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How small hydrodynamics can go

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冨嶽三十六景
神奈川沖浪裏
L
a
Hydrodynamics = effective field theory = continuum description
valid when L >>> a

Hydrodynamics in a slide

slowest

faster

much faster

+ ...

largest

smaller

much smaller

+ ...

Until which scale can we trust the hydrodynamic expansion ?

Which is the physical scale setting the breakdown of the hydrodynamic expansion ?

Hydrodynamic modes $\omega(k \to 0) = 0$ [conserved quantities, Goldstones]

Non-Hydrodynamic modes $\omega(k \to 0) \neq 0$ [relaxing modes, transient modes]

$$\mathcal{D}_{AB}(\omega, k)\, \boxed{\delta\psi^A} = 0$$

Dynamical matrix

Field fluctuations (e.g. energy, momentum)

Example:

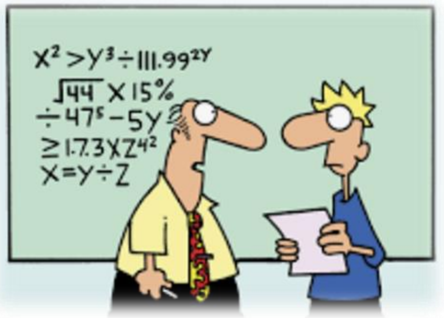
$$\mathcal{D}^{\perp}_{AB}(\omega, k) = \begin{pmatrix} \frac{\eta\, k^2}{\chi_{\pi\pi}} - i\,\omega & -i\,G\,k \\ -\frac{i\,k}{\chi_{\pi\pi}} & G\,k^2\,\xi_\perp - i\,\omega \end{pmatrix}$$

$$\omega_{\text{diff}}(z \equiv \mathbf{q}^2) = -i \sum_{n=1}^{\infty} c_n z^n,$$

$$\omega^{\pm}_{\text{sound}}(z \equiv \sqrt{\mathbf{q}^2}) = -i \sum_{n=1}^{\infty} a_n e^{\pm \frac{i\pi n}{2}} z^n,$$

Hydrodynamics = theory of slow and large-scale processes (expansion in gradients)

[Withers, Grozdanov, Starinets, Kovtun, Tadic, and many more ….]

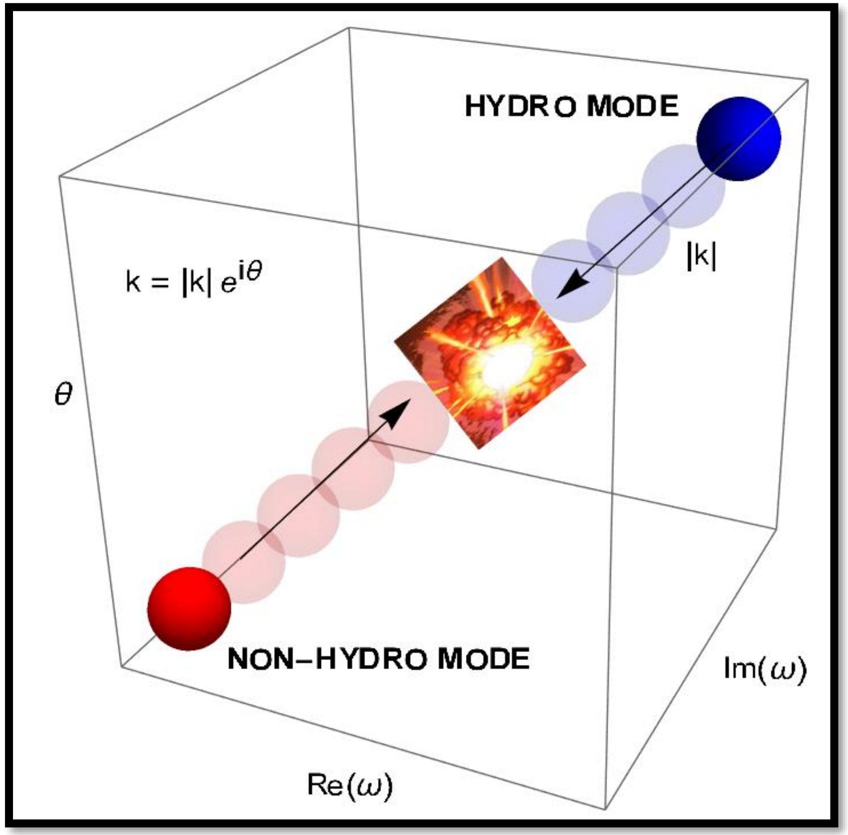


$$\mathrm{Det}\mathcal{D}_{ab} = F(\omega, k^2) = 0$$

Critical points

$$\omega_c, k_c^2 \in \mathbb{C}$$

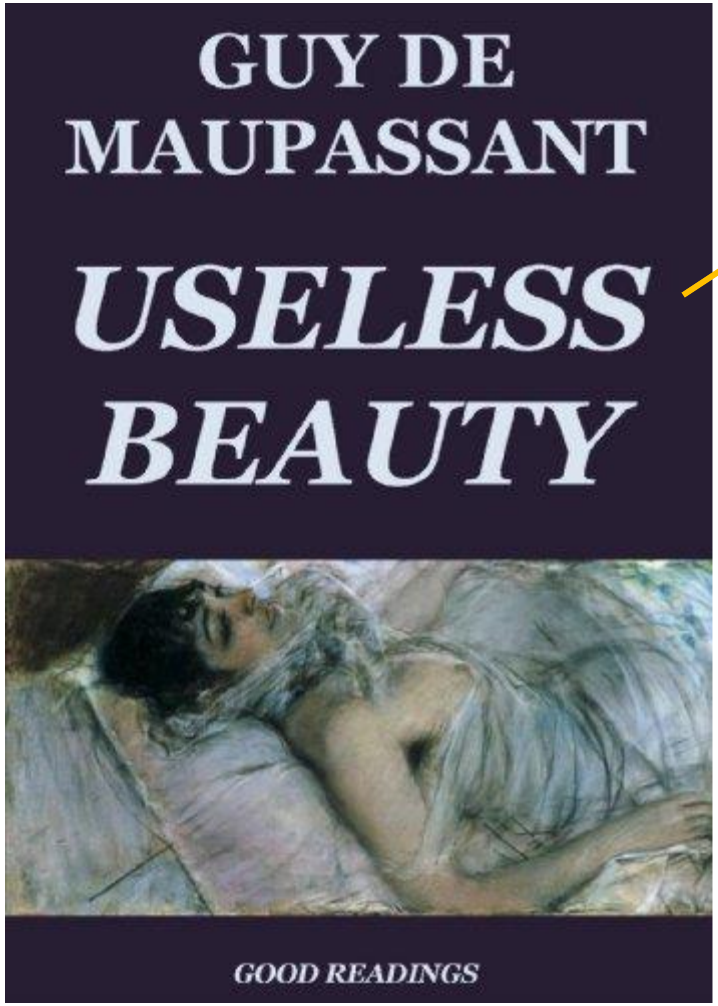
$$F(\omega_c, k_c^2) = 0\,, \qquad \frac{\partial F(\omega_c, k_c^2)}{\partial \omega} = 0\,,$$



$$\mathcal{R} \equiv |k_c|$$

Position of the lowest critical point determines the regime of applicability of linearized hydrodynamics (in momentum space)

“until when non-hydro modes are harmless”



Or not ?

