JT gravity - a review

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Iberian Strings 2021, January 19-22

Outline

Introduction: 2d dilaton gravity models

JT classical gravity
- Classical solution
- Schwarzian quantum mechanics

JT quantum gravity: disk amplitudes
- Partition function
- Boundary correlators and its gravitational physics

JT quantum gravity: higher topology
- Gravitational amplitudes
- JT gravity as a matrix integral

Some recent developments

Generalities of 2d gravity

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We need different coupling to matter: dilaton gravity

$S = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\left(\Phi R + V(\Phi)\right) + S_{bdy} + S_{matter}$
Φ is dilaton field

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Jackiw-Teitelboim (JT) 2d dilaton gravity where $V(\Phi) = -\Lambda\Phi$:
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- Dimensional reduction (s-wave) of 3d pure $\Lambda < 0$ gravity
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Geometry fixed as AdS_2: $ds^2 = \frac{-dF^2+dZ^2}{Z^2}, \quad Z \geq 0$
Poincaré patch (frame) of AdS_2, boundary at $Z = 0$

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Lightcone coordinates $U = F + Z$ and $V = F - Z$
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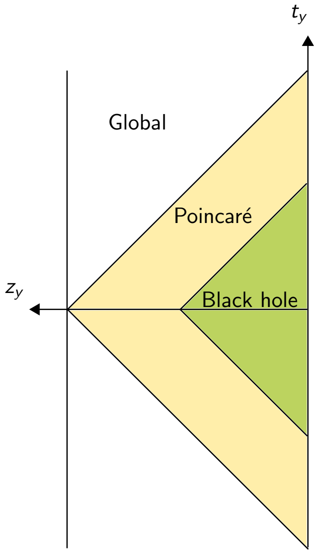
  Using radial coordinate $r \sim \coth\frac{2\pi}{\beta}(u-v)$:
  $ds^2 = -(r^2 - r_h^2)dt^2 + \frac{dr^2}{r^2-r_h^2}$
  is black hole solution with ADM mass $E \sim r_h^2$, and Hawking temperature $T \sim r_h \longrightarrow$ first law $T \sim \sqrt{E}$

Important frames in AdS_2 (2)

Penrose diagram



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Path-integrate out Φ:

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  Cannot compare spacetimes with different asymptotics

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Using $\sqrt{-\gamma} = 1/\epsilon, \quad K = 1 + \epsilon^2\{F,t\} + \dots$

$$\boxed{\Rightarrow S = -C\int dt\,\{F,t\}}, \quad C = \frac{a}{16\pi G}, \ \{F,t\} = \frac{F'''}{F'} - \frac{3}{2}\left(\frac{F''}{F'}\right)^2$$

Almheiri-Polchinski '15, Jensen '16, Maldacena-Stanford-Yang '16, Engelsöy-TM-Verlinde '16

$F(t) =$ time reparametrization in terms of proper time t

Schwarzian equation of motion and coupling to matter

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Generalization to include coupling to matter (CFT for simplicity):

$-C \frac{d}{dt} \{F, t\} = \frac{dE}{dt} = T_{vv}(t) - T_{uu}(t)$

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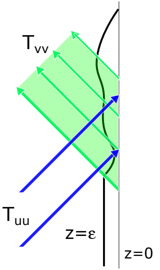
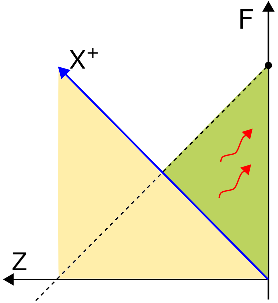
in terms of boundary values of matter energy
Total energy is changed by injection and extraction from holographic boundary
Wiggly boundary curve $(F(t), Z(t) = \epsilon F'(t))$



Consider an infalling matter pulse in Poincaré frame
$T_{vv}(t) = E_0\delta(t)$, matter (= 2d CFT) quantum effects included

Engelsöy-TM-Verlinde '16

$\frac{dE}{dt} = E_0\delta(t) + T_{vv}(t) - T_{uu}(t), \qquad E(t) = -C\{F, t\}$



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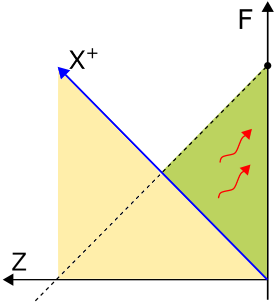

Boundary conditions on matter:

No outgoing matter in initial state:

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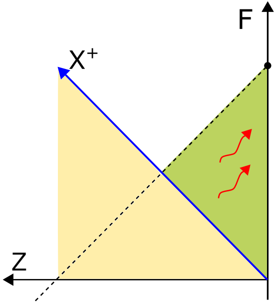

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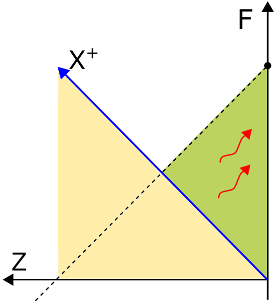

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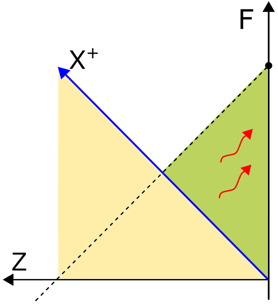

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where $A = \frac{c}{24\pi C}, \; \alpha = \frac{24\pi}{c}\sqrt{2CE_0}$

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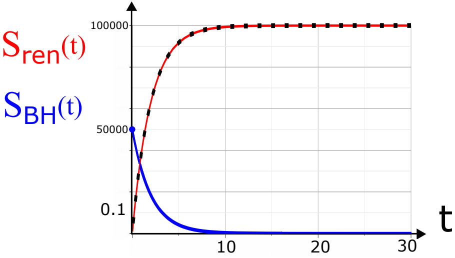

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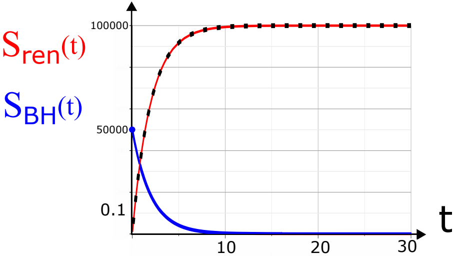

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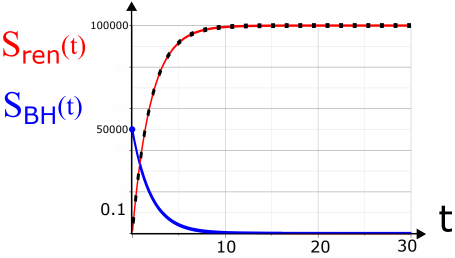

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We need to take this seriously Almheiri-Engelhardt-Marolf-Maxfield '19, Penington '19

$\rightarrow$ Jump in minimal quantum extremal (RT) surface

(Requires non-perturbative (in G_N) knowledge)

$\Rightarrow$ Island rule (see Lárus Thorlacius talk on Friday!)

JT Quantum Gravity

Next: Transfer to Euclidean thermal theory and obtain boundary correlation functions of JT gravity / Schwarzian QM:

$$\boxed{\left\langle \mathcal{O}_{h_1}\mathcal{O}_{h_2}\dots\right\rangle_\beta = \frac{1}{Z}\int_{\mathcal{M}}[\mathcal{D}f]\mathcal{O}_{h_1}\mathcal{O}_{h_2}\dots\ e^{C\int_0^\beta d\tau\,\{F,\tau\}}}$$

with $F \equiv \tan\left(\frac{\pi f(\tau)}{\beta}\right)$, $\qquad \{F,\tau\} = \{f,\tau\} + \frac{2\pi^2}{\beta^2}f'^2$

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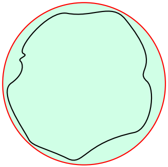
$f(\tau)$ is reparametrization of S^1:



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Application to partition function: Kitaev '15, Maldacena-Stanford-Yang '16, ...

$$Z_{\text{tree+1-loop}} = (\det \mathcal{O})^{-1/2} e^{-S_{\text{on-shell}}} \sim \left(\frac{2\pi^2 C}{\beta}\right)^{3/2} e^{\frac{2\pi^2 C}{\beta}}$$

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$$Z_{\text{tree+1-loop}} = (\det \mathcal{O})^{-1/2} e^{-S_{\text{on-shell}}} \sim \left(\frac{2\pi^2 C}{\beta}\right)^{3/2} e^{\frac{2\pi^2 C}{\beta}}$$

Stanford and Witten demonstrated that Schwarzian partition function is one-loop exact Stanford-Witten '17

$$Z(\beta) = \left(\frac{2\pi C}{\beta}\right)^{3/2} \exp\left(\frac{2\pi^2 C}{\beta}\right) \sim \int_0^{+\infty} dE \sinh\left(2\pi\sqrt{2CE}\right) e^{-\beta E}$$

