Novel higher-curvature variations of R^2 inflation

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Introduction

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- **Inflationary cosmology** is the place to look for new UV physics
- **Our question:** could we observe modifications of gravity? In particular, **higher-derivative corrections**?
- **String Theory** predicts the appearance of these corrections near Planck scale

Higher-derivative corrections can produce inflation on their own: **R^2 inflation**

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R+\frac{\alpha}{12}\ell^2R^2\right]$$

Simplified version of **Starobinsky's inflation**, based on the trace anomaly of matter fields

- Elegant model, containing only the metric
- Consistent with current experimental data on inflation

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In this talk I will consider recently identified of higher-derivative Lagrangians which are suitable for cosmology and will study the effect on the R^2 model

Note: one may also consider higher-curvature corrections to inflaton-based models.

1. "Geometric" curvature corrections
2. Slow-roll inflation in the Geometric-R^2 model
3. Holographic constraints
4. Primordial fluctuations
5. Conclusions

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Recently:

“Geometric” higher-derivative Lagrangians $\mathcal{L}(R_{\mu\nu\rho\sigma})$ Arciniega, Edelstein, Jaime '18; Cisterna, Grandi, Oliva '18; Arciniega, Bueno, PAC, Edelstein, Hennigar, Jaime '18

- 2nd order Friedmann equations
- Linearized EOMs on FLRW background are of 2nd order in time derivatives
- Black hole solutions of the form $ds^2 = -f(r)dt^2 - dr^2/f(r) + dr^2 d\Omega^2_{(2)}$ (Generalized Quasi-topological gravity, see Angel Murcia's talk)

Change dynamics without introducing additional dof → This is what you expect from an EFT

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$$\mathcal{R}_{(3)} = -\frac{3}{2} R_{\mu\ \ \nu}^{\ \rho\ \ \sigma} R_{\rho\ \ \sigma}^{\ \delta\ \ \gamma} R_{\delta\ \ \gamma}^{\ \mu\ \ \nu} - \frac{1}{8} R_{\mu\nu}^{\ \ \ \rho\sigma} R_{\rho\sigma}^{\ \ \ \alpha\beta} R_{\alpha\beta}^{\ \ \ \mu\nu} - \frac{5}{4} R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma}$$
$$- R_{\mu}^{\ \nu} R_{\nu}^{\ \rho} R_{\rho}^{\ \mu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho}{}_{\lambda} R^{\sigma\lambda} - \frac{1}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} R + \frac{1}{2} R_{\mu\nu} R^{\mu\nu} R\,.$$

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At quartic order there are three independent densities

$$\mathcal{R}_{(4)} = \mathcal{R}^{A}_{(4)} + \nu \mathcal{R}^{B}_{(4)} + \gamma \mathcal{R}^{C}_{(4)}$$

- Only $\mathcal{R}^{A}_{(4)}$ contributes to the Friedmann equations
- $\mathcal{R}^{B}_{(4)}$ contributes to the linearized equations
- $\mathcal{R}^{C}_{(4)}$ is trivial for cosmology

Observation: $\forall$ n there is a unique way in which these theories modify Friedmann equations Arciniega, Bueno, PAC, Edelstein, Hennigar, Jaime '18

"Geometric" curvature corrections

$$\mathcal{R}^{A}_{(4)} = \frac{3}{32}R^{\mu\nu\rho\sigma}R_{\mu\nu}{}^{\delta\gamma}R_{\delta\gamma}{}^{\chi\xi}R_{\rho\sigma\chi\xi} - \frac{1}{16}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^2 + \frac{1}{6}RR_{\mu}{}^{\rho}{}_{\nu}{}^{\sigma}R_{\rho}{}^{\delta}{}_{\sigma}{}^{\gamma}R_{\delta}{}^{\mu}{}_{\gamma}{}^{\nu} - \frac{3}{4}R^{\mu\nu}R^{\rho\sigma}R^{\delta\gamma}{}_{\mu\rho}R_{\delta\gamma\nu\sigma}$$
$$+ \frac{1}{16}R^2R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \frac{1}{2}RR_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} - \frac{3}{4}R_{\mu}{}^{\nu}R_{\nu}{}^{\rho}R_{\rho}{}^{\sigma}R_{\sigma}{}^{\mu} + \frac{5}{8}(R_{\mu\nu}R^{\mu\nu})^2 + RR_{\mu}{}^{\nu}R_{\nu}{}^{\rho}R_{\rho}{}^{\mu}$$
$$- \frac{7}{8}R^2R_{\mu\nu}R^{\mu\nu} + \frac{3}{32}R^4\,,$$

