Kaluza-Klein spectra and consistent truncations

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Iberian Strings 2021

January 21, 2021. *e*Lisbon

Based on:

GL, P. Ntokos, O.Varela – arXiv:1907.02087

GL, O. Varela – arXiv:1907.11027

K. Dimmitt, GL, P. Ntokos, O.Varela – arXiv:1911.12202

M. Cesàro, GL, O. Varela – arXiv:2007.05172

KK spectrum of Spin-2 modes

We consider a perturbed geometry

$$d\hat{s}^2_{d+4} = e^{2A(y)}\Big[\big(\bar{g}^{\mathrm{AdS}_4}_{\mu\nu}(x) + h_{\mu\nu}(x,y)\big)dx^\mu dx^\nu + d\bar{s}^2_d(y)\Big] \,,$$

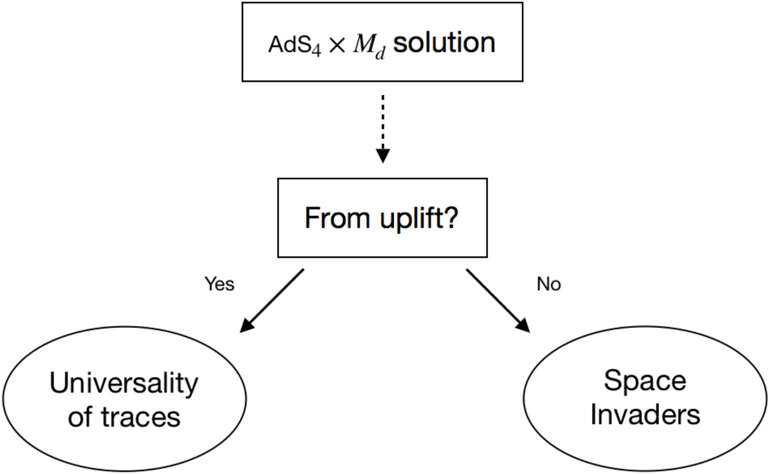
with $h_{\mu\nu}(x,y) = h^{[tt]}_{\mu\nu}(x)\mathcal{Y}(y)$ and $\bar{\Box}h^{[tt]}_{\mu\nu}(x) = (L^2M^2-2)h^{[tt]}_{\mu\nu}(x)$.

The $D = d+4$ Einstein equations reduce to [C.Bachas, J.Estes '11]

$$-\frac{e^{-(d+2)A}}{\sqrt{\bar{g}}}\partial_M\Big(e^{(d+2)A}\sqrt{\bar{g}}\,\bar{g}^{MN}\partial_N\mathcal{Y}\Big) = L^2M^2\mathcal{Y} \,,$$

with the spectrum organised in terms of the isometries of $d\bar{s}^2_d$.

Overview


$AdS_4 \times M_d$ solution
From uplift?
Yes
No
Universality of traces
Space Invaders

Plan

- Solutions from uplift
  - Gaugings and uplifts
  - Kaluza-Klein gravitons
  - Universality of traces
  - Duality covariant approach
- Other solutions
  - G-structures and the GMPS classification
  - The dual of the $\mathcal{Z}^3$-deformed ABJM
  - (numerical) Kaluza-Klein gravitons
  - The space invaders scenario

In the SL(8) basis,

$$\begin{array}{ccccccc} \mathbf{912} & \longrightarrow & \mathbf{36} & + & \mathbf{36'} & + & \underbrace{\mathbf{420} \;+\; \mathbf{420'}} \\ \Theta_{\mathbb{M}}{}^{\alpha} & \mapsto & \big(\Theta^{[AB]C}{}_{D} & , & \Theta_{[AB]}{}^{C}{}_{D} & , & \Theta_{\mathbb{M}}{}^{[CDEF]}\big) \end{array}$$

where

$$\Theta_{[AB]}{}^{C}{}_{D} = 2\delta^{C}_{[A}\theta_{B]D} \; , \qquad \Theta^{[AB]C}{}_{D} = 2\delta^{[A}_{D}\xi^{B]C} \; .$$

We consider

$$\mathrm{SO}(8)_e \quad : \quad \theta_{AB} = \mathrm{diag}(1,\ldots,1) \; , \quad \xi^{AB} = 0 \; ,$$

$$\mathrm{ISO}(7) \quad : \quad \theta_{AB} = \mathrm{diag}(1,\ldots,1,0) \; , \quad \xi^{AB} = \mathrm{diag}(0,\ldots,0,1) \; ,$$

$$\big[\mathrm{SO}(6)\times\mathrm{SO}(1,1)\big] \ltimes \mathbb{R}^{12} \quad : \quad \theta_{AB} = \mathrm{diag}(1,\ldots,1,0,0) \; , \quad \xi^{AB} = \mathrm{diag}(0,\ldots,0,1,-1) \; .$$