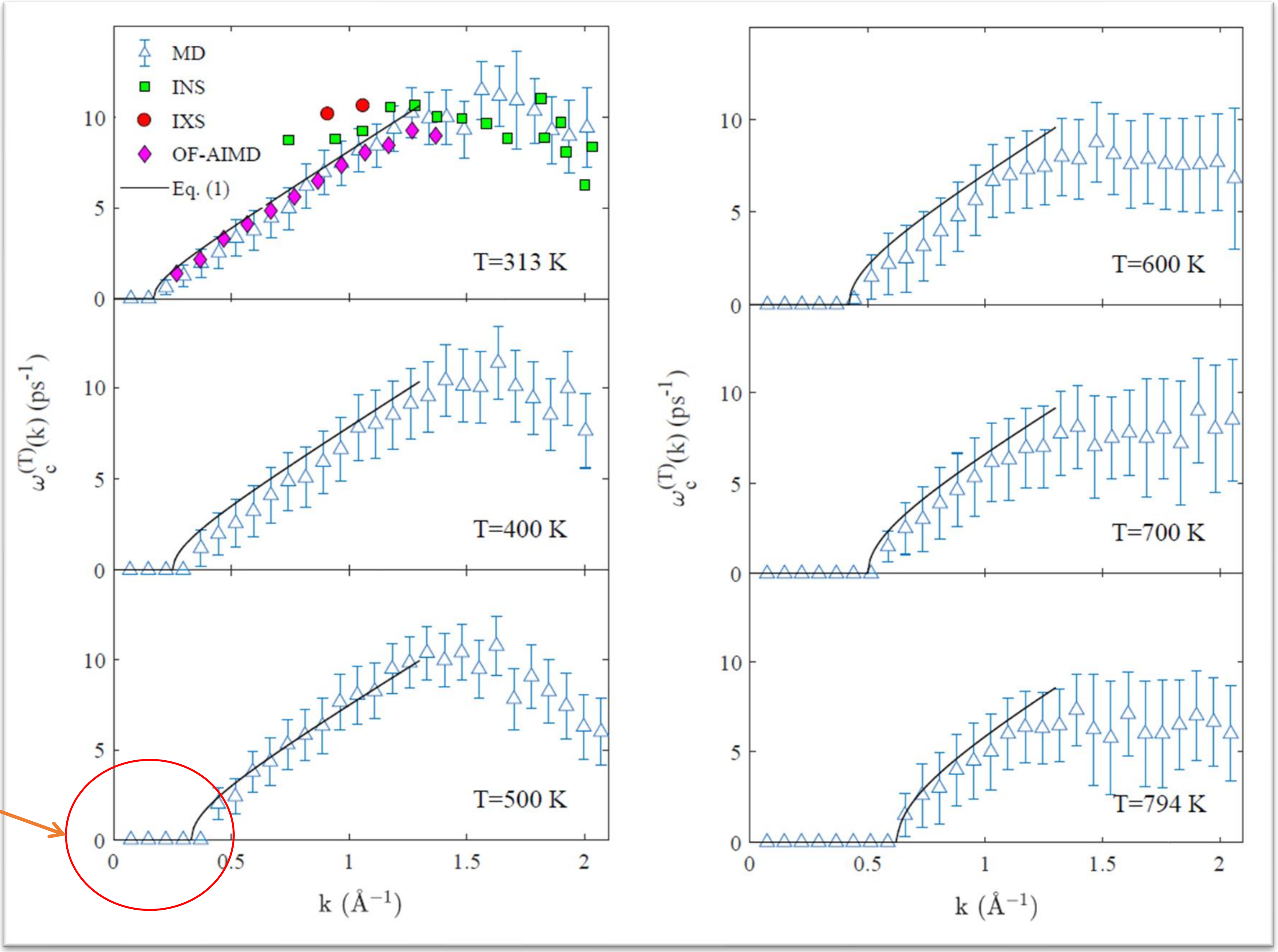
For the moment, a very beautiful mathematical result and few computations in holographic models, SYK model and kinetic theory

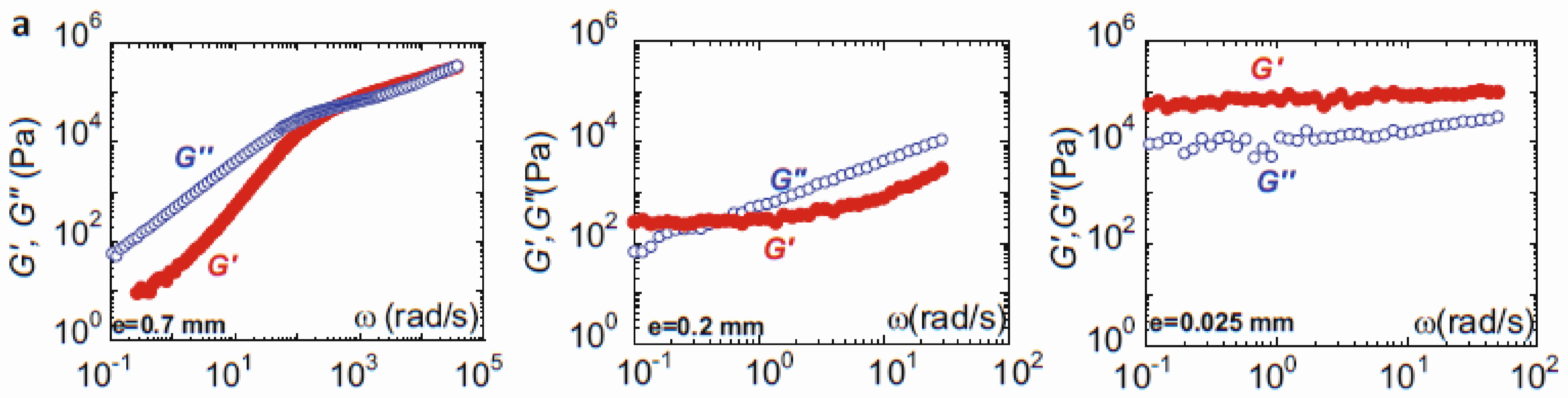
Can we apply these concepts to realistic liquids ?

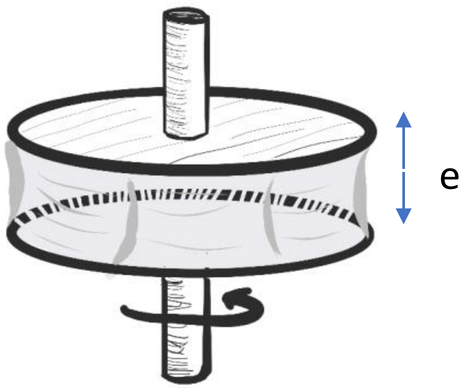
Dispersion relation of shear waves in real liquids

[Trachenko et Al,
+ many more]

Diffusion to propagation crossover







Smaller distance between the plates

→

Larger momentum probed

There is a liquid to solid crossover going to small sizes (or large momenta)

Identification of a low-frequency elastic behaviour in liquid water

Laurence Noirez[1] and Patrick Baroni[1]

