A New Spin on the WGC

Based on: arXiv:2011.05337
with A. Cole, G. Loges and G. Shiu

Lars Aalsma
University of Wisconsin-Madison

Iberian Strings 2021

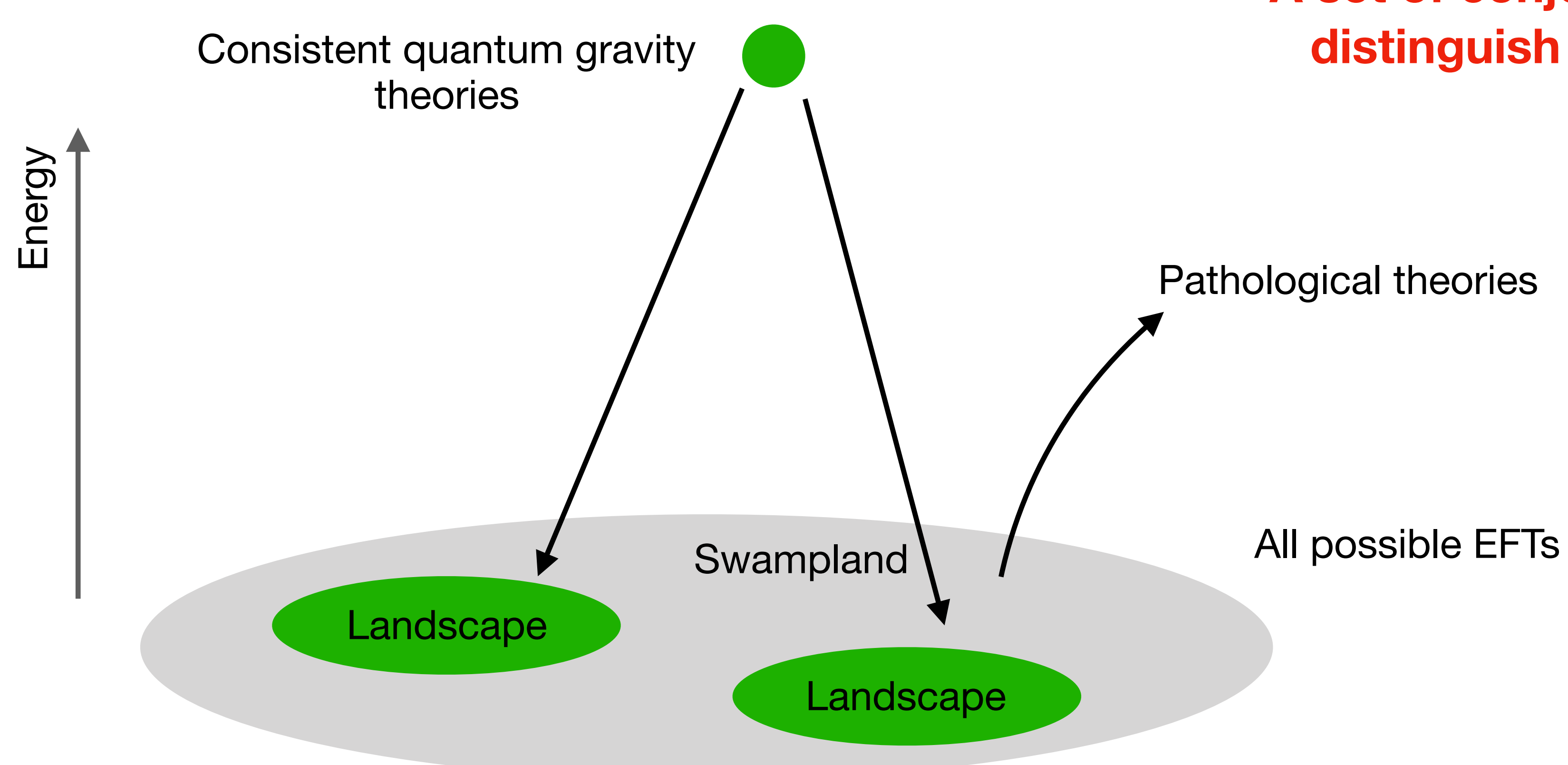
WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

Motivation

- It has been appreciated for a long time that UV physics constrains IR EFTs.