$$\mathcal{R}^{B}_{(4)} = \frac{3}{16}R^{\mu\nu\rho\sigma}R_{\mu\nu}{}^{\delta\gamma}R_{\delta\gamma}{}^{\chi\xi}R_{\rho\sigma\chi\xi} - \frac{3}{64}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^2 - \frac{1}{4}RR_{\mu}{}^{\rho}{}_{\nu}{}^{\sigma}R_{\rho}{}^{\delta}{}_{\sigma}{}^{\gamma}R_{\delta}{}^{\mu}{}_{\gamma}{}^{\nu} - \frac{9}{4}R^{\mu\nu}R^{\rho\sigma}R^{\delta\gamma}{}_{\mu\rho}R_{\delta\gamma\nu\sigma}$$
$$- \frac{3}{2}R^{\mu\nu}R_{\nu}{}^{\rho}R^{\sigma\delta\gamma}{}_{\mu}R_{\sigma\delta\gamma\rho} + \frac{3}{16}R^2R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \frac{15}{4}RR_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} - \frac{15}{4}R_{\mu}{}^{\nu}R_{\nu}{}^{\rho}R_{\rho}{}^{\sigma}R_{\sigma}{}^{\mu} + \frac{21}{8}(R_{\mu\nu}R^{\mu\nu})^2$$
$$+ 7RR_{\mu}{}^{\nu}R_{\nu}{}^{\rho}R_{\rho}{}^{\mu} - 6R^2R_{\mu\nu}R^{\mu\nu} + \frac{47}{64}R^4\,,$$

$$\mathcal{R}^{C}_{(4)} = -4R^{\mu\nu}R_{\nu}{}^{\rho}R^{\sigma\delta\gamma}{}_{\mu}R_{\sigma\delta\gamma\rho} + R_{\delta\gamma}R^{\delta\gamma}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + 4RR_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} + \frac{16}{3}RR_{\mu}{}^{\nu}R_{\nu}{}^{\rho}R_{\rho}{}^{\mu} - 5R^2R_{\mu\nu}R^{\mu\nu}$$
$$+ \frac{2}{3}R^4\,.$$

“Geometric” curvature corrections

We consider R^2 gravity enhanced with the geometric terms

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R + \frac{\alpha}{12}\ell^2 R^2 + \sum_{n=3}^{\infty}\lambda_n \ell^{2n-2}\mathcal{R}_{(n)}\right]$$

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This can be recast as a scalar-tensor theory

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R(1 + 2\alpha\ell^2\psi) - 12\alpha\ell^2\psi^2 + \sum_{n=3}^{\infty}\lambda_n \ell^{2n-2}\mathcal{R}_{(n)}\right]$$

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Comments

- The R^2 and $\mathcal{R}_{(n)}$ terms are of different nature and may appear at different scales
- We often have in mind $\lambda_n << \alpha$
- One may consider more general setups, *i.e.* adding R^n terms as well
- The geometric terms also fit well with standard single-field models Edelstein, Vázquez, Vilar-López '20; Edelstein, Mann, Vázquez, Vilar-López '20

Slow-roll inflation in the Geometric-R^2 model

Consider a flat cosmological FLRW ansatz

$$ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right)$$

and $\psi(t)$. Inserting this in the EOMs we obtain

$$F\left(H^2\right) = 2\alpha\ell^2\left[\psi(\psi - H^2) - H\dot{\psi}\right]$$

$$\dot{H}F'\left(H^2\right) = -\alpha\ell^2\left[\ddot{\psi} - H\dot{\psi} + 2\dot{H}\psi\right]$$

$$6\dot{H} + 12H^2 - 12\psi = 0$$

where $H = \dot{a}/a$, $\dot{\psi} = d\psi/dt$, etc, and F is the function defined as

$$F(x) = x + \ell^{-2}\sum_{n=3}^{\infty}(-1)^n\lambda_n\left(\ell^2 x\right)^n$$

Observation: this is surprisingly similar to the EOMs in stringy duality-invariant cosmology at all orders in α' Hohm, Zwiebach '19

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Slow-roll inflation

$$\epsilon = \frac{1}{H}\frac{dH}{dN}\,, \qquad dN = -Hdt$$

When $\epsilon << 1$ we have

$$\frac{d\psi}{dN} \approx \frac{2F(\psi)}{3\alpha\ell^2\psi}\,, \qquad \epsilon \approx \frac{F(\psi)}{3\alpha\ell^2\psi^2}$$

