A Non-Relativistic Limit of NS-NS Gravity

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Work in progress w/ E. Bergshoeff, L. Romano, J. Rosseel, and C. Simsek

A. Introduction and Motivation

B. Non-Relativistic Geometry

C. Gomis-Ooguri String Theory

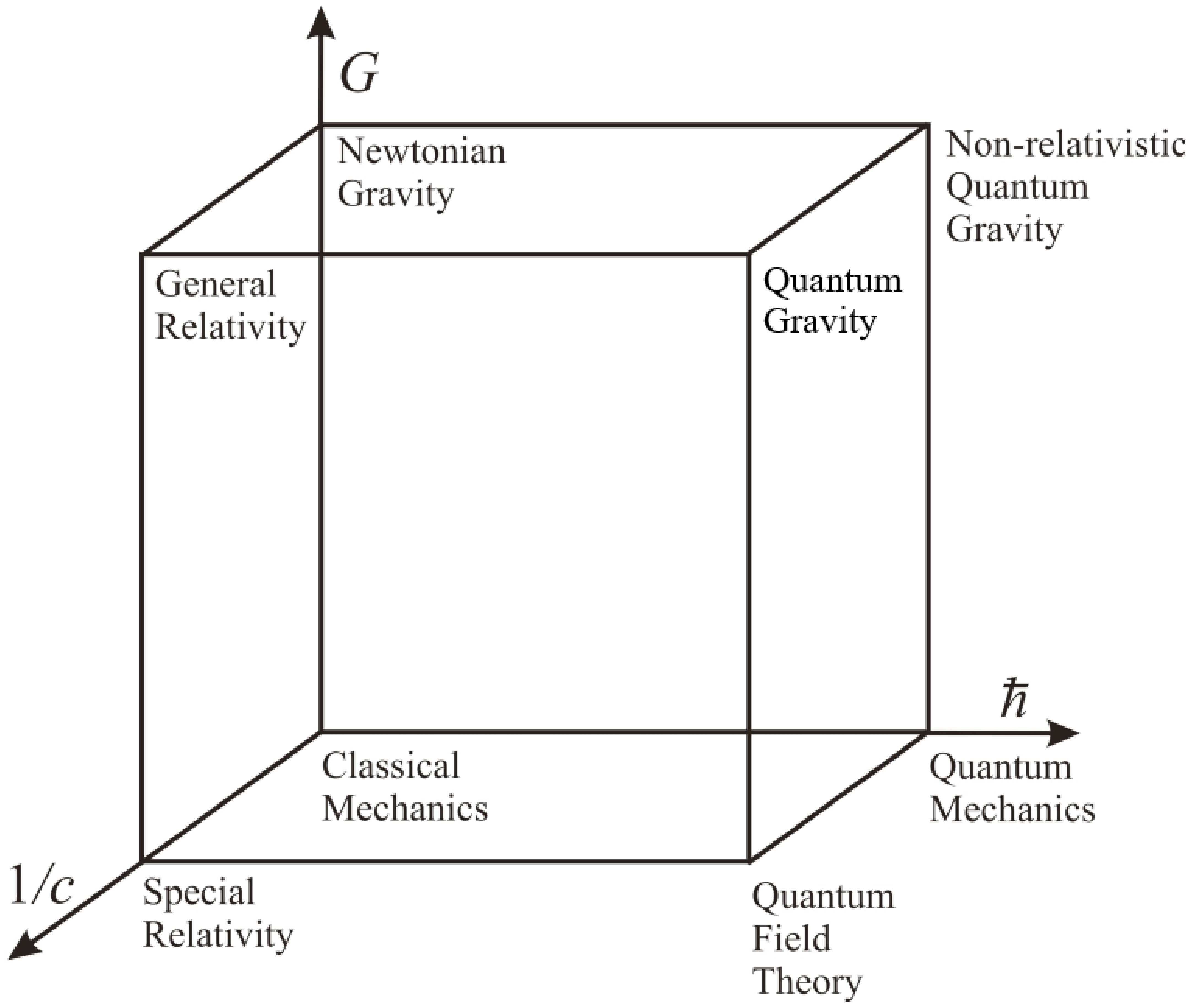
D. Non-Relativistic NS-NS Gravity

E. Open Questions and Outlook

A. Introduction and Motivation

Introduction and Motivation

Quantum Gravity through the Back Door?



[Bronstein ’33]

Non-Relativistic Holography: Bottom Up?