The observation

Vacua of $D = 4$ $\mathcal{N} = 8$ supergravities that preserve same susy and bosonic symmetry tend to exhibit the same universal spectrum of masses, *irrespectively of the gauging* considered

Critical point	SO(8)	ISO(7)	$[\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes \mathbb{R}^{12}$	same spectrum?
$\mathcal{N} = 8$ SO(8)	✓	×	×	–
$\mathcal{N} = 2$ U(3)	✓	✓	×	✓
$\mathcal{N} = 1$ G_2	✓	✓	×	✓
$\mathcal{N} = 1$ SU(3)	×	✓	✓	✓
$\mathcal{N} = 0$ SO(7)	✓	✓	×	✓
$\mathcal{N} = 0$ SO(6)	✓	✓	✓	✓
$\mathcal{N} = 0$ G_2	×	✓	×	–
$\mathcal{N} = 0$ SU(3)	×	✓	✓	×

[N.P.Warner '83] [A.Guarino, O.Varela '15] [A.Guarino, C.Sterckx '19]

The question

However, these gaugings enjoy different uplifts:

$$\mathrm{SO}(8) \quad \longhookrightarrow \quad \text{M-theory on } \mathrm{AdS}_4 \times S^7$$

[B.de Wit, H.Nicolai '87]

$$\mathrm{ISO}(7) \quad \longhookrightarrow \quad \text{mIIA on } \mathrm{AdS}_4 \times S^6$$

[D.Jafferis, A.Guarino, O.Varela '15]

$$\big[\mathrm{SO}(6) \times \mathrm{SO}(1,1)\big] \ltimes \mathbb{R}^{12} \quad \longhookrightarrow \quad \text{IIB on } \mathrm{AdS}_4 \times S^1 \times S^5 \text{ S-fold}$$

[G.Inverso, H.Samtleben, M.Trigiante '16]

Is universality preserved in the KK spectrum upon reduction?

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Is universality preserved in the KK spectrum upon reduction?

Today we will show that the answer is yes, *to some extent*:

The spectra of KK gravitons in general differ,
but the trace over masses at definite KK level is preserved.

Comparing gaugings

Inserting the uplifted metrics in the BE equation, find KK spectra which evaluate to [K.Dimmitt, GL, P.Ntokos, O.Varela '19]:

Solution	L^2M^2 @ SO(8)	L^2M^2 @ ISO(7)
$\mathcal{N}=8$ SO(8)	$\frac{1}{4}n(n+6)$	–
$\mathcal{N}=2$ U(3)$_c$	$\frac{1}{2}n(n+6)-\frac{1}{3}\ell(\ell+4)-\frac{1}{9}(\ell-2p)^2 + \frac{1}{18}[3(n-2r)+4(\ell-2p)]^2$	$\frac{2}{3}k(k+5)-\frac{1}{3}\ell(\ell+4)+\frac{1}{9}(\ell-2p)^2$
$\mathcal{N}=1$ G$_2$	$\frac{5}{8}n(n+6)-\frac{5}{12}k(k+5)$	$\frac{5}{12}k(k+5)$
$\mathcal{N}=1$ SU(3)	–	$\frac{5}{6}k(k+5)-\frac{5}{12}\ell(\ell+4)-\frac{5}{36}(\ell-2p)^2$
$\mathcal{N}=0$ SO(7)$_v$	$\frac{3}{4}n(n+6)-\frac{3}{5}k(k+5)$	$\frac{2}{5}k(k+5)$
$\mathcal{N}=0$ SO(7)$_c$	$\frac{3}{10}n(n+6)$	–
$\mathcal{N}=0$ SU(4)$_c$	$\frac{3}{8}n(n+6)-\frac{3}{16}(n-2r)^2$	–
$\mathcal{N}=0$ SO(6)$_v$	–	$k(k+5)-\frac{3}{4}\ell(\ell+4)$
$\mathcal{N}=0$ G$_2$	–	$\frac{1}{2}k(k+5)$

and similarly for the type IIB uplift of $\big[\mathrm{SO}(6)\times\mathrm{SO}(1,1)\big]\ltimes\mathbb{R}^{12}$.

$\Longrightarrow$ Individual eigenvalues are not preserved.

Universality of the traces

The sum of masses corresponding to a specific SO(8) level is universal in the U(3) solutions on S^6 and S^7 [Y.Pang, J.Rong, O.Varela '17].

Same occurs for all solutions if the $D = 4$ spectra already matches!

E.g.: for the SU(4)/SO(6) solutions of M-theory and mIIA, we find:

$$L^2 \operatorname{tr} M^2_{(n)}[\mathrm{D11}] \equiv L^2 \sum_{r=0}^{n} M^2_{n,r}\, d_{n,r} = \frac{39}{2} D_{n-1,10} \ ,$$

$$L^2 \operatorname{tr} M^2_{(n)}[\mathrm{mIIA}] \equiv L^2 \sum_{k=0}^{n} \sum_{\ell=0}^{k} M^2_{k,\ell}\, d_{k,\ell} = \frac{39}{2} D_{n-1,10} \ ,$$

from $[n,0,0,0]_{\mathrm{SO}(8)} \xrightarrow{\mathrm{SU}(4)\times\mathrm{U}(1)} \bigoplus_{r=0}^{n} [r,0,n-r]_{2r-n}$, etc.

