# Deep learning generation of holographic geometries from $q\bar{q}$ potential

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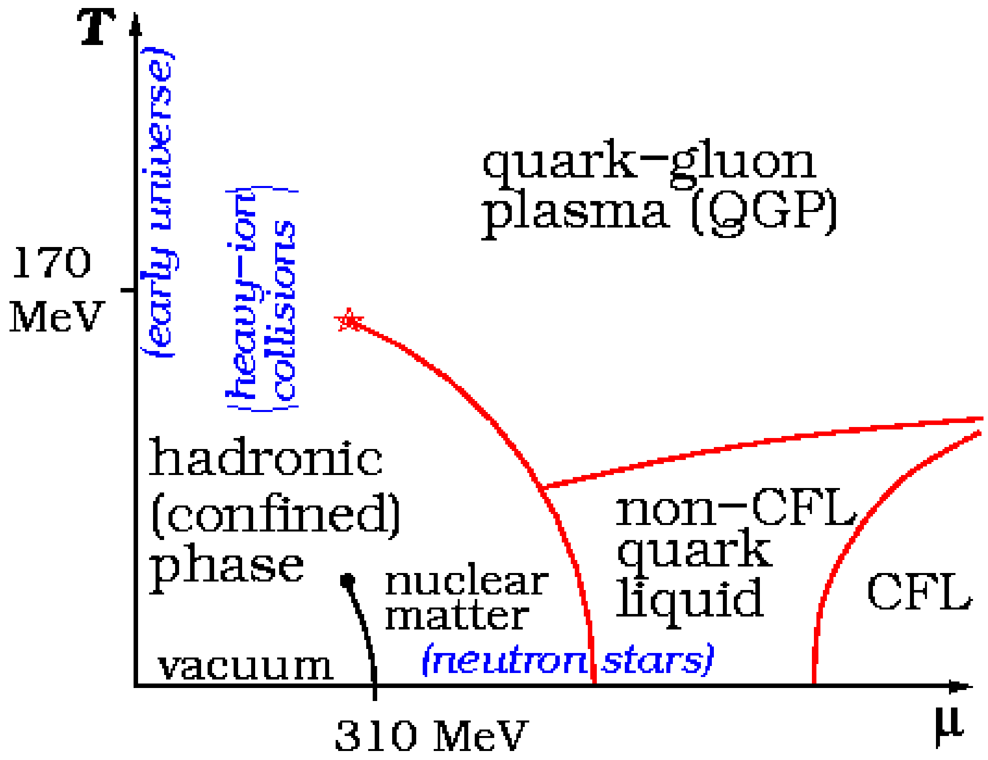
# Motivation




- QCD phase diagram. Source: Wikipedia

# Motivation

## Achievements of holographic theories:

•Confinement

•Shear viscosity calculation

•Probe into the finite temperature and chemical potential region

## Construction of dual theories to QCD

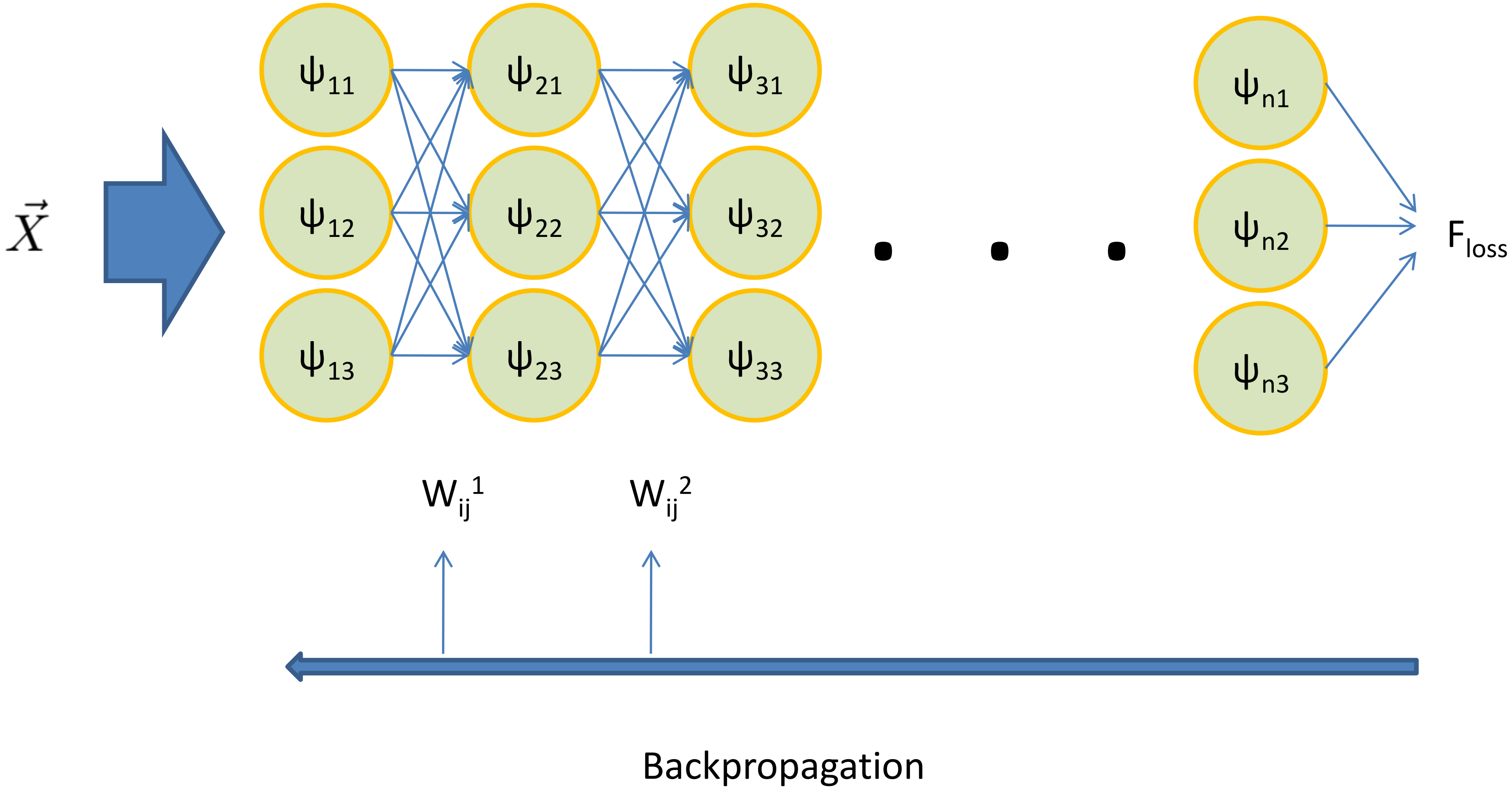
• Top down approach: ej Sakai-Sugimoto model [0412141,0507073 ]