Want to Understand:

A. Non-relativistic supergravity in 10 D
B. Relation to open/closed string dynamics
C. Brane solutions
D. Dualities
E. Non-relativistic M-Theory

Disclaimer: *Non-relativistic* refers to Galilean-type spacetime symmetries

$$[\text{Time Translation}, \text{Boost}] = \text{Space Translation}$$

$$[\text{Space Translation}, \text{Boost}] = \text{Time Translations}$$

$$[\text{Boost}, \text{Boost}] = \text{Spatial Rotations}$$

References:

A. Early work: Gomis-Ooguri '00, Danielsson et.al. '00, Klebanov-Maldacena '01, Gomis-Gomis-Kamimura '05, …

B. Geometric Structure: Cartan '24, Trautman et.al. '60s, Bergshoeff et.al. '12, Hartong-Obers et.al. '13

C. Quantum Consistency: Gomis-Oh-Wu-Yan '19, Gürsoy-Gallegos-Zinnato '19

D. Double Field Theory: Park '15, Blair '19, Gallegos-Gürsoy-Verma-Zinnato '20

A. Introduction and Motivation

B. Non-Relativistic Geometry

Non-Relativistic Geometry

Newton Cartan

$$S[X; G, A] \quad = \quad T_0 \int \mathrm{d}\sigma \sqrt{-\dot{X}^\mu \dot{X}^\nu \, G_{\mu\nu}} \quad + \quad Q_0 \int \mathrm{d}\sigma \, \dot{X}^\mu A_\mu$$

Newton Cartan

$$S[X; G, A] \quad = \quad T_0 \int \mathrm{d}\sigma \sqrt{-\dot{X}^\mu \dot{X}^\nu\, G_{\mu\nu}} \quad + \quad Q_0 \int \mathrm{d}\sigma\, \dot{X}^\mu A_\mu$$

$$T_0 = \omega\, m \qquad Q_0 = \omega\, m\,,$$

$$G_{\mu\nu} = -\omega^2\, \tau_\mu \tau_\nu + \delta_{A'B'}\, e_\mu{}^{A'} e_\nu{}^{B'}\,, \qquad (A', B' = 1, \cdots, D-1)$$

$$A_\mu = -\omega\, \tau_\mu + \omega^{-1} a_\mu$$

Newton Cartan

$$S[X; G, A] \quad = \quad T_0 \int \mathrm{d}\sigma \sqrt{-\dot{X}^{\mu}\dot{X}^{\nu}\, G_{\mu\nu}} \quad + \quad Q_0 \int \mathrm{d}\sigma\, \dot{X}^{\mu} A_{\mu}$$

$$= \quad \omega^2 m \int \mathrm{d}\sigma\, \dot{X}^{\mu}\tau_{\mu} \quad - \quad \omega^2 m \int \mathrm{d}\sigma\, \dot{X}^{\mu}\tau_{\mu} \quad + \quad \mathcal{O}(\omega^0)$$

Newton Cartan

$$S[X; G, A] = T_0 \int \mathrm{d}\sigma \sqrt{-\dot{X}^\mu \dot{X}^\nu G_{\mu\nu}} + Q_0 \int \mathrm{d}\sigma \, \dot{X}^\mu A_\mu$$

$$= \omega^2 m \int \mathrm{d}\sigma \, \dot{X}^\mu \tau_\mu - \omega^2 m \int \mathrm{d}\sigma \, \dot{X}^\mu \tau_\mu + \mathcal{O}(\omega^0)$$

$$\Big\downarrow \; \omega \to \infty$$

$$S_{NR}[X; \tau, e, a] = m \int d\sigma \left\{ (\tau_\mu \dot{X}^\mu)^{-1} \dot{X}^\nu \dot{X}^\rho e_\nu{}^{A'} e_\rho{}^{B'} \delta_{A'B'} + \dot{X}^\mu a_\mu \right\}$$

Newton Cartan

$$S[X;G,A] \quad = \quad T_0 \int \mathrm{d}\sigma \sqrt{-\dot{X}^\mu \dot{X}^\nu G_{\mu\nu}} \quad + \quad Q_0 \int \mathrm{d}\sigma \, \dot{X}^\mu A_\mu$$

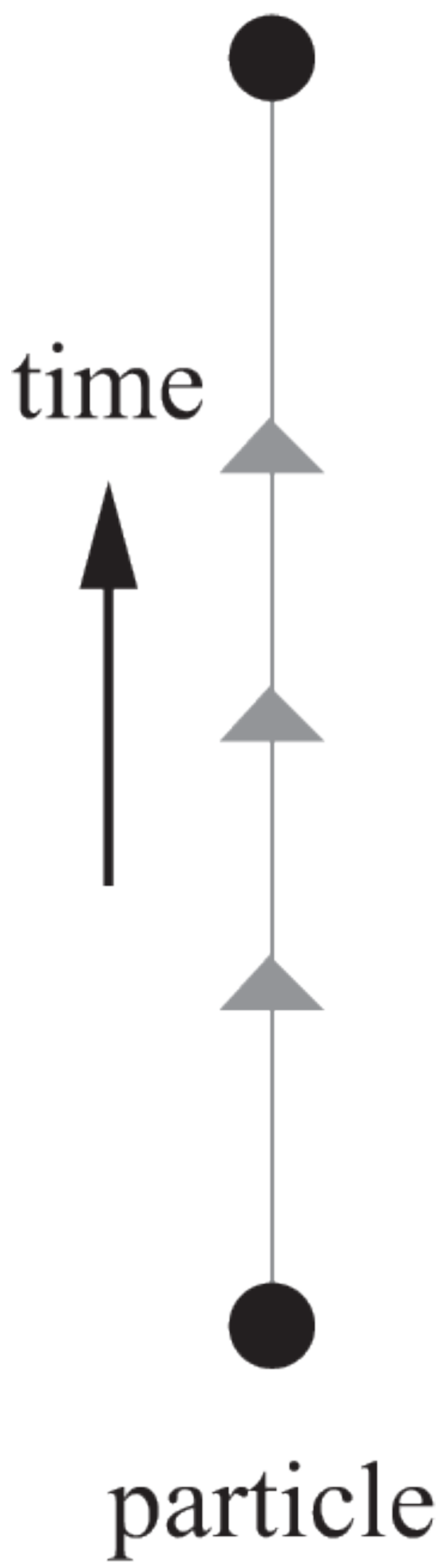
$$= \quad \omega^2 m \int \mathrm{d}\sigma \, \dot{X}^\mu \tau_\mu \quad - \quad \omega^2 m \int \mathrm{d}\sigma \, \dot{X}^\mu \tau_\mu \quad + \quad \mathcal{O}(\omega^0)$$