Density of states $\rho(E) = \sinh\left(2\pi\sqrt{2CE}\right)$

Disk partition function

How to compute with this action? $S = -C \int d\tau \left\{ \tan \frac{\pi}{\beta} f(\tau), \tau \right\}$

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Thermodynamic limit (saddle): found by $\frac{\partial S}{\partial E} = \beta$ at large E

$\implies T \sim \sqrt{E}$

$\rightarrow$ matches semi-classical JT black hole first law, saddle approximation invalid at small E

Boundary bilocal operator

Take massive scalar field in bulk, asymptotic expansion (AdS_2/CFT_1):

$$\phi(Z,F) \quad \to Z^{1-h}\tilde{\phi}_b(F) = \epsilon^{1-h}F'^{1-h}\tilde{\phi}_b(F(\tau)) = \epsilon^{1-h}\phi_b(\tau)$$

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Generating functional:

$$\begin{aligned} I &\sim \int dF_1 \int dF_2 \frac{1}{(F_1-F_2)^{2h}}\tilde{\phi}_b(F_1)\tilde{\phi}_b(F_2) \\ &= \int d\tau_1 \int d\tau_2 \frac{F'(\tau_1)^h F'(\tau_2)^h}{(F(\tau_1)-F(\tau_2))^{2h}}\phi_b(\tau_1)\phi_b(\tau_2) \end{aligned}$$

Bilocal operator:

$$\boxed{\mathcal{O}_h(\tau_1,\tau_2) \equiv \left(\frac{F'(\tau_1)F'(\tau_2)}{(F(\tau_1)-F(\tau_2))^2}\right)^h \equiv \left(\frac{f'(\tau_1)f'(\tau_2)}{\frac{\beta}{\pi}\sin^2\frac{\pi}{\beta}[f(\tau_1)-f(\tau_2)]}\right)^h}$$

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Other origins of this operator:

- Boundary-anchored Wilson line in 1^{st} order $SL(2,\mathbb{R})$ gauge formulation of JT gravity Blommaert-TM-Verschelde '18, Iliesiu-Pufu-Verlinde-Wang '19

Approaches to JT disk boundary correlators: an overview

Several approaches to obtain JT disk amplitudes exist:

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- 1d Liouville Bagrets-Altland-Kamenev '16, '17
- 2d Liouville CFT between ZZ-branes TM-Turiaci-Verlinde '17, TM '18
- 2d BF bulk Blommaert-TM-Verschelde '18, Iliesiu-Pufu-Verlinde-Wang '19
  Using first-order formulation of JT gravity in terms of an $SL(2,\mathbb{R})$ BF gauge theory
- Particle in infinite B-field in AdS_2 Yang '18, Kitaev-Suh '18, Suh '20
- Liouville gravity / minimal string TM-Turiaci '19, '20, TM '20

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Real-time two-point function $G_h(t) = \langle \mathcal{O}_h(t,0) \rangle$

$= \frac{1}{Z(\beta)} \int d\mu(E_1) d\mu(E_2) e^{it(E_1 - E_2) - \beta E_2} \frac{\Gamma\left(h \pm i\sqrt{2CE_1} \pm i\sqrt{2CE_2}\right)}{\Gamma(2h)}$

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Behavior full quantum expression:

Always decaying, where late-time behavior of correlator gets strong QG fluctuations (power-law decay $\sim 1/t^3$ instead of exponential decay)

Application: Complexity = volume conjecture

Complexity $\mathcal{C}$ of boundary theory at time t
= volume of extremal (maximal) surface anchored at the boundary at points at time t Susskind '14 . . .

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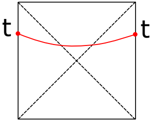
TFD: 

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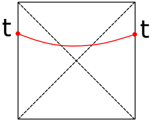
TFD: 

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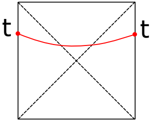

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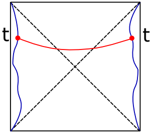

Geometry fixed to AdS_2: (renormalized) geodesic length of wormhole $= \ln \frac{(F(t_1)-F(t_2)^2}{F'(t_1)F'(t_2)}$

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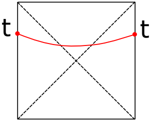
TFD: 

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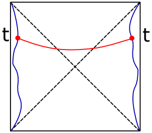

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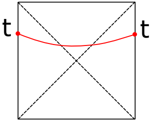
TFD: 

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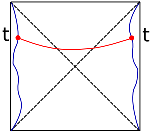

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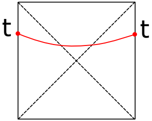
TFD: 

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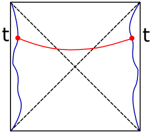

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Q: Complexity plateau? Higher topology / random matrices?