Same value obtained for IIB S-fold solution modulo small provisos.

In $\mathcal{N} = 8$ $D = 4$ gauged supergravities, there exist closed formulae for the mass matrices of the fields [A. Le Diffon, H.Samtleben, M.Trigiante '11].

E.g., for vectors

$$(M_v^2)_{\mathbb{M}}{}^{\mathbb{N}} = -\frac{g^2}{24}\left[\,\mathrm{tr}(X_{\mathbb{M}}\, X_{\mathbb{P}}) + \mathrm{tr}(\mathcal{M}^{-1}\, X_{\mathbb{M}}\, \mathcal{M}\, X_{\mathbb{P}}^T)\right]\mathcal{M}^{\mathbb{P}\mathbb{N}}\ .$$

Very lately, this has been extended up the KK tower via ExFT (see [E.Malek, H.Samtleben '19], [O.Varela '20], [O.Varela, M.Cesàro '20]).

But for gravitons we do not need that much machinery!

A duality covariant approach

For the gaugings considered here, $G \subset \mathrm{SL}(8) \subset E_{7(7)}$, and the graviton eigenfunctions are polynomials in $\mathbb{R}^8$ coordinates,

$$\mathcal{Y}^{A_1 \dots A_m} = \mu^{(A_1} \dots \mu^{A_m)} - \text{traces} \, , \qquad m = 0, 1, 2, \dots \, ,$$

with μ^A in the $\mathbf{8}$ of SL(8) constrained as $\theta_{AB}\mu^A\mu^B = 1$.

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with μ^A in the **8** of SL(8) constrained as $\theta_{AB} \mu^A \mu^B = 1$.

Then, BE PDE becomes an algebraic eigenvalue problem with

$$\mathcal{M}^2 = \mathrm{diag}\big(M^2_{(0)} \, , M^2_{(1)} \, , \dots \, , \; M^2_{(m)} \, , \dots \big) \, ,$$

with the blocks being [K.Dimmitt, GL, P.Ntokos, O.Varela '20]:

$$M^2_{(0)} = 0 \, , \qquad\qquad (M^2_{(1)})_A{}^B = -g^2 \mathcal{M}^{\mathbb{M}\mathbb{N}} \, \Theta_{\mathbb{M}}{}^B{}_C \, \Theta_{\mathbb{N}}{}^C{}_A \, ,$$

$$(M^2_{(m)})_{A_1 \dots A_m}{}^{B_1 \dots B_m} = - \, m \, g^2 \mathcal{M}^{\mathbb{M}\mathbb{N}} \Big[\Theta_{\mathbb{M}}{}^{(B_1|}{}_C \; \Theta_{\mathbb{N}}{}^C{}_{(A_1} \delta_{A_2}{}^{|B_2} \dots \delta_{A_m)}{}^{|B_m)} + (m-1) \Theta_{\mathbb{M}}{}^{(B_1}{}_{(A_1} \; \Theta_{\mathbb{N}}{}^{B_2}{}_{A_2} \delta_{A_3}{}^{B_3} \dots \delta_{A_m)}{}^{B_m)} \Big] \, .$$

Reducing SL(8) representations to those of G_{iso}, this recovers the values for the masses we obtained before.

Moreover, the universal coefficient in the traces is simply:

$$L^2 \mathrm{tr}\, M^2_{(1)} = \frac{6g^2}{V_0} \mathcal{M}^{\mathbb{M}\mathbb{N}}\, \Theta_{\mathbb{M}}{}^{A}{}_{B}\, \Theta_{\mathbb{N}}{}^{B}{}_{A}\ .$$

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The G-structures route

Demanding that the internal metric preserves $\mathcal{N}=2$ and at least two isometries, the line element takes on the local form [M.Gabella, D.Martelli, A.Passias, J.Sparks '12]

$$ds^2_{11} = \left(\frac{m_{FR}}{48}\right)^{2/3}(1+r^2+\alpha^{-2})\Big\{ds^2_4 + \frac{f\cdot\alpha}{\sqrt{1+(1+r^2)\alpha^2}}\,ds^2(\mathrm{KE}_4)$$
$$+\frac{\alpha^2}{4}\Big[dr^2 + \frac{r^2 f^2}{1+r^2}(d\tilde{\tau}+\sigma)^2 + \frac{1+r^2}{1+(1+r^2)\alpha^2}\Big(d\tilde{\psi}+\frac{f}{1+r^2}(d\tilde{\tau}+\sigma)\Big)^2\Big]\Big\}\,,$$

with $f(r)$ and $\alpha(r)$ obeying, from the torsion conditions,

$$\frac{f'}{f} = -\frac{1}{2}r\alpha^2\,, \qquad\qquad \frac{(r\alpha' - r^2\alpha^3)f}{\sqrt{1+(1+r^2)\alpha^2}} = -3\,.$$

For $ds^2(\mathrm{KE}_4) = ds^2(\mathbb{CP}_2)$, the isometry group enhances to $\mathrm{SU}(3)\times\mathrm{U}(1)_{\tilde{\psi}}\times\mathrm{U}(1)_{\tilde{\tau}}$, with $\mathrm{U}(1)_{\tilde{\tau}}$ broken by fluxes.