$$\Big\downarrow \; \omega \to \infty$$

$$S_{NR}[X;\tau,e,a] = m \int d\sigma \left\{ (\tau_\mu \dot{X}^\mu)^{-1} \dot{X}^\nu \dot{X}^\rho \, e_\nu{}^{A'} e_\rho{}^{B'} \delta_{A'B'} \quad + \quad \dot{X}^\mu \, a_\mu \right\}$$

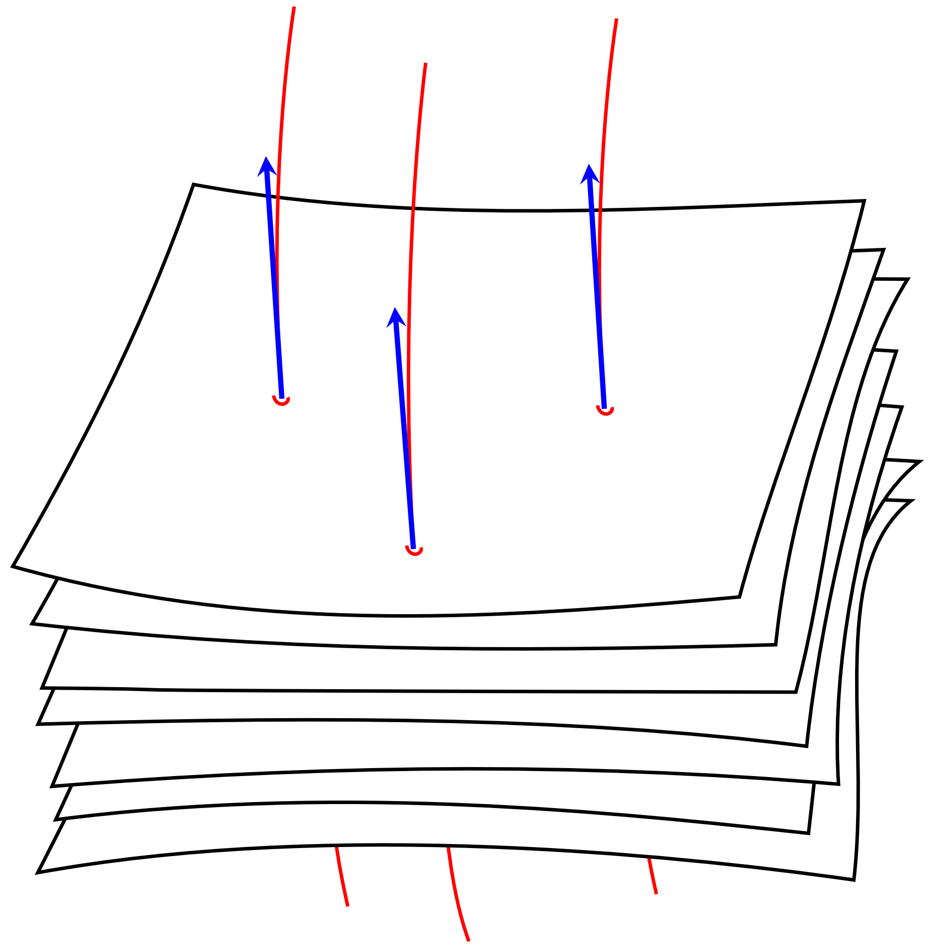
Newton Potential $\phi = -\tau^\mu a_\mu$