•Bottom up: ej: Improved Holographic QCD
[0707.1324 Gursoy,Kiristis; 0707.1349 Gursoy, Kiristis, Nitti; 1112.1261 Jarvinen, Kiristis]

## Possibility of precission holography

• Recent work by N.Jokela and A.Ponni [ 2007.00010] suggest that no interpolation of the data is required to obtain results.

# Neural network structure



# Metric reconstruction

We try to reconstruct a dual gravity metric using a bottom up approach

- Deep learning has a natural interpretation as holographic RG flow
  [1705.05750] Wen-Cong Gan, Fu-Wen Shu

- Data from the QFT: heavy quark potential

- Our parameters (weights) are the metric components in the dual gravity model

- We use a Deep Learning algorithm (pytorch) to fit a metric into the data.

# Wilson loops and metric reconstruction

Wilson loop definition

$$\mathcal{W}(\mathcal{C}) = \frac{1}{N} \mathrm{Tr}\, \mathcal{P} \left( e^{i \oint_{\mathcal{C}} d\tau \dot{x}^{\mu} A_{\mu}} \right)$$

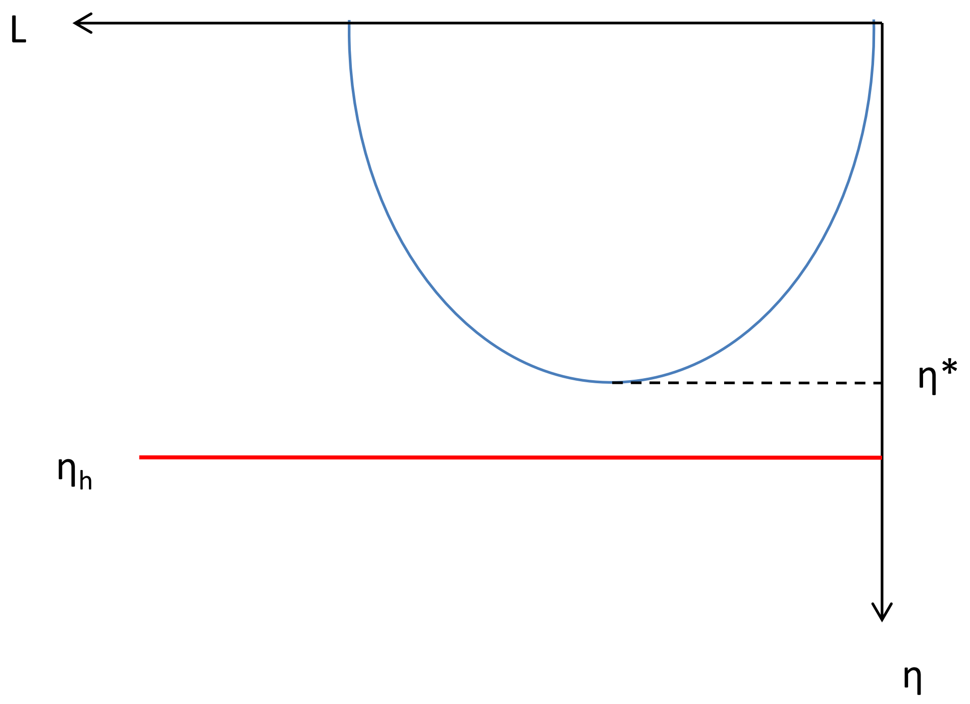
A rectangular wilson loop can be related to the potential between quarks

$$\langle W \rangle = e^{-iTV(L)}$$

# Wilson loops and metric reconstruction

Gravity dual

- String attached to the trajectory in the boundary [9803002] Maldacena
- Black Hole in the IR to make the theory have a finite temperature (Witten)



