Slow scrambling in extremal BTZ and microstate geometries

Kévin Nguyen

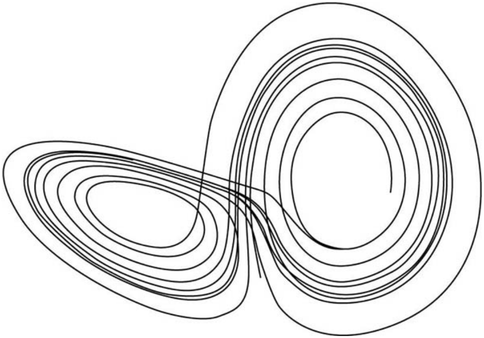
based on arXiv:2009.08518
in collaboration with B. Craps, M. De Clerck, P. Hacker and C. Rabideau

Iberian Strings

20th January 2021

KING'S College LONDON

Chaos and the butterfly effect

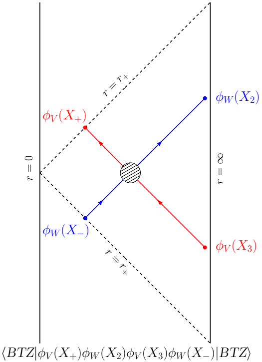


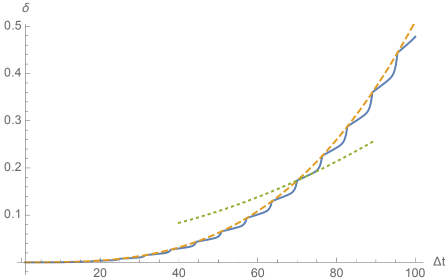
$$\{q(t), p(0)\} = \frac{\partial q(t)}{\partial q(0)} \sim e^{\lambda_L t}$$ Lyapunov growth

Overview

$$\langle [W(t), V(0)]^2 \rangle \approx \langle WW \rangle \langle VV \rangle - \text{Re} \underbrace{\langle V(0)W(t)V(0)W(t) \rangle}_{OTOC(t)} \approx e^{2\lambda_L t}$$

In extremal BTZ:



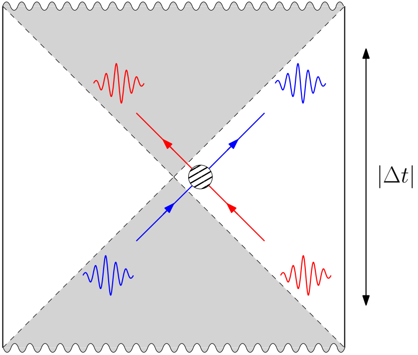


- ▶ WKB/geodesic approximation
- ▶ λ_L alternates between T_L and T_R with overall cubic law

OTOCs in gravity

Butterfly effect in AdS/CFT

AdS black hole

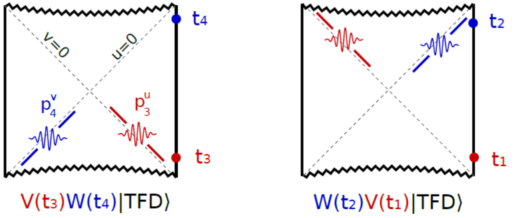


$$E^2_{collision} \sim e^{\lambda_L |\Delta t|} E_{in} E_{out} \qquad \text{with} \qquad \lambda_L = 2\pi T_{Hawking}$$

Original gravity formula

[Shenker-Stanford '14]

$$OTOC = \langle W(t,x)V(0)W(t,x)V(0)\rangle_\beta$$

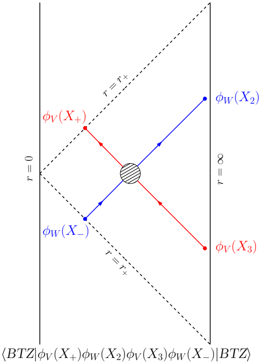


$$OTOC = \int e^{i\delta(t,x'-x'')}\left[p^U\Psi_1^*(p^U,x')\Psi_3(p^U,x')\right]\left[p^V\Psi_2^*(p^V,x'')\Psi_4(p^V,x'')\right]$$