Newton Cartan Geometry

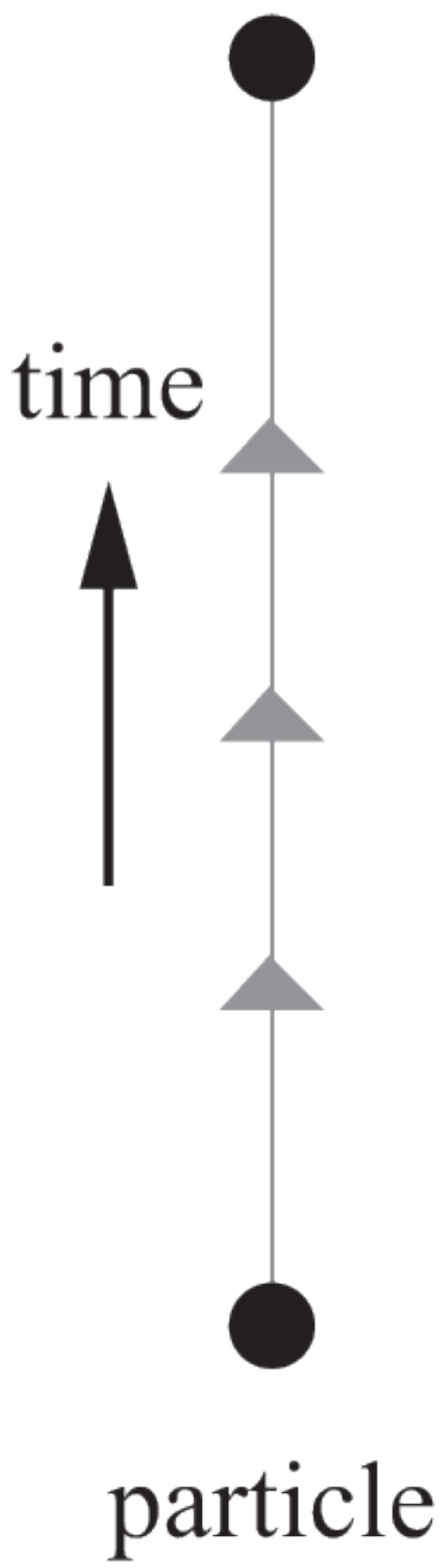


Background fields: $\left(\tau_\mu, e_\mu{}^{A'}, a_\mu\right)$

Preferred foliation orthogonal to the clock 1-form τ_μ



Newton Cartan Geometry

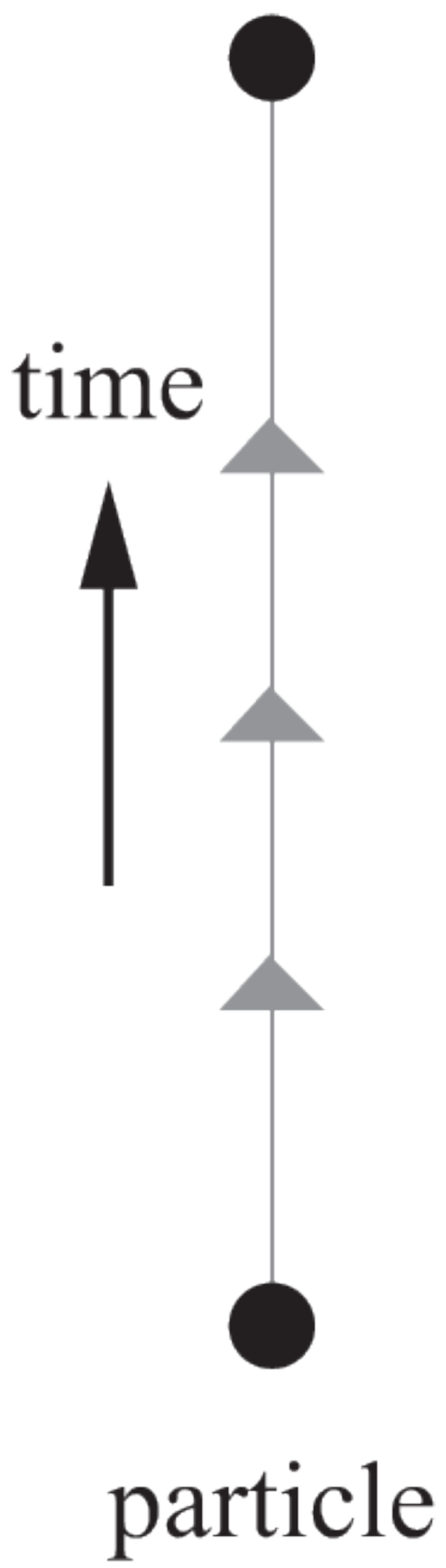


Background fields: $\left(\tau_\mu, e_\mu{}^{A'}, a_\mu\right)$

Preferred foliation orthogonal to the clock 1-form τ_μ
Repected by local Galilei transformations $(\lambda^{A'}, \lambda^{A'B'})$

$$\delta\tau_\mu = 0\,,$$
$$\delta e_\mu{}^{A'} = \lambda^{A'}{}_{B'}\, e_\mu{}^{B'} + \lambda^{A'}\tau_\mu\,,$$
$$\delta a_\mu = \lambda^{A'} e_\mu{}^{A'}\,.$$

Newton Cartan Geometry



Geometries: $\left(\tau_\mu, e_\mu{}^{A'}, a_\mu\right)$

Internal torsion as restrictions of
$\mathrm{d}\tau = \tau_{[A'B']} \oplus \tau_{A'0}$