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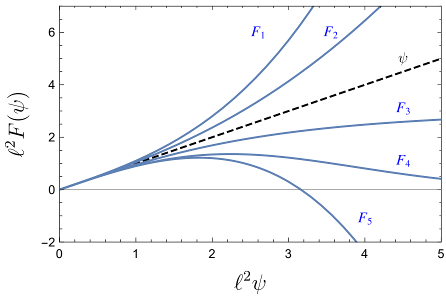
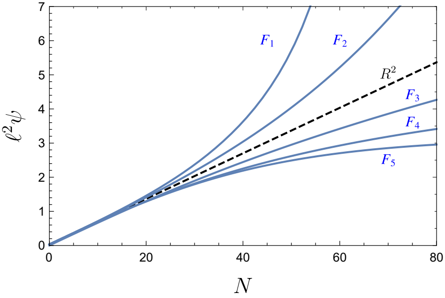
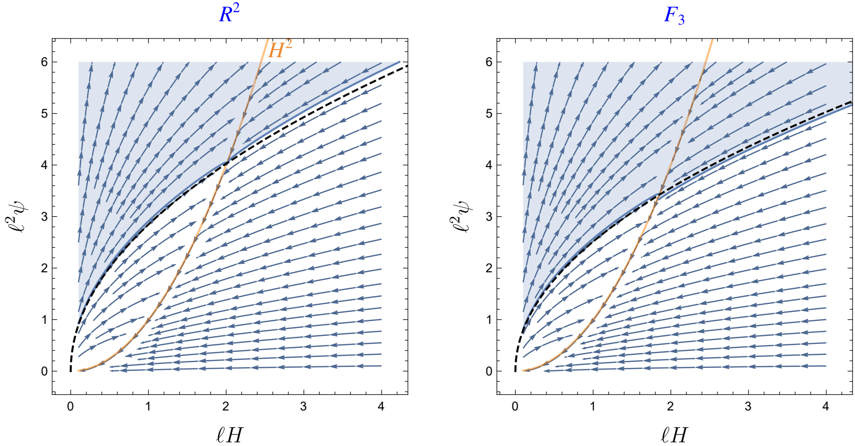




Attractor towards inflation

$$\frac{d\psi}{dN} = \frac{1}{H^2}\left[-\psi(\psi - H^2) + \frac{1}{2\alpha\ell^2}F(H^2)\right], \quad \frac{dH}{dN} = -\frac{2}{H}(\psi - H^2)$$



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2. Compute the holographic **2-point function** $\langle TT \rangle$ and impose **unitarity**

$$\langle T_{ab}(x) T_{cd}(0) \rangle_{\mathbb{R}^3} = \frac{C_T}{|x|^6} \mathcal{I}_{ac,bd}(x) \, , \quad C_T = C_T^E \left(F'(-\chi) + 2\alpha \ell^2 \psi \right)$$

where $R_{\mu\nu\rho\sigma} = -2\chi g_{\mu[\rho} g_{\sigma]\nu}$

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$$F'(H^2) + 2\alpha \ell^2 H^2 > 0$$

This is indeed a meaningful constraint on dS: $G_{\text{eff}} = G(F'(H^2) + 2\alpha \ell^2 H^2)^{-1}$
The constraint is necessary to avoid $G_{\text{eff}} < 0$ and negative-energy gravitons!

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3-point function $\langle TTT \rangle$ determined by C_T and a dimensionless coefficient t_4

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This is known for these theories in AdS Bueno, PAC, Ruipérez '18; Bueno, PAC, Hennigar, Penas, Ruipérez '20

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$$t_4 = -\frac{210\chi F''(-\chi)}{F'(-\chi) + 2\alpha\ell^2\psi}$$

We translate the holographic bound to dS

$$-4 \leq \frac{210 H^2 F''(H^2)}{F'(H^2) + 2\alpha\ell^2 H^2} \leq 4$$

We conjecture this is a physically meaningful bound in the bulk dS theory

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Inflation = quasi-de Sitter phase

These constraints must be satisfied $\forall H$ during inflation $\rightarrow$ at least $N \leq 60$

