi(P+1)\,. \end{cases} , \qquad u = \frac{b_0}{2\pi}\rho.$$

\+ the functions vanish at $\rho = 0$ and $\rho = 2\pi(P+1)$

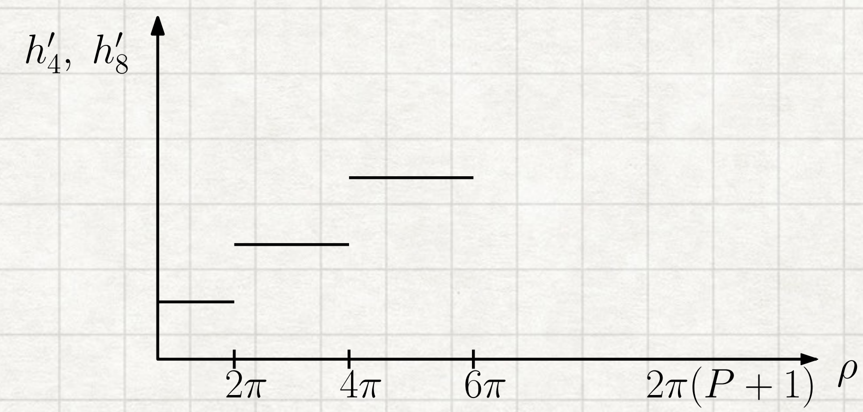
continuity of the NS sector



continuity of the functions h_4, h_8

↓

their derivatives present jumps



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discontinuities in the RR sector

D4 and D8 sources are located

The geometry

Page Charges

$$Q_{D8} = \nu_{k-1} - \nu_k \qquad Q_{D4} = \beta_{k-1} - \beta_k$$

→ D4, D8 branes will play the role of flavour branes

$$Q_{D6} = \mu_k \qquad Q_{D2} = \alpha_k$$

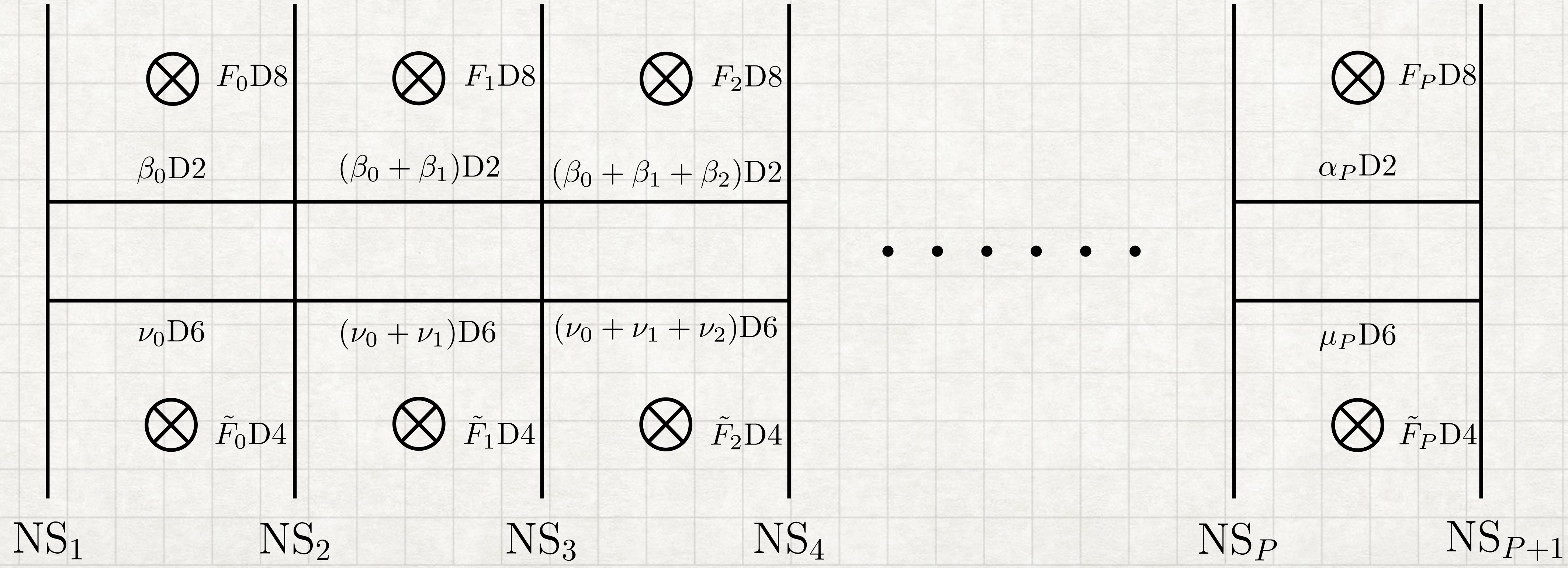
→ D2, D6 branes will play the role of colour branes

$$h_4^{(k)} = \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k), \qquad h_8^{(k)} = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k),$$

Supported by the analysis of the **Bianchi identities**

Hanany-Witten brane set-up

(Hanany, Witten'97)