Torsionless NC -G: $\tau_{A'B'} = 0\,, \tau_{A'0} = 0$

Twistless Torsional NC-G: $\tau_{A'B'} = 0, \tau_{A'0} \neq 0$

Torsional NC-G: $\tau_{A'B'} \neq 0\,, \tau_{A'0} \neq 0$

[Hartong-Obers '13, ..., Figueroa-O'Farrill '20]

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Gomis-Ooguri String Theory

From Particle to String

Clock 1-form

$$\tau_\mu$$

Defining a 1-dimensional foliation. The geometry

$$(\tau_\mu, e_\mu{}^{A'}, a_\mu)$$

Can have internal torsion

$$\mathrm{d}\tau = \tau_{[A'B']} \oplus \tau_{A'0}$$

Clock 1-forms

$$\tau_\mu{}^A \qquad (A = 0,1)$$

Defining a 2-dimensional foliation. The geometry

$$(\tau_\mu{}^A, e_\mu{}^{A'}, b_{\mu\nu})$$

Can have internal torsion

$$\mathrm{d}\tau^A = \tau_{[A'B']}{}^A \oplus \tau_{A'AB}\eta^{AB} \oplus \tau_{A'\{AB\}}$$

Limit

$$S_P[X; G, B, \Phi]$$

$$\Big\downarrow \; \omega \to \infty$$

$$S = T \int \mathrm{d}^2\sigma \left\{ \left(e_{\mu\nu} + b_{\mu\nu} \right) \partial X^\mu \bar{\partial} X^\nu + \lambda\, \tau_\mu\, \bar{\partial} X^\mu + \bar{\lambda}\, \bar{\tau}_\mu\, \partial X^\mu \right\} + S_\phi$$

$(e_{\mu\nu} = \delta_{A'B'}\, e_\mu{}^{A'} e_\nu{}^{B'})$

[Gomis-Ooguri '00, Gomis-Bergshoeff -Yan '18]

Symmetries of the Sigma model

A. worldsheet **diffeomorphisms** $\sigma^\alpha \to \tilde{\sigma}^\alpha(\sigma)$

B. worldsheet **Weyl** transformations $w(\sigma)$

C. targetspace **diffeomorphisms** $X^\mu(\sigma) \to \tilde{X}^\mu(\sigma)$

D. targetspace **1-form** symmetries $\delta\, b = d\theta$

} Expected

E. targetspace **string Galilei** transformations $(\lambda^{AB}, \lambda^{AA'}, \lambda^{A'B'})$

F. **Emergent targetspace dilatation** symmetry λ_D

The theory is unitary, UV complete, and has a spectrum of excitations with Galilei-invariant dispersion relations.

[Gomis-Oh-Yan-Yu '19] have studied the quantum consistency of the full curved space theory with the assumption of zero torsion

$$\tau_{A'B'}{}^{A} = 0\,, \qquad \tau_{A'AB}\,\eta^{AB} = 0\,, \qquad \tau_{A'\{AB\}} = 0$$

And found a set of consistent beta-function constraint on the background geometry. Including a Poisson equation.

Question: quantum consistency w/o any prior assumption on torsion?

Note: the limit $\omega \to \infty$ is a formal contraction leading to a theory with appropriate non-relativistic properties.

[Gomis-Ooguri '00] have shown that the limit is physically equivalent to a limit

$$T_p / T_{eff} \to \infty$$

decoupling all states in the spectrum that are not critically charged under $C^{(p+1)}$.

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Non-Relativistic NS-NS Gravity

Recall

Cancellation of the Weyl anomaly in relativistic string theory

$$T_\alpha{}^\alpha = 0$$

leads to the following beta functions

$$\beta^G_{\mu\nu} = \alpha'\Big(\mathrm{R}_{\mu\nu} + 2\,\nabla_\mu\partial_\nu\Phi - \frac{1}{4}\,H_\mu{}^{\rho\sigma}H_{\nu\rho\sigma}\Big)$$

$$\beta^B_{\mu\nu} = -\,2\alpha'\,\nabla^\rho H_{\rho\mu\nu} + \alpha'(\partial^\rho\Phi)H_{\rho\mu\nu}$$

$$\beta^\Phi = \alpha'\Big(\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}\,\nabla^\mu\partial_\mu\Phi - \frac{1}{4}\,H^2\Big)$$

Which also follow as E.O.M. of $S[G,B,\Phi] = \dfrac{1}{2\kappa^2}\displaystyle\int \mathrm{d}^Dx\,\mathrm{e}^{-2\Phi}\Big(\mathrm{R} + 4\,\partial\Phi^2 - \frac{1}{2}\,H^2\Big)$