The G-structures route

There are at least two choices of (f, α) for which the metric extends globally on S^7 in terms of the angles

$$\psi = \frac{1}{p}\,\tilde{\psi}\ , \qquad \tau = \tilde{\tau} + \frac{1}{3}\big(1 - \frac{1}{p}\big)\tilde{\psi}\ , \qquad 0 \leq r \leq r_0\ , \qquad p = 2, 3\ ,$$

with ψ and τ of period 2π and r_0 a solution-dependent constant.

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with ψ and τ of period 2π and r_0 a solution-dependent constant.

Regularity, S^7-topology and AdS/CFT require the asymptotics

$$f \xrightarrow[r\to 0]{} \frac{3p}{p-1}\ , \qquad \alpha \xrightarrow[r\to 0]{} w r^{-1+1/p}\ , \quad \text{with } w > 0\ ,$$

$$f \xrightarrow[r\to r_0]{} \frac{2\sqrt{1+r_0^2}}{r_0}\,(r_0 - r)\ , \qquad \alpha \xrightarrow[r\to r_0]{} \sqrt{\frac{2}{r_0(r_0-r)}}\ .$$

Aside: ABJM and some deformations

ABJM is the superconformal CS-matter theory describing the worldvolume of a stack of M2s on a $\mathbb{C}^4/\mathbb{Z}_k$ orbifold singularity.

For $k = 1$, susy enhances to $\mathcal{N} = 8$, and the field content includes four chiral superfields $(\mathcal{Z}^1, \mathcal{Z}^2, \mathcal{Z}^3, \mathcal{Z}^4)$ with a quartic superpotential $W \sim \mathcal{Z}^1\mathcal{Z}^2\mathcal{Z}^3\mathcal{Z}^4$.

ABJM admits two manifestly (SU(3), $\mathcal{N} = 2$)-preserving relevant deformations of the superpotential (schematically)

$$\Delta W = (\mathcal{Z}^4)^p \,, \quad \begin{cases} p = 2: \ \text{CPW} \\ \qquad \text{[R.Corrado, K.Pilch, N.P.Warner '02]} \\ p = 3: \ \text{GMPS} \\ \qquad \text{[M.Gabella, D.Martelli, A.Passias, J.Sparks '12]} \end{cases}$$

leading to IR R-charges

$$R_1 \equiv R(\mathcal{Z}^A) = \tfrac{2(p-1)}{3p} \,, \ A = 1,2,3 \,, \qquad R_2 \equiv R(\mathcal{Z}^4) = \tfrac{2}{p} \,.$$

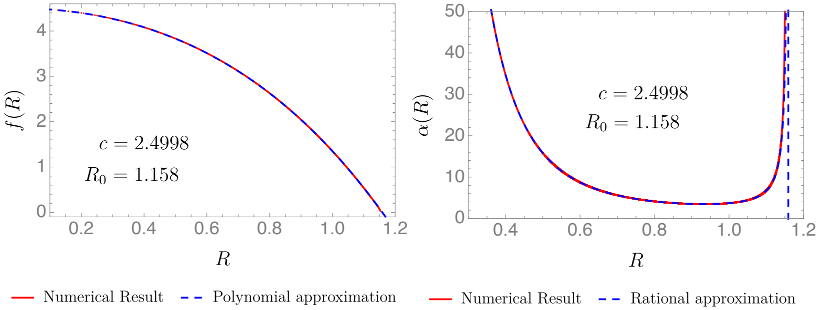
For $p = 2$, the CPW solution is recovered for

$$f = 6\left(1 - \frac{r}{r_0}\right) , \qquad \alpha = \sqrt{\frac{2}{r(r_0 - r)}} , \qquad r_0 = 2\sqrt{2} .$$

For $p = 3$, the solution is only known numeric-/perturbatively:

$$f(R) = \frac{9}{2} - cR^2 - \frac{c^2}{9}R^4 + \frac{(2187 - 128c^3)}{3888}R^6 + \frac{(19683c - 1264c^4)}{104976}R^8 + \mathcal{O}(R^{10}) ,$$

in terms of $R = r^{1/3}$ and a constant c.