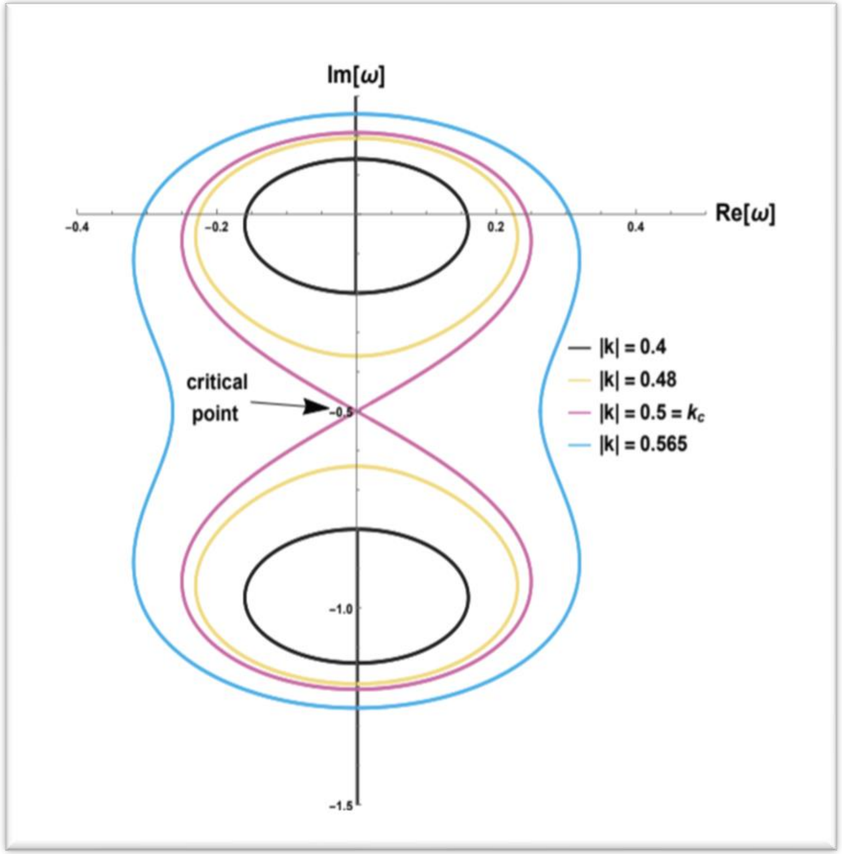
TELEGRAPHER EQUATION (Heaviside)

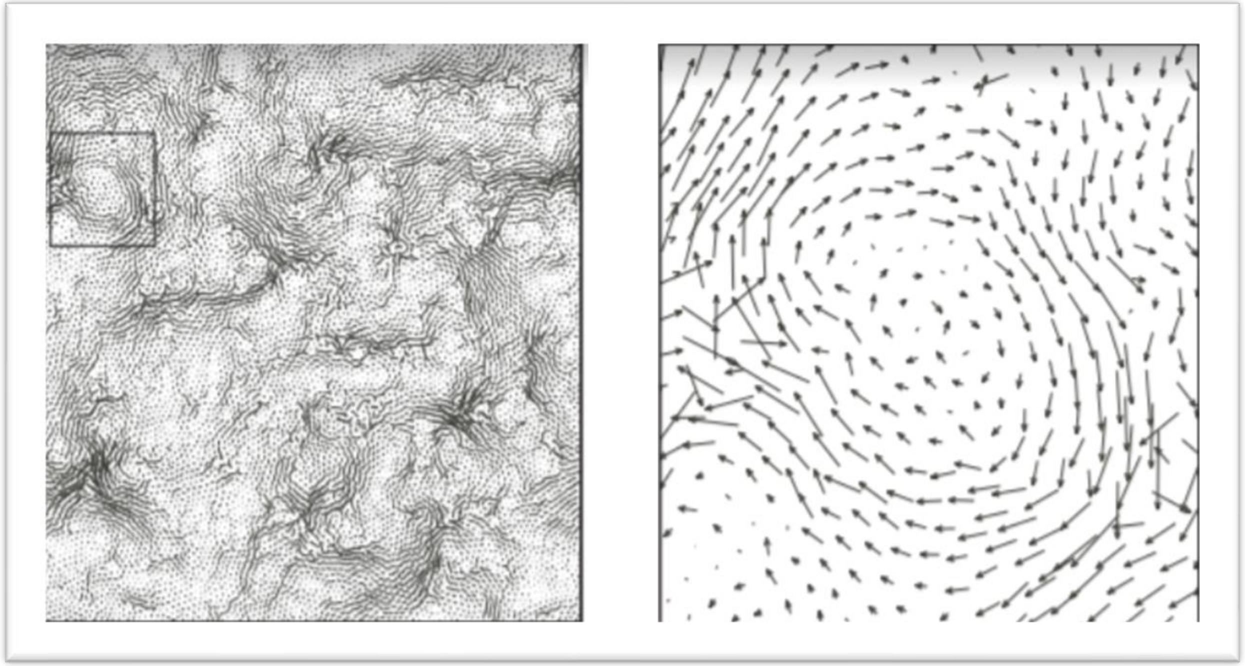
$$\omega^2 + i\,\omega/\tau - v^2\,k^2 = 0\,,$$

Several simulations and (few) experiments confirm this is a good description for shear waves in liquids

