Latest News on Resurgence applied to Topological Strings

Ricardo Couso-Santamaría

Instituto Superior Técnico, Lisbon

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Resurgence in Gauge and String Theories

Based on:
RCS, J. Edelstein, R. Schiappa, M. Vonk 1308.1695, 1407.4821
RCS 1507.04013
RCS, R. Schiappa, R. Vaz 1605.07473
Work in progress with M. Mariño and R. Schiappa.

General introduction

- Topological string theory describes strings probing the geometry of a Calabi–Yau.

  Observables only depend on the complex or Kähler structure and the string coupling g_s.

- The main observable is the perturbative free energy: an asymptotic series in g_s whose coefficients are functions of the moduli

$$F^{(0)}(g_s; z, \bar{z}) = \sum_{g=0}^{\infty} g_s^{2g-2} F_g^{(0)}(z, \bar{z})$$

- A longstanding problem is to define a nonperturbative completion for string theory.

- Our approach is resurgent:

$$F^{(0)} \longrightarrow \text{transseries for } F\left[g_s, \mathrm{e}^{-1/g_s}\right] \longrightarrow \text{transseries resummation}$$

- We also look at:
  - New terms in the transseries with monomial e^{-1/g_s^2}.
  - Resurgent properties of Gromov–Witten invariants: $F_g^{(0)} = \sum_d N_{g,d} Q^d$.

Structure

- From $F_g^{(0)}$ to a transseries in the B-model.
- Hidden monomials in the transseries: $\mathrm{e}^{-A_{\mathrm{NS}}/g_s^2}$.
- Resurgent properties of Gromov–Witten invariants: $N_{g,d}$.
- Resumming the transseries: $F = \sum_n \sigma^n \, \mathrm{e}^{-nA/g_s} \, \mathcal{S} F^{(n)}$.

Part I

- From $F_g^{(0)}$ to a transseries in the B-model.

  RCS, J. Edelstein, R. Schiappa, M. Vonk 1308.1695, 1407.4821

From $F_g^{(0)}$ to a transseries in the B-model.

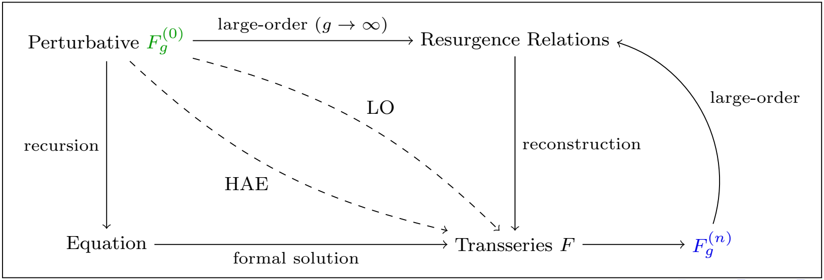
- Our goal is to extend

$$\sum_{g=0}^{\infty} g_s^{2g-2} F_g^{(0)} \quad \text{with} \quad \sum_{n=1}^{\infty} \sigma^n \, \mathrm{e}^{-nA/g_s} \sum_{h=0}^{\infty} g_s^h \, F_h^{(n)}$$

$$\text{or} \quad \sum_{n_1,n_2,\ldots=1}^{\infty} \sigma_1^{n_1} \sigma_2^{n_2} \ldots \, \mathrm{e}^{-(n_1 A_1 + n_2 A_2 + \ldots)/g_s} \sum_{h=0}^{\infty} g_s^h \, F_h^{(n_1,n_2,\ldots)}$$

assuming resurgent properties for the free energy.

- The schematic approach is: [LO: Large-order; HAE: Holomorphic anomaly equations]