Limit

$$S[G,B,\Phi] = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\, E\, \mathrm{e}^{-2\Phi}\Big(\mathrm{R}+4\,(\partial\Phi)^2 - \frac{1}{2}\,H^2\Big)$$

$$\mathrm{e}^{\Phi} = \omega\, \mathrm{e}^{\phi}$$

$$G_{\mu\nu} = \omega^2\, \eta_{AB}\, \tau_\mu{}^A \tau_\nu{}^B + \delta_{A'B'}\, e_\mu{}^{A'} e_\nu{}^{B'}\,,$$

$$B_{\mu\nu} = -\omega^2\, \epsilon_{AB}\, \tau_\mu{}^A \tau_\nu{}^B + b_{\mu\nu}\,,$$

$A, B = 0, 1,$

$A', B' = 2, \cdots, D-1$

Limit

$$S[G,B,\Phi] = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\, E\, \mathrm{e}^{-2\Phi}\left(\mathrm{R}+4\,(\partial\Phi)^2 - \frac{1}{2}\,H^2\right)$$

$$= +\frac{\omega^2}{2\kappa^2}\int \mathrm{d}^D x\, e\, \mathrm{e}^{-2\phi}\left(\tau_{A'B'}{}^{A}\tau^{A'B'B}\,\eta_{AB}\right)$$

from R

$$-\frac{\omega^2}{2\kappa^2}\int \mathrm{d}^D x\, e\, \mathrm{e}^{-2\phi}\left(\tau_{A'B'}{}^{A}\tau^{A'B'B}\,\eta_{AB}\right)$$

from H^2

$$+\mathcal{O}(\omega^0)$$

Limit

$$S[G, B, \Phi]$$

$$\Big\downarrow \ \omega \to \infty$$

$$S[\tau^A, e^{A'}, b, \phi] = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}x \left(\mathrm{R}(J) + 4\, \nabla_{A'} \phi\, \nabla^{A'} \phi - \frac{1}{12}\, h_{A'B'C'}\, h^{A'B'C'} - 4\, \tau_{A'\{AB\}} \tau^{A'\{AB\}} \right)$$

The equations of motion can be organized as irreps of SO(1,1)xSO(8):

$$(\mathbf{1},\mathbf{1})_\phi : \qquad \langle \phi \rangle = 0\,,$$

$$(\mathbf{2},\mathbf{1}) : \qquad \langle g \rangle_{\{AB\}} = 0\,,$$

$$(\mathbf{2},\mathbf{8})_- \oplus (\mathbf{2},\mathbf{8})_+ : \qquad \langle v_\pm \rangle_{AA'} = 0\,,$$

$$(\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{35}) : \qquad \langle g \rangle_{(A'B')} = 0\,,$$

$$(\mathbf{1},\mathbf{1})_- : \qquad \langle s_- \rangle = 0\,,$$

$$(\mathbf{1},\mathbf{28}) : \qquad \langle b \rangle_{[A'B']} = 0\,.$$

Limit of equations of motion gives additional singlet $(\mathbf{1},\mathbf{1})_+$

$$(\mathbf{1},\mathbf{1})_\phi : \qquad [\phi] = 0\,,$$

$$(\mathbf{2},\mathbf{1}) : \qquad [g]_{\{AB\}} = 0\,,$$

$$(\mathbf{2},\mathbf{8})_- \oplus (\mathbf{2},\mathbf{8})_+ : \qquad [v_\pm]_{AA'} = 0\,,$$

$$(\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{35}) : \qquad [g]_{(A'B')} = 0\,,$$

$$(\mathbf{1},\mathbf{1})_+ \oplus (\mathbf{1},\mathbf{1})_- : \qquad [s_\pm] = 0\,,$$

$$(\mathbf{1},\mathbf{28}) : \qquad [b]_{[A'B']} = 0\,.$$

Note:

Missing one equation $\langle s_+ \rangle$: The Poisson equation!

Reason: emergent dilatation symmetry

$$\delta_D \tau_\mu{}^A = \lambda_D \, \tau_\mu{}^A \,, \qquad \delta_D \phi = \lambda_D \,, \qquad \rightarrow \delta_D S = 0$$

for which part of torsion acts as a gauge field $\delta_D \, \tau_{A'AB} \, \eta^{AB} = \partial_{A'} \lambda_D$ **.**

The full set of constraints on the background geometry

$$\langle \phi \rangle = \nabla^{A'} \nabla_{A'} \phi - \left(\nabla_{A'} \phi\right)^2 + \frac{1}{4} \mathrm{R}(J) - \frac{1}{48} h_{A'B'C'} h^{A'B'C'} - \tau_{A'\{AB\}} \tau^{A'\{AB\}} \,,$$

$$\langle g \rangle_{\{AB\}} = 2 \left(\nabla_{B'} - 2 \left(\nabla_{B'} \phi\right)\right) \tau^{B'}{}_{\{AB\}}$$

$$\langle v_- \rangle_{AA'} = \mathrm{R}_{C'A}(J)_{A'}{}^{C'} + 2 \nabla_A \nabla_{A'} \phi + 2 \nabla^B \tau_{A'\{AB\}}$$

$$\langle g \rangle_{A'B'} = \mathrm{R}_{C'(A'}(J)_{B')}{}^{C'} + 2 \nabla_{(A'} \nabla_{B')} \phi - \frac{1}{4} h_{A'C'D'} h_{B'}{}^{C'D'} - 4 \tau_{A'\{AB\}} \tau_{B'}{}^{\{AB\}}$$