- This idea has been formulated most sharply in the context of the swampland. [Ooguri, Vafa '06]

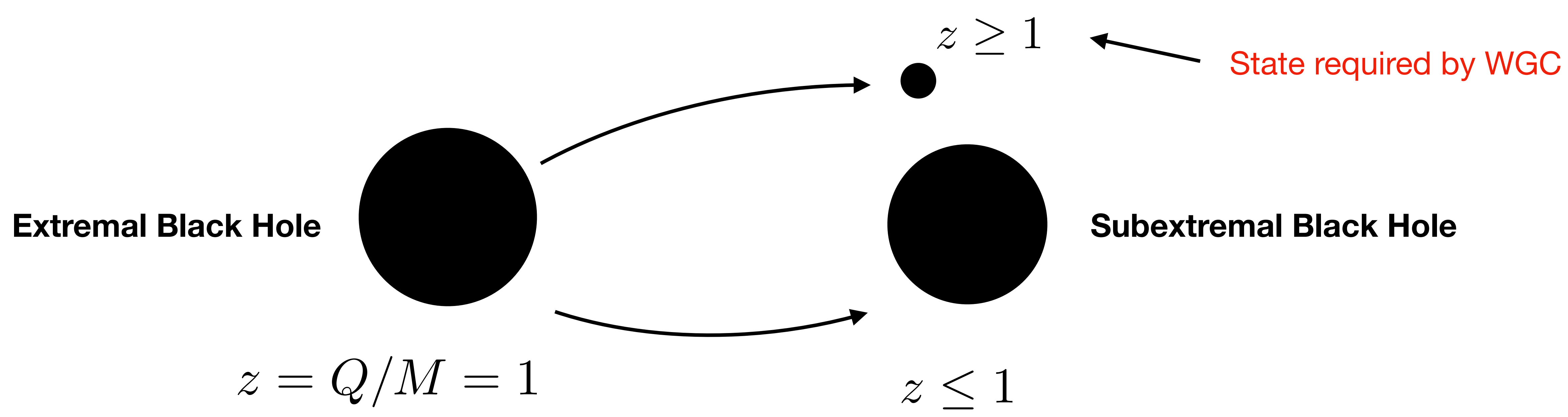
A set of conjectures (and some proofs) distinguish the Landscape from the Swampland



Timo Weigand's review on Friday!

Weak Gravity Conjecture

- Loosely formulated, the WGC requires extremal charged black holes to decay. [Arkani-Hamed, Motl, Nicolis, Vafa '06]



- In the "mild" form, the WGC-satisfying state can be a black hole.

- Possible when higher-derivative corrections are taken into account.

WGC from Positivity

- Higher-derivative corrections modify the extremality bound of extremal Reissner-Nordström black holes. [Kats, Motl, Padi '06]

$$\mathcal{L} = R - \frac{1}{4}F_{ab}F^{ab} + \frac{a_1}{4}(F_{ab}F^{ab})^2 + \frac{a_2}{2}F_{ab}F_{cd}W^{abcd}$$

$$\Delta z \sim \frac{2a_1 - a_2}{Q^2} \quad \longleftrightarrow \quad \Delta S \sim \sqrt{\frac{2a_1 - a_2}{Q^2}}$$

WGC is satisfied when $2a_1 - a_2 \geq 0$

- Unitarity and causality constrain Wilson coefficients, but additional UV assumptions are needed to prove WGC. [Hamada, Noumi, Shiu '18][Bellazzini, Lewandowski, Serra '19][Alberte, de Rham, Jaitly, Tolley '20] [Alberte, de Rham, Jaitly, Tolley '20] …

- What is the minimal set of assumptions?

Main results

- We reformulate the (mild) WGC as a condition on the stress tensor that applies to any extremal black hole. [LA, Cole, Loges, Shiu '20]

$$\Delta z \geq 1 \quad \longleftrightarrow \quad \int_\Sigma d^{d-1}x\sqrt{h}\,\delta T_{ab}\xi^a n^b \leq 0$$

- What underlying principle leads to positivity?

- We proof positivity for BTZ; implies stronger bounds on black objects with BTZ near-horizon geometry.