Kaluza-Klein gravitons on GMPS

For the $p = 3$ solution, the eigenvalue problem $\mathcal{L}\mathcal{Y} = L^2 M^2 \mathcal{Y}$ depends on the operator [M.Cesàro, GL, O.Varela '20]

$$\begin{aligned}\mathcal{L} = &- \frac{4}{r\alpha^2 f^3}\partial_r\left[r f^3 \partial_r\right] - \frac{\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha}\Box_{S^5} \\ &- \frac{4}{9}\left(1+\frac{1}{r^2\alpha^2}\right)\partial_\psi^2 - \frac{8}{3}\left[\frac{2}{9}\left(1+\frac{1}{r^2\alpha^2}\right) - \frac{1}{r^2\alpha^2 f}\right]\partial_\psi\partial_\tau \\ &- \left[-\frac{\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha} + \frac{16}{81}\left(1+\frac{1}{r^2\alpha^2}\right) + \frac{4(1+r^2)}{r^2\alpha^2 f^2} - \frac{16}{9r^2\alpha^2 f}\right]\partial_\tau^2 \,,\end{aligned}$$

and the SU(3)×U(1)$_\psi$×U(1)$_\tau$ isometry can be exploited as

$$\mathcal{Y} = \sum_{\ell,m,j} \xi_{\ell,m,j}(r)\, Y_{\ell,m}(z,\bar{z},\tau)\, e^{ij\psi} \,,$$

with

$$\Box_{S^5} Y_{\ell,m} = -\ell(\ell+4) Y_{\ell,m} \,, \qquad \partial_\tau Y_{\ell,m} = i m Y_{\ell,m} \,.$$

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$$
\begin{aligned}
L^2 M^2 \xi = & -\frac{4}{r\alpha^2 f^3}\frac{d}{dr}\left[r f^3 \frac{d\xi}{dr}\right] + \frac{\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha}\ell(\ell+4)\xi \\
& + \frac{4}{9}\left(1+\frac{1}{r^2\alpha^2}\right) j^2 \xi + \frac{8}{3}\left[\frac{2}{9}\left(1+\frac{1}{r^2\alpha^2}\right) - \frac{1}{r^2\alpha^2 f}\right] jm\,\xi \\
& + \left[-\frac{\sqrt{1+(1+r^2)\alpha^2}}{f\cdot\alpha} + \frac{16}{81}\left(1+\frac{1}{r^2\alpha^2}\right) + \frac{4(1+r^2)}{r^2\alpha^2 f^2} - \frac{16}{9r^2\alpha^2 f}\right] m^2 \xi \,,
\end{aligned}
$$

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$$
\mathcal{Y} = \sum_{\ell,m,j} \xi_{\ell,m,j}(r)\, Y_{\ell,m}(z,\bar{z},\tau)\, e^{ij\psi} \,,
$$

with

$$
\Box_{S^5} Y_{\ell,m} = -\ell(\ell+4) Y_{\ell,m} \,, \qquad \partial_\tau Y_{\ell,m} = im Y_{\ell,m} \,.
$$

The RG flow connecting the SO(8) and GMPS solutions implies that the towers of KK gravitons must be related through

$$[n,0,0,0] \xrightarrow{\mathrm{SU}(3)\times\mathrm{U}(1)_3} \bigoplus_{\ell=0}^{n}\bigoplus_{t=0}^{n-\ell}\bigoplus_{p=0}^{\ell}[p,\ \ell-p]_{-R_1(\ell-2p)+R_2(n-\ell-2t)}\ ,$$

with the two sets of quantum numbers related as

$$n=2k+|j|+\ell\,,\qquad m=2p-\ell\,,\qquad j=n-\ell-2t\,.$$

Thus, we can sweep over the complete mass spectrum, and the schematic form of the dual operators, can be similarly inferred.

n	$[p,\ell-p]_{\frac{4}{9}(2p-\ell)+\frac{2}{3}(n-\ell-2t)}$	$d_{p,\,\ell-p}$	L^2M^2	Δ	Dual operator	Short?
0	$[0,0]_0$	1	0	3	$\mathcal{T}^{(0)}_{\alpha\beta}\vert_{s=2}$	✓
1	$[0,0]_{\pm\frac{2}{3}}$	1	$\frac{22}{9}$	$\frac{11}{3}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^4\vert_{s=2}$, c.c.	✓
	$[1,0]_{\frac{4}{9}}, [0,1]_{-\frac{4}{9}}$	3	1.76	3.50	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\vert_{s=2}$, c.c.	
2	$[0,0]_{\pm\frac{4}{3}}$	1	$\frac{52}{9}$	$\frac{13}{3}$	$\mathcal{T}^{(0)}_{\alpha\beta}(\mathcal{Z}^4)^2\vert_{s=2}$, c.c.	✓
	$[1,0]_{-\frac{2}{9}}, [0,1]_{\frac{2}{9}}$	3	4.68	4.13	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\bar{\mathcal{Z}}_4\vert_{s=2}$, c.c.	
	$[2,0]_{\frac{8}{9}}, [0,2]_{-\frac{8}{9}}$	6	3.88	3.97	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^{B)}\vert_{s=2}$, c.c.	
	$[1,0]_{\frac{10}{9}}, [0,1]_{-\frac{10}{9}}$	3	5.07	4.21	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\mathcal{Z}^4\vert_{s=2}$, c.c.	
	$[0,0]_0$	1	5.92	4.36	$\mathcal{T}^{(0)}_{\alpha\beta}(1-4a^2\mathcal{Z}^4\bar{\mathcal{Z}}_4+b\mathcal{Z}^A\bar{\mathcal{Z}}_A)\vert_{s=2}$	
	$[1,1]_0$	8	4	4	$\mathcal{T}^{(0)}_{\alpha\beta}(\mathcal{Z}^A\bar{\mathcal{Z}}_B-\frac{1}{3}\delta^A_B\mathcal{Z}^C\bar{\mathcal{Z}}_C)\vert_{s=2}$	