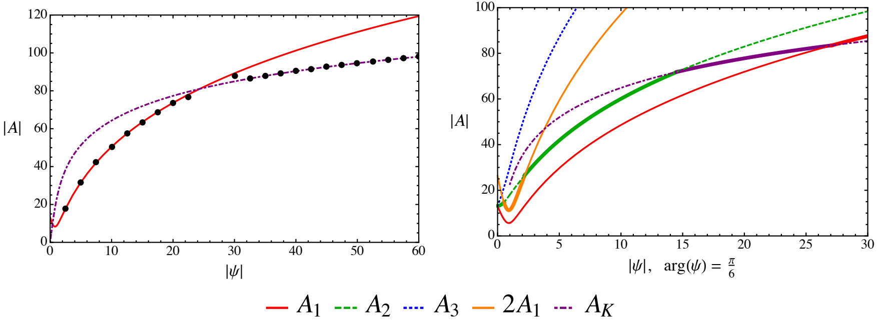
LO: large-order route

To leading order

$$F_g^{(0)} \sim \sum_{h=0}^{\infty} \frac{\Gamma(2g-1-h)}{(A_{\text{dom}})^{2g-1-h}} \frac{S_1}{\pi i} F_h^{(1)} + \ldots \Rightarrow {A_{\text{dom}}}^2(z, \not{\bar{z}}) = \lim_{g\to\infty} 4g^2 \frac{F_g^{(0)}(z,\bar{z})}{F_{g+1}^{(0)}(z,\bar{z})}.$$

A_{dom} must be the instanton action of smallest absolute value.

Example: local $\mathbb{P}^2$



$A_{\text{K}} = 2\pi t$; A_1, A_2, A_3 : associated to conifold point.

We collapse the tower of equations for $F_g^{(0)}$ into one and use a new transseries ansatz:

$$\frac{\partial F_g^{(0)}}{\partial S} = \frac{1}{2}\left(D_z \partial_z F_{g-1}^{(0)} + \sum_{h=1}^{g-1} \partial_z F_h^{(0)}\, \partial_z F_{g-h}^{(0)} \right), \quad g \geq 2$$

$$\Downarrow$$

$$\partial_S F^{(0)} + U\, \partial_z F^{(0)} - \frac{1}{2} g_s^2 \left(D_z \partial_z F^{(0)} + \left(\partial_z F^{(0)} \right)^2 \right) = V + \frac{1}{g_s^2} W$$

$$\Uparrow$$

$$\left[\partial_S - \frac{1}{2} \left(\partial_z A^{(\boldsymbol{n})} \right)^2 \right] F_g^{(\boldsymbol{n})} = - \sum_{h=1}^{g} \mathcal{D}_h^{(\boldsymbol{n})} F_{g-h}^{(\boldsymbol{n})}$$

$$+ \frac{1}{2} \sum_{\boldsymbol{m} < \boldsymbol{n}} \sum_{h=0}^{g} \left(\partial_z F_{h-1}^{(\boldsymbol{m})} - \partial_z A^{(\boldsymbol{m})}\, F_h^{(\boldsymbol{m})} \right) \left(\partial_z F_{g-1-h}^{(\boldsymbol{n}-\boldsymbol{m})} - \partial_z A^{(\boldsymbol{n}-\boldsymbol{m})}\, F_{g-h}^{(\boldsymbol{n}-\boldsymbol{m})} \right)$$

where $A^{(\boldsymbol{n})} = n_1 A_1 + n_2 A_2 + \ldots$ and $(z, \bar{z}) \to (z, S = S(z, \bar{z}))$ is a useful change of variables.

Summary

- There is a transseries extension of the perturbative free energy $F^{(0)}$ of the form

$$\sum_{g=0}^{\infty} g_s^{2g-2} F_g^{(0)} + \sum_{n_1,n_2,\dots}^{\infty} \sigma_1^{n_1} \sigma_2^{n_2} \dots \mathrm{e}^{-(n_1 A_1 + n_2 A_2 + \dots)/g_s} \sum_{h=0}^{\infty} g_s^h F_h^{(n_1,n_2,\dots)}$$

  that can be obtained from a combination of numerical large-order analysis (LO) and integration of the holomorphic anomaly equations (HAE).

- Instanton actions are holomorphic (no $\bar{z}$ or S) and come in pairs $\pm A_i$.

- There is room for resonance between different sectors ($\boldsymbol{n} \cdot \boldsymbol{A} = 0$, $\boldsymbol{n} \neq \boldsymbol{0}$).

- There can be transseries monomials like $\log g_s$.

Part II

- Hidden monomials in the transseries: $\mathrm{e}^{-A_{\mathrm{NS}}/g_s^2}$.

  RCS 1507.04013

Can there be other monomials in g_s?

- When $z \to 0$ (i.e. $Q \to 0$ or $t \to \infty$) all the instanton actions move off to infinity, so there are no more poles in the Borel plane.

  If the transseries is complete with monomials e^{-A/g_s} we should expect a convergent series for $F^{(0)}(g_s; \bar{z})$ for any value of $\bar{z}$ (or S).