Overview

1. **Sketch of Derivation.**

2. Proof for BTZ black holes.

3. Application to black strings.

Sketch of Derivation

- Using the Iyer-Wald formalism, we relate the Hamiltonian to the off-shell variation of the equations of motion.

Correction to entropy

Killing vector

$$(\star) \quad \left(\int_{S_{\infty}^{d-2}} - \int_{S_{\text{hor}}^{d-2}} \right) \delta \mathbf{H} = \int_{\Sigma} \mathrm{d}^{d-1}x \, \sqrt{h} \, \delta T^{\text{eff}}_{ab} n^a \xi^b \ .$$

Variation of asymptotic charge

Stress tensor containing corrections

- To evaluate the left-hand side, we need to specify an extremal black hole background.

Example: Charged Black Hole

[LA, Cole, Loges, Shiu '20]

Uncorrected metric: $$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2$$

Uncorrected gauge field: $$A = \left(-\frac{Q}{4\pi r} + \Phi_+\right) dt$$

→ $$f(r) = \frac{(r-r_-)(r-r_+)}{r^2}$$

Two conserved charges:

$$\delta H_{\partial_t} = \int_{S^2_\infty} \delta \mathbf{H}_{\partial_t} = \delta M_4$$

$$\delta H_\lambda = \int_{S^2_\infty} \delta \mathbf{H}_\lambda = -\Phi_+ \delta Q$$

- Evaluating $(\star)$ with fixed asymptotic charges:

$$\Delta S \sim -\frac{r_+ \delta f(r_+)}{2G_4} = -\int_\Sigma \mathrm{d}^3x \sqrt{h}\, \delta T^{\text{eff}}_{ab} n^a \xi^b \geq 0 \quad \longleftrightarrow \quad \Delta z \geq 1$$

Charged Black Holes and Beyond

[LA, Cole, Loges, Shiu '20]

- Now we choose a particular form of the corrections.

$$\mathcal{L} = R - \frac{1}{4}F_{ab}F^{ab} + \frac{a_1}{4}(F_{ab}F^{ab})^2 + \frac{a_2}{2}F_{ab}F_{cd}W^{abcd}$$

- Evaluating the integrated condition, we find (as before):

WGC satisfied when:

$$\int_\Sigma \mathrm{d}^3x\,\sqrt{h}\,\delta T^{\mathrm{eff}}_{ab}n^a\xi^b \sim -(2a_1 - a_2) \leq 0$$

- This approach can be adapted to any stationary black hole solution that has a Killing horizon.

- What property of the corrections leads to this positivity?

Overview

1. Sketch of Derivation.

2. **Proof for BTZ black holes.**

3. Application to black strings.

Correction to BTZ Black Hole

- Consider the following corrections to three-dimensional AdS gravity:

$$\mathcal{L} = R + \frac{2}{\ell^2} + \alpha_1 \ell R^2 + \alpha_2 \ell R_{ab} R^{ab}$$

AdS$_3$ Gravity **CFT$_2$**

$$\frac{J_3}{\ell M_3} = 1 + \frac{48\pi G_3 (3\alpha_1 + \alpha_2)}{\ell} \quad \longleftrightarrow \quad c = \frac{3\ell}{2G_3}\left(1 - \frac{48\pi G_3 (3\alpha_1 + \alpha_2)}{\ell}\right)$$

- A positive correction to the extremality bound decreases the central charge.

Positivity from RG flow

- In CFT_2, the central charge decreases along an RG flow by the c-theorem. [Zamolodchikov '86]

CFT_2

AdS_3 Gravity

$\mathrm{CFT}_{\mathrm{UV}}(c)$ + relevant deformation ⟷ $\frac{J_3}{\ell M_3} = 1$ BTZ black hole + light field

$\mathrm{CFT}_{\mathrm{IR}}(c')$ + irrelevant deformation ⟷ $\frac{J_3}{\ell M_3} = 1 + (\dots)(c - c')$ BTZ black hole + corrections

When c-theorem holds: (c-c')>0

When NEC holds: $\int_\Sigma \mathrm{d}^2 x \sqrt{h}\, \delta T_{ab} \xi^a n^b \leq 0$

- For relevant perturbations, a spinning WGC holds.

