Insights from real-time dynamics

Yago Bea

Queen Mary University of London

Work in progress with Hans Bantilan and Pau Figueras

Outline

We use together:

- Relativistic Hydrodynamics
- Holography
- Numerical Relativity

⟶ To obtain new insights

Outline

We use together:

- Relativistic Hydrodynamics
- Holography
- Numerical Relativity

⟶ To obtain new insights

We go beyond the state of the art:

1 – New holographic solution

2 – Evolutions of hydrodynamics

3 – Evolutions in BDNK

4 – Fluid/gravity: new examples?

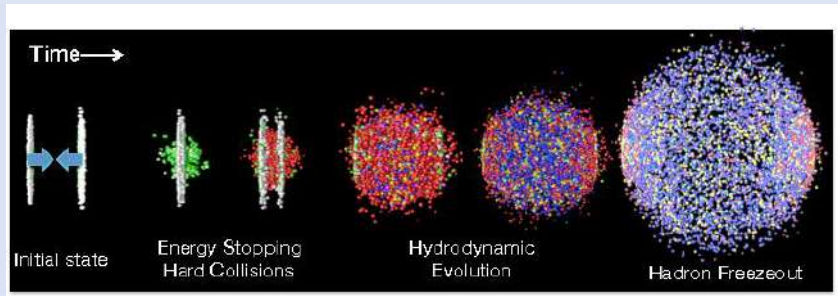
Hydrodynamics

Why hydrodynamics? ⟶ It describes interesting phenomena:

Hydrodynamics

Why hydrodynamics? ⟶ It describes interesting phenomena:

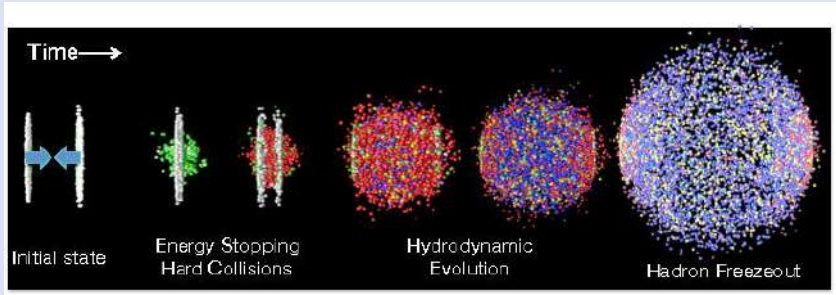
Quark-Gluon Plasma



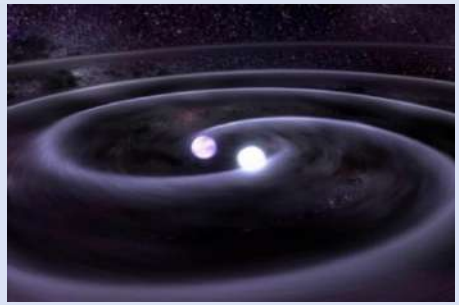
Hydrodynamics

Why hydrodynamics? ⟶ It describes interesting phenomena:

Quark-Gluon Plasma



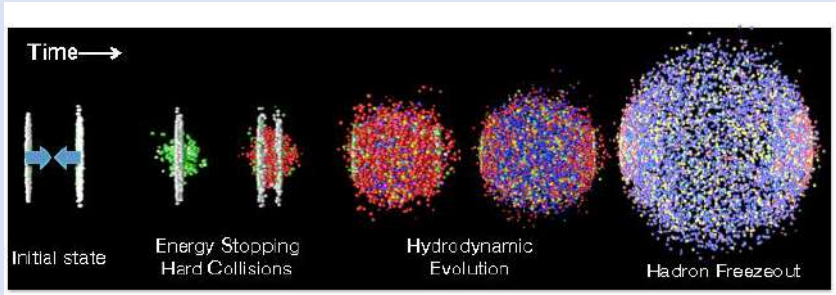
Neutron star mergers



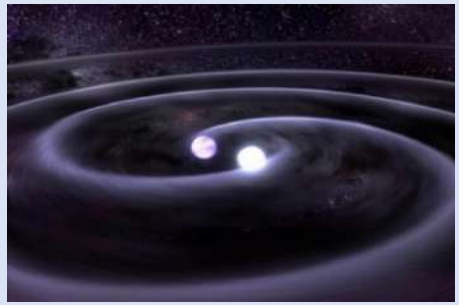
Hydrodynamics

Why hydrodynamics? ⟶ It describes interesting phenomena:

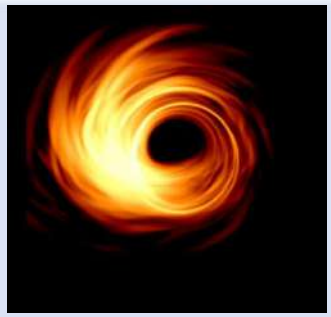
Quark-Gluon Plasma

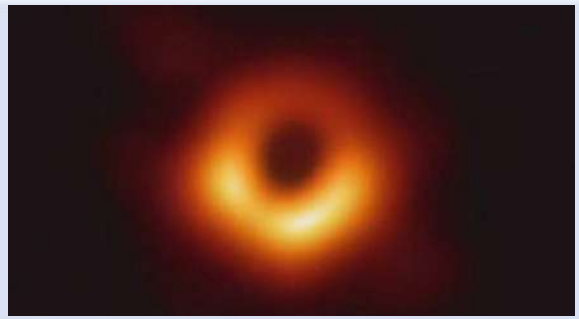


Neutron star mergers



Black hole accretion disk

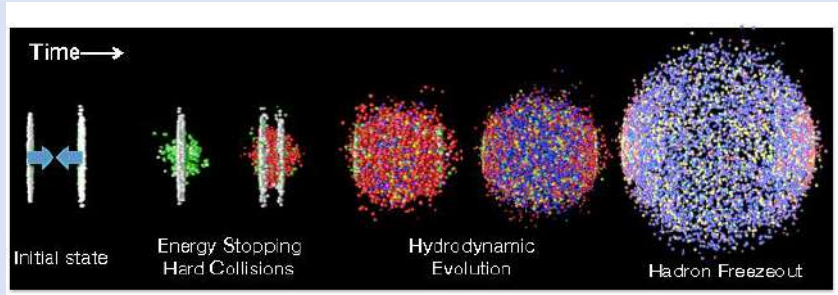




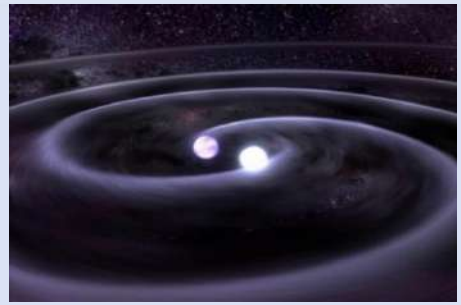
Hydrodynamics

Why hydrodynamics? ⟶ It describes interesting phenomena:

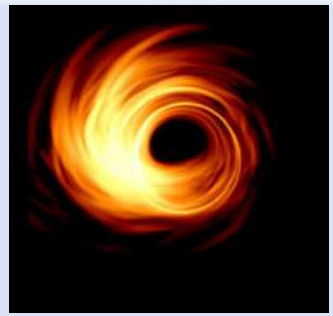
Quark-Gluon Plasma

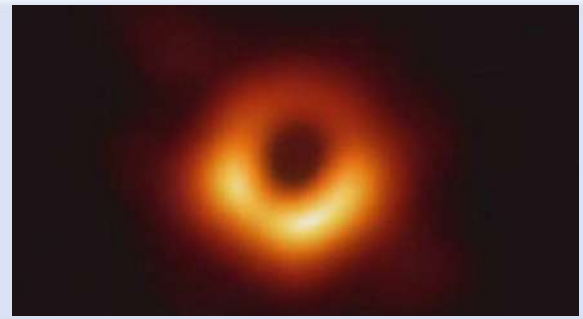


Neutron star mergers



Black hole accretion disk



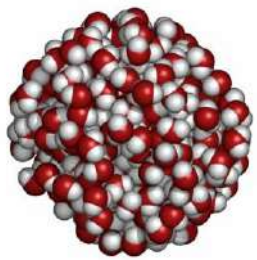


⟶ Relevant for groundbreaking research!

Hydrodynamics

Effective theory

Water



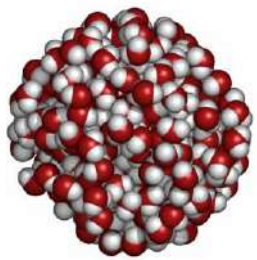
Complicated molecular dynamics

Collective description: hydrodynamics

Hydrodynamics

Effective theory

Water



l_m

L

Complicated molecular dynamics

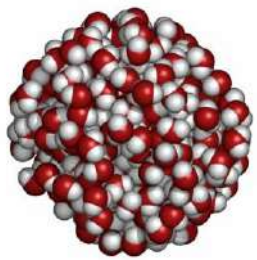
Collective description: hydrodynamics

- Two scales well separated: $l_m \ll L$

Hydrodynamics

Effective theory

Water



l_m

L

Complicated molecular dynamics

Collective description: hydrodynamics

- Two scales well separated: $l_m \ll L$

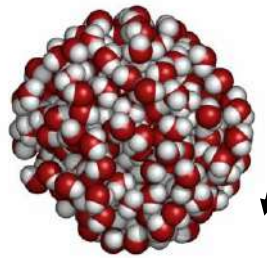
Universality

- Theories satisfying $\nabla_\mu T^{\mu\nu} = 0$ have a hydro regime.

Hydrodynamics

Effective theory

Water



l_m

L

Complicated molecular dynamics

Collective description: hydrodynamics

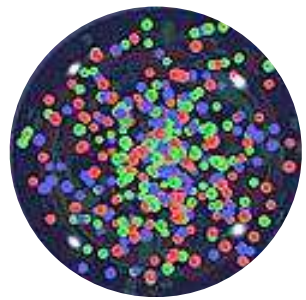
- Two scales well separated: $l_m \ll L$

Universality

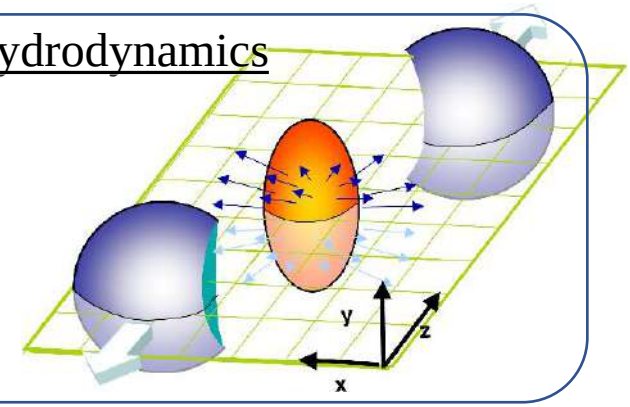
- Theories satisfying $\nabla_\mu T^{\mu\nu} = 0$ have a hydro regime.

Quark-gluon plasma

Complicated partonic dynamics



Hydrodynamics



Causal hydrodynamics

Constitutive relations

$$T_{\mu\nu} = T_{\mu\nu}^{ideal} + \partial + \partial^2 + \ldots$$

Gradient expansion l_m/L

0th order 1st order 2nd order

Causal hydrodynamics

Constitutive relations

$T_{\mu\nu} = T^{ideal}_{\mu\nu} + \partial + \partial^2 + \ldots$ **Gradient expansion** l_m/L

0th order 1st order 2nd order

$\nabla_\mu T^{\mu\nu} = 0$ Dynamical equations (non linear)

Causal hydrodynamics

Constitutive relations

$$T_{\mu\nu} = T_{\mu\nu}^{ideal} + \partial + \partial^2 + \ldots$$ **Gradient expansion** l_m/L

0th order 1st order 2nd order

$$\nabla_\mu T^{\mu\nu} = 0$$ Dynamical equations (non linear)

Causal evolutions are required!

Causal hydrodynamics

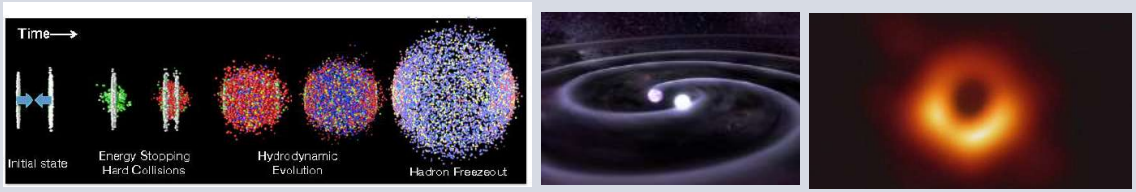
Constitutive relations

$$T_{\mu\nu} = T^{ideal}_{\mu\nu} + \partial + \partial^2 + \ldots$$ **Gradient expansion** l_m/L

0th order 1st order 2nd order

$$\nabla_\mu T^{\mu\nu} = 0$$ Dynamical equations (non linear)

Causal evolutions are required!



Ideal hydro ⟶ Well posed

Causal hydrodynamics

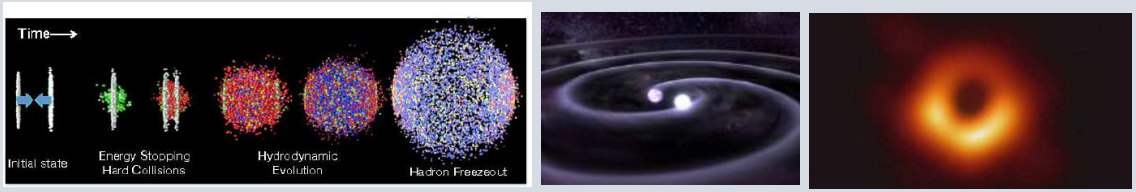
Constitutive relations

$$T_{\mu\nu} = T_{\mu\nu}^{ideal} + \partial + \partial^2 + \ldots$$ **Gradient expansion** l_m/L

0th order 1st order 2nd order

$$\nabla_\mu T^{\mu\nu} = 0$$ Dynamical equations (non linear)

Causal evolutions are required!