- This is **not** what we find:

$$\lim_{z \to 0} F^{(0)} = \sum_{g=0}^{\infty} g_s^{2g-2} H_g^{(0)}(S) \quad \text{is still asymptotic!}$$

  But the asymptotic growth is not $\Gamma(2g-1)$ but $\Gamma(g-1)$.

- Moreover, the limiting free energy $H^{(0)}(g_s; u)$ is universal in the appropriate variable $u := S - \text{const}$.

Is the transseries complete?

- The first few free energies are:

$$H_2^{(0)} = u\left(\frac{5}{24}u^2 + \frac{1}{2}u + \frac{1}{2}\right),$$

$$H_3^{(0)} = u^3\left(\frac{5}{16}u^3 + \frac{5}{8}u^2 + \frac{1}{2}u + \frac{1}{6}\right),$$

$$H_4^{(0)} = u^5\left(\frac{1105}{1152}u^4 + \frac{15}{8}u^3 + \frac{25}{16}u^2 + \frac{2}{3}u + \frac{1}{8}\right),$$

$$H_5^{(0)} = u^7\left(\frac{565}{128}u^5 + \frac{1105}{128}u^4 + \frac{15}{2}u^3 + \frac{175}{48}u^2 + u + \frac{1}{8}\right).$$

Geometries checked:
Local $\mathbb{P}^2$
Local del Pezzo $E_{5,\ldots,8}$
Mirror quintic
Local $\mathbb{P}^1 \times \mathbb{P}^1$

- Alim, Yau, Zhou (1506.01375) showed that the leading coefficients are determined by the Airy equation, and this result can be extended to include u,

$$(\partial_x^2 - x)v(x) = 0, \quad \text{where} \quad x = (2\tau_s)^{-2/3}, \quad v = 2^{-1/3}\mathrm{e}^{\frac{\zeta-1/3}{3\tau_s}}\tau_s^{1/6}\exp H^{(0)}, \quad \tau_s = u^3 g_s^2.$$

Also $\zeta = 1 - \tau_s/u$ is kept fixed.

- Alternatively, from the holomorphic anomaly equations (HAE)

$$\partial_u H^{(0)} - \frac{3}{2}g_s^2 u^3\left(\partial_u H^{(0)} + \frac{u}{3}\partial_u^2 H^{(0)} + \frac{u}{3}\left(\partial_u H^{(0)}\right)^2\right) = \frac{1}{2u} + \frac{1}{u^2}.$$

The transseries at the large-radius point

- Either the Airy equation (in g_s) or the HAE (in u) admit a transseries

$$H = \sum_{g=0}^{\infty} \tau_s^{g-1} H_g^{(0)}(u) + \sum_{n=1}^{\infty} \sigma^n \mathrm{e}^{-nA_u(u)/\tau_s} \sum_{g=0}^{\infty} \tau_s^g H_g^{(n)}(u)$$

where $A_u = \frac{2}{3u^3}$, $\tau_s = u^3 g_s^2$.

- The resurgent properties are inherited from those of the Airy function.

$$H_g^{(0)} \sim \frac{S_1}{2\pi i} \sum_{h=0}^{\infty} \frac{\Gamma(g-1-h)}{{A_u}^{g-1-h}} H_h^{(1)}, \quad \text{as } g \to \infty,$$

$$H_g^{(1)} \sim -\frac{1}{2\pi} \frac{\Gamma(g)}{(-A_u)^g} + \frac{S_1}{\pi i} \sum_{h=0}^{\infty} \frac{\Gamma(g-h)}{{A_u}^{g-h}} H_h^{(2)}, \quad \text{as } g \to \infty,$$

where $S_1 = -i$.

- The HAE could be analyzed from the point of view of parametric resurgence since the differential equation is wrt u but the transseries variable is τ_s.

The transseries out of the large-radius point

- This transseries at $z = 0$ is the limit of

$$\sum_{n=1}^{\infty} \sigma_{\rm NS}^n \mathrm{e}^{-n A_{\rm NS}(z,\bar{z})/g_s^2} \sum_{g=0}^{\infty} (g_s^2)^g F_g^{{\rm NS},(n)}(z,\bar{z}).$$

- The action $A_{\rm NS}$ is not holomorphic, but satisfies

$$\partial_{\bar{z}} A_{\rm NS} + \frac{1}{2} \overline{C}_{\bar{z}}^{zz} \left(\partial_z A_{\rm NS}\right)^2 = 0.$$

- An approximate solution near the large-radius point is

$$A_{\rm NS} \simeq -i \mathrm{e}^{-K},$$

  where K is the Kähler potential for the moduli space.