The geometry

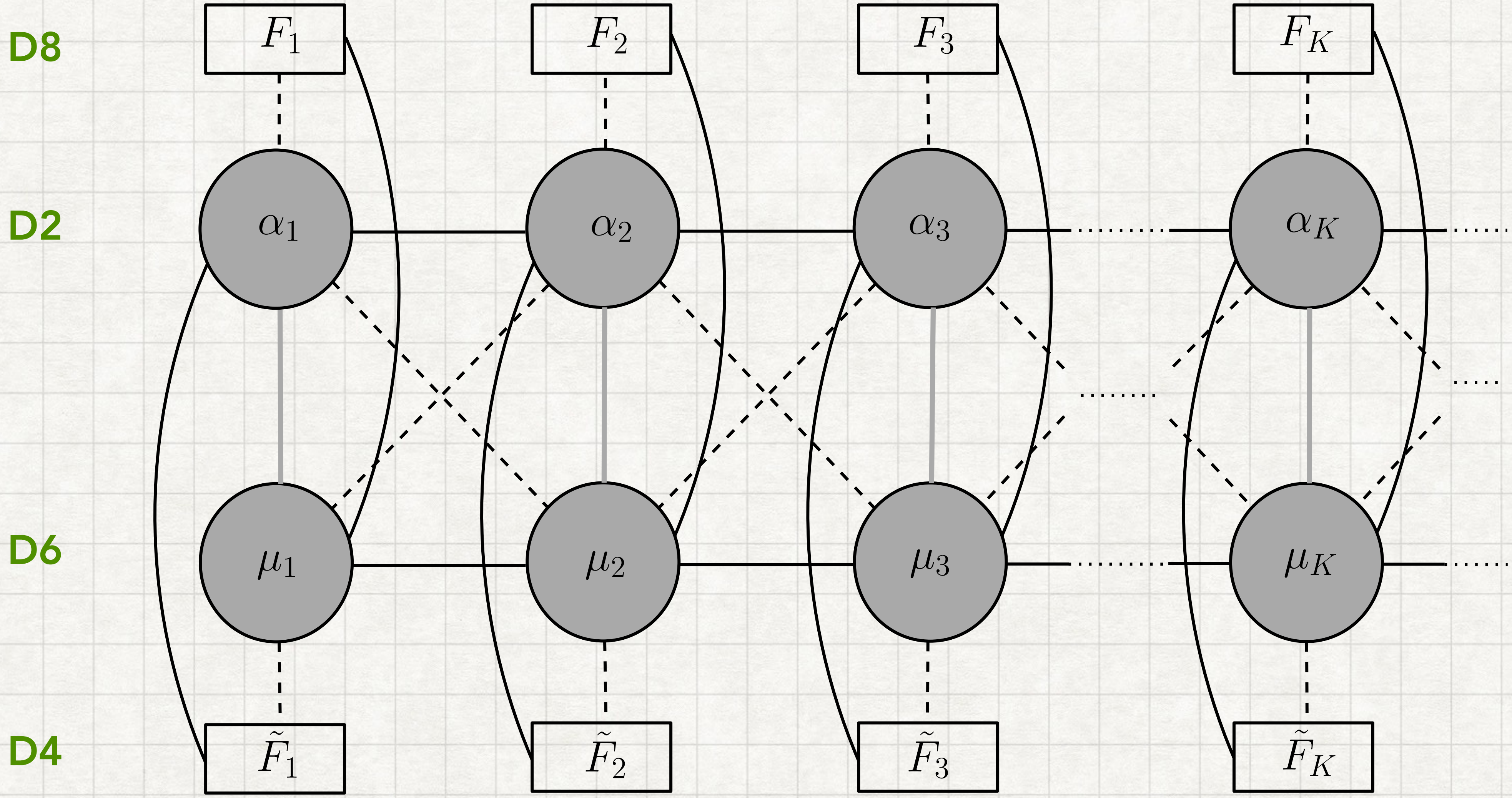
We compute the Holographic Central Charge where the idea is to compute the volume of the internal space, thus for our background we get, (Freedman, Gubser, Pilch, Wagner; Krauss, Larsen; Klebanov, Kutasov, Murugan; Macpherson, Nunez, et.al)

$$c_{hol} = \frac{3\pi}{2G_N} \mathrm{Vol}(\mathrm{CY}_2) \int_0^{2\pi(P+1)} h_4 h_8 d\rho.$$

Observables calculated using the geometry are trustable as long as the numbers ν_k, β_k and P are large.