Ideal hydro ⟶ Well posed

Viscous hydro ⟶ Ill posed

Causal hydrodynamics

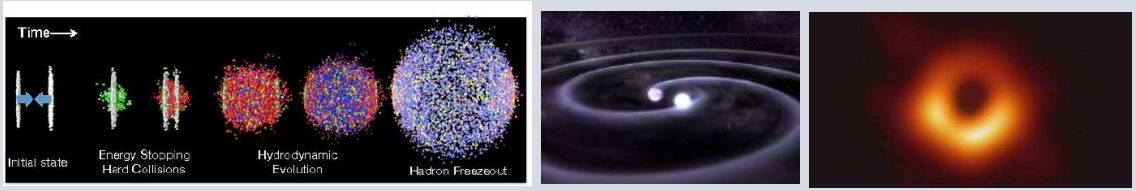
Constitutive relations

$$T_{\mu\nu} = T^{ideal}_{\mu\nu} + \partial + \partial^2 + \ldots$$ **Gradient expansion** l_m/L

0th order 1st order 2nd order

$$\nabla_\mu T^{\mu\nu} = 0$$ Dynamical equations (non linear)

Causal evolutions are required!



Ideal hydro ⟶ Well posed

Viscous hydro ⟶ Ill posed

Usual fix

MIS (BRSSS) ⟶ Well posed

Holography

Holography

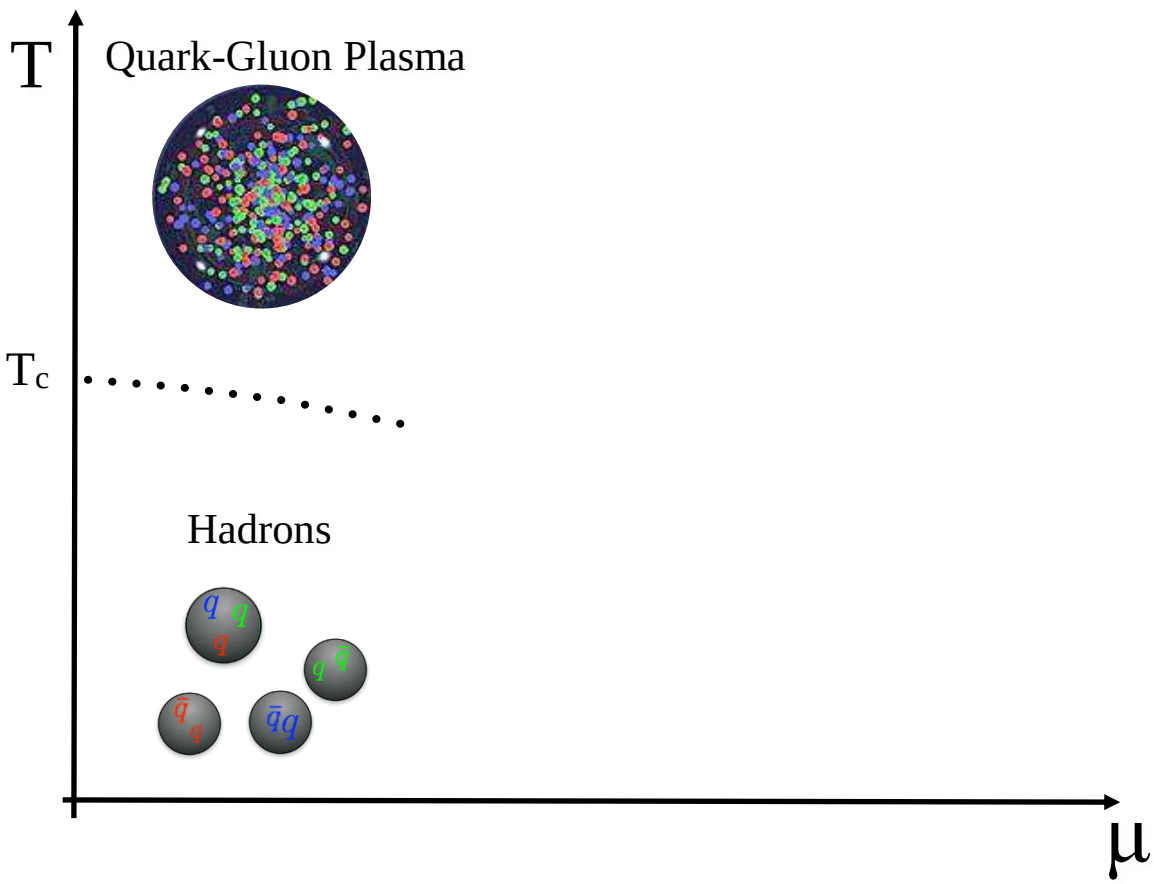
→ Excellent framework to study hydrodynamics.

Holography

→ Excellent framework to study hydrodynamics.

→ Strongly coupled field theories far from equilibrium from first principles.