$$\langle v_+ \rangle_{AA'} = -2 \left(\nabla_{B'} - 2 \left(\nabla_{B'} \phi\right)\right) \tau^{B'}{}_{A'A} - 4 \tau^{B'}{}_{\{AB\}} \tau_{B'A'}{}^{B} - \epsilon_{AB} h_{A'B'C'} \tau^{B'C'B}$$

$$\langle s_- \rangle = 4 \tau_{A'B'C} \tau^{A'B'C}$$

$$\langle b \rangle_{A'B'} = \left(\nabla_{C'} - 2 \left(\nabla_{C'} \phi\right)\right) h^{C'}{}_{A'B'} + 2 \epsilon^{AB} \nabla_A \tau_{A'B'B}$$

$$\langle s_+ \rangle = - \mathrm{R}_{AA'}(G)^{AA'} - \epsilon^{AB} \mathrm{R}_{AB}(M)$$

Some of which can be read as pure constraints on internal torsion $\mathrm{d}\tau^{A} = \tau_{A'B'A} \oplus \tau_{A'\{AB\}}$

$$\langle\phi\rangle = \nabla^{A'}\nabla_{A'}\phi - \left(\nabla_{A'}\phi\right)^{2} + \frac{1}{4}\,\mathrm{R}(J) - \frac{1}{48}\,h_{A'B'C'}h^{A'B'C'} - \tau_{A'\{AB\}}\tau^{A'\{AB\}}\,,$$

$$\langle g\rangle_{\{AB\}} = 2\left(\nabla_{B'} - 2\,(\nabla_{B'}\phi)\right)\tau^{B'}{}_{\{AB\}}$$

$$\langle v_{-}\rangle_{AA'} = \mathrm{R}_{C'A}(J)_{A'}{}^{C'} + 2\,\nabla_{A}\nabla_{A'}\phi + 2\,\nabla^{B}\tau_{A'\{AB\}}$$

$$\langle g\rangle_{A'B'} = \mathrm{R}_{C'(A'}(J)_{B')}{}^{C'} + 2\,\nabla_{(A'}\nabla_{B')}\phi - \frac{1}{4}\,h_{A'C'D'}\,h_{B'}{}^{C'D'} - 4\,\tau_{A'\{AB\}}\tau_{B'}{}^{\{AB\}}$$

$$\langle v_{+}\rangle_{AA'} = -\,2\left(\nabla_{B'} - 2\,(\nabla_{B'}\phi)\right)\tau^{B'}{}_{A'A} - 4\,\tau^{B'}{}_{\{AB\}}\tau_{B'A'}{}^{B} - \epsilon_{AB}\,h_{A'B'C'}\tau^{B'C'B}$$

$$\langle s_{-}\rangle = 4\,\tau_{A'B'C}\,\tau^{A'B'C}$$

$$\langle b\rangle_{A'B'} = \left(\nabla_{C'} - 2\,(\nabla_{C'}\phi)\right)h^{C'}{}_{A'B'} + 2\,\epsilon^{AB}\,\nabla_{A}\tau_{A'B'B}$$

$$\langle s_{+}\rangle = -\,\mathrm{R}_{AA'}(G)^{AA'} - \epsilon^{AB}\,\mathrm{R}_{AB}(M)$$

Upon imposing zero torsion: $\tau_{A'B'}{}^{A} = 0\,,\, \tau_{A'AB}\,\eta^{AB} = 0\,,$ **and** $\tau_{A'\{AB\}} = 0$

$$\langle\phi\rangle = \nabla^{A'}\nabla_{A'}\phi - \left(\nabla_{A'}\phi\right)^2 + \frac{1}{4}\,\mathrm{R}(J) - \frac{1}{48}\,h_{A'B'C'}h^{A'B'C'}\,,$$

$$\langle g\rangle_{\{AB\}} = 0$$

$$\langle v_{-}\rangle_{AA'} = \mathrm{R}_{C'A}(J)_{A'}{}^{C'} + 2\,\nabla_{A}\,\nabla_{A'}\,\phi$$

$$\langle g\rangle_{A'B'} = \mathrm{R}_{C'(A'}(J)_{B')}{}^{C'} + 2\,\nabla_{(A'}\nabla_{B')}\,\phi - \frac{1}{4}\,h_{A'C'D'}\,h_{B'}{}^{C'D'}$$

$$\langle v_{+}\rangle_{AA'} = 0$$

$$\langle s_{-}\rangle = 0$$

$$\langle b\rangle_{A'B'} = \left(\nabla_{C'} - 2\,(\nabla_{C'}\phi)\right)h^{C'}{}_{A'B'}$$

$$\langle s_{+}\rangle = -\,\mathrm{R}_{AA'}(G)^{AA'} - \epsilon^{AB}\,\mathrm{R}_{AB}(M)$$

The non-trivial equations

$$\langle s_+ \rangle \,, \qquad \langle v_- \rangle_{AA'} \,, \qquad \langle g \rangle_{A'B'} \,, \qquad \langle b \rangle_{A'B'} \,, \qquad \langle \phi \rangle$$

are in agreement with the results of [Gomis-Oh-Wu-Yan ’19].