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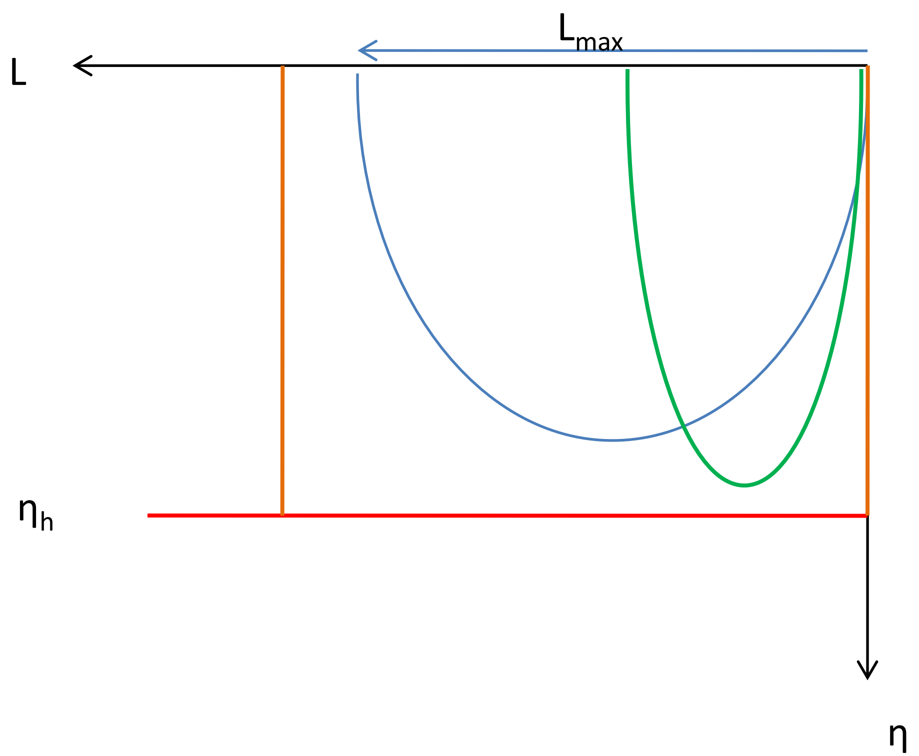


# Wilson loops and metric reconstruction

- Wilson loops data can be used to reconstruct a dual metric [2008.10883] Hashimoto

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)d\mathbf{x}^2$$

$$L = 2\int_{\eta_*}^{\Lambda} d\eta \frac{p}{g^{1/2}\sqrt{fg - p^2}}$$

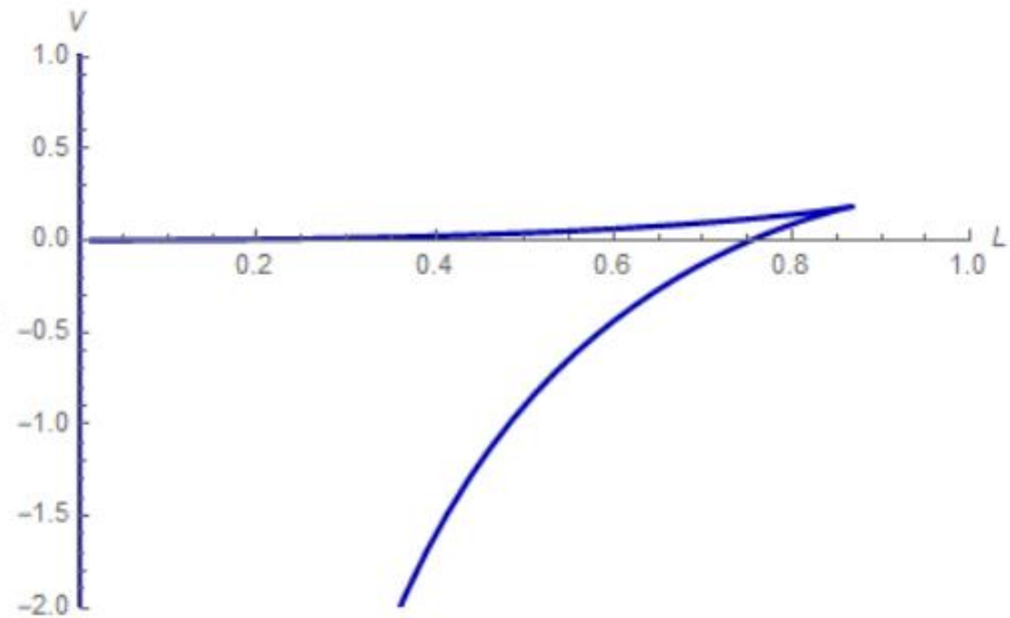
$$V = 2\int_{\eta_*}^{\Lambda} d\eta \left(\frac{fg^{1/2}}{\sqrt{fg-p^2}} - 2R\sqrt{f(\eta)}\right) - 2\int_0^{\eta_*} d\eta\sqrt{f}$$

$$p^2 = f(\eta_*)g(\eta_*)$$

- This was used to reconstruct, for example, the near horizon geometry of a confining theory

# Wilson loops and metric reconstruction

Full reconstruction can be hard due to non-dominant configurations appearing in the IR.