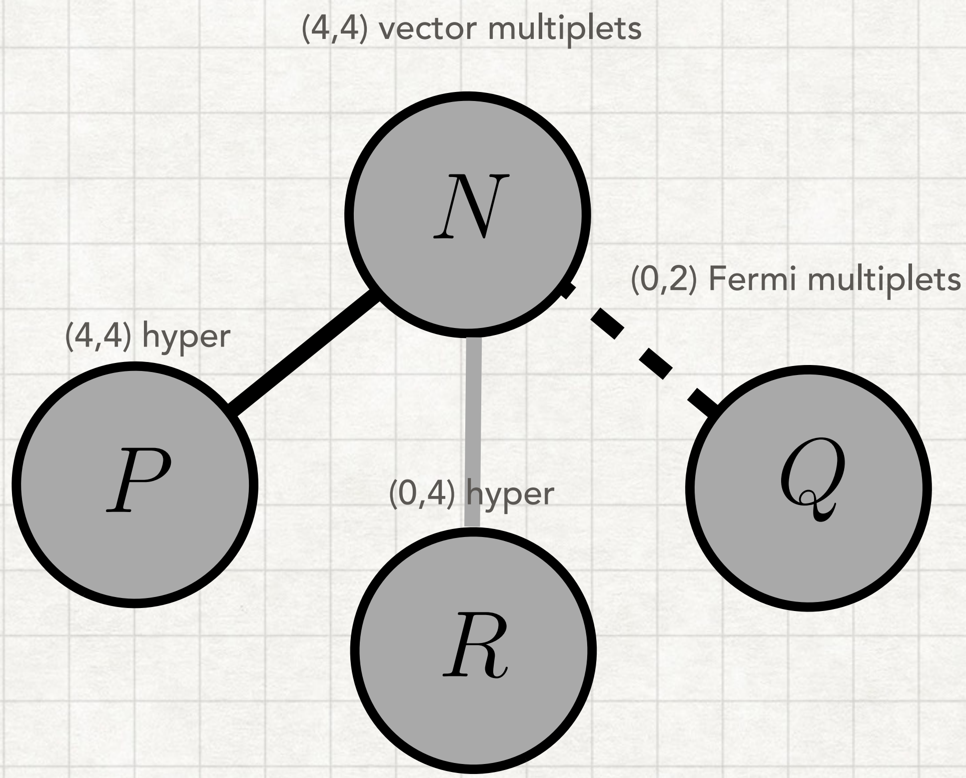
The 2d SCFTs

In usual cases, the Hanany-Witten brane set-ups have associated linear quivers. Our proposal relates our backgrounds with $\mathcal{N} = (0,4)$ SUSY quiver field theories.



The 2d SCFTs

The field theory is described in terms of (0,2) multiplets that combine into (0,4) and (4,4) multiplets. It is obtained by assembling the building block:



Using the contribution to the gauge anomaly coming from each multiplet, we find that for each SU(N) gauge group the cancellation of the anomaly imposes,

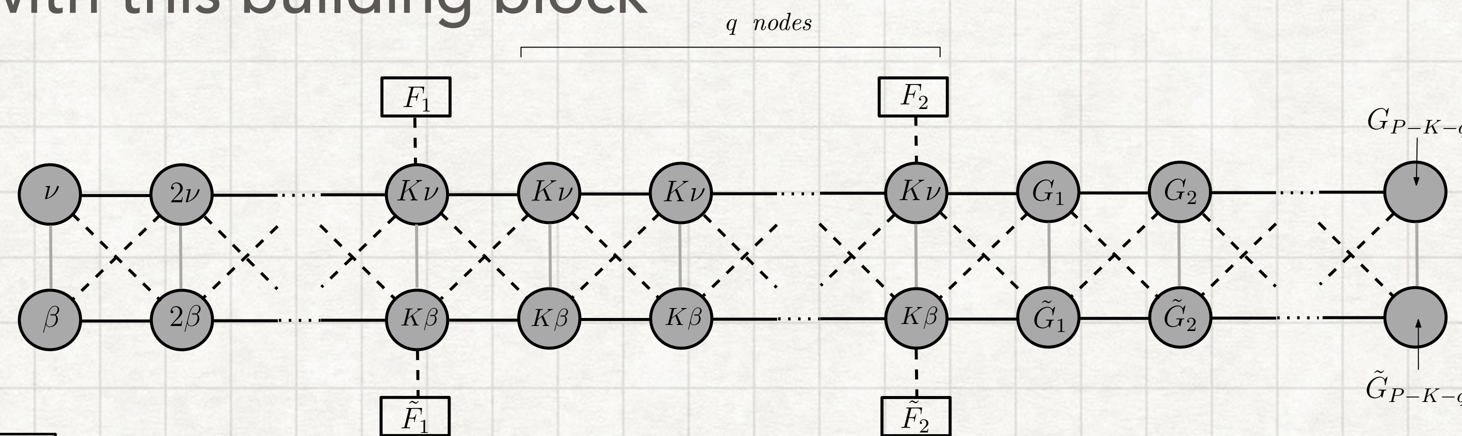
$$2R = Q$$

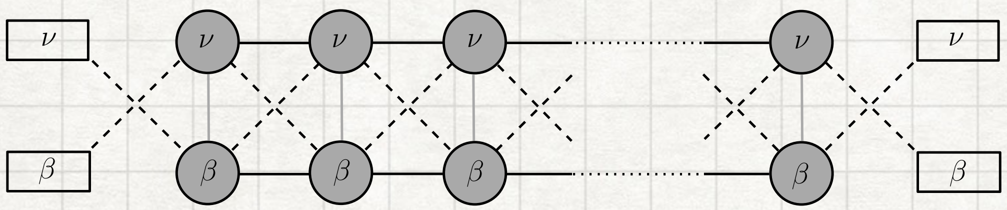
We proposed that our quivers become conformal in the IR and then the central charge of the quiver and R-Symmetry anomaly get related by the (0,4) superconformal algebra, (Putrov, Song, Yan 2016)