$$\omega = -\frac{i}{2\,\tau} \pm \sqrt{v^2\,k^2 - \frac{1}{4\,\tau^2}}\,,$$

$$\mathcal{R} \equiv |k_c| = \frac{1}{2\,v\,\tau} = k_g\,.$$



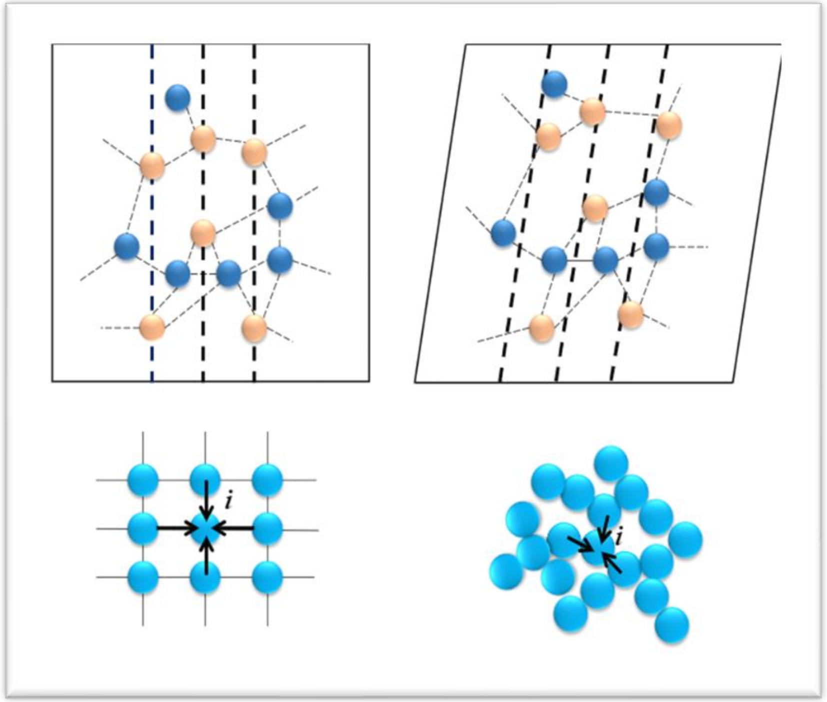


Non-affine displacements

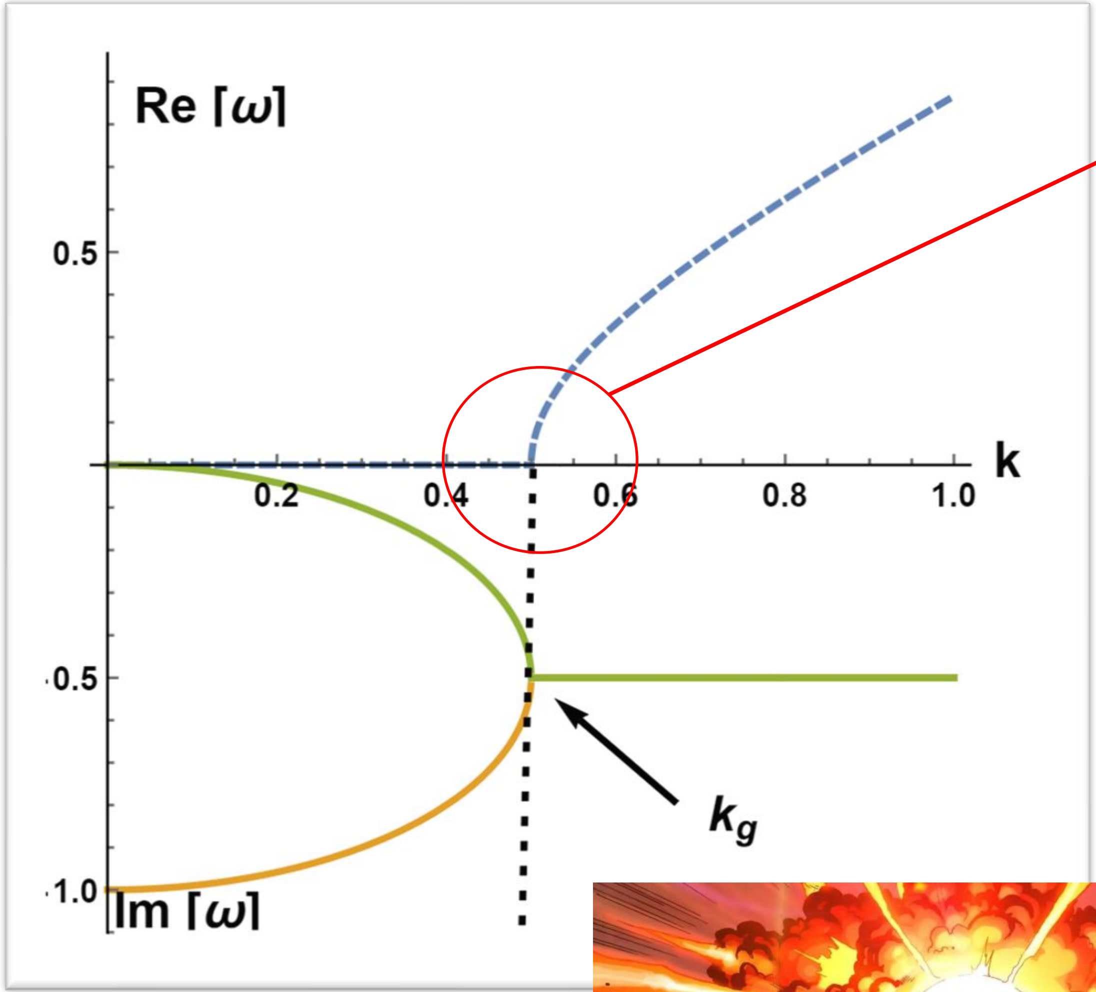
Singular parts of the displacements (cf.dislocations and vortices)

[MB et Al] arXiv:2101.05015

$$u_i(\mathbf{x}) = \underbrace{\gamma_{ij}\, x^j}_{\text{affine}} + \underbrace{u'_i(\mathbf{x})}_{\text{non-affine}} \ ,$$



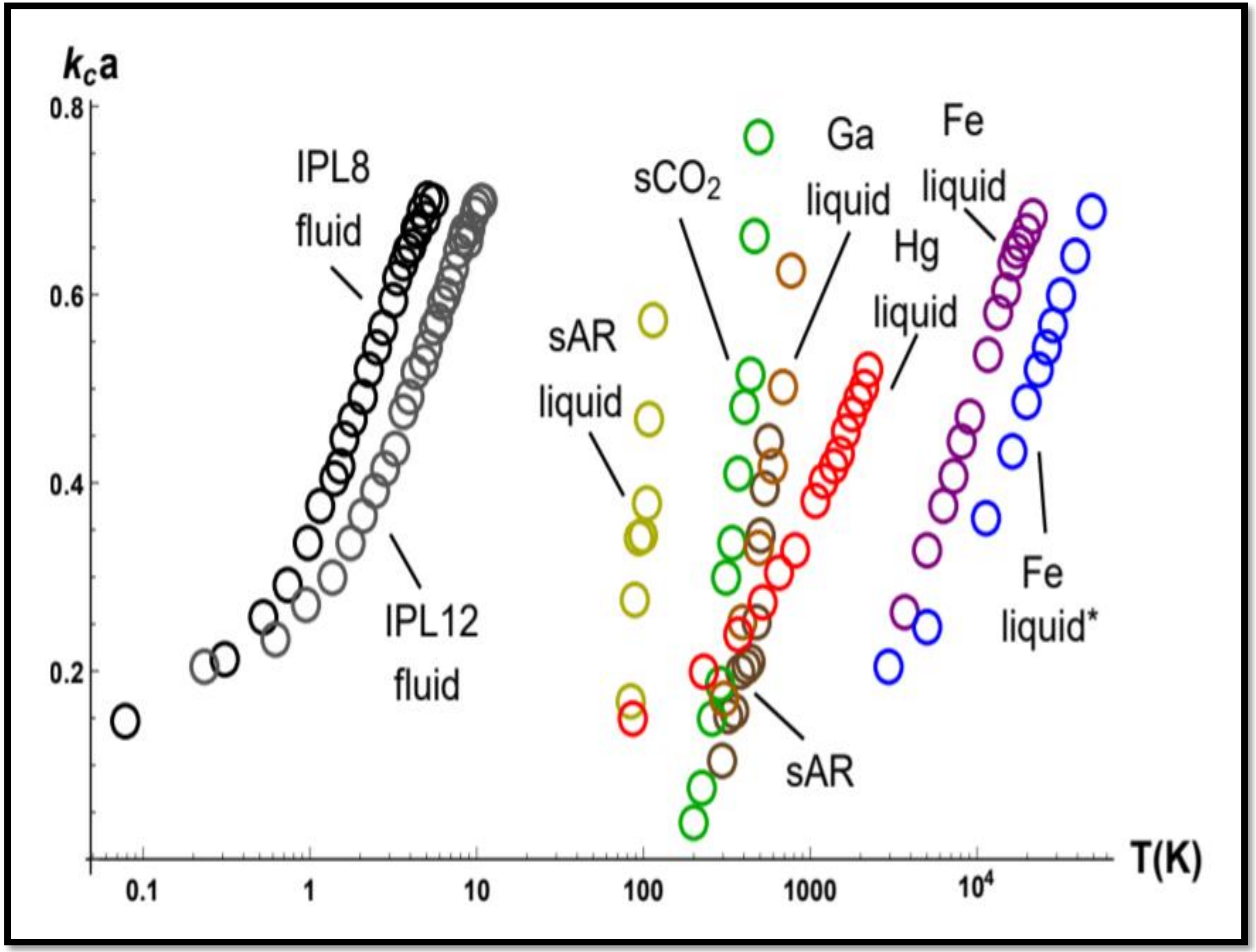
Macroscopic phase relaxation (symmetry restoration)



We can extract the radius of convergence from real data (MD simulations + experiments)

- Simple because the collision happens at real values of momentum (unfortunately not the same in the longitudinal sector)
- The simple telegrapher equation would certainly receive corrections but it fits very well the data around the critical point (=corrections & higher order modes negligible)

[MB] arXiv:2010.05916



Hydrodynamics works worse and worse going towards small temperatures!