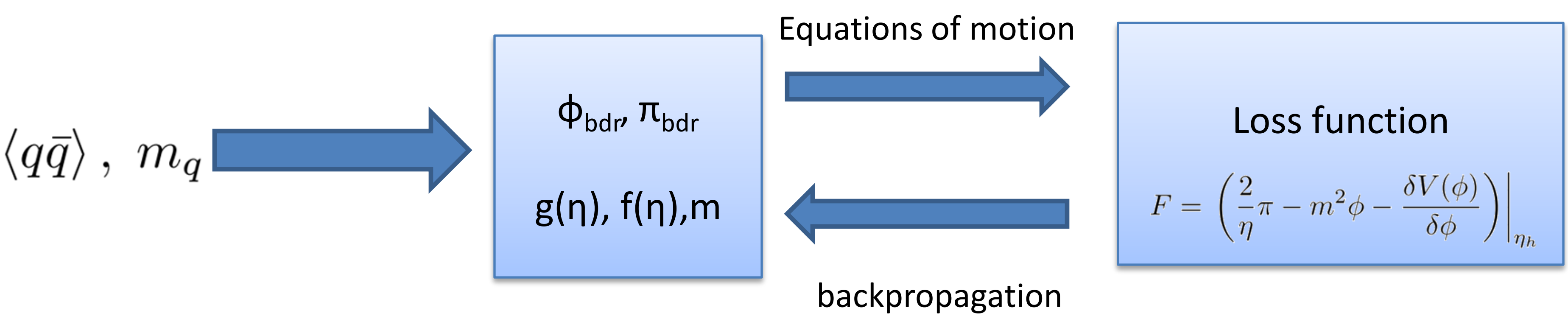



These kind of configurations make a full inversion not feasible

# AdS-DL background

The proposed way for using Deep Learning to reconstruct the metric of the gravity dual is the following (we follow [1802.08313] Hashimoto et al. method):

$$S=\int dx^{d+1}\sqrt{-det(h)}\left[-\frac{1}{2}\left(\partial^{\mu}\phi\right)^{2}+m\phi^{2}-V(\phi)\right]$$



# AdS-DL background

This method was used in [1809.1053]Hashimoto, Sugishita, Tanaka, Tomiya to reproduce an a toy model candidate for a gravity dual to QCD from a metric of the form:

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)d\mathbf{x}^2$$

- Computed from the quark condensate and quark mass
- However this observables are only related through the magnitude g(η)*f(η).
- The dependence of the Wilson Loop on the metric is more complex and we attempt to achieve more accurate results using it.

# Complex potential in QCD.

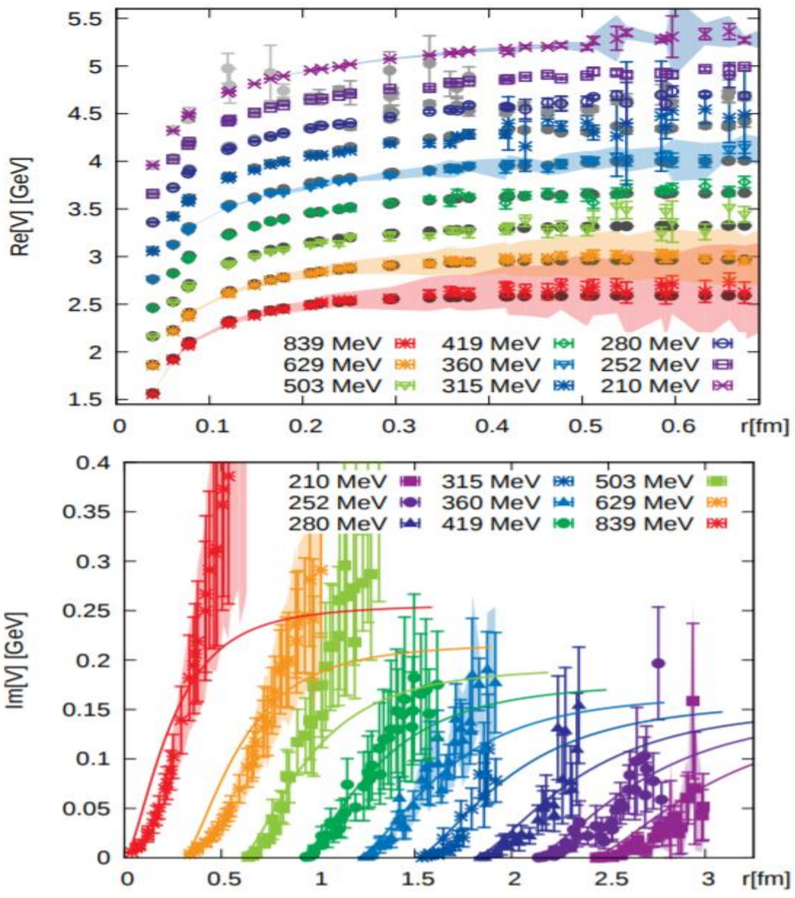
Potential from a Wilson Loop in a thermal bath:

$$V(r) = \lim_{t\to\infty} \frac{i\partial_t W(t,r)}{W(t,r)}$$

- This potential was first found in perturbation theory
- Can also be obtained as a result from Lattice [1410.2546] Burnier, Kaczmarek, Rothkopf
- This potential is a complex quantity