Kaluza-Klein gravitons on GMPS

Analytics: Short multiplets

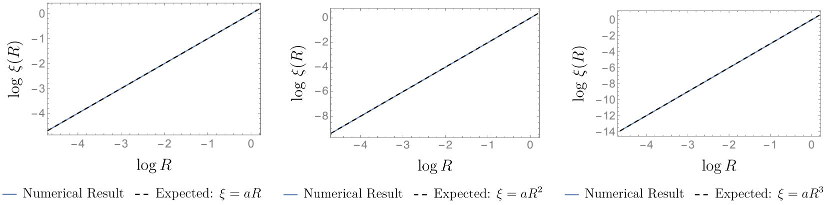
For every n, and both $p = 2, 3$, the modes $[0, 0]_{\pm R_2 n}$ have mass

$$L^2 M_n^2 = R_2 n \left(R_2 n + 3 \right) .$$

These states are short, with $\Delta_n = R_2 n + 3$, and dual to $\mathcal{T}^{(0)}_{\alpha\beta} (\mathcal{Z}^4)^n$.

The corresponding eigenfunctions can be found analytically to be

$$\mathcal{Y}_j = (\xi_1)^j e^{ij\psi} = R^j e^{ij\psi} .$$



$j = 1$ $\qquad$ $j = 2$ $\qquad$ $j = 3$

For $n \geq 2$, and both $p = 2, 3$, the modes $[1, 1]_{\pm R_2(n-2)}$ are dual to

$$\mathcal{T}^{(0)}_{\alpha\beta} \left(\mathcal{Z}^A \mathcal{Z}_B - \frac{1}{3} \delta^A_B \mathcal{Z}^C \mathcal{Z}_C \right) (\mathcal{Z}^4)^{n-2} .$$

These operators are long, but $\Delta_n = (n-2) R_2 + 4$
(they are shadows of the massless vector [M.Billo et al. '00]).

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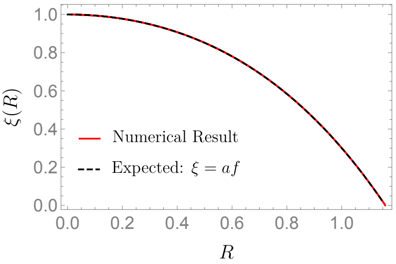
The corresponding eigenfunctions have the form

$$\mathcal{Y}_j = \xi_8 R^j Y_{2,0} e^{ij\psi} , \quad j = 0, 1, \ldots$$

with $\xi_8 \propto f(r)$.



The relation $\xi_8 \propto f(r)$ holds on CPW too, and in both cases, as well

$$\xi_8 = (\xi_3)^2 \, .$$

Also, for CPW, $(\xi_3)^2 + (\xi_1)^2 = 1$ is the constraint defining S^7 in $\mathbb{R}^8$.

On the GMPS solution, that would mean

$$f = \frac{9}{2}\left[1 - \left(\frac{r}{r_0}\right)^{2/3}\right] \quad \Rightarrow \quad \alpha^2 = \frac{4}{3r^2\left[\left(\frac{r}{r_0}\right)^{-2/3} - 1\right]} \, .$$

The relation $\xi_8 \propto f(r)$ holds on CPW too, and in both cases, as well

$$\xi_8 = (\xi_3)^2 \, .$$

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On the GMPS solution, that would mean

$$f = \frac{9}{2}\left[1 - \left(\frac{r}{r_0}\right)^{2/3}\right] \quad \Rightarrow \quad \alpha^2 = \frac{4}{3r^2\left[\left(\frac{r}{r_0}\right)^{-2/3} - 1\right]} \, .$$

Sadly, $\nexists\ r_0$ such that

$$\frac{(r\alpha' - r^2\alpha^3)f}{\sqrt{1 + (1 + r^2)\alpha^2}} = -3 \, .$$

This means that our S^7 is not isometrically embedded in $\mathbb{R}^8$!
(c.f. the squashed S^7 of [M.Awada, M.Duff, C.Pope '83])

Another similarity with the ADP squashed S^7 is that it neither allows an $\mathcal{N} = 8$ consistent truncation.