  This quantity is proportional to the volume of the Calabi–Yau → possible interpretation in terms of NS5-branes?

- Resurgent properties of Gromov–Witten invariants: $N_{g,d}$.

  RCS, R. Schiappa, R. Vaz 1605.07473

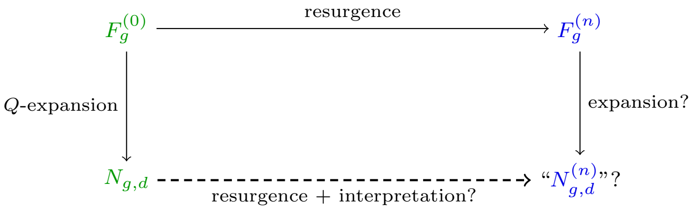
Gromov–Witten invariants

The perturbative free energy of the B-model has a mirror dual expansion around $Q = 0$,

$$F^{(0)} = \sum_{g=0}^{\infty} g_s^{2g-2} F_g^{(0)}(z) = \sum_{g=0}^{\infty} g_s^{2g-2} \sum_{d=1}^{\infty} N_{g,d} Q^d.$$

The Gromov–Witten invariants, $N_{g,d}$, are rational numbers with enumerative and geometric information about the (mirror) Calabi–Yau.

Question: If the perturbative free energy can be completed to a nonperturbative transseries, are there nonperturbative analogues of the Gromov–Witten invariants?



The simplest example: resolved conifold

- The free energies and GW invariants are:

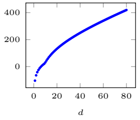
$$F_g^{(0)} = \frac{(-1)^{g-1} B_{2g}}{2g(2g-2)!} \, \mathrm{Li}_{3-2g}(Q), \qquad N_{g,d} = \frac{(-1)^{g-1} B_{2g}}{2g(2g-2)!} d^{2g-3}.$$

- For fixed g or d there is no factorial growth of $N_{g,d}$ in the other index.

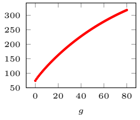
  Only when $d \propto g$ do we get the combination $n^n \simeq n!$ when $n \to \infty$ (Stirling).