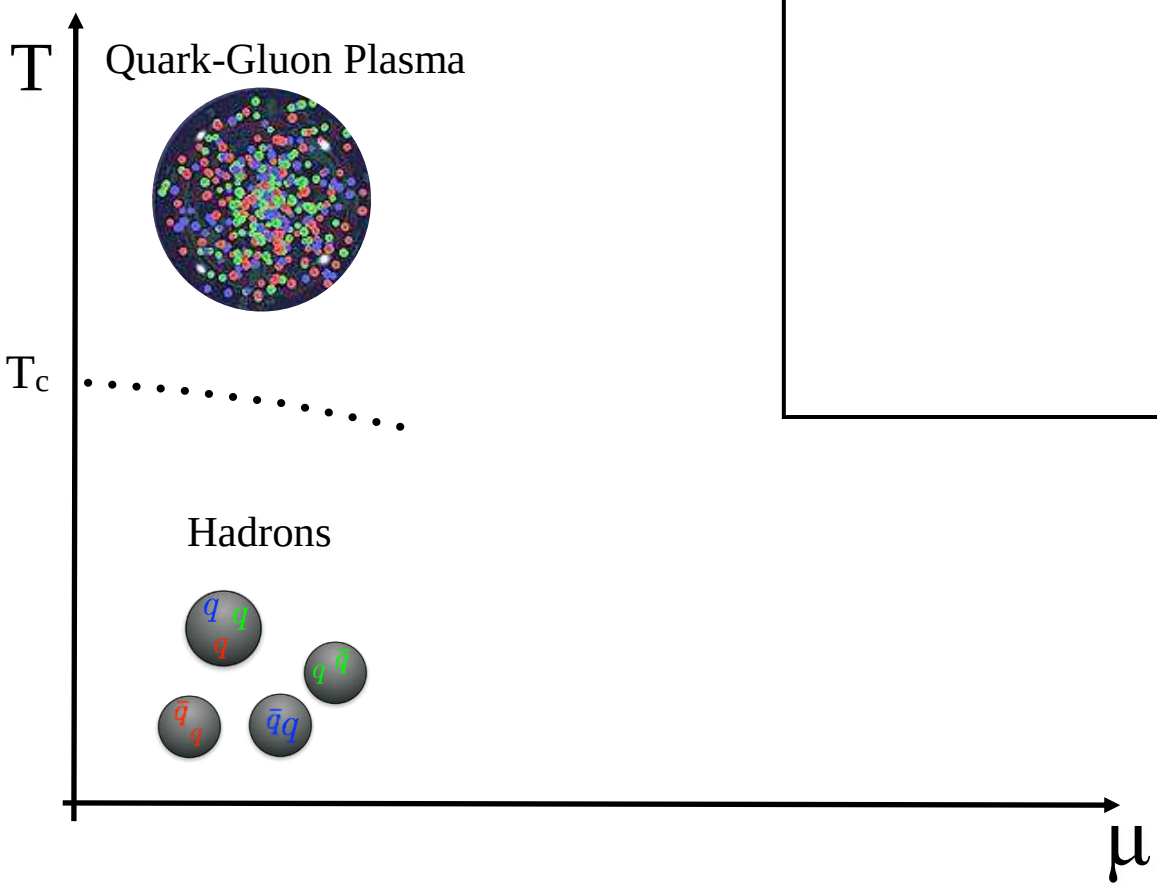
Holography



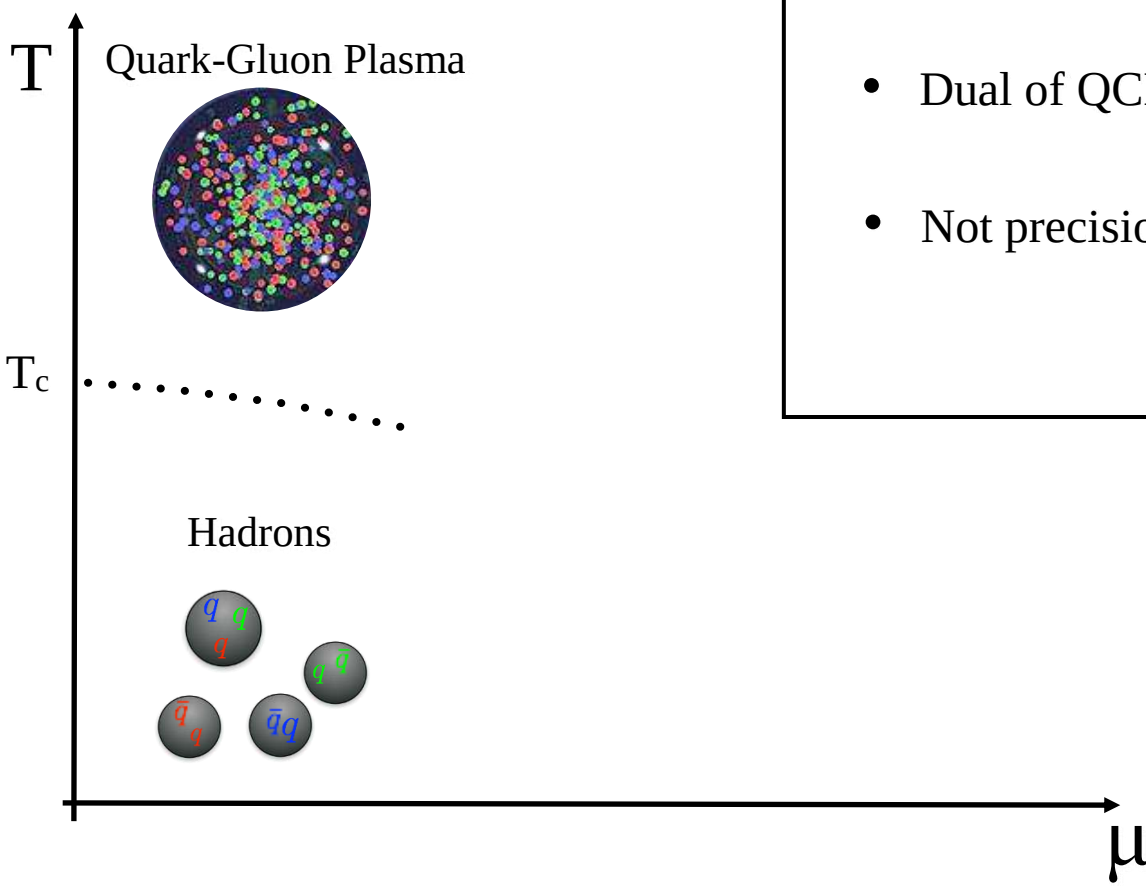
Holography

Holography

- Strongly coupled QFT
- Out of equilibrium physics



Holography

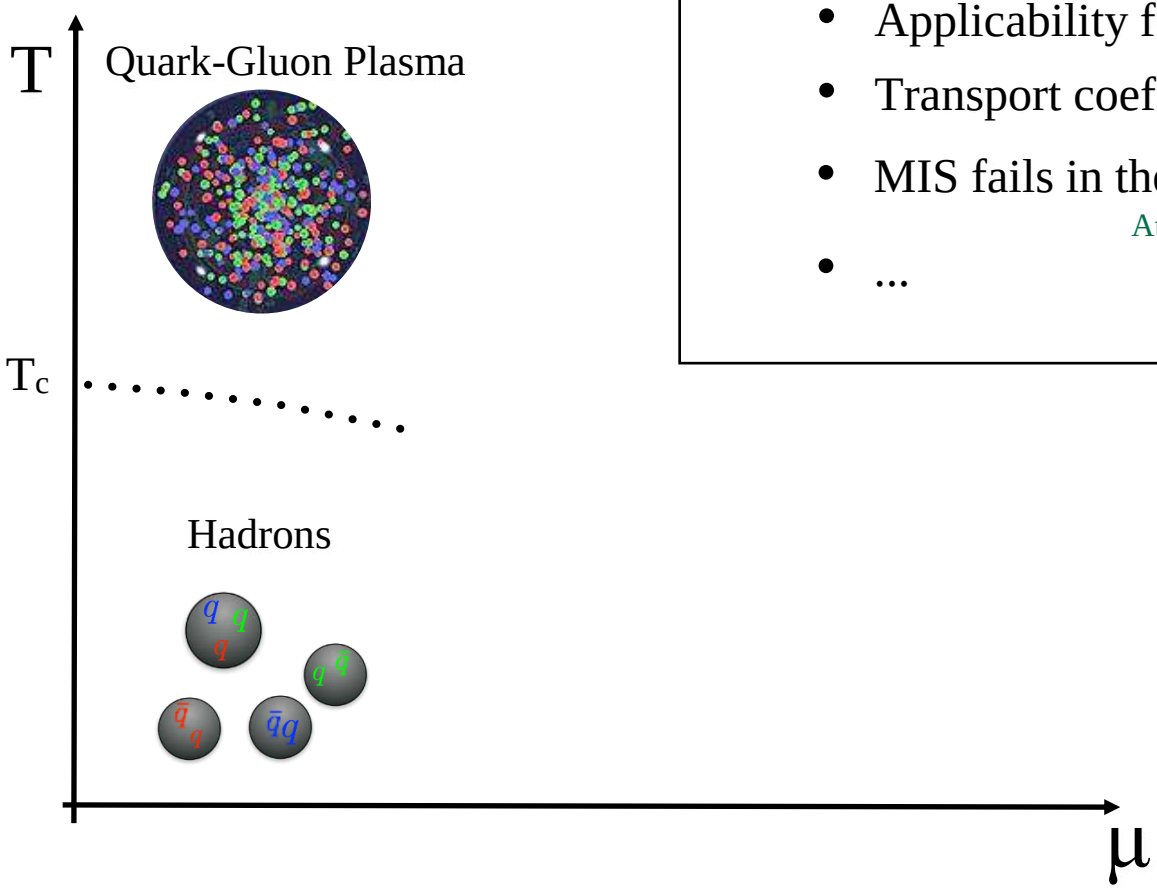


Holography

- Strongly coupled QFT
- Out of equilibrium physics
- Dual of QCD not known…
- Not precision holography

→ Qualitative aspects

Holography



What have we learned from holography so far?

Chesler, Yaffe, Casalderrey, Mateos, Heller, van der Schee, …

- Early hydrodynamization times
- Applicability with large gradients
- Applicability for small systems
- Transport coefficients
- MIS fails in the presence of a phase transition

  Attems, Bea, Mateos, Casalderrey, Triana, Zilhao ’19, ‘20
- ...

1 - New holographic solution

Holography: Our model

- Einstein-Hilbert with cosmological constant in 3+1 dimensions.