Overview

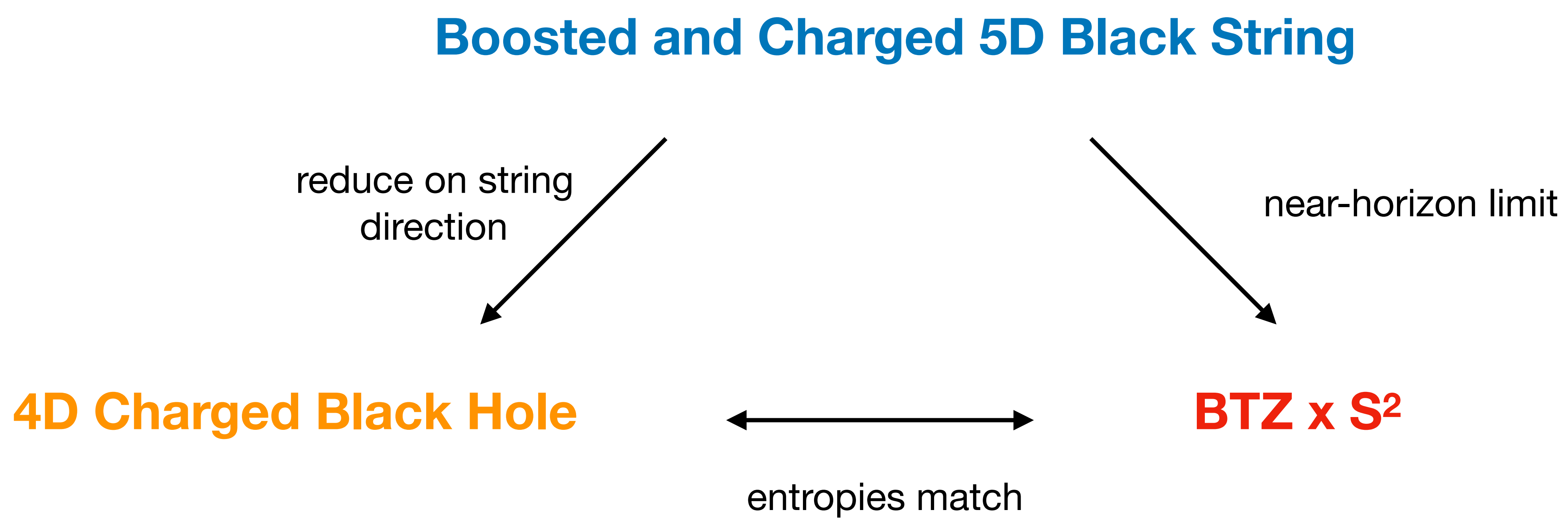
1. Sketch of Derivation.

2. Proof for BTZ black holes.

3. **Application to black strings.**

Boosted Black Strings

- When BTZ arises as a near-horizon limit, entropy can be computed using Cardy's formula. [Strominger '98]
- Can we use the BTZ geometry to infer the charged WGC?



Extremality and Entropy

[LA, Cole, Loges, Shiu '20]

- The black string is described by:

$$\mathcal{L} = R - \frac{3}{4} F_{ab}F^{ab} + \alpha_1 \left(F_{ab}F^{ab}\right)^2 + \alpha_2 F_{ab}F_{cd}W^{abcd} + \alpha_3 R_{\text{GB}}$$