$\log\|N_{g,d}\|$	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
20	111.4	117.1	122.6	128.0	133.2	138.3	143.4	148.3	153.2	157.9	162.6	167.3	171.8	176.3	180.8	185.2	189.5	193.9	198.1	202.3	206.5
21	113.7	119.6	125.3	130.8	136.2	141.4	146.6	151.6	156.6	161.5	166.3	171.0	175.7	180.3	184.8	189.3	193.7	198.1	202.5	206.8	211.1
22	116.0	122.0	127.9	133.5	139.1	144.5	149.7	154.9	160.0	165.0	169.9	174.7	179.5	184.2	188.8	193.4	197.9	202.4	206.8	211.2	215.5
23	118.2	124.4	130.4	136.2	141.9	147.4	152.8	158.1	163.3	168.4	173.4	178.4	183.2	188.0	192.8	197.4	202.0	206.6	211.1	215.6	220.0
24	120.4	126.7	132.9	138.9	144.7	150.3	155.9	161.3	166.6	171.8	177.0	182.0	187.0	191.8	196.7	201.4	206.1	210.7	215.3	219.9	224.4
25	122.5	129.0	135.3	141.5	147.4	153.2	158.9	164.4	169.9	175.2	180.4	185.6	190.6	195.6	200.5	205.4	210.1	214.9	219.5	224.2	228.7
26	124.5	131.2	137.7	144.0	150.1	156.0	161.8	167.5	173.1	178.5	183.9	189.1	194.3	199.3	204.3	209.3	214.1	219.0	223.7	228.4	233.0
27	126.5	133.4	140.0	146.5	152.7	158.8	164.8	170.6	176.2	181.8	187.2	192.6	197.8	203.0	208.1	213.1	218.1	223.0	227.8	232.6	237.3
28	128.5	135.5	142.3	148.9	155.3	161.5	167.6	173.5	179.3	185.0	190.6	196.0	201.4	206.7	211.9	217.0	222.0	227.0	231.9	236.8	241.6
29	130.4	137.6	144.6	151.3	157.9	164.2	170.4	176.5	182.4	188.2	193.9	199.4	204.9	210.3	215.6	220.8	225.9	231.0	236.0	240.9	245.8
30	132.2	139.6	146.7	153.7	160.4	166.9	173.2	179.4	185.4	191.3	197.1	202.8	208.4	213.8	219.2	224.5	229.7	234.9	240.0	245.0	249.9
31	134.0	141.6	148.9	155.9	162.8	169.5	175.9	182.2	188.4	194.4	200.3	206.1	211.8	217.4	222.9	228.2	233.6	238.8	244.0	249.1	254.1
32	135.7	143.5	151.0	158.2	165.2	172.0	178.6	185.1	191.4	197.5	203.5	209.4	215.2	220.9	226.4	231.9	237.3	242.7	247.9	253.1	258.2
33	137.4	145.4	153.0	160.4	167.6	174.5	181.3	187.8	194.3	200.5	206.6	212.7	218.5	224.3	230.0	235.6	241.1	246.5	251.8	257.1	262.3
34	139.0	147.2	155.0	162.6	169.9	177.0	183.9	190.6	197.1	203.5	209.7	215.9	221.9	227.7	233.5	239.2	244.8	250.3	255.7	261.0	266.3
35	140.6	149.0	157.0	164.7	172.2	179.4	186.4	193.3	199.9	206.4	212.8	219.0	225.1	231.1	237.0	242.8	248.4	254.0	259.5	265.0	270.3
36	142.2	150.7	158.9	166.8	174.4	181.8	188.9	195.9	202.7	209.3	215.8	222.2	228.4	234.5	240.4	246.3	252.1	257.7	263.3	268.8	274.3
37	143.7	152.4	160.7	168.8	176.6	184.1	191.4	198.5	205.5	212.2	218.8	225.3	231.6	237.8	243.8	249.8	255.7	261.4	267.1	272.7	278.2
38	145.1	154.0	162.6	170.8	178.7	186.4	193.9	201.1	208.2	215.0	221.8	228.3	234.7	241.0	247.2	253.3	259.2	265.1	270.9	276.5	282.1
39	146.6	155.6	164.3	172.7	180.8	188.7	196.3	203.7	210.8	217.8	224.7	231.4	237.9	244.3	250.6	256.7	262.8	268.7	274.6	280.3	286.0
40	147.9	157.2	166.1	174.6	182.9	190.9	198.6	206.2	213.5	220.6	227.6	234.3	241.0	247.5	253.9	260.1	266.3	272.3	278.3	284.1	289.9

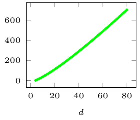
$\log|N_{30,d}|$



$\log|N_{g,30}|$



$\log|N_{d,d}|$



The simplest example: resolved conifold

- For $d = a_0 + a_1 g$ going to infinity,

$$N_{g,d}\Big|_{d=a_0+a_1 g} \sim \frac{\Gamma(2g-3/2)}{\left(\frac{4\pi}{e a_1}\right)^{2g-3/2}} \frac{\left(\frac{2e}{a_1}\right)^{3/2} e^{2a_0/a_1}}{2\pi^2}.$$

- We can connect with the B-model resurgence via the mirror map:

$$F_g^{(0)} = \sum_{d=1}^{\infty} N_{g,d} Q^d.$$

- To connect with the asymptotics of $F_g^{(0)}$ we find value of d that maximizes $N_{g,d}Q^d$,

$$d = \frac{2g-3}{t}, \quad \text{where} \quad t = -\log Q.$$

$$F_g^{(0)} \sim \frac{\Gamma(2g-1)}{(2\pi t)^{2g-1}} \frac{t}{\pi} \quad \longleftrightarrow \quad N_{g,d}Q^d\Big|_{d=\frac{2g-3}{t}} \sim \frac{\Gamma(2g-3/2)}{(2\pi t)^{2g-3/2}} \frac{t^{3/2}}{2\pi^2}.$$

The precise relation would come from a saddle point expansion around $d = \frac{2g-3}{t}$.

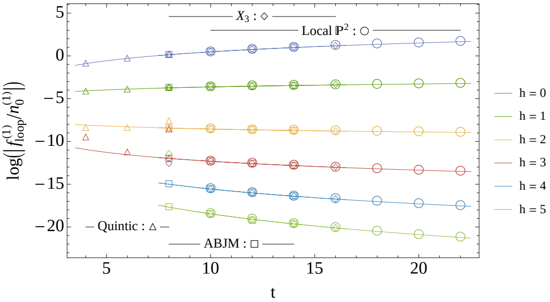
Universal growth related to $A = 2\pi t$

- All geometries have a factorial growth of $N_{g,d}$ at $d = (2g-3)/t$ that connects with the (Kähler) instanton action $A = 2\pi t$.