# Complex potential in QCD.

Real and imaginary parts of the potential
Source : [1410.2546] Burnier, Kaczmarek, Rothkopf



# Complex potential in holography

The potential obtained from a Wilson loop can be extended to complex values at greater lengths [0807.4747] Albacete, Kovchegov, Taliotis

$$L = 2\int_{\eta_*}^{\Lambda} d\eta \frac{p}{g^{1/2}\sqrt{fg - p^2}}$$

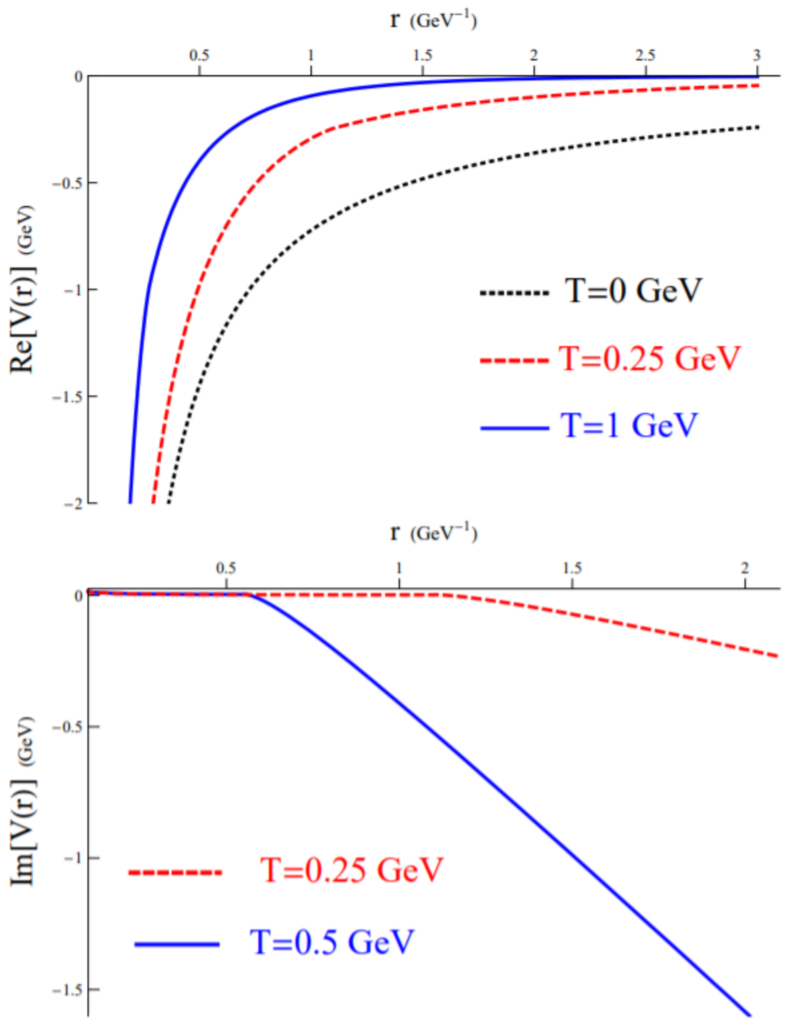
$$V = 2\int_{\eta_*}^{\Lambda} d\eta \left( \frac{f g^{1/2}}{\sqrt{fg - p^2}} - 2R\sqrt{f(\eta)} \right) - 2\int_0^{\eta_*} d\eta \sqrt{f}$$

Anti-deSitter space admits an analytic solution for L

$$L(\eta_*) = 2\pi^{3/2}\eta_h \frac{\frac{\eta_*^4}{\eta_h^4}\sqrt{2 - 2\frac{\eta_*^4}{\eta_h^4}}\,{}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{5}{4}; \frac{\eta_*^4}{\eta_h^4}\right)}{\Gamma\left(\frac{1}{4}\right)^2}$$

- For greater L, we need complex values of the deepest point of the string.
- This is identified with a saddle point of the Nambu-Goto action.
- This values also lead to complex-valued potential

