R^2 corrected AdS_2 holography

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Outline

Main goal

Find the holographic dictionary of a specific subset of 4-derivative $\mathcal{N} = 2$ low energy effective actions of gravity in 4D.

Extremal static near horizon backgrounds factorize into $\mathrm{AdS}_2 \times S^2$

$$\downarrow$$

Apply $\mathrm{AdS}_2/\mathrm{CFT}_1$ to compute degeneracy of ground states

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Apply $\mathrm{AdS}_2/\mathrm{CFT}_1$ to compute degeneracy of ground states

Constant scalar field: $\mathbf{AdS_3} \rightarrow \mathbf{AdS_2}$ [Cvetič, Papadimitriou '16]

- holographic stress tensor vanishes identically
- dual operator to constant scalar field is non-trivial and transforms anomalously with Brown-Henneaux central charge
- microstates accounting for black hole entropy survive and should be related to the expectation value of the dual operator

Theory

Four derivative, $\mathcal{N} = 2$ low energy effective actions in 4D obtained from CY_3 compactification of superstring theory [Cardoso, de Wit, Mahapatra '07]

- Gravity coupled to Abelian gauge fields and scalars;
- Extremal static near horizon backgrounds;
- BPS configurations;
- 2D reduced theory encodes R^2 corrections and exhibits EM duality;
- Given in terms of symplectic functions.

2D effective action

4D metric $ds^2 = ds_2{}^2 + v_2 d\Omega^2 \implies R_4 = R_2 - 2/v_2$.

AdS$_2$ background $v_1 R_2 = 2$ with $v_1 \sim L^2$. Locally, use **FG gauge**:

$$ds_2{}^2 = dr^2 + h_{tt} dt^2 \,, \quad \sqrt{-h} = \alpha(t) e^{r/\sqrt{v_1}} + \beta(t) e^{-r/\sqrt{v_1}} \,.$$

4D background supported by $e^I, p^I,\ Y^I, \Upsilon$ and holomorphic function $F(Y^I)$ homogeneous of degree 2. Incorporate 4-derivative terms W^2 by including Weyl multiplet into F and preserving its homogeneity

$$F(\lambda Y^I, \lambda^2 \Upsilon) = \lambda^2 F(Y^I, \Upsilon)$$

Legendre transform 2D Lagrangian w.r.t p^I to make EM duality manifest

$$H(e^I, f_I) = \mathcal{L}(e^I, p^I) + p^I f_I$$

2D effective action

$$H = \tfrac{1}{4}\left(\sqrt{-h_2}/v_2\right)^{-1}(e^I, f_I)\begin{bmatrix} N_{IJ} + R_{IK}N^{KL}R_{LJ} & -2R_{IK}N^{KJ} \\ -2N^{IK}R_{KJ} & 4N^{IJ}\end{bmatrix}\begin{pmatrix} e^J \\ f_J \end{pmatrix}$$
$$+ (e^I, f_I)\left[2i\begin{pmatrix} F_I - \bar{F}_I \\ -(Y^I - \bar{Y}^I)\end{pmatrix} + 4\Upsilon\begin{pmatrix} \bar{F}_{IK}N^{KL}F_{\Upsilon L} \\ -N^{IJ}F_{\Upsilon J}\end{pmatrix} + 4\bar{\Upsilon}\begin{pmatrix} F_{IK}N^{KL}\bar{F}_{\Upsilon L} \\ -N^{IJ}\bar{F}_{\Upsilon J}\end{pmatrix}\right]$$
$$+ \frac{\sqrt{-h_2}}{v_2}\Big\{\frac{8\mathrm{i}}{\sqrt{-\Upsilon}}(\bar{Y}^I F_I - Y^I\bar{F}_I) - 2i(\bar{Y}^I F_I - Y^I\bar{F}_I)$$
$$-2i(\Upsilon F_\Upsilon - \bar{\Upsilon}\bar{F}_\Upsilon) + 8\Upsilon\bar{\Upsilon}\,\bar{F}_{\Upsilon I}N^{IJ}F_{\Upsilon J}$$
$$+ 2\Upsilon F_{\Upsilon I}N^{IJ}(F_J - \bar{F}_{JL}Y^L) + 2\bar{\Upsilon}\bar{F}_{\Upsilon I}N^{IJ}(\bar{F}_J - F_{JL}\bar{Y}^L)$$
$$+ 2\mathrm{i}(F_\Upsilon - \bar{F}_\Upsilon)\left(32 - 8\sqrt{-\Upsilon}\right)\Big\} - \sqrt{-h_2}\,P(R_2)$$