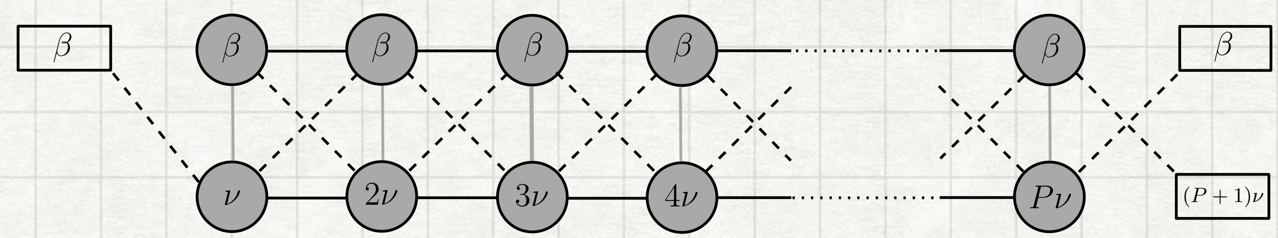
$$c = 6(n_{hyp} - n_{vec})$$

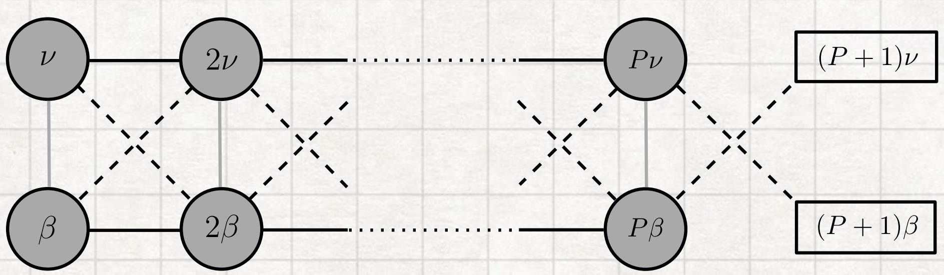
The 2d SCFTs

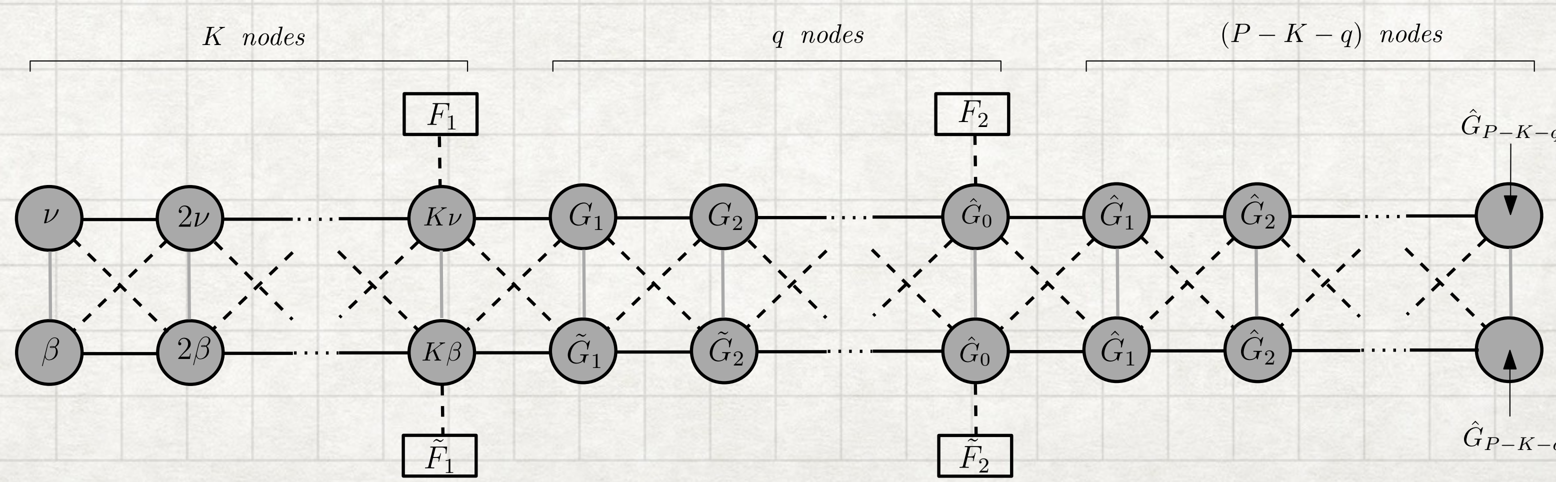
We construct several examples with this building block