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Constraints on the cubic term $F(H^2) = H^2 - \lambda_3 \ell^4 H^6$

$$C_T > 0 \quad \Rightarrow \quad \frac{\lambda_3}{\alpha^2} < 1$$

$$-4 \leq t_4 \leq 4 \quad \Rightarrow \quad -1.5 \cdot 10^{-4} \leq \frac{\lambda_3}{\alpha^2} \leq 1.7 \cdot 10^{-4}$$

Constraints on the quartic term $F(H^2) = H^2 + \lambda_4 \ell^6 H^8$

$$C_T > 0 \quad \Rightarrow \quad \frac{\lambda_4}{\alpha^3} > -\frac{8}{27}$$

$$-4 \leq t_4 \leq 4 \quad \Rightarrow \quad -2.1 \cdot 10^{-6} \leq \frac{\lambda_4}{\alpha^3} \leq 1.9 \cdot 10^{-6}$$

Primordial fluctuations

Primordial fluctuations

- The study of perturbations is facilitated by the 2nd order linearized equations $\Rightarrow$ no additional modes or ghosts

- We work perturbatively in λ_3 and λ_4

- Expansion parameters: $\frac{1}{N}, \quad \frac{\lambda_3 N^2}{\alpha^2}, \quad \frac{\lambda_4 N^3}{\alpha^3}$

Primordial fluctuations

Power spectra at super-Hubble scales

$$\mathcal{P}_T(k) = rA_S^2\left(\frac{k}{aH}\right)^{n_T}, \quad \mathcal{P}_S(k) = A_S^2\left(\frac{k}{aH}\right)^{n_s-1}$$

$$n_T \approx -\frac{3}{2N^2} - \frac{10\lambda_3}{9\alpha^2} + \frac{2\lambda_4 N}{\alpha^3}$$

$$n_s \approx 1 - \frac{2}{N} - \frac{32\lambda_3 N}{27\alpha^2} + \frac{4\lambda_4 N^2}{3\alpha^3}$$

$$r \approx \frac{12}{N^2} - \frac{64\lambda_3}{9\alpha^2} + \frac{16\lambda_4 N}{3\alpha^3}$$

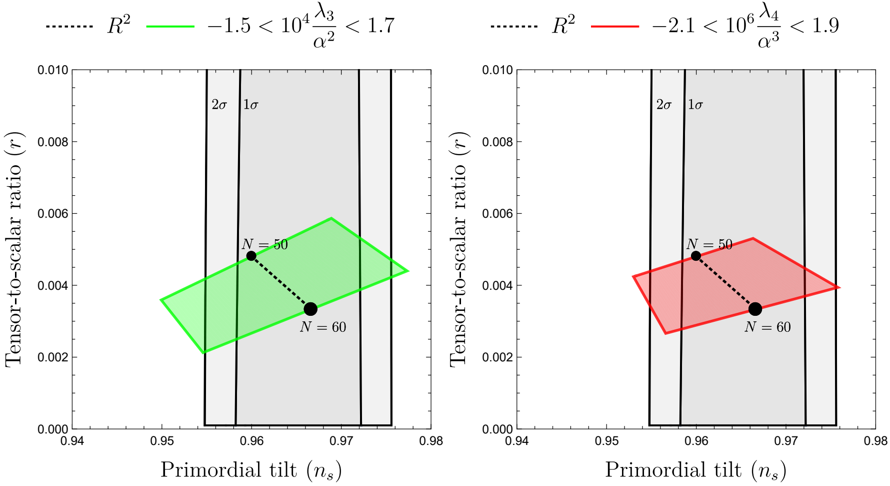
Note that

$$\frac{r}{n_T} = -8 + \frac{32N^2\lambda_3}{3\alpha^2} - \frac{128N^3\lambda_4}{9\alpha^3} \neq -8$$

⇒ distinguishes these theories from standard single-field models

Primordial fluctuations

Comparison with experimental bounds Planck '18



Conclusions

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- New set of higher-derivative corrections to R^2 inflation
- These models give rise to a well-behaved cosmological evolution, with an attractor towards inflation
- We have constrained the parameters of these models applying AdS/CFT methods. Important to understand the meaning of these constraints on dS
- Predictions for n_s are within current experimental bounds
- Future experiments [LiteBIRD, CORE, PIXIE] will tell if r is close to the R^2 prediction $r \sim 10^{-3}$. Will we observe deviations as well?

Future work

- How to distinguish this model from others?
- More precise characterization of the pheno (running of n_s, reheating, non-Gaussianities)
- More general higher-derivative models (EFT?)

Conclusions

Thank you for your attention

Bonus

Experimental vs holographic bounds on (λ_3, λ_4)