$$\frac{P(R_2)}{4i} = \frac{(\bar{Y}^I F_I - Y^I\bar{F}_I)}{\sqrt{-\Upsilon}}\,R_2 - \frac{(F_\Upsilon - \bar{F}_\Upsilon)}{2}\left[8\,v_2\,{R_2}^2 - 32R_2 - 4R_2\sqrt{-\Upsilon}\right]$$

Dynamical fields: $h_{ij}, A^I_i, \tilde{A}_{i\,I}, Y^I, \Upsilon, v_2$

$$e^I \equiv F^I_{rt} = \partial_r A^I_t - \partial_t A^I_r\,, \quad f_I \equiv G_{rt\,I} = \partial_r \tilde{A}_{t\,I} - \partial_t \tilde{A}_{r\,I}\,.$$

Steps to take into account

1. Metric in **FG gauge** and $A^I_r = \tilde{A}_{r\,I} = 0$.
2. Add counterterms to impose **Dirichlet boundary conditions** at $r \to \infty$.
   - Compatible with the symplectic structure on the space of solutions
   - Compatible with gauge symmetries of the symplectic variables

   $\Longrightarrow$ **$\mathbf{A_t}$ requires special care**

[Papadimitriou '10]

[Cvetič, Papadimitriou '16]

[Castro, Larsen, Papadimitriou '18]

[Castro, Mühlmann '20]

Variational principle for A^I and $\tilde{A}_I$

$$\begin{pmatrix} \tilde{A}_{tI} \\ -A^I_t \end{pmatrix} = \sqrt{v_1}\,\frac{\alpha(t)\, e^{r/\sqrt{v_1}}}{\sqrt{-h_2}} \left(1 - \frac{\beta}{\alpha}\, e^{-2r/\sqrt{v_1}}\right) \begin{pmatrix} f_I \\ -e^I \end{pmatrix} + \begin{pmatrix} \tilde{\mu}_I(t) \\ -\mu^I(t) \end{pmatrix} ,$$

with $f_I, e^I \propto \sqrt{-h_2} \implies$ leading mode is the one $\propto \alpha(t)$.

For the canonical momenta we have $\pi_I = -q_I$ and $\tilde{\pi}^I = p^I$.

Add counterterms

$$-\int_{\partial M} dt \left(\pi_I A^I_t + \tilde{\pi}^I \tilde{A}_{t\,I}\right) + S' \left[\pi_I, \tilde{\pi}^I\right] + \int_{\partial M} dt \left(\pi_I A^{\mathrm{ren}\,I}_t + \tilde{\pi}^I \tilde{A}^{\mathrm{ren}}_{t\,I}\right) ,$$

inducing the canonical transformations

$$\begin{pmatrix} A^I_t \\ \pi_I \end{pmatrix} \to \begin{pmatrix} -\pi_I \\ A^{\mathrm{ren}\,I}_t \end{pmatrix} = \begin{pmatrix} -\pi_I \\ A^I_t - \frac{\delta S'}{\delta \pi_I} \end{pmatrix} , \quad \begin{pmatrix} \tilde{A}_{t\,I} \\ \tilde{\pi}^I \end{pmatrix} \to \begin{pmatrix} -\tilde{\pi}^I \\ \tilde{A}^{\mathrm{ren}}_{t\,I} \end{pmatrix} = \begin{pmatrix} -\tilde{\pi}^I \\ \tilde{A}_{tI} - \frac{\delta S'}{\delta \tilde{\pi}^I} \end{pmatrix} .$$