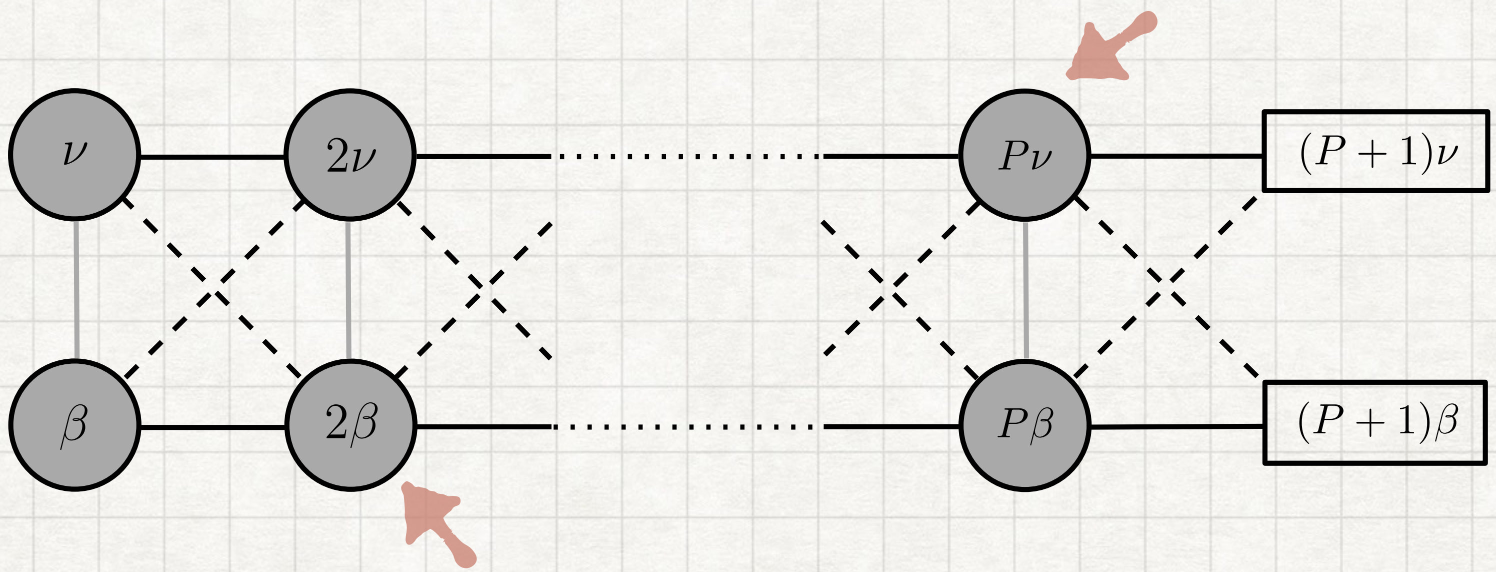


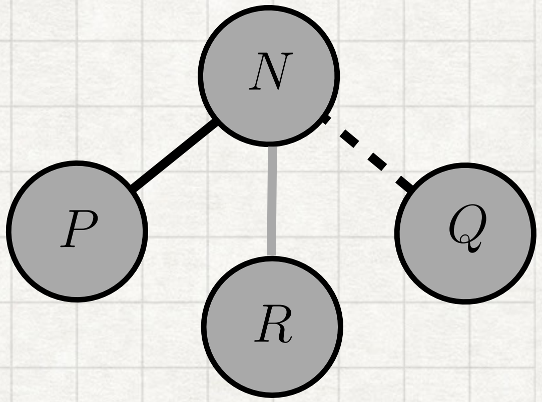




The 2d SCFTs

An illustrative example





Gauge anomaly $2R = Q$

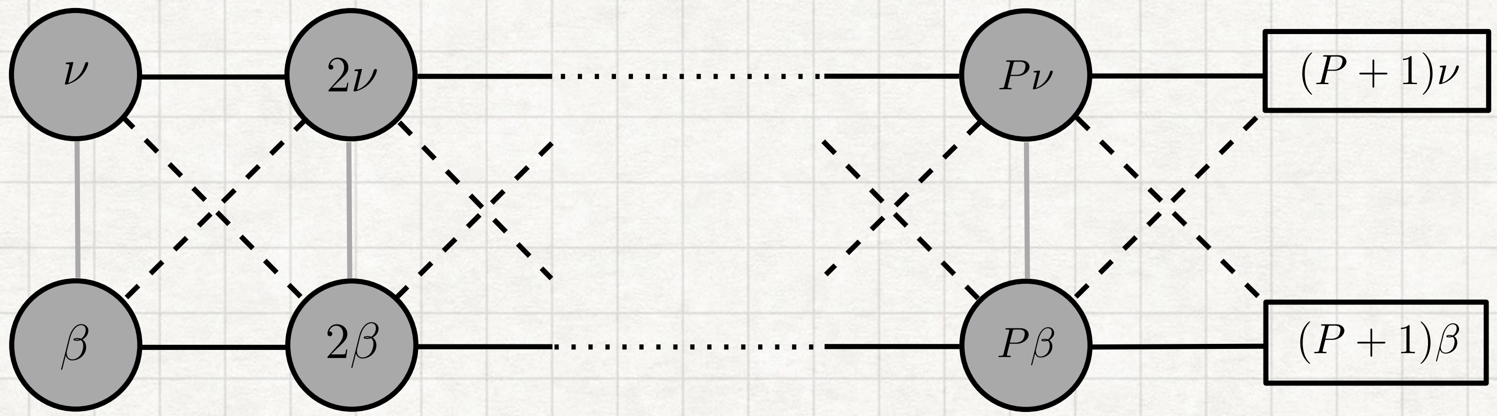
$$R = 2\nu$$
$$Q = \nu + 3\nu$$

$$R = P\beta$$
$$Q = (P-1)\beta + (P+1)\beta$$

The anomalies of each of the gauge groups vanish

The 2d SCFTs

An illustrative example



Central Charge

$$n_{vec} = \sum_{j=1}^{P} (j^2(\beta^2 + \nu^2) - 2)$$

$$n_{hyp} = \sum_{j=1}^{P} j^2\nu\beta + \sum_{j=1}^{P-1} j(j+1)(\nu^2 + \beta^2)$$

$$c = 6(n_{hyp} - n_{vec})$$

$$c = 6\nu\beta(\frac{P^3}{3} + \frac{P^2}{2} + \frac{P}{6}) - 3(\nu^2 + \beta^2)(P^2 + P) + 12P$$

$$\sim 2\nu\beta P^3$$

Holographic Central Charge

The holographic description of this system

$$h_4(\rho) = \begin{cases} \frac{\beta}{2\pi}\rho & 0 \leq \rho \leq 2\pi P \\ \frac{\beta P}{2\pi}(2\pi(P+1) - \rho) & 2\pi P \leq \rho \leq 2\pi(P+1) \end{cases}$$

$$h_8(\rho) = \begin{cases} \frac{\nu}{2\pi}\rho & 0 \leq \rho \leq 2\pi P \\ \frac{\nu P}{2\pi}(2\pi(P+1) - \rho) & 2\pi P \leq \rho \leq 2\pi(P+1) \end{cases}$$

$$c_{hol} = \frac{3}{\pi}\int_0^{2\pi(P+1)} h_4 h_8 d\rho \quad \sim 2\nu\beta P^3$$

exactly as the field theory!

AdS_2/SCQM

The geometry

$$\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{M}_4 \times \mathrm{I} \xrightarrow{\text{T-duality}} \mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{M}_4 \times \mathrm{I} \times \mathrm{S}^1$$