Cf. naïve argument

$$\omega/T \ll 1,\ \ k/T \ll 1$$

arXiv:2010.05916

Liquid	$k_c\, a$
2D Yukawa (MD) [20]	0.25
dusty plasma (MD) [21, 22]	0.3-1.2
2D Yukawa (EXP) [23]	0.16-0.31
Liquid Fe (EXP) [24, 25]	0.3
Liquid Cu (EXP) [24, 25]	0.4
Liquid Zn (EXP) [24, 25]	0.3
3D LJ fluid (MD) [26]	0.2-0.7
Liquid Fe (MD) [26]	0.2-0.7
IPL8-IPL12 fluid (MD) [26]	0.2-0.7
Liquid Hg (MD) [26]	0.15-0.55
Supercritical Ar (MD) [27]	0.05-0.8
Subcritical liquid Ar (MD) [27]	0.2-0.7
Supercritical CO_2 [27]	0.1-0.5
Liquid Ga (EXP, MD) [28]	0.25-0.6
2D Coulomb classical fluids (MD) [29]	0.3-2
Quark Gluon Plasma [30]	3.3

$$k_c\, a \approx \mathcal{O}(1)$$

Breakdown scale given by the only microscopic scale available: the inter-molecular distance a

Below "a", the continuum clearly stops to make sense because we start seeing the individual fluid particles (cf. phonons with wavelength comparable to lattice spacing)