OTOC four-point correlator

Schwarzian out-of-time ordered (OTOC) 4-point function:
$\langle V_1 W_3 V_2 W_4 \rangle = \langle \mathcal{O}_h(t_1, t_2) \mathcal{O}_h(t_3, t_4) \rangle_{\text{OTO}}$

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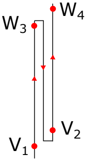
Gravitational semi-classical interpretation: OTOC $\langle V_1 W_3 V_2 W_4 \rangle$ in boundary theory captures gravitational shockwave behavior Shenker-Stanford '15



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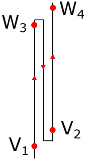

Large C limit of complete OTOC with light h, gives full Dray-'t Hooft eikonal shockwave expressions Lam-TM-Turiaci-Verlinde '18

Multiboundary amplitudes (1)

Path integral (in Euclidean signature):

$$\int \frac{[\mathcal{D}g_{\mu\nu}]}{\text{Vol(Diff)}} [\mathcal{D}\Phi] \, e^{-S_{\text{JT}} - S_{bdy}} = \int \frac{[\mathcal{D}g_{\mu\nu}]}{\text{Vol(Diff)}} \delta(R - \Lambda) e^{-S_{bdy}}$$

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- Manifold with holographic boundary: non-trivial boundary dynamics, described by Schwarzian action
- Manifold with only geodesic boundaries ($K=0, S_{bdy}=0$) : remaining path integral $\int_{R=\Lambda} \frac{1}{\text{Vol(Diff)}} [\mathcal{D}g_{\mu\nu}] \times 1$ is computing volume integral of all inequivalent metrics one can put on a 2d manifold with given $R=\Lambda$ and geodesic boundaries
  $\rightarrow$ Mathematicians call this the volume of moduli space of Riemann surfaces = Weil-Petersson volume

Multiboundary amplitudes (2)

In order to analyze higher topology, we augment the action by the Einstein-Hilbert action:

$S = -\frac{S_0}{2\pi}\left[\frac{1}{2}\int R + \oint K\right] + S_{JT}$

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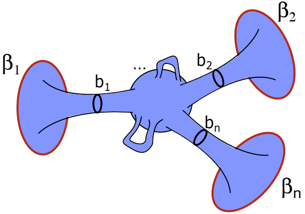
Motivation:

Comes from zero'th order term in near-extremal near-horizon expansion of higher dim. black holes where S_0 is the extremal entropy

Multiboundary amplitudes (3)

Multiboundary amplitudes Saad-Shenker-Stanford '19:



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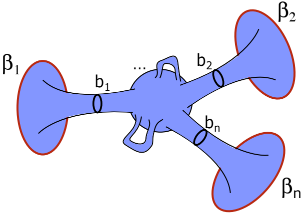

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Ingredients:

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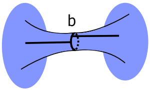
- measure $d\mu(b) = dbb$

  Gluing tubes with twist: $0 \dots b$



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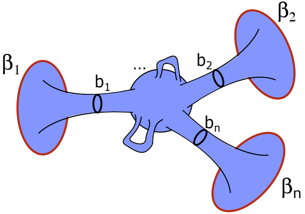

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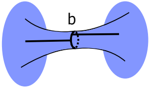

- Weil-Petersson (WP) volume $V_{g,n}(\mathbf{b}) \equiv V_{g,n}(b_1 \dots b_n)$
  volume of moduli space of Riemann surfaces of genus g with
  n geodesic boundaries of length b_i
  multivariate polynomials in b_i^2

JT gravity as a matrix integral

Mirzakhani '07:

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Eynard-Orantin '07:

$\rightarrow$ match with topological recursion relations of a double-scaled matrix model with leading (large L) spectral density
$\langle\rho(E)\rangle_0 = \rho_0(E) = L \sinh 2\pi\sqrt{E}$

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Holographic interpretation:
→ Boundary Hamiltonian ≡ random matrix H
→ Ensemble-averaged holography (cfr. SYK has some averaging)

Application: Maldacena's version of the information paradox in AdS/CFT

Boundary two-point function in thermal AdS/CFT decays exponentially at late times due to bulk quasinormal modes:

E.g. in 1+1d: $\langle \mathcal{O}(t)\mathcal{O}(0)\rangle_\beta = \frac{1}{\left(\sinh \frac{\pi}{\beta} t\right)^{2h}} \sim e^{-\frac{2\pi}{\beta} ht}$

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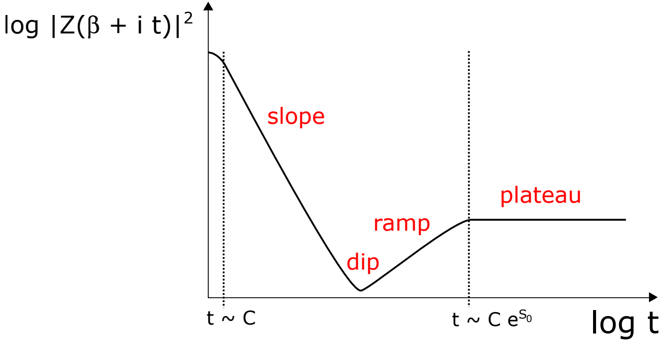
Spectral form factor: Cotler et al. '16, Saad-Shenker-Stanford '18
$|Z(\beta+it)|^2 = \sum_{n,m} e^{-\beta(E_n+E_m)} e^{it(E_m-E_n)}$
Late-time low mean $\sim Z(2\beta) \neq 0$

Typical form of spectral form factor in (averaged) chaotic systems Cotler-Gur-Ari-Hanada-Polchinski-Saad-Shenker-Stanford-Streicher-Tezuka '16:



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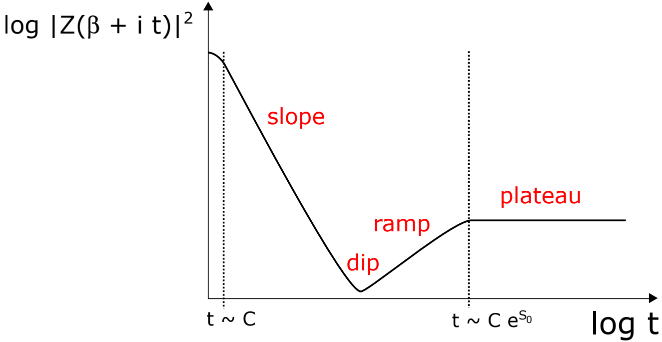

JT gravity has such late-time behavior for its spectral form factor

→ easily proved using matrix description

→ has gravitational interpretation in terms of higher topology

Spectral form factor in RMT - late time behavior (2)

In matrix integral, the spectral form factor is $\langle Z(\beta + it)Z(\beta - it)\rangle$ (two analytically continued macroscopic loop operators inserted in matrix integral)

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Interpretation: disconnected piece, sine kernel, and contact term

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Factorized term dominates as $\sim e^{2S_0}$ ($\rho_0(E) \sim e^{S_0}\sinh 2\pi\sqrt{E}$)

However, at very late times $t \sim e^{S_0}$, one has $E - E' \sim e^{-S_0}$ and this can compensate the suppression and give important effects!

Spectral form factor in JT - late time behavior (1)

Now we can understand the late-time behavior of the spectral form factor in JT gravity Saad-Shenker-Stanford '19

- Slope: Factorized contribution $\rho_0(E)\rho_0(E')$:

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  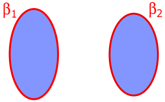

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  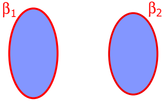

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  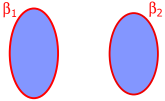

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  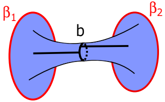

  $\int_0^{+\infty} dbb \left(\frac{1}{\beta_1^{1/2}} e^{-\frac{C}{2\beta_1}b^2}\right)\left(\frac{1}{\beta_2^{1/2}} e^{-\frac{C}{2\beta_2}b^2}\right)$

  $\sim \frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2}$ where $\beta_1 = \beta + it$ and $\beta_2 = \beta - it$

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Reinstating $\langle n| \mathcal{O} |m\rangle$ = study boundary two-point function
Leads to similar picture Blommaert-TM-Verschelde '19, Saad '19

Generalizations

Deformation of JT gravity Maxfield-Turiaci '20, Witten '20

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  $\mathcal{N} = 4$ SYM dual to $AdS_5 \times S^5$ type IIB $\rightarrow$ fixed Hamiltonian
  (no ensemble averaging) $\rightarrow$ no connected topologies ?

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JT gravity is ideal test case to study conceptual questions about quantum gravity