$$\mathrm{d}s^2_{\mathrm{AdS}_3} = \frac{1}{4}\left[\left(\mathrm{d}\tilde{\psi} + \eta\right)^2 + \mathrm{d}s^2_{\mathrm{AdS}_2}\right]$$

$$\mathrm{d}s^2 = \frac{u}{\sqrt{\hat{h}_4 h_8}}\left(\frac{1}{4}\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{\hat{h}_4 h_8}{4\hat{h}_4 h_8 + (u')^2}\mathrm{d}s^2_{\mathrm{S}^2}\right) + \sqrt{\frac{\hat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\hat{h}_4 h_8}}{u}(\mathrm{d}\rho^2 + \mathrm{d}\psi^2)\,,$$

$$e^{-2\Phi} = \frac{h_8}{4\hat{h}_4}\left(4\hat{h}_4 h_8 + (u')^2\right), \qquad H_3 = \frac{1}{2}\mathrm{d}\left(-\rho + \frac{uu'}{4\hat{h}_4 h_8 + (u')^2}\right)\wedge \mathrm{vol}_{\mathrm{S}^2} + \frac{1}{2}\mathrm{vol}_{\mathrm{AdS}_2}\wedge \mathrm{d}\psi,$$

$$\hat{F}_1 = h_8'\,\mathrm{d}\psi\,, \qquad \hat{F}_3 = \frac{1}{2}\left(h_8'(\rho - 2\pi k) - h_8\right)\mathrm{vol}_{\mathrm{S}^2}\wedge \mathrm{d}\psi + \frac{1}{4}\left(\frac{u'(\hat{h}_4 u' - u\hat{h}_4')}{2\hat{h}_4^2} + 2h_8\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge \mathrm{d}\rho,$$

$$\hat{F}_5 = \frac{1}{16}\left(\frac{\left(u - (\rho - 2\pi k)u'\right)(u\hat{h}_4' - \hat{h}_4 u')}{\hat{h}_4^2} + 4(\rho - 2\pi k)h_8\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge \mathrm{vol}_{\mathrm{S}^2}\wedge \mathrm{d}\rho - \hat{h}_4'\mathrm{vol}_{\mathrm{CY}_2}\wedge \mathrm{d}\psi.$$

