$T\overline{T}$-deformation of q-Yang-Mills theory

Joint work with Richard J. Szabo and Miguel Tierz:

JHEP (2019) [1810.05404], JHEP (2020) [2009.00657](today)

Leonardo Santilli

Iberian Strings, IST Lisbon via Zoom

20/01/2021

Grupo de Física Matemática & Departamento de Matemática

Faculdade de Ciências, Universidade de Lisboa

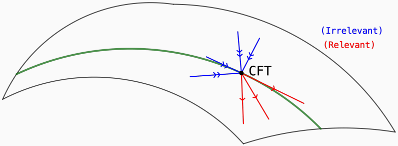
Overview

- Introduction: perturbing a QFT with the operator $T\overline{T}$.
- Lightening review: The family of two-dimensional Yang-Mills theories.
- $T\overline{T}$-deformed q-Yang-Mills.
- Outlook.

Introduction

Perturbing QFTs

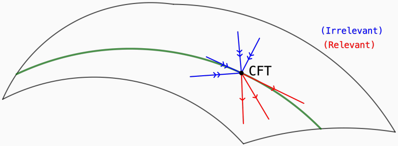
Understanding all possible deformations of a CFT allows to explore its neighbourhood in the space of theories.



Eventually, we want to chart the space of all consistent QFTs.

Perturbing QFTs

Understanding all possible deformations of a CFT allows to explore its neighbourhood in the space of theories.



Eventually, we want to chart the space of all consistent QFTs. *Vaste programme!*

More instructive approach: characterize deformations that preserve a distinguished property, e.g. integrability.

Perturbing with $T\overline{T}$: Definition

Irrelevant deformations of two-dimensional QFTs by the composite operator $T\overline{T}$ *[Cavaglià-Negro-Szécsényi-Tateo,Smirnov-Zamolodchikov]* have attracted considerable attention in recent years.

$$T = -2\pi T_{zz}, \ \overline{T} = -2\pi T_{\bar{z}\bar{z}}, \ \Theta = 2\pi T_{z\bar{z}}$$

$$T\overline{T} = \lim_{(z',\bar{z}')\to(z,\bar{z})} \left(T(z,\bar{z})\overline{T}(z',\bar{z}') - \Theta(z,\bar{z})\Theta(z',\bar{z}') \right) + (\text{total deriv.})$$

That is, $T\overline{T} = \det[\text{stress-energy } T]$ in complex coordinates, regularized with point splitting.

Perturbing with $T\overline{T}$: Generalities

Original motivation: it is the lowest spin representative of a family of integrability preserving deformations *[Smirnov-Zamolodchikov]*.

The deformation is solvable: follow quantities along the irrelevant flow. The energy levels evolve according to a Burgers equation.

Perturbing with $T\overline{T}$: Generalities

Original motivation: it is the lowest spin representative of a family of integrability preserving deformations *[Smirnov-Zamolodchikov]*.

The deformation is solvable: follow quantities along the irrelevant flow. The energy levels evolve according to a Burgers equation.

Regardless of the original QFT being integrable, $T, \overline{T}, \Theta$ are conserved $\Longrightarrow$ $T\overline{T}$ exists in a QFT under very mild assumptions.

Perturbing with $T\overline{T}$: Applications

Broad variety of applications in 2d QFT, including

- integrability,
- holography (both $\mathrm{AdS}_3/\mathrm{CFT}_2^{T\overline{T}}$ and $\mathrm{AdS}_2^{T\overline{T}}/\mathrm{CFT}_1$),
- SCFTs,
- perturbation by generalized irrelevant operators, etc.

Deformed theory: How to get it

Turn on the deformation: plug $\tau T\overline{T}$ in the Lagrangian.

Direct approach: obtain deformed quantities solving the flow equation. "Solvability" is at work: this can often be done to all orders, obtaining closed form expressions *[Many authors, including: Bonelli,Conti,Doroud,HernándezChifflet,Iannella,Jiang,Negro,Sfondrini,Tateo,Zhu,...]*.

Deformed theory: How to get it

Turn on the deformation: plug $\tau T\overline{T}$ in the Lagrangian.

Direct approach: obtain deformed quantities solving the flow equation. "Solvability" is at work: this can often be done to all orders, obtaining closed form expressions *[Many authors, including: Bonelli,Conti,Doroud,HernándezChifflet,Iannella,Jiang,Negro,Sfondrini,Tateo,Zhu,...]*.