In the GMPS case, it only accommodates minimal $\mathcal{N} = 2$, $D = 4$ sugra [GL, O.Varela '19], as expected from [J.Gauntlett, O.Varela '07].

Also related to supersymmetry, we must allocate modes with different spins and same SU(3) charges in supermultiplets of $\mathrm{OSp}(4|2)_3 \times \mathrm{SU}(3)$, which is the preserved supergroup.

The space invaders scenario

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However, as for the ADP S^7 again,
this allocation cannot be made level by level!

Spin	SO(8)	SU(3) × U(1)$_3$											
2	$\mathbf{1}$	$\mathbf{1}_0$											
$\frac{3}{2}$	$\mathbf{8}_s$	$\mathbf{1}_{+1}$ $\mathbf{1}_{-1}$	$\mathbf{3}_{\frac{1}{9}}$	$\bar{\mathbf{3}}_{-\frac{1}{9}}$									
1	$\mathbf{28}$	$\mathbf{1}_0$	$\mathbf{3}_{-\frac{8}{9}}$ $\mathbf{3}_{\frac{10}{9}}$	$\bar{\mathbf{3}}_{\frac{8}{9}}$ $\bar{\mathbf{3}}_{-\frac{10}{9}}$	$\mathbf{8}_0$	$\mathbf{3}_{-\frac{2}{9}}$	$\bar{\mathbf{3}}_{\frac{2}{9}}$	$\mathbf{1}_0$					
			$\mathbf{3}_{-\frac{8}{9}}$	$\bar{\mathbf{3}}_{\frac{8}{9}}$									
$\frac{1}{2}$	$\mathbf{56}_s$		$\mathbf{3}_{\frac{1}{9}}$ $\mathbf{3}_{\frac{1}{9}}$	$\bar{\mathbf{3}}_{-\frac{1}{9}}$ $\bar{\mathbf{3}}_{-\frac{1}{9}}$	$\mathbf{8}_{+1}$ $\mathbf{8}_{-1}$	$\mathbf{3}_{\frac{7}{9}}$ $\mathbf{3}_{-\frac{11}{9}}$	$\bar{\mathbf{3}}_{-\frac{7}{9}}$ $\bar{\mathbf{3}}_{\frac{11}{9}}$	$\mathbf{1}_{+1}$ $\mathbf{1}_{-1}$	$\mathbf{6}_{-\frac{1}{9}}$	$\bar{\mathbf{6}}_{\frac{1}{9}}$	$\mathbf{1}_{\frac{1}{3}}$	$\mathbf{1}_{-\frac{1}{3}}$	
			$\mathbf{3}_{-\frac{17}{9}}$	$\bar{\mathbf{3}}_{\frac{17}{9}}$		$\mathbf{3}_{\frac{7}{9}}$	$\bar{\mathbf{3}}_{-\frac{7}{9}}$	$\mathbf{1}_{+1}$ $\mathbf{1}_{-1}$					$\mathbf{3}_{\frac{1}{9}}$ $\bar{\mathbf{3}}_{-\frac{1}{9}}$
0	$\mathbf{35}_v$				$\mathbf{8}_0$	$\mathbf{3}_{-\frac{2}{9}}$	$\bar{\mathbf{3}}_{\frac{2}{9}}$	$\mathbf{1}_0$	$\mathbf{6}_{\frac{8}{9}}$	$\bar{\mathbf{6}}_{-\frac{8}{9}}$	$\mathbf{1}_{\frac{4}{3}}$	$\mathbf{1}_{-\frac{4}{3}}$	$\mathbf{3}_{\frac{10}{9}}$ $\bar{\mathbf{3}}_{-\frac{10}{9}}$
	$\mathbf{35}_c$		$\mathbf{3}_{-\frac{8}{9}}$	$\bar{\mathbf{3}}_{\frac{8}{9}}$	$\mathbf{8}_0$	$\mathbf{3}_{-\frac{2}{9}}$	$\bar{\mathbf{3}}_{\frac{2}{9}}$	$\mathbf{1}_0$	$\mathbf{6}_{-\frac{10}{9}}$	$\bar{\mathbf{6}}_{\frac{10}{9}}$	$\mathbf{1}_{-\frac{2}{3}}$	$\mathbf{1}_{\frac{2}{3}}$	
						$\mathbf{3}_{\frac{16}{9}}$	$\bar{\mathbf{3}}_{-\frac{16}{9}}$	$\mathbf{1}_0$ $\mathbf{1}_{+2}$ $\mathbf{1}_{-2}$					$\mathbf{3}_{-\frac{8}{9}}$, $\bar{\mathbf{3}}_{\frac{8}{9}}$ $\mathbf{3}_{-\frac{8}{9}}$, $\bar{\mathbf{3}}_{\frac{8}{9}}$ $\mathbf{3}_{-\frac{2}{9}}$, $\bar{\mathbf{3}}_{\frac{2}{9}}$ $\mathbf{1}_0$
		Massless graviton	Short gravitino	Short gravitino	Massless vector	Short vector	Short vector	Long vector	Massive hyper	Massive hyper	Massive hyper	Massive hyper	Eaten modes