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda)$$

Holography: Our model

- Einstein-Hilbert with cosmological constant in 3+1 dimensions.

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda)$$

- Dual to the decoupled sector of the stress tensor of a CFT in 2+1 dim.
- CFT on Minkowski space and we focus on the Poincare patch of AdS.

Holography: Our model

- Einstein-Hilbert with cosmological constant in 3+1 dimensions.

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda)$$

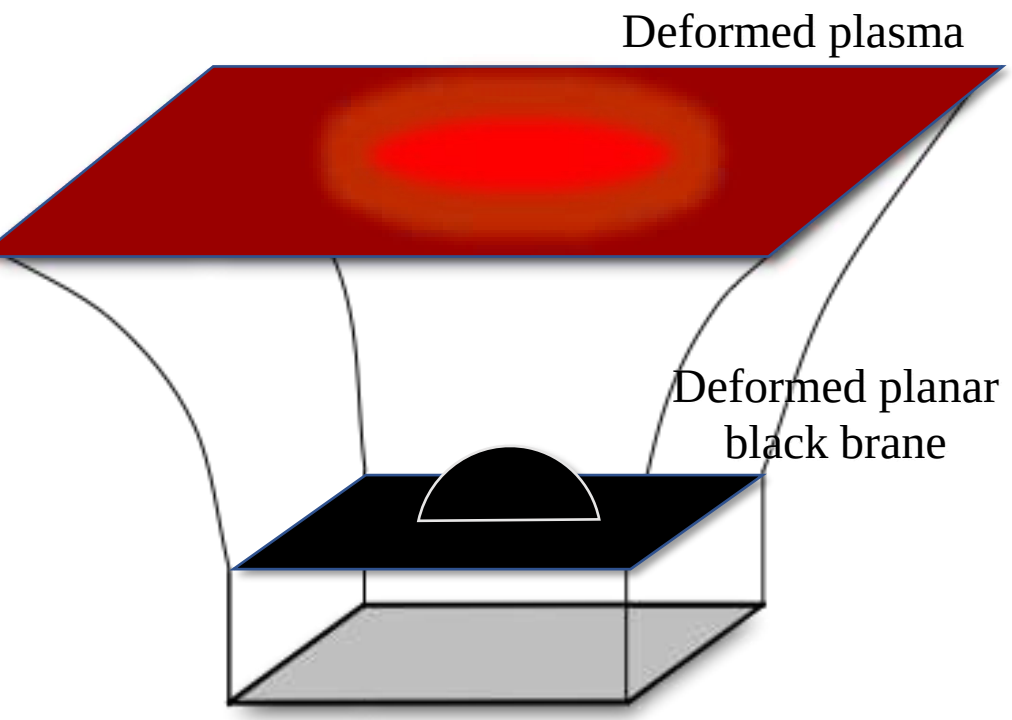
- Dual to the decoupled sector of the stress tensor of a CFT in 2+1 dim.
- CFT on Minkowski space and we focus on the Poincare patch of AdS.

We obtain solutions:

- Far from equilibrium physics
- No symmetry assumptions
- Numerical relativity to obtain the real-time evolution
- Dynamics in 3+1 dimensions (2+1 in the CFT)