# Complex potential in holography

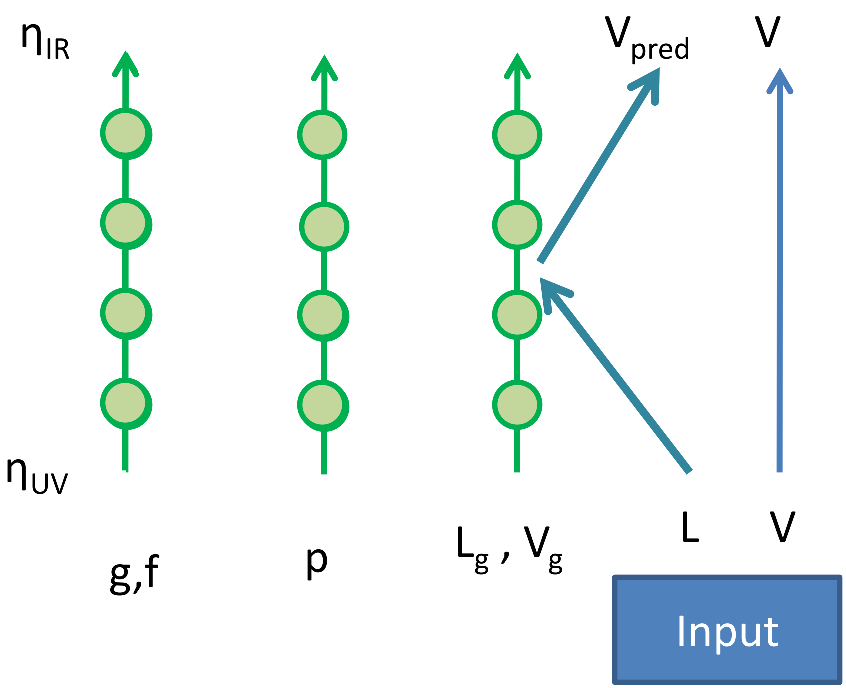



Source: Albacete, Kovchegov, Taliotis [0807.4747]

# Neural network: Reconstruction of $AdS_5$

•We identify the layers of the network with the depth of the bulk. We use 40 layers

•We use both an IR and UV cutoff




•We compute the length and potential for a string ending at each Layer

•Each step in the net performs the integral discussed before for V and L until the tip

• The curves L(r), V(r) are reconstructed with a linear interpolation between each point.

• Our data sample consists of pairs of L and V. The network uses L to determine r, and then computes the predicted V for such value.

• The loss function is determined from the difference between the predicted $V_{pred}$ and V.

# Neural network: Reconstruction of $AdS_5$

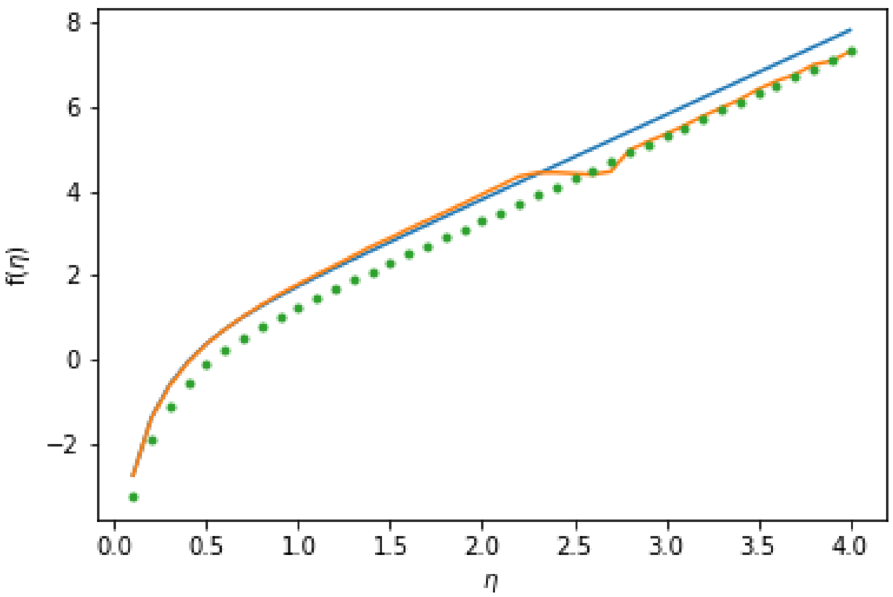
Preliminary results

Additional loss functions.