- The background is a SUSY solution of supergravity IIB if the functions satisfy, $u'' = 0, \quad h_8'' = 0, \quad h_4'' = 0$.
- The three functions are lineal and piecewise, $h_4^{(k)} = \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k),\; h_8^{(k)} = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k),\; u = \frac{b_0}{2\pi}\rho$

The geometry

T-duality

$AdS_3 \times S^2 \times M_4 \times I \longrightarrow AdS_2 \times S^2 \times M_4 \times I \times S^1$

$\hat{F}_0$	**D8**	$Q_{D8} = \nu_{k-1} - \nu_k$	$\hat{F}_1$	**D7**	$Q_{D7} = \nu_{k-1} - \nu_k$
$\hat{F}_2$	**D6**	$Q_{D6} = \mu_k$	$\hat{F}_3$	**D5**	$Q_{D5} = \mu_k$
$\hat{F}_4$	**D4**	$Q_{D4} = \beta_{k-1} - \beta_k$	$\hat{F}_5$	**D3**	$Q_{D3} = \beta_{k-1} - \beta_k$
$\hat{F}_6$	**D2**	$Q_{D2} = \alpha_k$	$\hat{F}_7$	**D1**	$Q_{D1} = \alpha_k$

The Bianchi identities allow to determine which branes are actually present

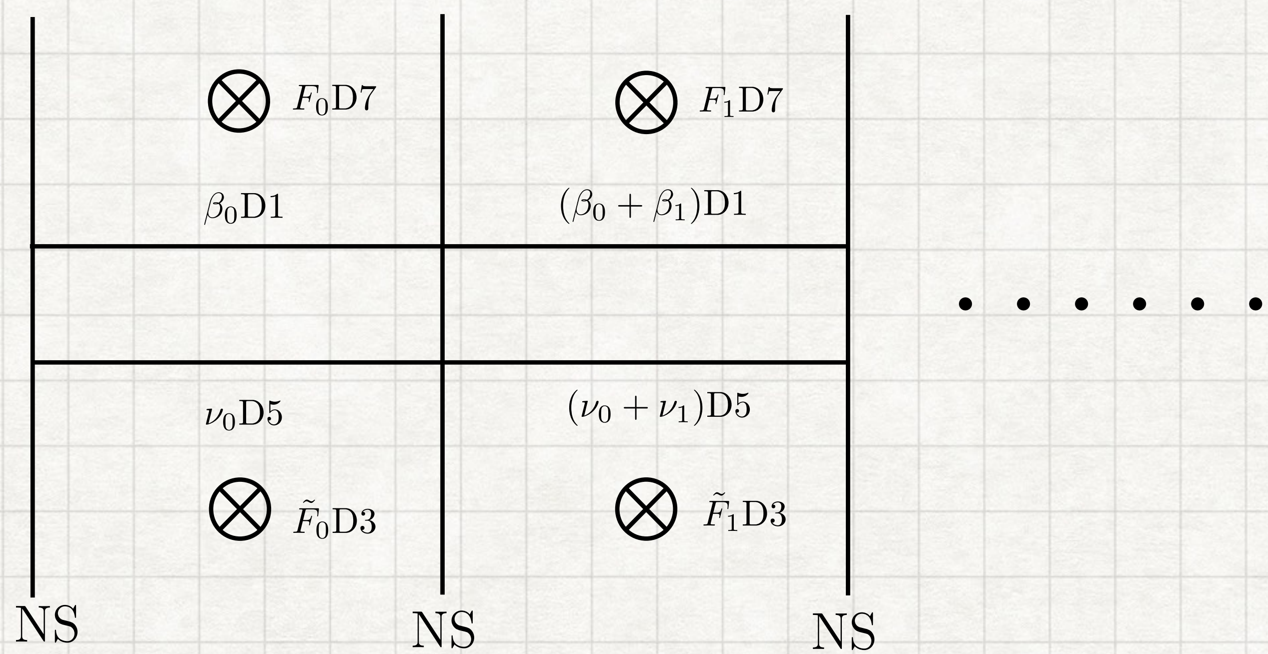
→ D1, D5 branes will play the role of color branes

→ D3, D7 branes will play the role of flavour branes

The geometry

Hanany-Witten brane set-up

(Hanany, Witten'97)



$$h_4^{(k)} = \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k),$$
$$h_8^{(k)} = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k),$$

"Holographic Central Charge"