Breakthrough insight *[Dubovsky-Gorbenko-HernándezChifflet]*:

$$T\overline{T}\text{-def. } \mathrm{QFT}_2 \; \cong \; \mathrm{QFT}_2 + \mathrm{JT \ gravity}$$

We use Euclidean formulation *[Coleman-AguileraDamia-Freedman-Sony]*. It is equivalent to a field-dependent change of variables *[Conti-Negro-Tateo]*.

The family of two-dimensional Yang-Mills theories

Yang-Mills theory in 2d

Yang-Mills action on a Riemann surface Σ:

$$S_{\mathrm{YM}} = \frac{1}{2g_{\mathrm{YM}}^2} \int_\Sigma \mathrm{Tr}\, F^A * F^A = \int_\Sigma \mathrm{Tr}\left(\mathrm{i}\, \phi\, F^A + \frac{g_{\mathrm{YM}}^2}{2} \phi^2\, \omega \right)$$

with the second equality true in 2d inside the path integral.

Yang-Mills theory in 2d

Yang-Mills action on a Riemann surface Σ:

$$S_{\mathrm{YM}} = \frac{1}{2g_{\mathrm{YM}}^2} \int_{\Sigma} \mathrm{Tr}\, F^A * F^A = \int_{\Sigma} \mathrm{Tr}\Big(\mathrm{i}\, \phi\, F^A + \frac{g_{\mathrm{YM}}^2}{2} \phi^2\, \omega \Big)$$

with the second equality true in 2d inside the path integral.

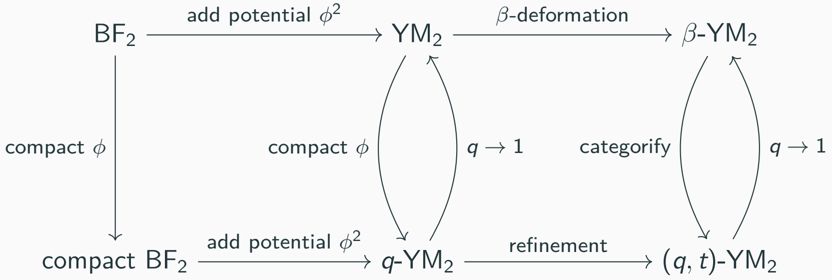
Noteworthy aspects:

- YM_2 (or def.) $\cong$ topological BF_2 (or def.) $+ V(\phi)$.
- Dependence only on volume form ω, not on metric.
- Abelianization *[Blau-Thompson]* $\Rightarrow$ path integral reduces to a finite-dimensional expression:

$$\mathcal{Z}_{q\text{-}\mathrm{YM}}[\Sigma] = \sum_R (\dim_q R)^{\chi(\Sigma)}\, q^{\frac{p}{2} C_2(R)}.$$

Two-dimensional Yang-Mills and its deformations

Web of Yang-Mills theories in 2d:



Type IIA string theory on threefold

$$X = \mathrm{Tot}\left(\mathcal{O}(p-\chi(\Sigma)) \oplus \mathcal{O}(-p) \longrightarrow \Sigma\right)$$

with N D4 branes on the 4-cycle

$$C_4 = \mathrm{Tot}\left(\mathcal{O}(-p) \longrightarrow \Sigma\right)$$

and any number of D0 branes, and possibly with D2 branes wrapping Σ.

q-Yang-Mills from string theory

Type IIA string theory on threefold

$$X = \mathrm{Tot}\left(\mathcal{O}(p - \chi(\Sigma)) \oplus \mathcal{O}(-p) \longrightarrow \Sigma\right)$$

with N D4 branes on the 4-cycle

$$C_4 = \mathrm{Tot}\left(\mathcal{O}(-p) \longrightarrow \Sigma\right)$$

and any number of D0 branes, and possibly with D2 branes wrapping Σ.

Theory on C_4: U(N) twisted 4d $\mathcal{N} = 4$ super-Yang-Mills. It reduces to Yang-Mills on Σ, with modified path integral measure, i.e. q-Yang-Mills *[Aganagic,Ooguri,Saulina,Vafa]*.

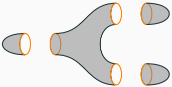
Corollary: $\mathcal{Z}_{q\text{-YM}}[\Sigma]$ counts black holes BPS states.