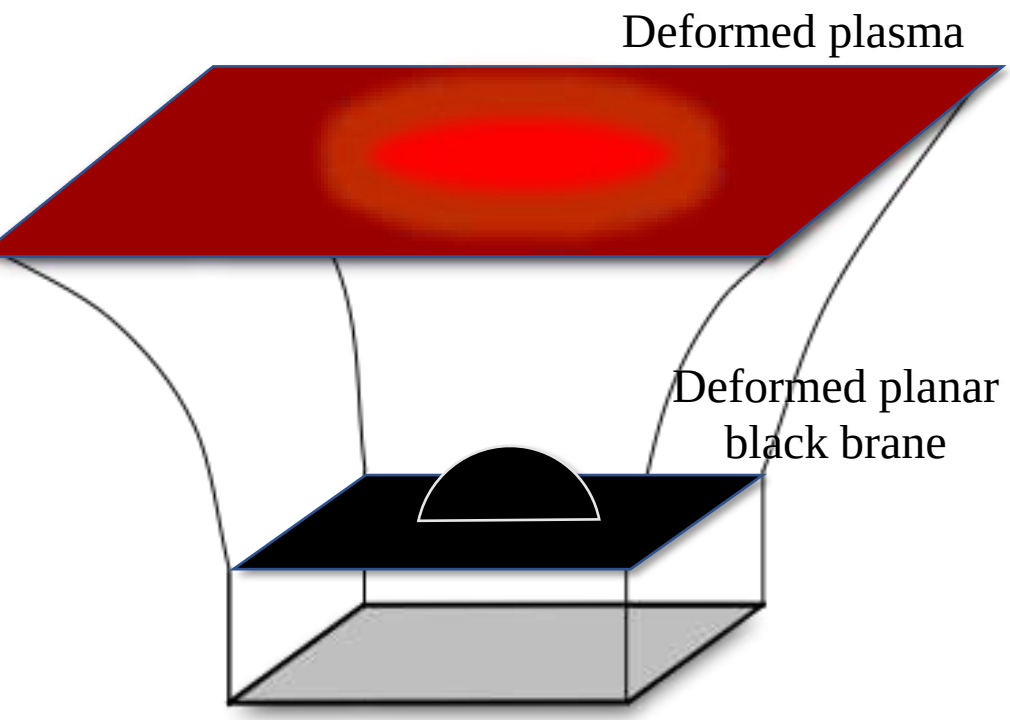
Holographic solutions

Studied in the literature



Holographic solutions

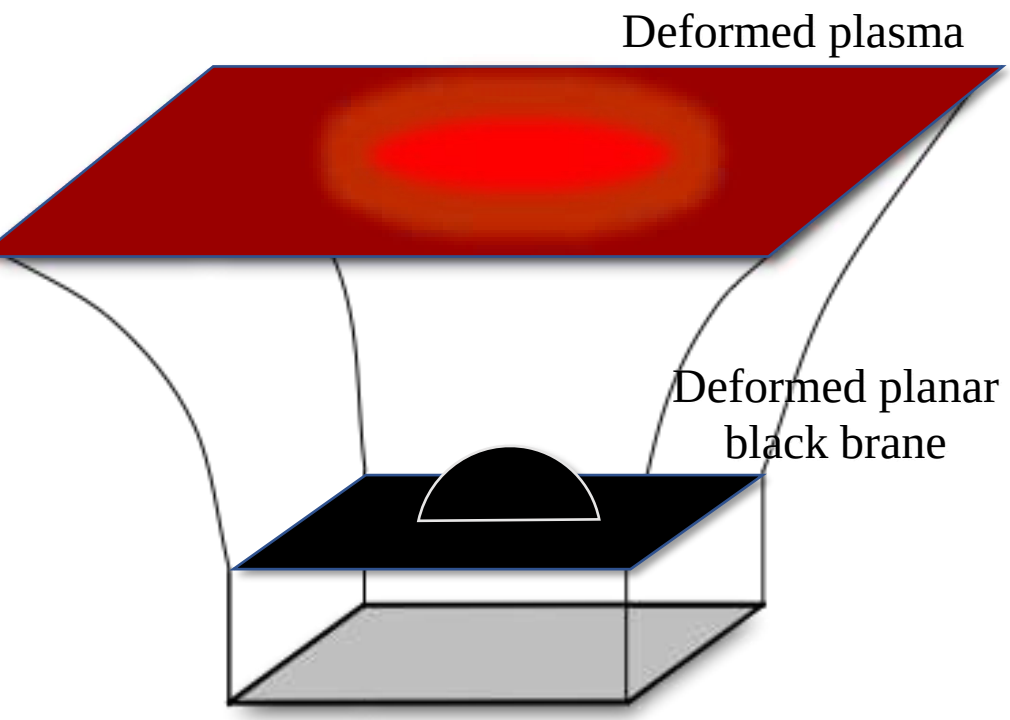
Studied in the literature



- $t \to \infty$ homogeneous black brane

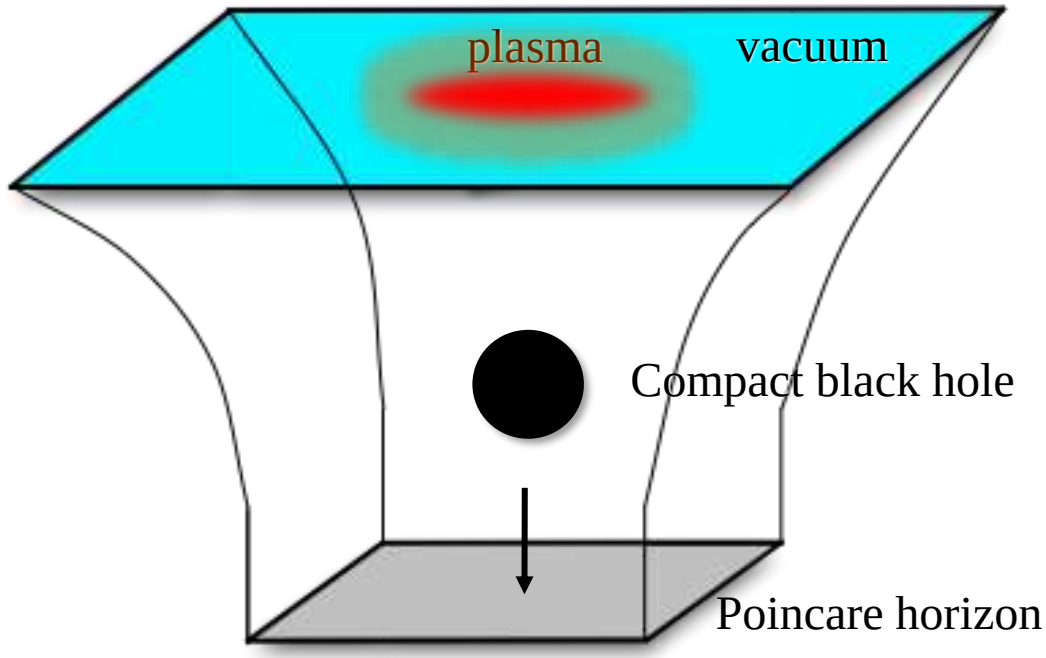
Holographic solutions

Studied in the literature



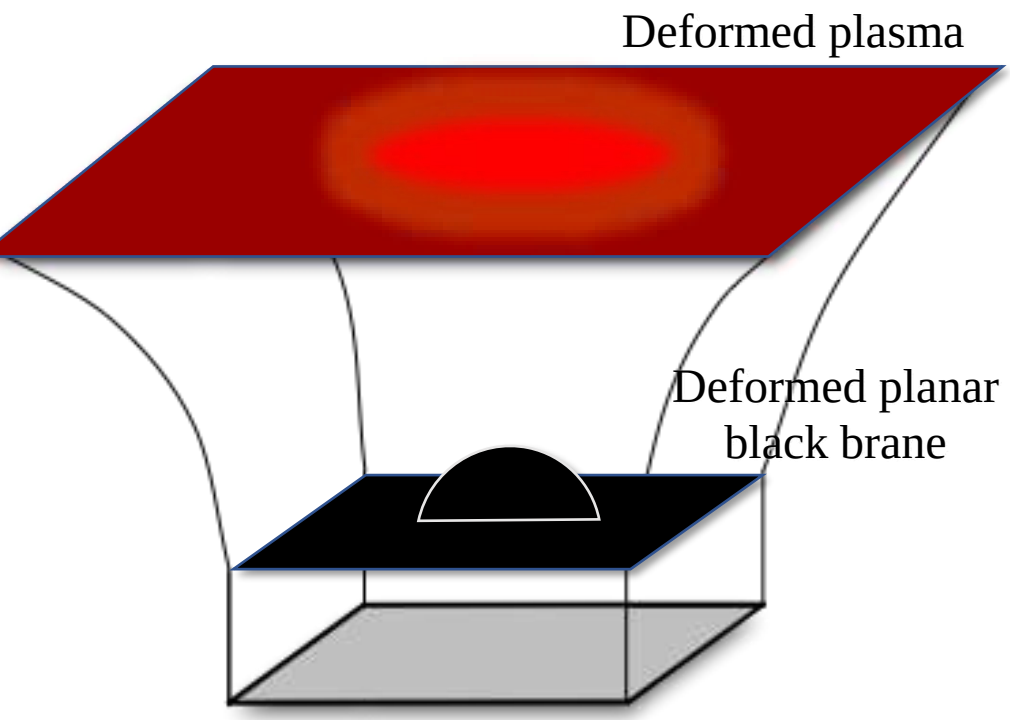
- $t \to \infty$ homogeneous black brane