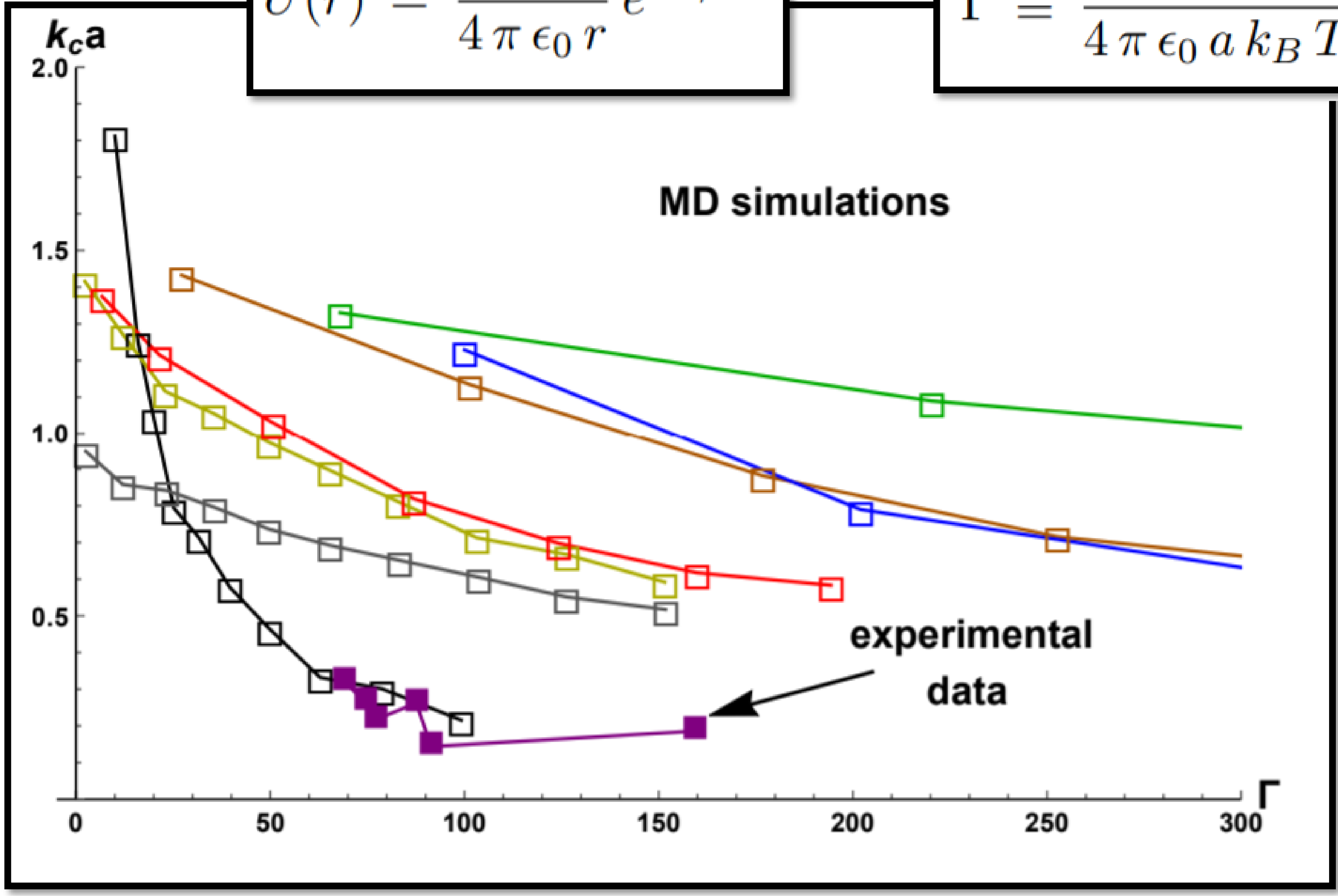
arXiv:2010.05916

Coulomb liquids

arXiv:2010.05916

$$U(r) = \frac{Q}{4\pi\epsilon_0 r} e^{-r/\lambda_D}$$

$$\Gamma \equiv \frac{Q^2}{4\pi\epsilon_0 a k_B T}$$



Holographic results

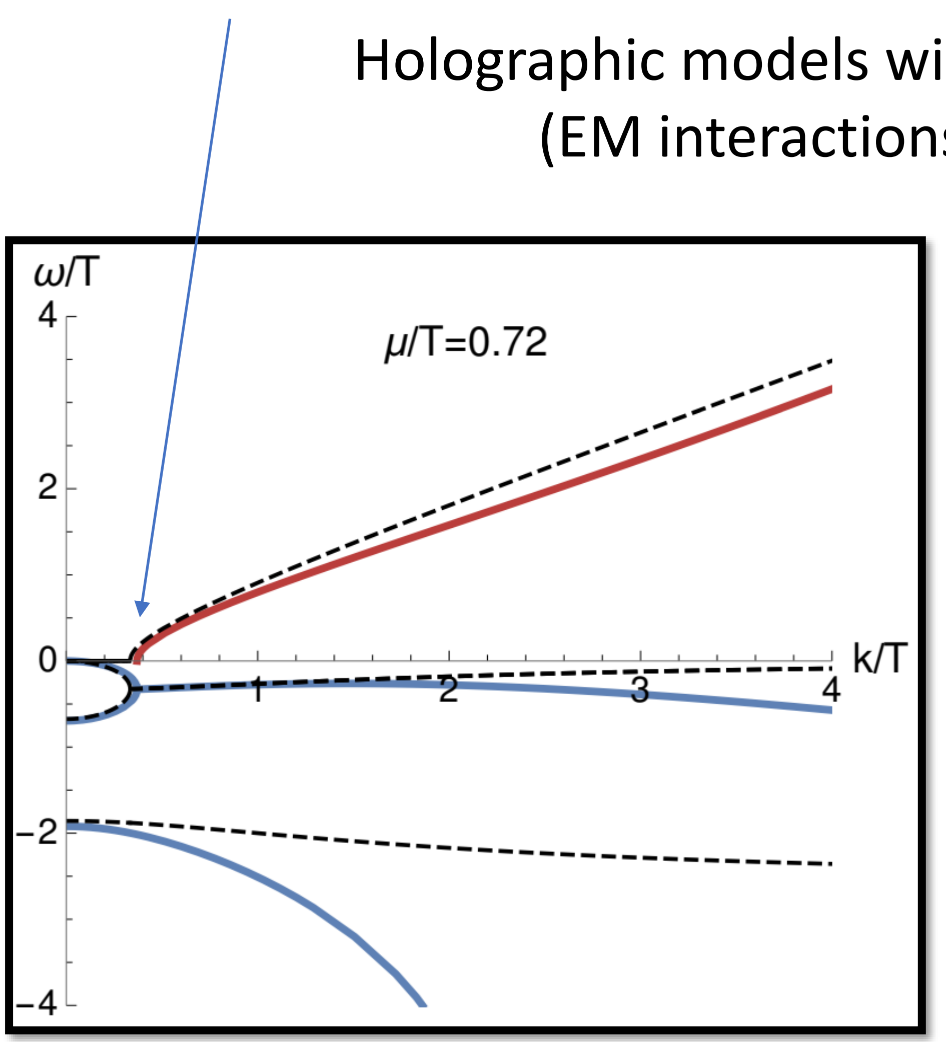
arXiv:1905.00804

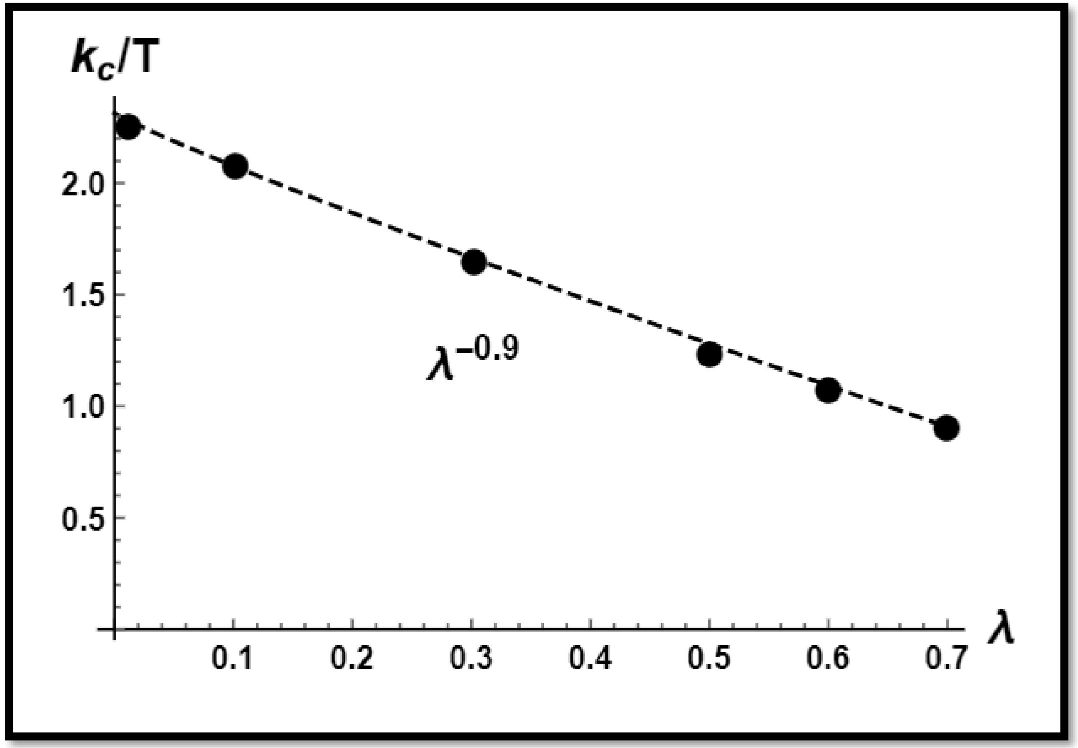
[Baggioli, Gran, Tornso 2020]

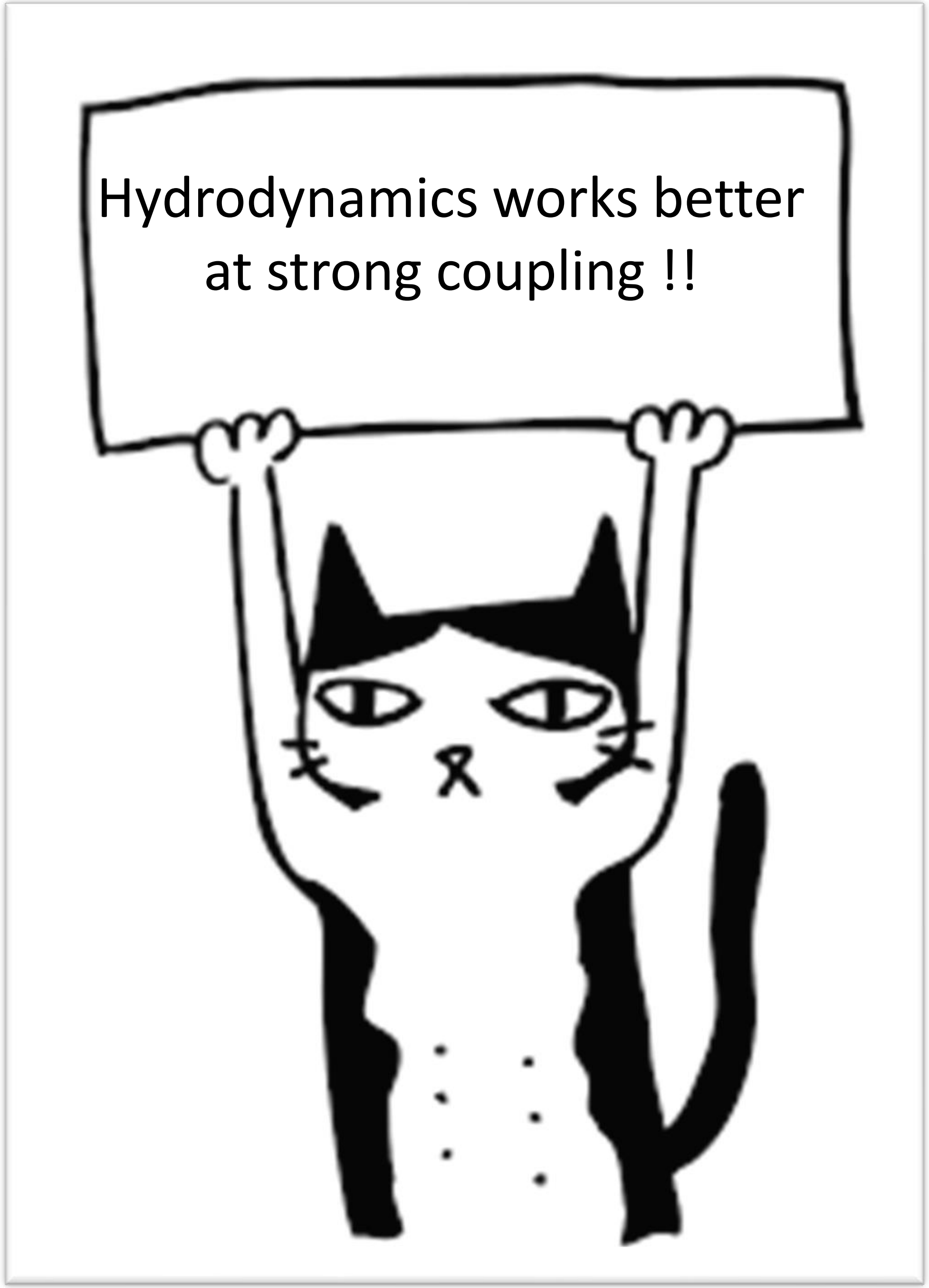
$$\omega^2 \mathcal{A}_x + \mathcal{J}_x = 0$$

Holographic models with dynamical gauge fields
(EM interactions at the boundary)

Mixed b.c.s.