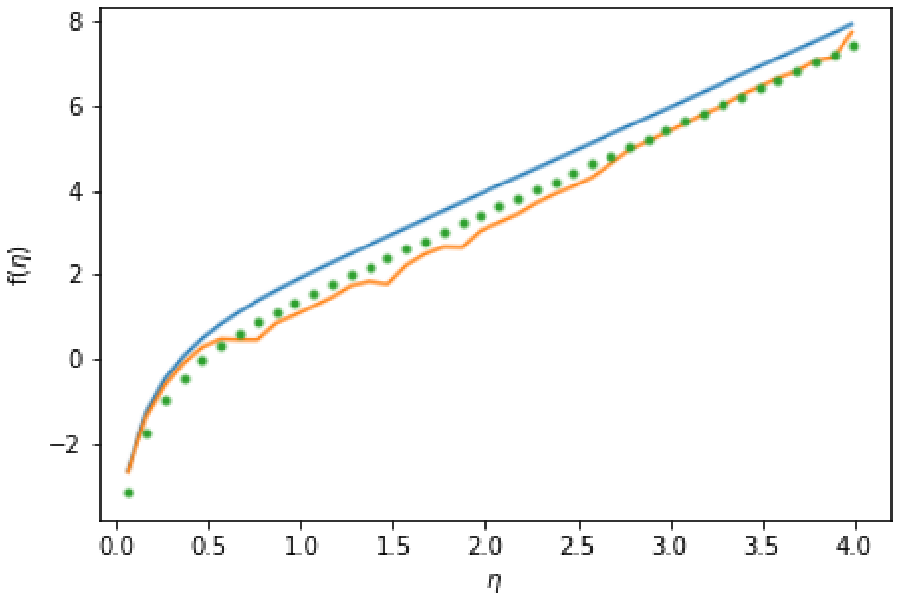
- $f(\eta)*g(\eta)$ monotonicity

- Fixed f and g boundary values at the UV cutoff

f(η) during learning



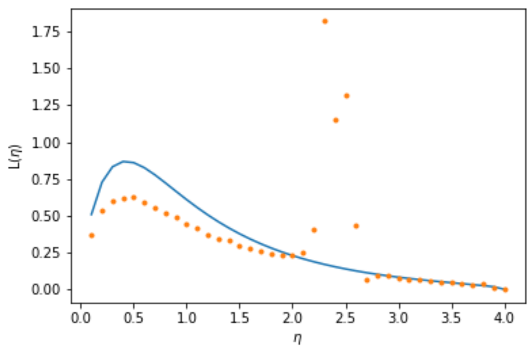

f(η) after learning



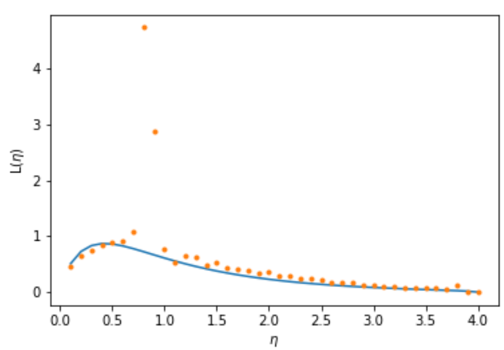
# Neural network: Reconstruction of $AdS_5$