- Ψ: Fourier transform of the boundary-to-horizon propagator
- $e^{i\delta}$: eikonal scattering amplitude along the horizon ($t \gg \beta$)
- evaluation by saddle point approximation

Geodesic approximation

[Balasubramanian-Craps-De Clerck-KN '19]



New alternative formula:

$$\frac{\text{OTOC}}{\langle WW\rangle\langle VV\rangle} \approx e^{i\delta}\Big|_{\text{saddle}}$$

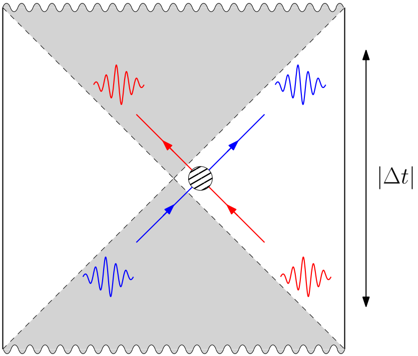
with the eikonal phase shift

$$\delta = \int \left(h^V_{\mu\nu}\, T^{\mu\nu}_W + h^W_{\mu\nu}\, T^{\mu\nu}_V\right)$$

During the early-time Lyapunov growth, we simply have

$$CS \equiv \frac{\langle[\phi_W,\phi_V]^2\rangle}{\langle\phi_V\phi_V\rangle\langle\phi_W\phi_W\rangle} \approx \delta^2 \sim (E^2_{collision})^2 \qquad (\delta \ll 1 \text{ or } t \ll t_*)$$

AdS black hole



$$E^2_{collision} \sim e^{\lambda_L |\Delta t|} E_{in} E_{out} \qquad \text{with} \qquad \lambda_L = 2\pi T_{Hawking}$$

Slow scrambling in vacuum

The original computational method of [Shenker-Stanford '14] assumes $t \gg \beta$, and is therefore not suited to study the zero temperature limit.

Our new method allows to recover *slow scrambling* at zero temperature [Balasubramanian-Craps-De Clerck-KN '19]

$$CS(t,x) \sim \left(G_N \left(t - |x|\right)^2 \right)^2 ,$$

in agreement with CFT computations [Roberts-Stanford '14].

Slow scrambling in extremal BTZ

Extremal BTZ

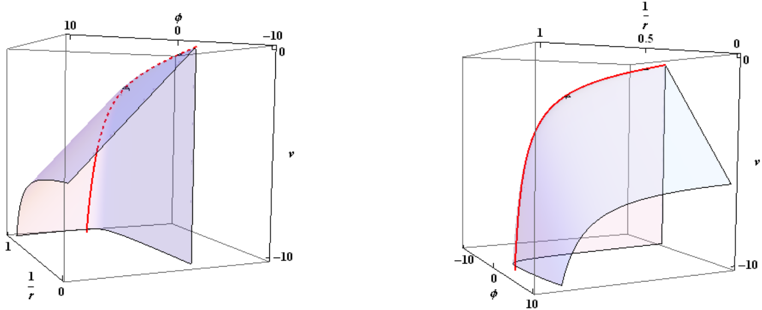
Metric

$$ds^2 = \ell_{AdS}^2 \left[-\left(r^2 - 2r_+^2\right) dt^2 + \frac{r^2 \ dr^2}{\left(r^2 - r_+^2\right)^2} - 2r_+^2 \ dt \, d\varphi + r^2 \ d\varphi^2 \right],$$

to which we can associate three temperatures

$$T_{\text{Hawking}} = 0, \qquad T_L = 0, \qquad T_R = \frac{r_+}{\pi}.$$

Shock wave of outgoing particle in the decompactified limit:



The solution in the compact case is obtained via a sum over images.

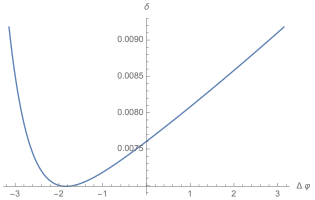
The shockwave decays exponentially to the left of the particle and polynomially to its right.