Boundary action

$$
\begin{aligned}
S_\partial = \int_{\partial M} dt\sqrt{-\gamma}\Bigg\{ & 2F'(R_2)K + \frac{64i}{\sqrt{v_1}}\left(F_\Upsilon - \bar{F}_\Upsilon\right)\left(v_2 R_2 - \frac{\sqrt{-\Upsilon}}{4}\right) \\
& + \frac{1}{4\sqrt{v_1}}\left(\pi_I, \tilde{\pi}^I\right)\begin{bmatrix} 4N^{IJ} & 2N^{IK}R_{KJ} \\ 2R_{IK}N^{KJ} & N_{IJ} + R_{IK}N^{KL}R_{LJ} \end{bmatrix}\begin{pmatrix}\pi_J \\ \tilde{\pi}^J\end{pmatrix} \\
& + \frac{4}{\sqrt{v_1}}\Re\left[\left(Y^I - 2i\Upsilon F_{\Upsilon J}N^{JI}, F_I - 2i\Upsilon F_{\Upsilon L}N^{LK}\bar{F}_{KI}\right)\right]\begin{pmatrix}\pi_I \\ \tilde{\pi}^I\end{pmatrix} \\
& - \frac{i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right)}{\sqrt{v_1}}\left(2 + \frac{8}{\sqrt{-\Upsilon}}\right) + \frac{8}{\sqrt{v_1}}\Upsilon\bar{\Upsilon}F_{\Upsilon J}N^{JI}\bar{F}_{\Upsilon I} \\
& - \frac{2}{\sqrt{v_1}}\left[\Upsilon F_{\Upsilon J}N^{JI}\left(F_I - \bar{F}_{IK}Y^K\right) + \mathrm{h.c.}\right]\Bigg\}
\end{aligned}
$$

Solution to the equations of motion

2D metric: ${ds_2}^2 = dr^2 + h_{tt} dt^2$

$$h_{tt} = -\left(\alpha(t) e^{r/\sqrt{v_1}} + \beta(t) e^{-r/\sqrt{v_1}}\right)^2$$

- $v_2 R_2 = 2$ on-shell $\Longrightarrow$ $v_2 = v_1$
- focus on class of solutions which includes BPS black hole solutions:

$$Y^I - \bar{Y}^I = i p^I\,, \quad F_I - \bar{F}_I = i q_I\,, \quad \Upsilon = -64\,, \quad \frac{i\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right)}{v_2} = {G_4}^{-1}$$

These imply

$$\begin{pmatrix} f_I \\ e^I \end{pmatrix} = \frac{\sqrt{-h_2}}{v_2} \begin{pmatrix} F_I + \bar{F}_I \\ Y^I + \bar{Y}^I \end{pmatrix}.$$

On-shell variation of the renormalized action

Renormalized action: $S_{\text{ren}} = I_{\text{bulk}} + I_{\text{ct}}$

$$\delta S_{\text{ren}} = \int_{\partial M} dt \left(\pi^{tt}_{\text{ren}} \delta h_{tt} + \pi_I \delta A_t^{\text{ren}\, I} + \tilde{\pi}^I \delta \tilde{A}^{\text{ren}}_{t\, I} + \pi^{\text{ren}}_{v_2} \delta v_2 \right)$$

- Consider variations in the **space** of **asymptotic solutions**

$$\delta h_{tt} = e^{2r/\sqrt{v_1}} \delta \left(-\alpha^2\right) , \quad \delta A_t^{\text{ren}\, I} = \delta \mu^I , \quad \delta \tilde{A}^{\text{ren}}_{t\, I} = \delta \tilde{\mu}_I$$

- Add a source $\nu(t)$ for the irrelevant operator dual to v_2

$$\delta v_2 = e^{r/\sqrt{v_1}} \delta \nu$$

$Y^I \propto v_2$ and $\Upsilon \propto {v_2}^2 \implies \pi^{\text{ren}}_{v_2}$ has contributions from these terms.