  The asymptotic behavior is universal, independent of the geometry.

$$N_{g,d}\, Q^d \Big|_{g=\frac{t}{2}d+q} \sim \sum_{n=1}^{+\infty} \sum_{h=0}^{+\infty} \frac{\Gamma\left(2g-\frac{3}{2}-h\right)}{(n 2\pi t)^{2g-\frac{3}{2}-h}} \frac{n_0^{(1)} t^{\frac{3}{2}-h}}{2^{2h+1}\pi^{h+2} n^{\frac{3}{2}+h}} \mathcal{P}_h(q),$$

  where we take $t \in 2\mathbb{N}$ and $q \in \mathbb{Z}$; $\mathcal{P}_h(q)$ is a polynomial. $n_0^{(1)}$ is the first Gopakumar–Vafa invariant.



$$\mathcal{P}_0 = 1,$$

$$\mathcal{P}_1 = -\frac{71}{12} + 12q - 4q^2,$$

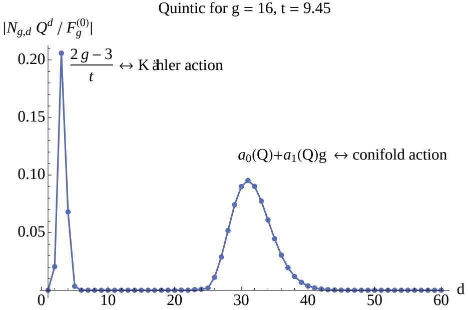
$$\mathcal{P}_2 = \frac{11545}{288} - 131q + \frac{419q^2}{3} - \frac{176q^3}{3} + 8q^4,$$

$$\mathcal{P}_3 = -\frac{17534803}{51840} + \frac{33553q}{24} - \frac{157393q^2}{72} + \frac{15220q^3}{9} - \frac{2062q^4}{3} + \frac{416q^5}{3} - \frac{32q^6}{3}.$$

Growth related to other instanton actions

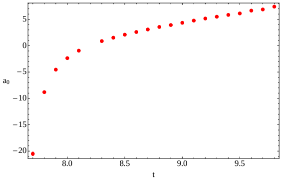
- Other instanton actions can control the growth of $F_g^{(0)}$.

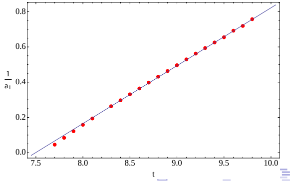
  Other slopes $d = a_0(Q) + a_1(Q)g$ can connect to them.



Geometries checked:
Local $\mathbb{P}^2$
Mirror quintic
Local $\mathbb{P}^1 \times \mathbb{P}^1$
$X_p = \mathcal{O}(p-2) \oplus \mathcal{O}(-p) \to \mathbb{P}^1$
Hurwitz

- We find $a_0(Q)$ and $a_1(Q)$ numerically but cannot identify them in closed form.





The problems

- We don't know the analytic form of $d = a_0 + a_1 g$ for actions other than $2\pi t$, although we have numerical values.

  We can confirm numerically that the actions for $F_g^{(0)}$ and $N_{g,d}$ are in correspondence.

- Resurgence relations for $N_{g,d}$ are not purely numerical: they depend on t.

  The regular expansion around $Q = 0$ of $F_g^{(0)}$ to obtain $N_{g,d}$ does NOT work at the asymptotic level,