Eikonal phase in extremal BTZ

$$\delta(t,\varphi) = \sum_{\substack{n\in\mathbb{Z} \\ |\varphi_n|\leq t}} f(t,\, \varphi_n), \qquad \varphi_n = \varphi + 2\pi n$$

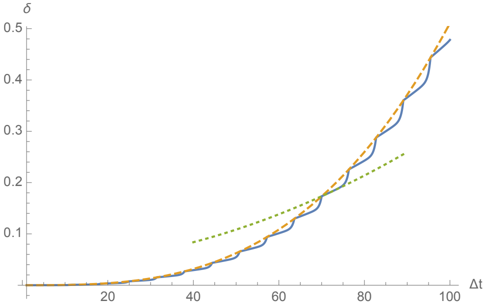
Each term in the sum corresponds to one interaction of a geodesic with the shockwave of the other geodesic. There are many such interactions due to the $\varphi \sim \varphi + 2\pi$ periodicity.

As the time separation t between the two insertions is increased, the number of interactions grows. We find $O(t)$ interactions with the polynomial sheet of the shockwave and $O(\ln t)$ with the exponential sheet.



Altogether we find that overall increase of the eikonal phase is

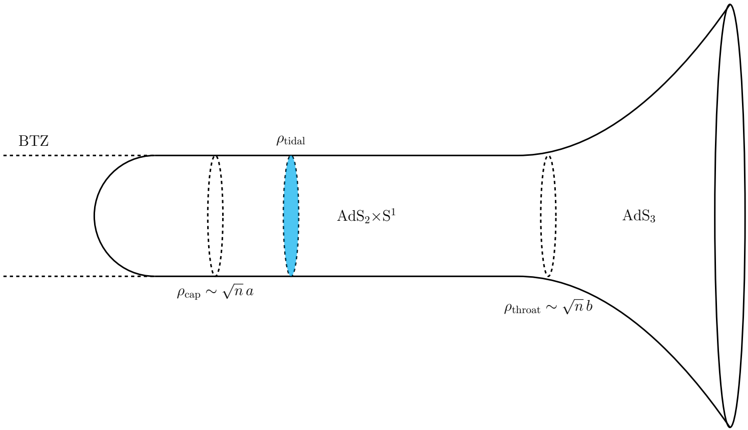
$$\delta(t) \sim G_N m_V m_W \lfloor t \rfloor t^2 \,.$$



Scrambling in microstate geometries

$(1, 0, n)$ superstratum geometries

[Bena-Martinec-Walker-Warner-... '16-'20]



Departures from BTZ computation

As we increase the time separation t between boundary insertions, the interaction takes place deeper in the bulk. We take the scrambling time as a reference timescale

$$t_s = \left(\frac{N_1 N_5}{h^2} \right)^{\frac{1}{3}} .$$

We expect departures from the BTZ computation when the process probes bulk regions

1. where tidal forces are strong such that the geodesic approximation breaks down

$$|\theta| \gtrsim m \qquad \longleftrightarrow \qquad t > t_{tidal} \sim \left(\frac{h^7 N_1 N_5}{T_R^3} \right)^{\frac{1}{6}} t_s$$

2. near the cap where the geometry significantly from the classical BTZ solution

$$t > t_{cap} \sim \frac{N_1 N_5}{T_R} \sim \left(\frac{h^2 (N_1 N_5)^2}{T_R^3} \right)^{\frac{1}{3}} t_s > t_{tidal}$$

Summary and open problems

OTOC by geodesic approximation

- *pros*: position space, applicable to any AlAdS spacetime, behavior at finite time (slow scrambling)
- *cons*: no account for late-time dissipation (OTOC late-time decay)

OTOC in extremal BTZ and superstratum geometries

- average slow scrambling $\sim t^3$
- alternation between $\sim t^2$ and $\sim e^{T_R t}$ behavior on small time scales
- departures occurring at t_{tidal} and t_{cap}

Open problems

- inclusion of dissipation within the geodesic approximation [Festuccia-Liu '08]
- quantify departures from classical BTZ solution
- ...