$\text{BTZ} \times S^2$	$T=0$	$z = 1 + \frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}$	
		$S = 2\pi Q\sqrt{\frac{M_3}{G_3}}\left(1 - \frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}\right)$	$= \frac{\pi Q^2}{G_4}\left(1 - \frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}\right)$
	$z=1$	$T = \sqrt{\frac{G_3 J_3(8\alpha_1+3\alpha_2-12\alpha_3)}{\pi Q^3}}$	
		$S = 2\pi Q\sqrt{\frac{M_3}{G_3}}\left(1 + \sqrt{\frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}}\right)$	$= \frac{\pi Q^2}{G_4}\left(1 + \sqrt{\frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}}\right)$
4D	$T=0$	$z = 1 + \frac{2a_1+a_2}{10}$	$= 1 + \frac{8\alpha_1+7\alpha_2+6\alpha_3}{40}$
		$S = \frac{\pi Q^2}{G_4}\left(1 - 4a_1 + 4a_3\right)$	$= \frac{\pi Q^2}{G_4}\left(1 - \frac{8\alpha_1+3\alpha_2-12\alpha_3}{2}\right)$
	$z=1$	$T = \frac{\pi}{Q}\sqrt{\frac{2(2a_1+a_2)}{5}}$	$= \frac{\pi}{Q}\sqrt{\frac{8\alpha_1+7\alpha_2+6\alpha_3}{10}}$
		$S = \frac{\pi Q^2}{G_4}\left(1 + \sqrt{\frac{2(2a_1+a_2)}{5}}\right)$	$= \frac{\pi Q^2}{G_4}\left(1 + \sqrt{\frac{8\alpha_1+7\alpha_2+6\alpha_3}{10}}\right)$

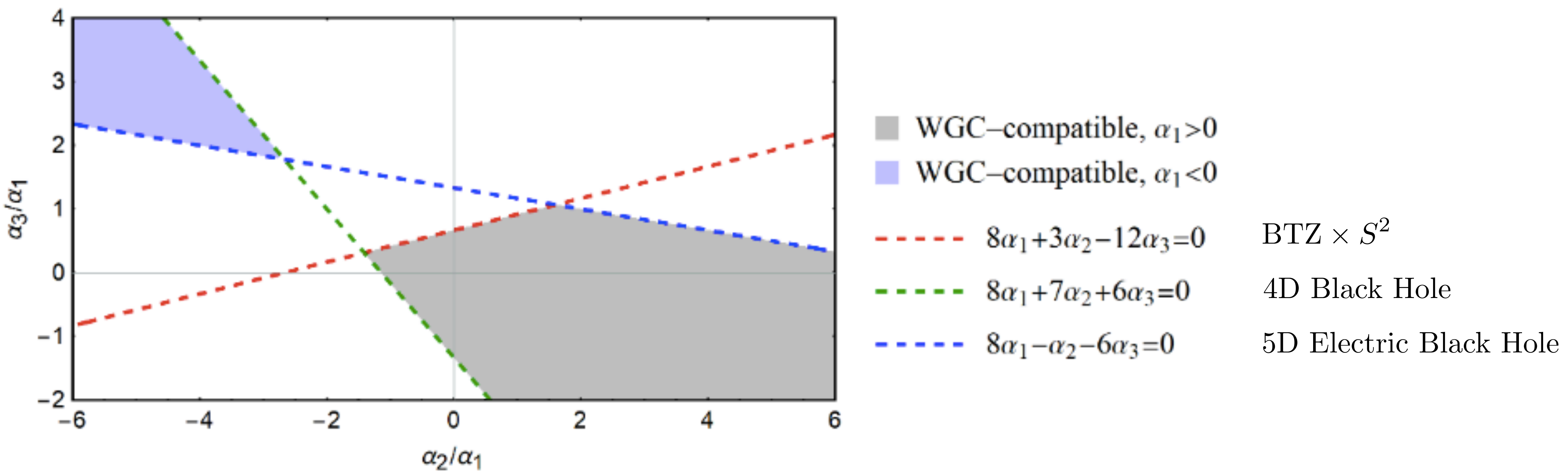
complementary extremality bounds

extremal entropy

Total Landscaping

[LA, Cole, Loges, Shiu '20]

- The extremal entropy of the 4D Black Hole and BTZ agree, but their extremality bounds do not.
- The spinning and charged WGC give complementary information; they strengthen positivity bounds.



Summary

- We rephrased the mild form of the WGC as a condition on the stress tensor.

$$\Delta z \geq 1 \quad \longleftrightarrow \quad \int_\Sigma d^{d-1}x\sqrt{h}\,\delta T_{ab}\xi^a n^b \leq 0$$

- This condition gives a computationally efficient and general way of computing extremality corrections.

- For BTZ black holes, we proved a spinning WGC.

- Combining the spinning and charged WGC, we found stronger positivity bounds.