$$F_g^{(0)} \sim \frac{\Gamma(2g-1)}{A^{2g-1}} F_0^{(1)}.$$

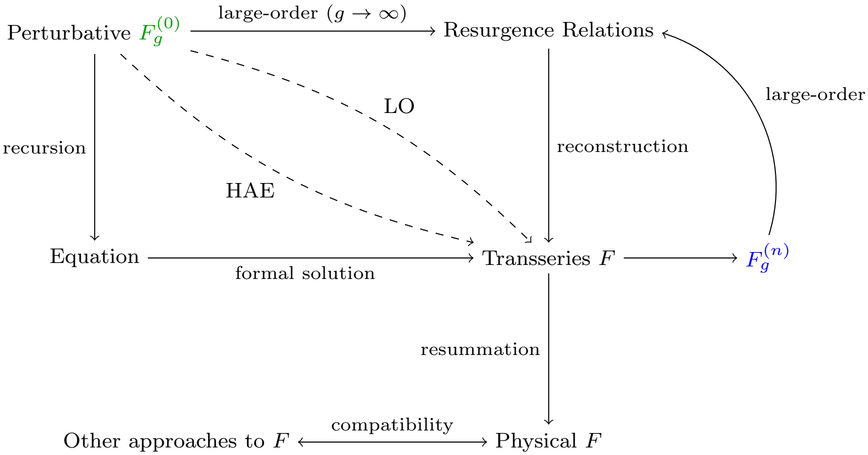
  Both A and $F_0^{(1)}$ have a $\log(Q)$-singularity.

- Is there any enumerative and nonperturbative information we can extract from $F_g^{(n)}$?

- Resumming the transseries: $F = \sum_n \sigma^n \, e^{-nA/g_s} \, \mathcal{S} F^{(n)}$.

  Work in progress with M. Mariño and R. Schiappa.

Resummation of the transseries



Resummation

Resummation is a process that takes a transseries

$$F(\boldsymbol{\sigma}, g_s; z, S) = \sum_{g=0}^{+\infty} g_s^{2g-2} F_g^{(\mathbf{0})}(z,S) + \sum_{\boldsymbol{n}\neq\mathbf{0}}^{+\infty} \boldsymbol{\sigma}_{\mathrm{D}}^{\boldsymbol{n}}\, \mathrm{e}^{-\boldsymbol{n}\cdot\boldsymbol{A}_{\mathrm{D}}(z)/g_s} \sum_{g=0}^{\infty} g_s^g F_g^{\mathrm{D},(\boldsymbol{n})}(z,S)$$

$$+ \sum_{n=1}^{\infty} \sigma_{\mathrm{NS}}^n\, \mathrm{e}^{-nA_{\mathrm{NS}}(z,S)/g_s^2} \sum_{g=0}^{+\infty} (g_s^2)^g F_g^{\mathrm{NS},(n)}(z,S).$$

into a number.

Asymptotic series are resummed with Borel-Écalle resummation

$$\sum_{g=0}^{+\infty} g_s^g F_g^{(\boldsymbol{n})} \longmapsto \int_0^{\infty} ds\, \mathrm{e}^{-s/g_s} \left(\sum_{g=0}^{+\infty} s^g \frac{F_g^{(\boldsymbol{n})}}{g!} \right)$$

or its numerical counterpart: Borel-Padé-Écalle resummation.

Transseries parameters $\boldsymbol{\sigma}$ must be fixed to numerical values.

Nonperturbation definition of topological strings

For toric Calabi–Yau geometries Grassi, Hatsuda, Mariño (1410.3382) proposed a nonperturbative free energy

$$F = \log Z.$$

Construction for local $\mathbb{CP}^2$:

Underlying Riemann surface: $\mathrm{e}^x + \mathrm{e}^y + \mathrm{e}^{-x-y} + z = 0$

Operator: $\hat{\mathcal{O}} := \mathrm{e}^{\hat{x}} + \mathrm{e}^{\hat{y}} + \mathrm{e}^{-\hat{x}-\hat{y}}, \quad [\hat{x}, \hat{y}] = i\hbar, \quad \hat{\rho} := \hat{\mathcal{O}}^{-1}$

Finite traces: $Z_\ell(\hbar) := \mathrm{Tr}\, \hat{\rho}^\ell, \quad \ell = 1, 2, \ldots$

Partition functions: $\displaystyle\sum_{N=0}^{\infty} \boxed{Z(N, \hbar)}\, \kappa^N := \exp\left(- \sum_{\ell=1}^{\infty} Z_\ell(\hbar) \frac{(-\kappa)^\ell}{\ell} \right)$