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Open Questions and Outlook

Supersymmetry?

A. There are emergent super-conformal symmetries as partners of the emergent dilatations

B. Role of torsion? Remarkably, the following a-priori constraints

$$\tau_{A'B'}{}^{-} = 0 \qquad\qquad \tau_{A'+}{}^{-} = 0\,,$$

are necessary for a well-defined limit <u>and</u> invariant under supersymmetry.

C. Role of the fermion chiralities? Five NR supergravities? 11 dimensions?

D. Non-relativistic Killing spinor equations? Restrictions?

Summary

I have presented an alternative approach to studying the background dynamics of non-relativistic (super)string theory.

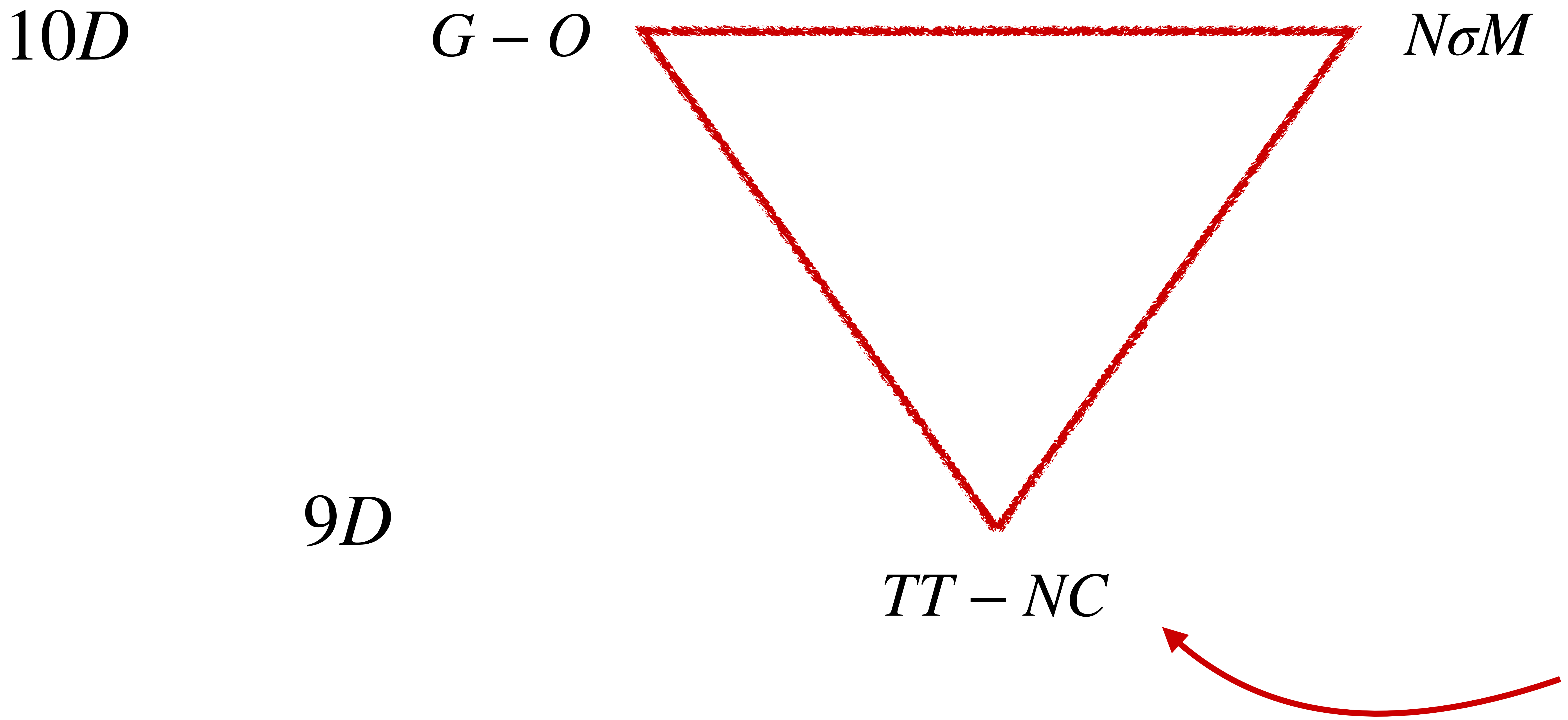
This opens the road for a number of possible applications and extensions:

- Branes
- Dualities
- Holography

Thank You!

T Duality

Gomis-Ooguri string theory with an spatial, longitudinal isometry $K_\mu = \tau_\mu{}^A K_A, K^2 > 0$ is T-dual to a relativistic string theory on a background with a null isometry $K^2 = 0$



[Harmark-Hartong-Obers ’18, Gürsoy-Gallegos-Zinnato ’19]