Conclusions

Today we showed:

- The complete spectra of KK gravitons around solutions of M-theory, mIIA and IIB from uplift and the universality of the mass traces.
- An SL(8)-covariant formula for their KK masses.
- The spectrum of gravitons around the dual of the cubic deformation of ABJM and some short supermultiplets.
- This solution can't be obtained from uplift of a maximal gauged supergravity, its metric isn't isometrically embedded in $\mathbb{R}^8$ and its spectrum displays *space invasion*.

Outlook

Many conundra remain!

- Mechanism behind the universal behaviour?
- Generalisation of $\mathcal{M}^2$ and $\mathcal{Y}$ for gaugings outside SL(8)?
- Embedding of GMPS in $\mathbb{CP}^4$?
- Correct invasion pattern?
- Exact relation between
  Uplift from $\mathcal{N} = 8 \leftrightarrow S^7 \hookrightarrow \mathbb{R}^8$ isometric $\leftrightarrow$ No space inv.?

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Muito obrigado!!

GMPS outside $\mathcal{N} = 8$ truncation

The IR-fixed point dual to the GMPS solution is at the end of the flow generated by $\Delta W = (\mathcal{Z}^4)^3$.

At the level of the lagrangian,

$$\Delta\mathcal{L} = (Z^4)^2(\bar{Z}^4)^2 + \frac{1}{2}\chi^4\chi^4 Z^4 + \frac{1}{2}\bar{\chi}^4\bar{\chi}^4\bar{Z}^4 \ ,$$

which belong to the $\mathbf{294}_v$ and $\mathbf{224}_{cv}$ of SO(8), and therefore outside the supergravity level.

We can scan through the quantum numbers ℓ, m, j to obtain the *complete spectrum* numerically:

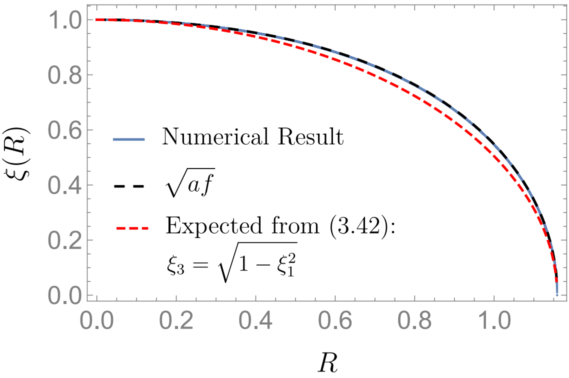
1. For given ℓ, m, j, we use the asymptotics for f and α to get the ODE near $R \to 0$ and $R \to R_0$, and keep the normalisable solution at each end.
2. We integrate numerically the complete ODE using the above solutions as seeds. This is done from left and right with a parameter λ labelling all possible masses.
3. The valid $\xi_\lambda^L(R)$ and $\xi_\lambda^R(R)$ must be linearly dependent over the whole range of R, i.e.:

$$W(\lambda, R) = \xi_\lambda^L(R)\,\dot{\xi}_\lambda^R(R) - \xi_\lambda^R(R)\,\dot{\xi}_\lambda^L(R) = 0\,, \quad \forall R$$

This selects an infinite discrete set of λ's that we label by $k = 0, 1, \ldots$.

Failure of the isometric embedding

We can check that $\xi_3 \propto \sqrt{\xi_8}$, but $\xi_3 \not\propto \sqrt{1-\xi_1^2}$:



The space invaders scenario

In contrast, for the $\mathcal{N} = 1$ G_2 solution

Spin	SO(8)	G_2					
2	$\mathbf{1}$	$\mathbf{1}$					
$\frac{3}{2}$	$\mathbf{8}_s$	$\mathbf{1}$	$\mathbf{7}$				
1	$\mathbf{28}$		$\mathbf{7}+\mathbf{7}$	$\mathbf{14}$			
$\frac{1}{2}$	$\mathbf{56}_s$		$\mathbf{7}$	$\mathbf{14}$	$\mathbf{1}$	$\mathbf{27}$	$\mathbf{7}$
0	$\mathbf{35}_v$				$\mathbf{1}$	$\mathbf{27}$	$\mathbf{7}$
	$\mathbf{35}_c$				$\mathbf{1}$	$\mathbf{27}$	$\mathbf{7}$
		Massless graviton	Massive gravitino	Massless vector	Wess-Zumino	Wess-Zumino	Eaten modes

And the same happens for the other solutions from uplift (see [I.R.Klebanov, T.Klose, A.Murugan, '09] for $U(3)_c$).