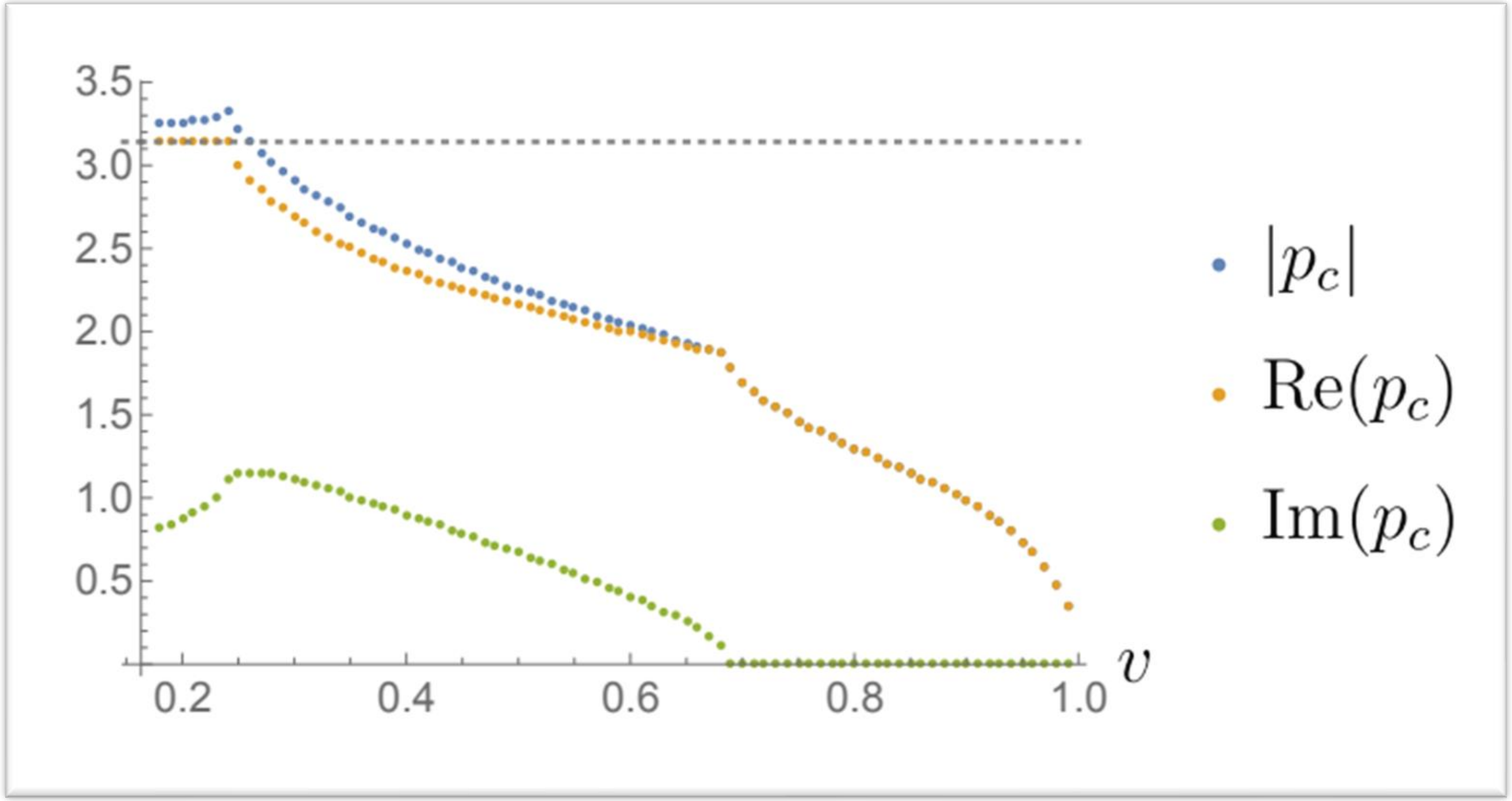




Not supported by any quantitative computation !
Wrong (or not so simple) ???

JUST DO IT.

Direct computations challenge this slogan



Results in SYK model

[Choi, Mezei, Sarosi, 2020]

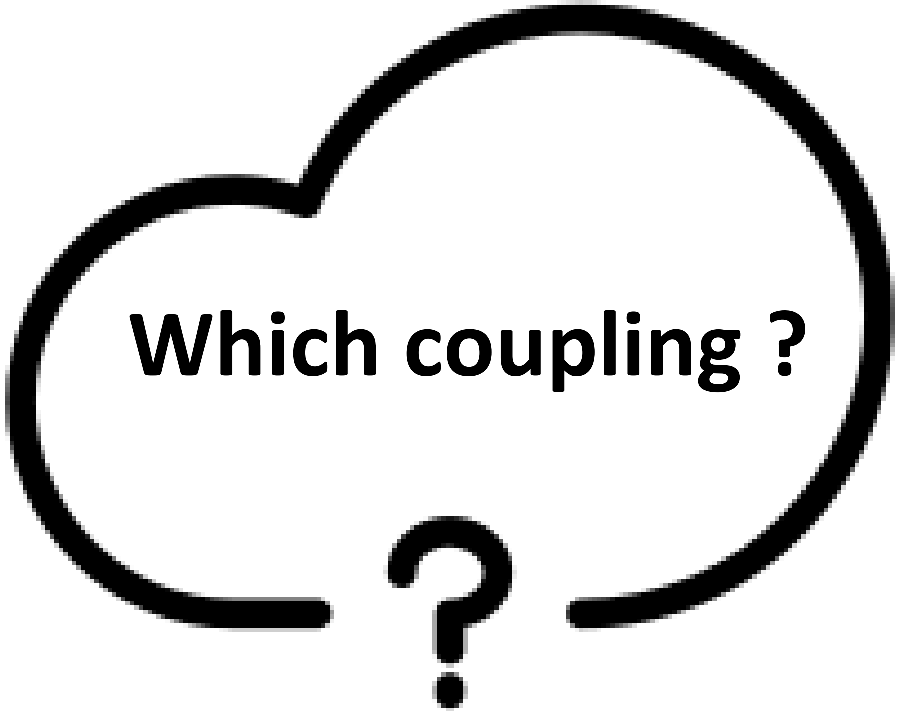
Do these results support our idea ?