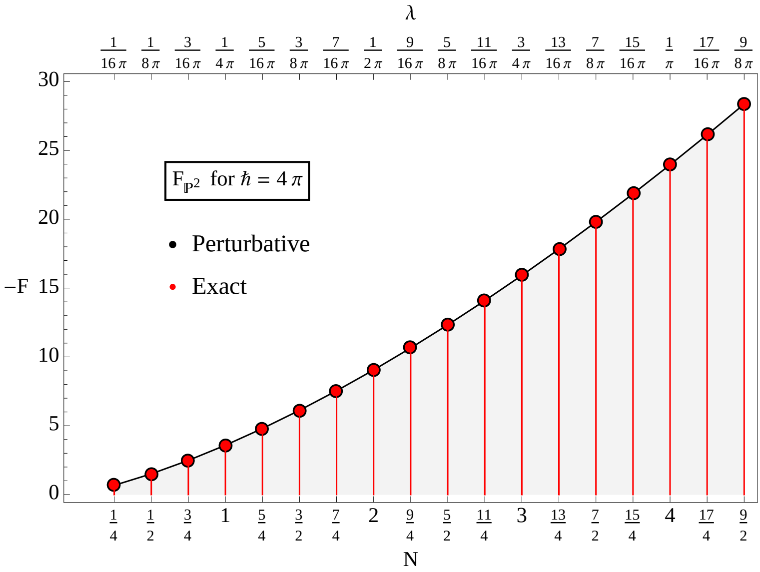
Dictionary:

$$\hbar = \frac{4\pi^2}{g_s}, \qquad N = \frac{t_c}{g_s}, \qquad t_c\text{: conifold flat coordinate.}$$

Examples:

$$Z(1, 2\pi) = \frac{1}{9}, \qquad Z(2, 4\pi) = \frac{5}{324} - \frac{1}{12\sqrt{3}\pi}.$$

Pertubative vs Exact



Resummation example: $N = 2$, $\hbar = 4\pi$ $(g_s = \pi)$

Exact result vs resummation of perturbation theory:

$$\text{exact } (-F) = \underline{9.049\,862\,10}2\,738\,02\ldots = -\log\left(\frac{5}{324} - \frac{1}{12\sqrt{3}\pi}\right)$$

$$\text{pert } (-F^{(0)}) = \underline{9.049\,862\,10}3\,051\,21\ldots$$

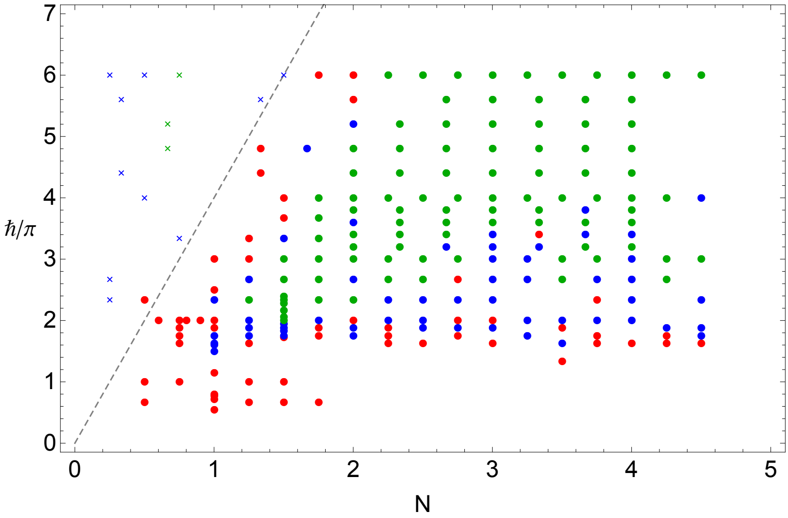
The difference is a nonperturbative effect, exponentially small, e^{-A/g_s}:

$$\Delta F = \underline{3.131\,8}\ldots \cdot 10^{-10}$$

$$\text{1-inst } F^{(1)} = \underline{3.131\,8}40\ldots \cdot 10^{-10}$$

Relevant instanton actions are A_{K} and A_2.

Resummation of the transseries



tl;dr

- Using large-order analysis and the holomorphic anomaly equations we can find a transseries in e^{-A/g_s} that extends $F^{(0)}$.

- At the a special point in moduli space where the Borel plane is empty the $F_g^{(0)}(z=0)$ grow slowlier, like $g!$ instead of $(2g)!$. Universality.

  There is a new sector of the transseries in $\mathrm{e}^{-A_{\mathrm{NS}}/g_s^2}$ that has NS5-brane features.

- Using the mirror map from B to A-model we can study the resurgence of Gromov–Witten invariants, $N_{g,d}$.

  Only when d and g are in linear combination do they grow factorially and there is a correspondence with growth of $F^{(0)}$. Universality for $d = (2g-3)/t$.

- We can resum the transseries and match against nonperturbative proposal of Grassi–Hatsuda–Mariño.

  We find good results to the right of a Stokes line. Work in progress.