On-shell variation in terms of the sources

$$\delta S_{\text{ren}} = \int_{\partial M} dt \left(\underbrace{\pi^{tt}_{\text{ren}} \delta h_{tt}}_{\hat{\pi}^{tt}\delta(-\alpha^2)} + \underbrace{\pi_I \delta A_t^{\text{ren}\, I}}_{\hat{\pi}_I \delta \mu^I} + \underbrace{\tilde{\pi}^I \delta \tilde{A}^{\text{ren}}_{t\, I}}_{\hat{\tilde{\pi}} \delta \tilde{\mu}_I} + \underbrace{\pi^{\text{ren}}_{v_2} \delta v_2}_{\hat{\pi}_{v_2} \delta \nu} \right) .$$

$$\hat{\pi}^{tt} = 0 \, , \quad \hat{\pi}_I = -\frac{q_I}{\alpha} \, , \quad \hat{\tilde{\pi}}^I = \frac{p^I}{\alpha} \, , \quad \hat{\pi}_{v_2} = -\frac{2}{G_4 \sqrt{v_1}} \frac{\beta}{\alpha} \, ,$$

with ${G_4}^{-1} = i \left(\bar{Y}^I F_I - Y^I \bar{F}_I \right) / v_2$. Renormalized action written in terms of the sources

$$S_{\text{ren}} = \int dt \left(-\frac{2}{G_4 \sqrt{v_1}} \beta \, \nu - q_I \mu^I + p^I \tilde{\mu}_I + \mathcal{O}\left(\nu^2\right) \right) .$$

- Obtained result similar to the one in [Cvetič, Papadimitriou '16];
- Functional form of result does not depend on R^2 corrections.

Residual gauge symmetries

PBH diffeos $(\xi^i \partial_i)$ **+ gauge transformations** $\left(\Lambda^I, \tilde{\Lambda}_I\right)$ preserving

$$\underbrace{\mathcal{L}_\xi h_{rr} = \mathcal{L}_\xi h_{rt} = 0}_{\text{FG gauge}} \quad \underbrace{\mathcal{L}_\xi A^I_r + \partial_r \Lambda^I = 0}_{A^I_r = 0} \quad \underbrace{\mathcal{L}_\xi \tilde{A}_{r\,I} + \partial_r \tilde{\Lambda}_I = 0}_{\tilde{A}_{r\,I} = 0}$$

Solution:

$$\xi^t = \varepsilon(t) + \partial_t \sigma(t) \int_r^\infty h^{tt}\left(r', t\right) dr' \,, \quad \xi^r = \sigma(t) \,,$$

$$\Lambda^I = \varphi^I(t) - \partial_t \sigma(t) \int_r^\infty h^{tt}(r', t)\, A^I_t(r', t)\, dr' \,,$$

$$\tilde{\Lambda}_I = \tilde{\varphi}_I(t) - \partial_t \sigma(t) \int_r^\infty h^{tt}(r', t)\, \tilde{A}_{t\,I}(r', t)\, dr' \,.$$

- $\varepsilon(t) \to$ generator of boundary diffeos
- $\sigma(t) \to$ generator of boundary Weyl transformations
- $\varphi^I(t)\,,\ \tilde{\varphi}_I(t) \to$ generators of residual boundary $U(1)$ transformations

Asymptotic symmetries

PBH acting on the sources α, μ^I, $\tilde{\mu}_I$, ν:

$$\delta_{\rm PBH}\alpha = \frac{\sigma}{\sqrt{v_1}}\,\alpha + \partial_t(\varepsilon\,\alpha) \qquad \delta_{\rm PBH}\,\nu = \varepsilon\,\partial_t\nu + \frac{\sigma}{\sqrt{v_1}}\nu$$

$$\delta_{\rm PBH}\,\mu^I = \partial_t(\varepsilon\,\mu^I + \varphi^I) \qquad \delta_{\rm PBH}\,\tilde{\mu}_I = \partial_t(\varepsilon\,\tilde{\mu}_I + \tilde{\varphi}_I)$$

$$\delta_{\rm PBH}\,\beta = \partial_t\,(\varepsilon\,\beta) - \frac{\sigma}{\sqrt{v_1}}\,\beta - \frac{\sqrt{v_1}}{2}\,\partial_t\left(\frac{\partial_t\sigma}{\alpha}\right)$$

Asymptotic symmetries at $\nu = 0$: $\delta\alpha = \delta\mu^I = \delta\tilde{\mu}_I = 0$

$$\varepsilon = \frac{\zeta(t)}{\alpha} \qquad \sigma = -\sqrt{v_1}\,\frac{\partial_t\zeta}{\alpha}$$

$$\varphi^I = -\varepsilon\,\mu^I + k^I \qquad \tilde{\varphi}_I = -\varepsilon\,\tilde{\mu}_I + \tilde{k}_I$$