$T\overline{T}$-deformed q-Yang-Mills

$T\overline{T}$-deformation of two-dimensional Yang-Mills: How to get it

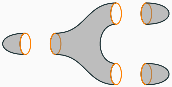
$T\overline{T}$-deform YM_2 (and its relatives):

- start with BF_2 and use TFT axioms to cut Σ in elementary pieces;



$T\overline{T}$-deform YM_2 (and its relatives):

- start with BF_2 and use TFT axioms to cut Σ in elementary pieces;

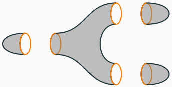


- turn on $V(\phi)$ (and q-deformation) on each component;

$T\overline{T}$-deformation of two-dimensional Yang-Mills: How to get it

$T\overline{T}$-deform YM_2 (and its relatives):

- start with BF_2 and use TFT axioms to cut Σ in elementary pieces;

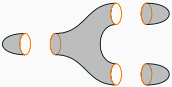


- turn on $V(\phi)$ (and q-deformation) on each component;
- couple each flat component to JT gravity;

$T\overline{T}$-deformation of two-dimensional Yang-Mills: How to get it

$T\overline{T}$-deform YM_2 (and its relatives):

- start with BF_2 and use TFT axioms to cut Σ in elementary pieces;



- turn on $V(\phi)$ (and q-deformation) on each component;
- couple each flat component to JT gravity;
- topological part is unaffected.

$T\overline{T}$-deformation of two-dimensional Yang-Mills: Results

We have shown that

- solving the gravitational path integral gives $V(\phi) \mapsto \frac{V(\phi)}{1-\frac{\tau}{N^3}V(\phi)}$,
- Abelianization persists, and
- we can glue back the pieces, obtaining the $T\overline{T}$-def. theory on Σ.

$T\overline{T}$-deformation of two-dimensional Yang-Mills: Results

We have shown that

- solving the gravitational path integral gives $V(\phi) \mapsto \frac{V(\phi)}{1-\frac{\tau}{N^3} V(\phi)}$,
- Abelianization persists, and
- we can glue back the pieces, obtaining the $T\overline{T}$-def. theory on Σ.

For closed Σ,

$$\mathcal{Z}_{q\text{-YM}}^{T\overline{T}}[\Sigma] = \sum_R \left(\dim_q R\right)^{\chi(\Sigma)} \; q^{\frac{p}{2} C_2^{T\overline{T}}(R,\tau)}$$

with $C_2^{T\overline{T}}(R,\tau) = \frac{C_2(R)}{1-\frac{\tau}{N^3} C_2(R)}$.

We have shown that

- solving the gravitational path integral gives $V(\phi) \mapsto \frac{V(\phi)}{1-\frac{\tau}{N^3} V(\phi)}$,
- Abelianization persists, and
- we can glue back the pieces, obtaining the $T\overline{T}$-def. theory on Σ.

For closed Σ,

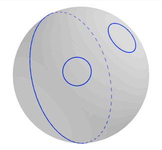
$$\mathcal{Z}_{q\text{-YM}}^{T\overline{T}}[\Sigma] = \sum_{R} \left(\dim_q R\right)^{\chi(\Sigma)} \ q^{\frac{p}{2} \, C_2^{T\overline{T}}(R,\tau)}$$

with $C_2^{T\overline{T}}(R,\tau) = \frac{C_2(R)}{1-\frac{\tau}{N^3} C_2(R)}$. (Derived earlier by *[Conti-Iannella-Negro-Tateo, Ireland-Shyam]*, but without proof of Abelianization.)

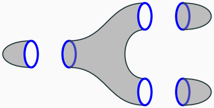
$T\overline{T}$-deformation of two-dimensional Yang-Mills: More results

Similar expressions can be obtained for: Σ with boundaries and/or punctures, Wilson loops, symmetry defects. For example:

$$\langle W_{R_1} \cdots W_{R_s} \rangle \propto \sum_{R} (\dim_q R)^{\chi(\Sigma)-s} q^{\frac{p-a}{2} C_2^{T\overline{T}}(R)} \prod_{i=1}^{s} \sum_{\widetilde{R}_i} (\dim_q \widetilde{R}_i) q^{\frac{a_i}{2} C_2^{T\overline{T}}(\widetilde{R}_i)} \mathrm{N}_{R R_i}^{\widetilde{R}_i}$$



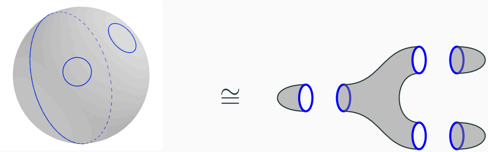
$\cong$



$T\overline{T}$-deformation of two-dimensional Yang-Mills: More results

Similar expressions can be obtained for: Σ with boundaries and/or punctures, Wilson loops, symmetry defects. For example:

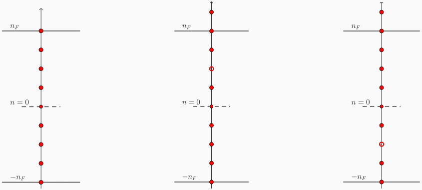
$$\langle W_{R_1} \cdots W_{R_s} \rangle \propto \sum_{R} (\dim_q R)^{\chi(\Sigma)-s} q^{\frac{p-a}{2} C_2^{T\overline{T}}(R)} \prod_{i=1}^{s} \sum_{\widetilde{R}_i} (\dim_q \widetilde{R}_i) q^{\frac{a_i}{2} C_2^{T\overline{T}}(\widetilde{R}_i)} \mathrm{N}_{R R_i}^{\widetilde{R}_i}$$



Motto: $T\overline{T}$-deformation preserves Abelianization.

However, perturbing with $T\overline{T}$ spoils

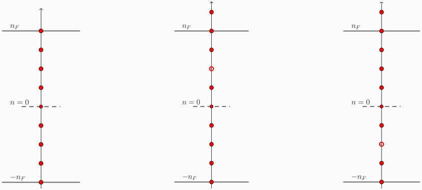
- Factorization of the partition function *[Many authors, including: Aganagic,Caporaso,Cirafici,Griguolo,Ooguri,Pasquetti,Saulina,Seminara,Szabo,Tanzini,Vafa...]*. In the free fermion formalism *[Douglas]* $T\overline{T}$ introduces non-local interactions.



$T\overline{T}$-deformation of two-dimensional Yang-Mills: No-go

However, perturbing with $T\overline{T}$ spoils

- Factorization of the partition function *[Many authors, including: Aganagic,Caporaso,Cirafici,Griguolo,Ooguri,Pasquetti,Saulina,Seminara,Szabo,Tanzini,Vafa...]*. In the free fermion formalism *[Douglas]* $T\overline{T}$ introduces non-local interactions.



- Truncation of sum to integrable R at $q \to 1$ and connection with Chern-Simons theory.

Large N phase transition

Large N phase transition without $T\overline{T}$

U(N) YM_2 on $\Sigma = \mathbb{S}^2$ undergoes a 3rd order phase transition *[Douglas-Kazakov]* induced by instantons *[Gross-Matytsin]*.

Large N phase transition without $T\overline{T}$

$U(N)$ YM_2 on $\Sigma = \mathbb{S}^2$ undergoes a 3rd order phase transition *[Douglas-Kazakov]* induced by instantons *[Gross-Matytsin]*.

The same is true for q- and (q,t)-YM_2 and $p > 2$ *[Caporaso-Cirafici-Griguolo-Pasquetti-Seminara-Szabo,Arsiwalla-Boels-Mariño-Sinkovics,Jafferis-Marsano,Kokenyesi-Sinkovics-Szabo]*. The q-deformation enhances the volume of the weak coupling phase in parameter space.

Large N phase transition with $T\overline{T}$ (but no q-deformation)

U(N) YM$_2^{T\overline{T}}$ on $\mathbb{S}^2$ undergoes a 3rd order transition induced by instantons *[LS-Tierz]*. $T\overline{T}$-deformation reduces the volume of the weak coupling phase in parameter space.

Idea of proof: expand $C_2^{T\overline{T}}$ as a geometric series in τ. Large N equilibrium measure $\rho(x)\mathrm{d}x$ determined by

$$\int \mathrm{d}u \frac{\rho(u)}{x-u} = \frac{A}{\chi} x \sum_{k=0}^{\infty} (k+1)\, \tau^k \left(\mu_2 - \frac{1}{12} \right)^k ,$$

$\mu_2 = \int \rho(u) u^2 \mathrm{d}u$. Solve it perturbatively: at each order in τ, simply get renormalized A.

Large N phase transition with $T\overline{T}$ (but no q-deformation)

U(N) YM$_2^{T\overline{T}}$ on $\mathbb{S}^2$ undergoes a 3rd order transition induced by instantons *[LS-Tierz]*. $T\overline{T}$-deformation reduces the volume of the weak coupling phase in parameter space.