Preliminary results

## Generated L

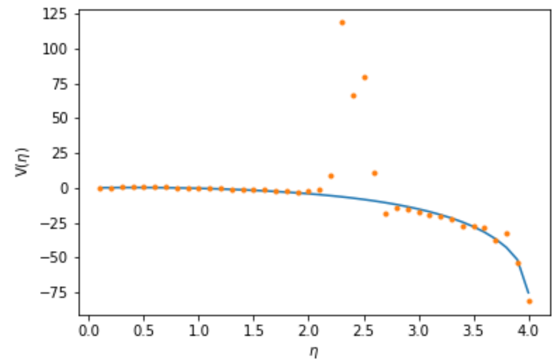



L during learning

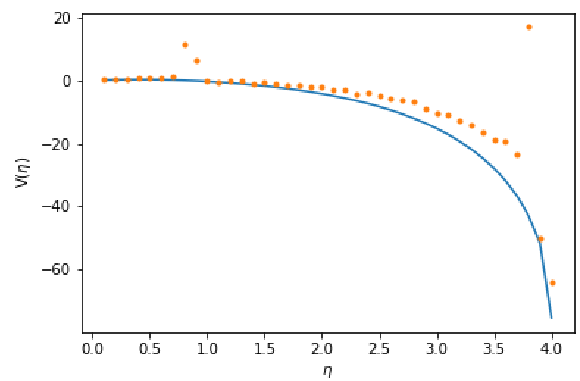



L after learning

## Generated V



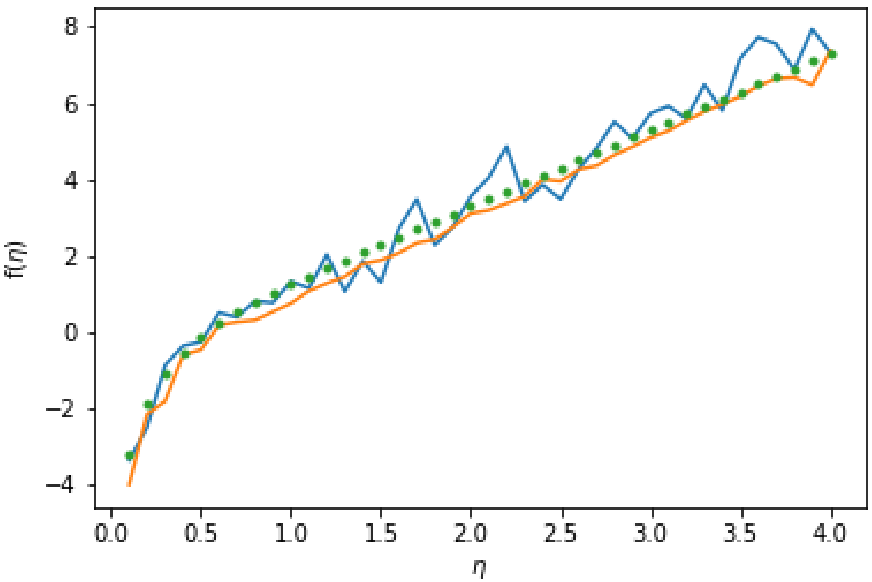

V during learning

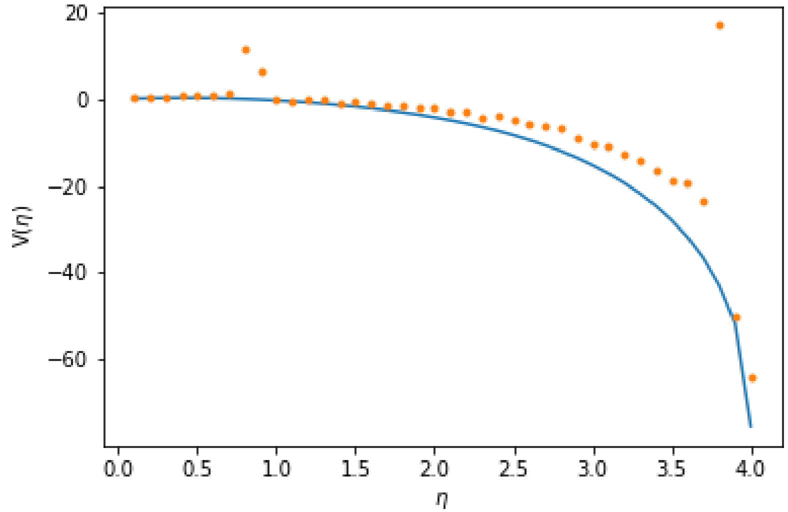


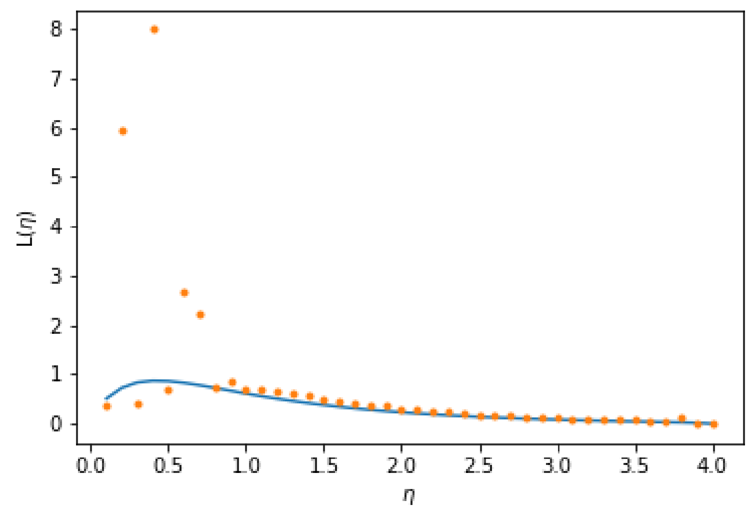

V after learning

# Neural network: Reconstruction of $AdS_5$

Preliminary results



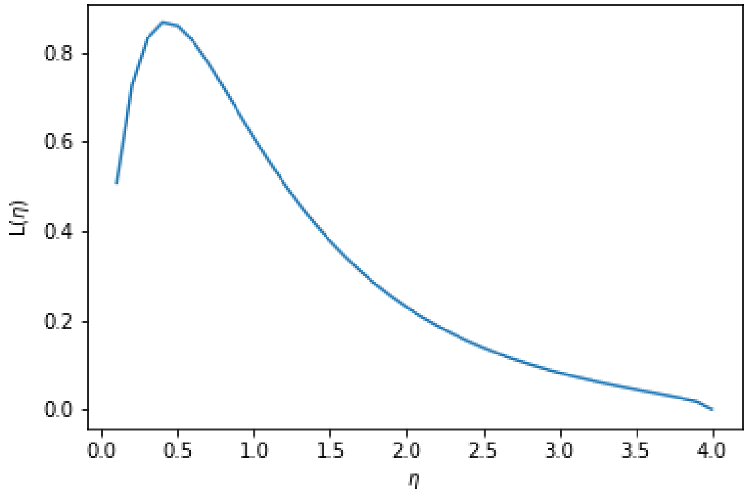
# Numerical complex potential

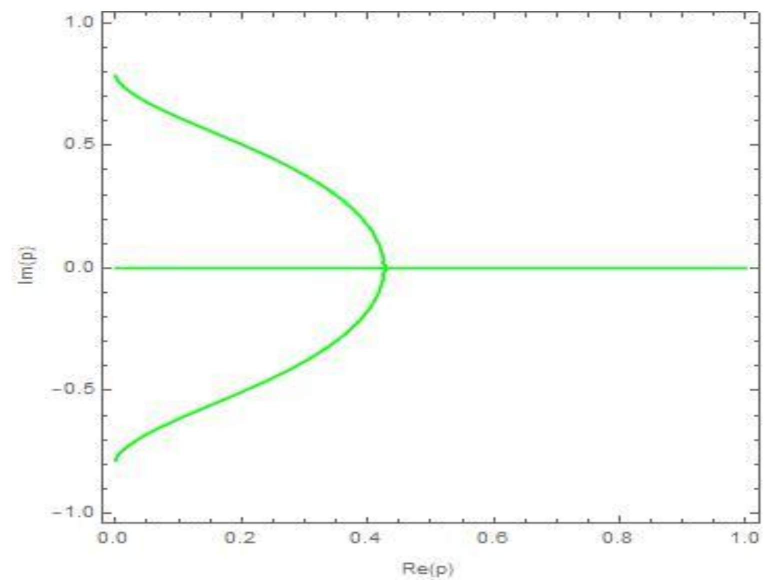
Probes into the IR:

- Analytic continuation of L and V for each metric

- Solve Cauchy-Riemann equations until a new solution for real L is found at another set of p*'s

- Solve CR-equations for the potential for new p*'s.



# Conclusions and future research

• Complex potential between quarks is a viable to construct holographic duals to a QFT

• The evaluation of the potential seems compatible with the method of gradient descent of deep neural networks.

•Future lines of research:

1. Improvement of the network efficiency with additional loss functions
2. Apply this method to data from QCD Lattice
3. New predictions from the obtained metric
4. Add gauge fields for small $\mu$ lattice comparison
5. Extension to out of equilibrium situations