Symmetry algebra: Witt $\oplus\, u(1)^{2n}$

Define $dx^+ = \alpha\, dt \implies \sigma = -\sqrt{v_1}\partial_+\zeta$

$$\underbrace{\delta_{\rm sym}\,\beta = \alpha\,\partial_+\left(\frac{\zeta}{\alpha}\,\beta\right) + \partial_+\zeta\;\beta + \frac{\alpha\, v_1}{2}\,\partial_+^3\zeta\,, \quad \hat{\pi}_{v_2} = -\frac{2}{G_4\sqrt{v_1}}\frac{\beta}{\alpha}}$$

$$\delta_{\rm sym}\hat{\pi}_{v_2} = \zeta\,\partial_+\hat{\pi}_{v_2} + 2\partial_+\zeta\,\hat{\pi}_{v_2} - \frac{\sqrt{v_1}}{G_4}\partial_+^3\zeta\,,$$

Anomalous transformation with

$$c \sim \frac{\sqrt{v_1}}{G_4} = \frac{\sqrt{v_1}}{2\sqrt{-h_2}}\left(p^I f_I - q_I e^I\right)$$

Composite scalar field

$$F(\lambda Y, \lambda^2 \Upsilon) = \lambda^2 F(Y, \Upsilon)\,, \quad \frac{8i}{v_2\sqrt{-\Upsilon}}\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) = {G_4}^{-1}\,.$$

Consider a composite scalar field $\hat{\Omega}$ with asymptotic variation

$$\delta_{\hat{\Omega}} = e^{r/\sqrt{v_1}}\delta\Omega\,\mathcal{D}\,, \quad \mathcal{D} \equiv Y^I\partial_{Y^I} + 2\Upsilon\partial_\Upsilon + \mathrm{h.c.}$$

S_{ren} transforms as

$$\delta S_{\mathrm{ren}} = \int_{\partial M} dt\,\alpha\,\hat{\Pi}\,\delta\Omega\,,$$

with $\hat{\Pi} = Y^I\hat{\Pi}_I + 2\Upsilon\hat{\Pi}_\Upsilon + \mathrm{h.c.}$

$$\hat{\Pi}_I = \lim_{r\to\infty}\left(\frac{e^{2r/\sqrt{v_1}}}{\sqrt{-\gamma}}\frac{\delta S_{\mathrm{ren}}}{\delta Y^I}\right)\,, \quad \hat{\Pi}_\Upsilon = \lim_{r\to\infty}\left(\frac{e^{2r/\sqrt{v_1}}}{\sqrt{-\gamma}}\frac{\delta S_{\mathrm{ren}}}{\delta\Upsilon}\right)\,.$$

Composite scalar field

Non-trivial contribution comes from $\mathbf{S_{GHY}} \propto P'(R_2)$ and we find $\mathcal{D}P'(R_2) = P'(R_2)$ on-shell. Consequently

$$\hat{\Pi} = -\frac{4}{\sqrt{v_1}} P'(R_2) \frac{\beta}{\alpha}\,, \quad \delta_{\rm sym}\hat{\Pi} = \zeta\,\partial_+\hat{\Pi} + 2\partial_+\zeta\,\hat{\Pi} - 2\sqrt{v_1}P'(R_2)\partial_+^3\zeta\,.$$

Setting $\alpha = 1$ and $\beta = 0$ (4D BPS solution)

$$\delta_{\rm sym}\hat{\Pi} = -\sqrt{v_1}\,2P'(R_2)\,\partial_t^3\varepsilon\,.$$

4D BPS entropy $\mathcal{S} = 2\pi P'(R_2)$ [Cardoso, de Wit, Mohaupt '99] Viewing the action as a combination of

$$\underbrace{z^A = \frac{Y^A}{Y^0}\,, \quad \frac{\Upsilon}{{Y^0}^2}}_{\text{invariant under } \mathcal{D}}\,, \quad \Upsilon\left(Y^0\right)^2 \implies \tilde{\Omega} \sim \ln\left[\Upsilon\left(Y^0\right)^2\right]$$

$4D \to 5D \to 3D$

Restrict to 4D solutions with charges (q_0, p^A) with $A = 1, \ldots, n$, i.e. no p^0 charge! Interpret A_0 gauge field as KK field.