Our new solution



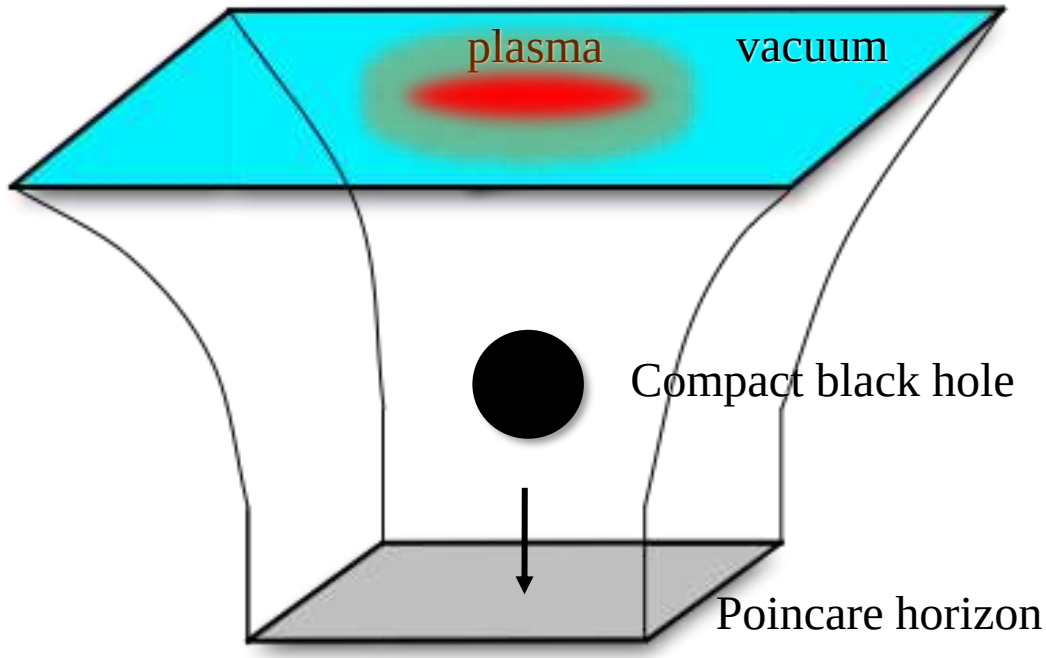
Holographic solutions

Studied in the literature



- $t \to \infty$ homogeneous black brane

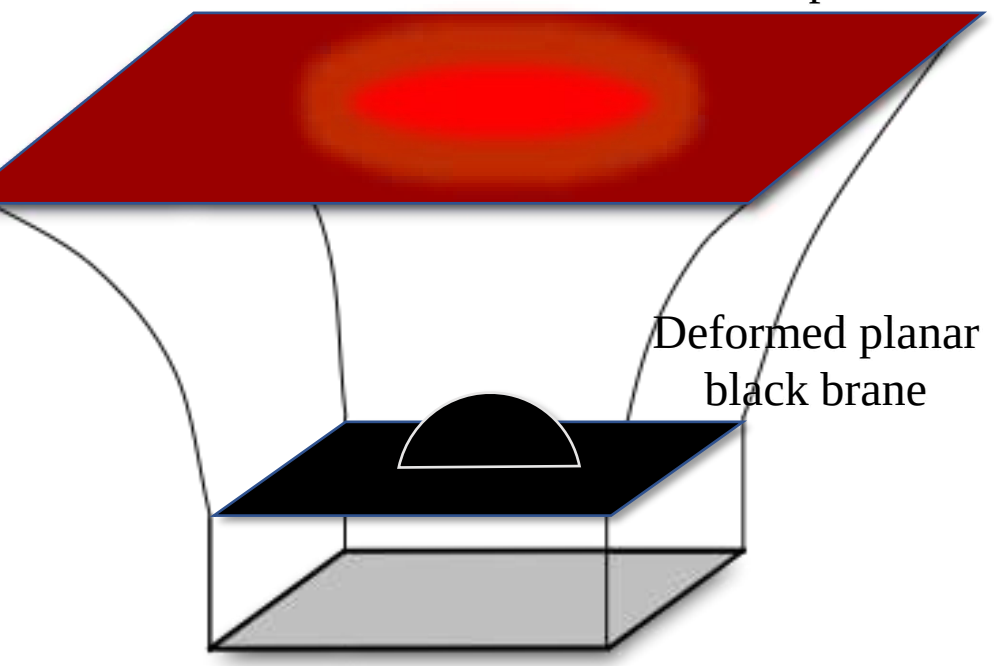
Our new solution



- $t \to \infty$ pure AdS
- Captures some physical aspects of QGP
- Cauchy formulation