Idea of proof: expand $C_2^{T\overline{T}}$ as a geometric series in τ. Large N equilibrium measure $\rho(x)\mathrm{d}x$ determined by

$$\int \mathrm{d}u \frac{\rho(u)}{x-u} = \frac{A}{\chi} x \sum_{k=0}^{\infty} (k+1)\,\tau^k \left(\mu_2 - \frac{1}{12}\right)^k ,$$

$\mu_2 = \int \rho(u) u^2 \mathrm{d}u$. Solve it perturbatively: at each order in τ, simply get renormalized A.

Renormalization of A by a positive coefficient reduces the weak coupling phase. Look at instantons: suppression factor vanishes earlier with $T\overline{T}$ turned on.

Large N phase transition with both $T\overline{T}$ and q-deformation

q- and (q,t)-$\mathrm{YM}_2^{T\overline{T}}$ experience again a instanton-induced 3rd order phase transition. Volume of weak coupling phase is

- reduced, compared to q-YM_2
- enhanced, compared to $\mathrm{YM}_2^{T\overline{T}}$

Proof as for ordinary $\mathrm{YM}_2^{T\overline{T}}$. Look at instantons: both effects combine in deforming the suppression factor.

Large N phase transition with both $T\overline{T}$ and q-deformation

q- and (q,t)-$\mathrm{YM}_2^{T\overline{T}}$ experience again a instanton-induced 3rd order phase transition. Volume of weak coupling phase is

- reduced, compared to q-YM_2
- enhanced, compared to $\mathrm{YM}_2^{T\overline{T}}$

Proof as for ordinary $\mathrm{YM}_2^{T\overline{T}}$. Look at instantons: both effects combine in deforming the suppression factor.

Transition extends below $p = 2$ *[LS-Szabo-Tierz]* all the way down to $p = 1$ (confirmed later in *[Gorsky-Pavshinkin-Tyutyakina]*).

Outlook

Yang-Mills-Higgs and integrable systems

Add Higgs field to the story $\Longrightarrow$ Wavefunctions of 2d U(N) Yang-Mills-Higgs correspond to those of non-linear Schrödinger *[Gerasimov-Shatashvili]*.

Yang-Mills-Higgs and integrable systems

Add Higgs field to the story $\Longrightarrow$ Wavefunctions of 2d U(N) Yang-Mills-Higgs correspond to those of non-linear Schrödinger *[Gerasimov-Shatashvili]*.

Reminder: $T\overline{T}$ preserves integrability.

Does the correspondence survive in the $T\overline{T}$-deformed theories?

Prospects

But why do we care?

But why do we care?

Eventually, we would like to gain analytic control on flows by irrelevant operators in higher dimensions.

$$
\begin{array}{ccc}
\mathcal{T}_A^{(d=2)} & \longleftrightarrow & \mathcal{T}_B^{(d>2)} \\
\quad T\overline{T}\text{-deformation} \Big\downarrow & & \vdots \ \blacksquare\text{-deformation (?)} \\
\mathcal{T}_A^{T\overline{T}(d=2)} & \leftarrow\text{-}\text{-}\text{-}\text{-}\text{-}\text{-}\rightarrow & \mathcal{T}_B^{\bullet(d>2)}
\end{array}
$$

Prospects

YM_2 is tightly related to four-dimensional SUSY theories:

- q-YM_2 descends from twisted 4d $\mathcal{N}=4$ super-YM in $C_4 = \mathrm{Tot}\left(\mathcal{O}(-p) \longrightarrow \Sigma\right)$ *[Aganagic-Ooguri-Saulina-Vafa]*;
- Class S theories *[Gaiotto,Gadde-Rastelli-Razamat-Yan]*;
- The 0-instanton sector of YM_2 on $\mathbb{S}^2$ appears from localization of $\mathcal{N}=2$ with defects *[Pestun,Giombi,Wang,Komatsu]*.

Prospects

YM_2 is tightly related to four-dimensional SUSY theories:

- q-YM_2 descends from twisted 4d $\mathcal{N}=4$ super-YM in $C_4 = \mathrm{Tot}\left(\mathcal{O}(-p) \longrightarrow \Sigma\right)$ *[Aganagic-Ooguri-Saulina-Vafa]*;
- Class S theories *[Gaiotto,Gadde-Rastelli-Razamat-Yan]*;
- The 0-instanton sector of YM_2 on $\mathbb{S}^2$ appears from localization of $\mathcal{N}=2$ with defects *[Pestun,Giombi,Wang,Komatsu]*.

What happens if we couple the 2d sector to JT gravity? Can we pull the $T\overline{T}$-deformation back to 4d?

This is the end of the presentation.

Thank you for your attention.