Can lift to solutions of 5D $\mathcal{N} = 2$ SUGRA with 4-derivative corrections and near-horizon $\mathrm{AdS}_3 \times \mathrm{S}^2$ [Castro, Davis, Kraus, Larsen '07]

1. rewrite reduced theory in units of ${k_2}^2 \equiv G_4 B^{-2}$;
2. Perform lift to 3D [Cvetič, Papadimitriou '16] ;
3. Compare with AdS_3 results of [Castro, Davis, Kraus, Larsen '07].

Define $v_2 = e^{-\psi} B^2\,, \quad {ds_2}^2 = e^{-\psi} d{\tilde{s}_2}^2$

$$\hookrightarrow \; \mathcal{O}_\psi = -\hat{\pi}_\psi = -\frac{2}{B}\left[\frac{1}{{k_2}^2} + 16i\left(F_\Upsilon - \bar{F}_\Upsilon\right)\sqrt{-\Upsilon}\right]\frac{\beta}{\alpha}$$

$\implies$ **Anomalous variation modified by R^2 corrections**

From the 5D point of view

$$ds_5{}^2 = \underbrace{d\tilde{s}_2{}^2 + B^2 d\Omega_2{}^2}_{e^{\psi}\left(ds_2{}^2 + v_2 d\Omega_2{}^2\right)} + e^{-2\psi}\left(dx^5 - A^0\right)^2$$

4D theory:

$$F(Y,\Upsilon) = -\frac{1}{6}\frac{C_{ABC}Y^A Y^B Y^C}{Y^0} - \frac{1}{24}\frac{1}{64}c_{2A}\frac{Y^A}{Y^0}\Upsilon\,,$$

with C_{ABC}, c_{2A} constants associated with CY3 data. Take $q_0 > 0$ and $p^A < 0$ and use [Castro, Davis, Kraus, Larsen '07]

$$\frac{c_L}{6} = p_L^3 = \frac{1}{6}\left(C\cdot p^3 + c_2\cdot p\right) > 0\,,\quad \frac{c_R}{6} = p_R^3 = \frac{1}{6}\left(C\cdot p^3 + \frac{1}{2}c_2\cdot p\right) > 0$$

From [Castro, Davis, Kraus, Larsen '07] we have

$$B\, e^{\psi} \frac{\mathcal{S}_{\rm Wald}}{\pi} = \sqrt{2\, G_4} p_L^3$$

Using [Cvetič, Papadimitriou '16], 2D CFT stress tensor τ_{++} contains term $\propto \mathcal{O}_\psi$:

$$\delta\tau_{++} = 2\tau_{++}\,\partial_+\zeta + \zeta\,\partial_+\tau_{++} - \underbrace{\frac{{k_2}^2}{{k_3}^2}\sqrt{2\, G_4}\, p_L^3}_{=\frac{c}{24\pi}}\,\partial_+^3\zeta$$

Since ${k_3}^2 = 2\pi R_5 {k_2}^2$

$$\frac{c}{24\pi} = \frac{\sqrt{2\, G_4}}{12\pi\, R_5} c_L$$

- CFT_1 naturally embeds in chiral half of CFT_2.
- For τ_{--} only L_0^R survives for constant dilaton $\to u(1)$ algebra

1. Introduction
2. Variational principle with R^2 terms and holographic renormalization
3. Asymptotic symmetries and anomalous transformations
4. Summary

Summary and Outlook

1. Obtained renormalized variational principle for constant scalar fields;
2. Identified the expectation value of irrelevant operators dual to scalar fields. Using asymptotic symmetries we found
   - Composite operator with $c \sim \mathcal{S}$;
   - Under a lift $AdS_2 \to AdS_3$, $\mathcal{O}_\psi$ has $c \propto c_L$ of AdS_3.
3. Hints that holographic dual of 2D QG encodes data that is embedded in chiral half of 2D CFT.

Outlook

1. Study near-BPS BHs in $nAdS_2/nCFT_1$
2. Investigate $AdS_2 \hookrightarrow AdS_3$ embedding in presence of R^2 terms