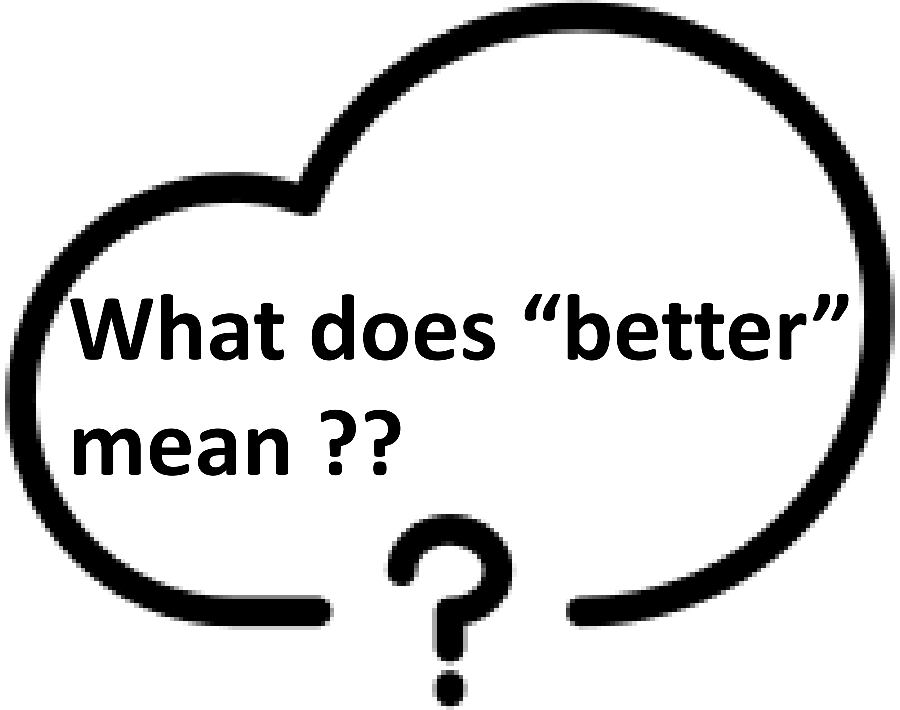
Conclusions

- Direct application of the recent mathematical framework to realistic systems
- "Experimental verification" of intuitive arguments :
  Hydro limited by microscopic inhomogeinities scale,
  Hydro work worse at low temperature
- Surprise : hydro does not work better at strong coupling
  (see more later)

Comments

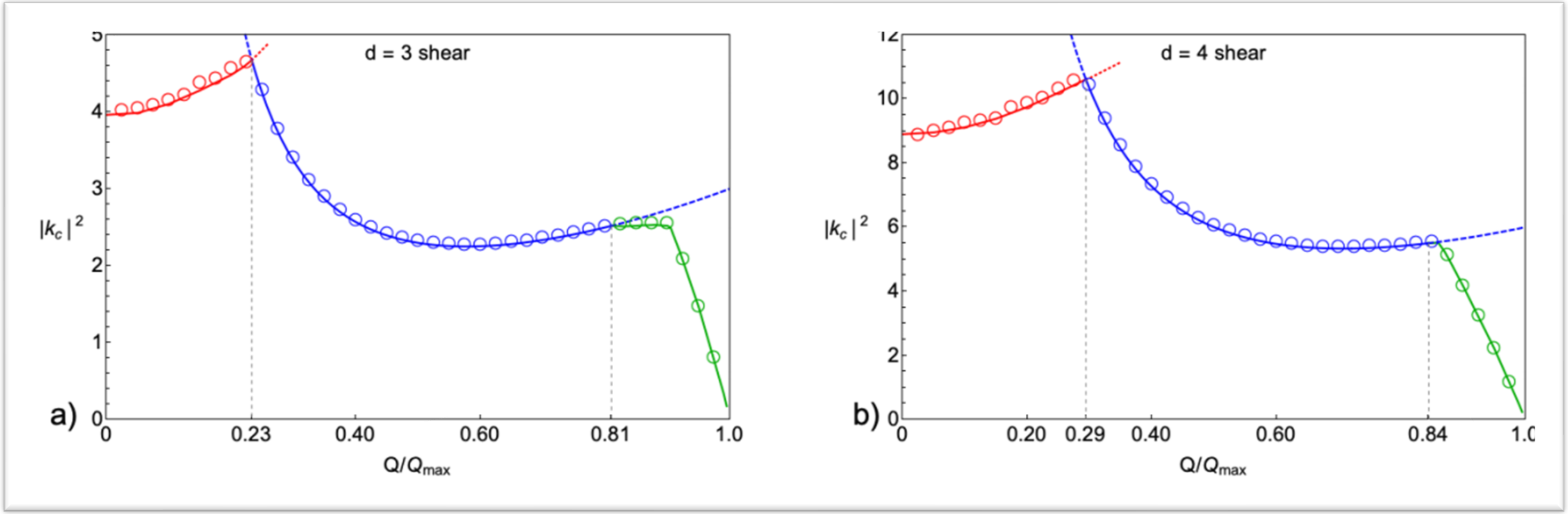
“Hydro works better at strong coupling”





Comments

[Pantelidou, Jansen]



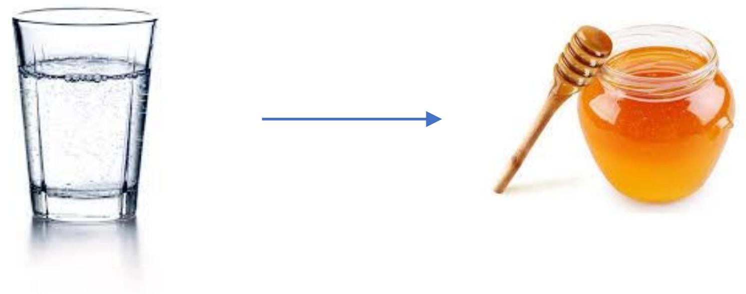
Hydro works better in large number of dimensions (true?).
Why ?

$$a = \left(\frac{2(D-1)}{D}\, n \right)^{-1/D}$$

Can we explain it from this ?

Simple (or not) questions

Given a liquid, is the radius of convergence growing if I make it more viscous and why ?

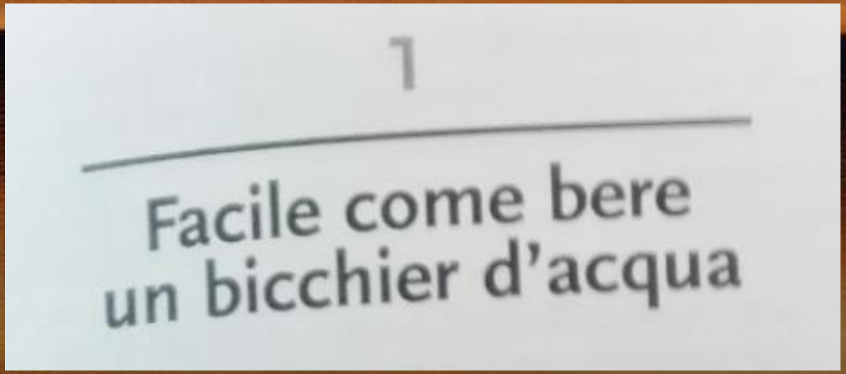


Which physical properties determine the radius of convergence ? And how the latter depends on them ?

Other expansions in physics (chiral perturbation theory, elasticity, cosmology)

Do we really need complex space ?
For experiments it’s a no-go!
Other ideas ?
See [Heller & friends]

CONCLUSIONS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Facile come bere
un bicchier d'acqua

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Thanks!