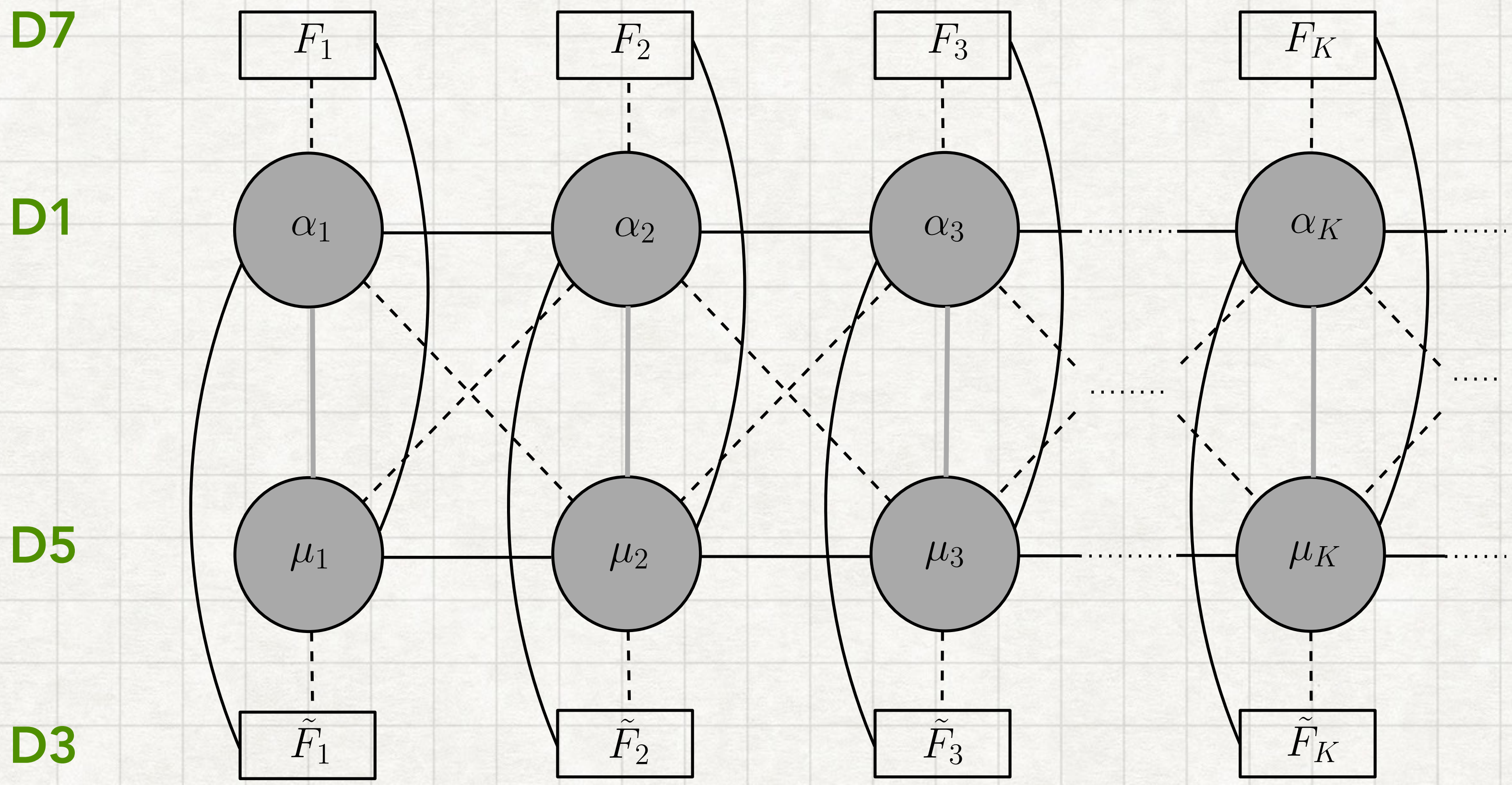
$$c_{hol} = \frac{3}{\pi}\int_0^{2\pi(P+1)} h_4 h_8 d\rho$$

(Freedman, Gubser, Pilch, Wagner;
Krauss, Larsen;
Klebanov, Kutasov, Murugan;
Macpherson, Nunez, et.al)

The same result as in AdS_3

SCQM

- The proposal is to provide a UV $\mathcal{N} = 4$ quantum mechanics, that conjecturally flows to a super conformal quantum mechanics dual to our AdS_2 geometries



- From the original $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ symmetry of the CFT_2, only one of the sectors is kept
- On field theoretical terms, the number of vacua of the SCQM is $c = 6(n_{hyp} - n_{vec})$

(Putrov, Song, Yan 2016)

Conclusions

- We present a new entry in the mapping between SCFTs and AdS-supergravity backgrounds, for the case the 2d $\mathcal{N}=(0,4)$ (small) SCFTs and backgrounds with $AdS_3\times S^2$ factors.
- We proposed explicit 2d dual SCFTs based in information obtained from the geometry. The SCFTs are defined as the IR fixed points of 2d QFTs built out of (0,2) multiplets.
- We discussed one example that constitutes a stringent test, where the duality is checked with the computation of the central charge
- We discussed that T-dualisation on AdS_3 is equivalent to starting with a given $\mathcal{N}=(0,4)$ $SCFT_2$ doing a dimensional reduction keeping the $\mathcal{N}=4$ SUSY right sector.
- We proposed a quiver quantum mechanics, that conjecturally flows in the IR to a $\mathcal{N}=4$ SCQM dual to our $AdS_2\times S^2$

Open problems

- Dual CFT of the solutions in class II?
- More checks of the duality

THANKS!