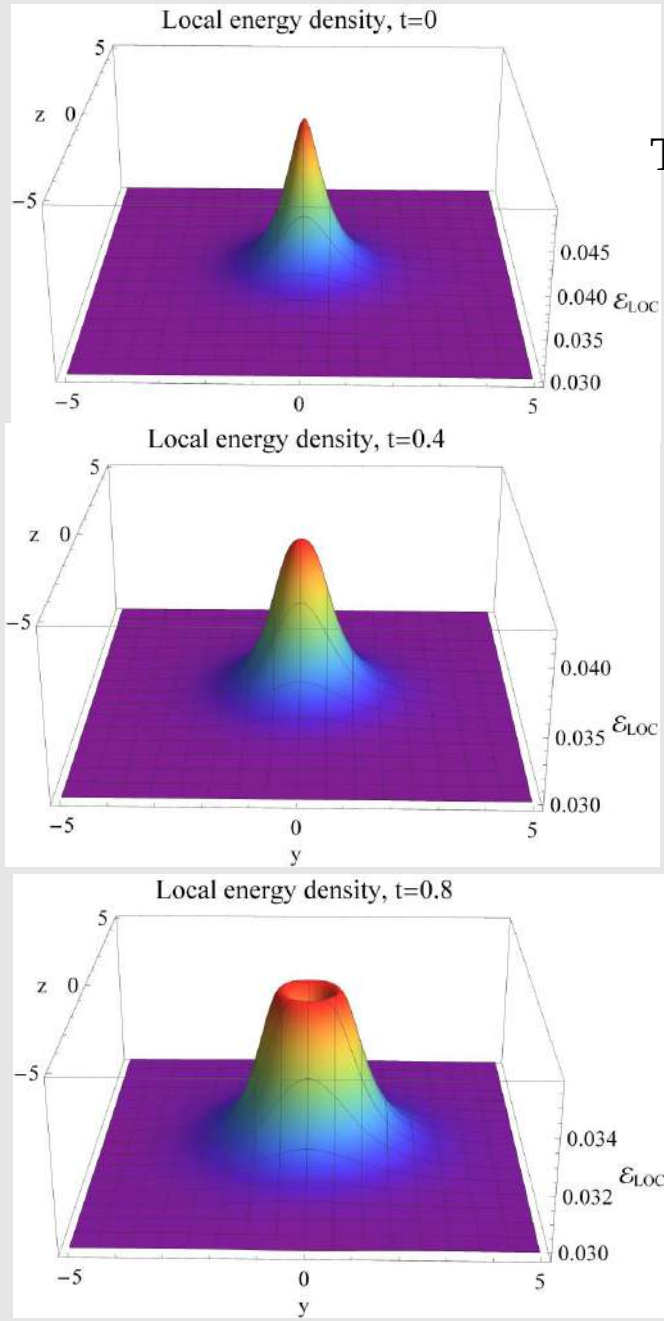
Black brane

Deformed plasma



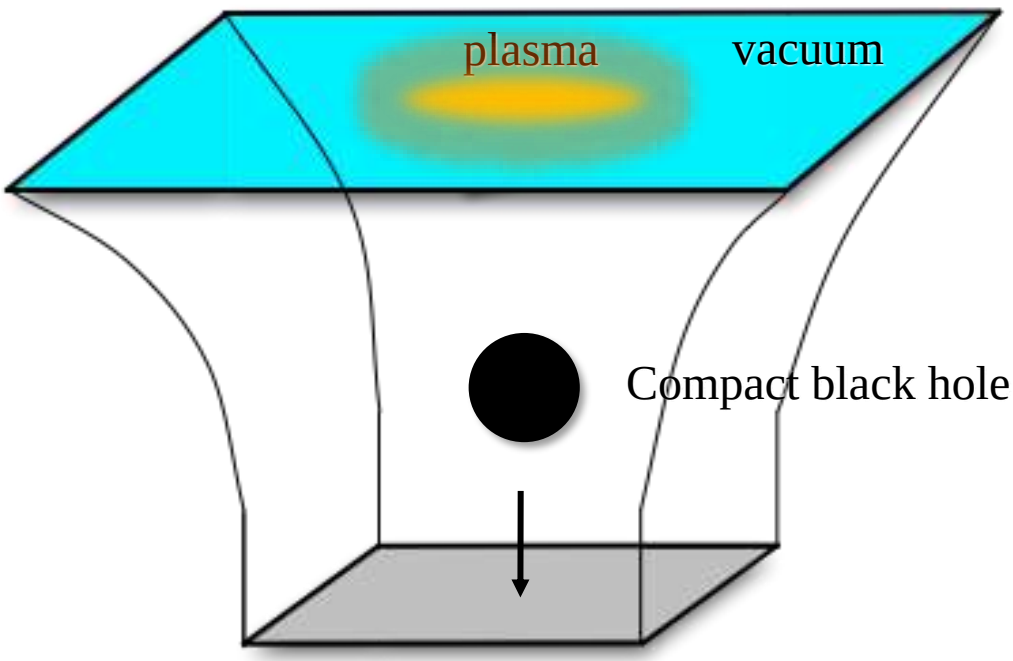
Deformed planar black brane

Local energy density

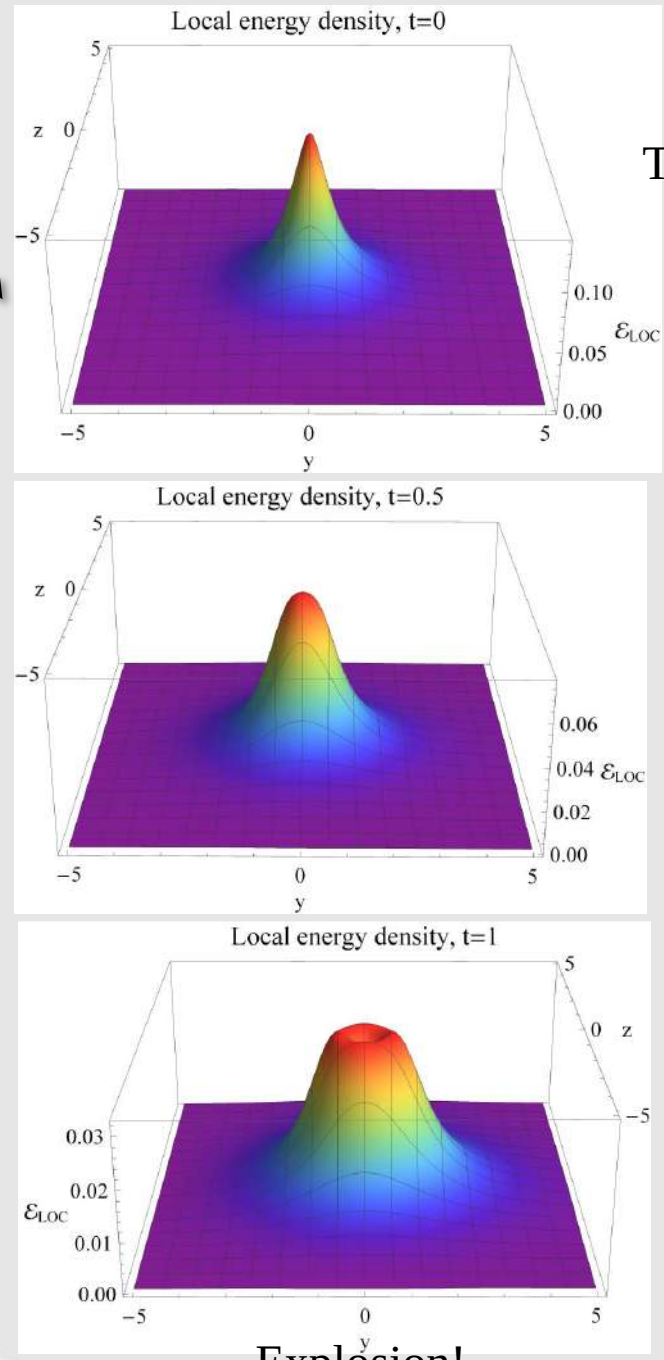


Explosion!

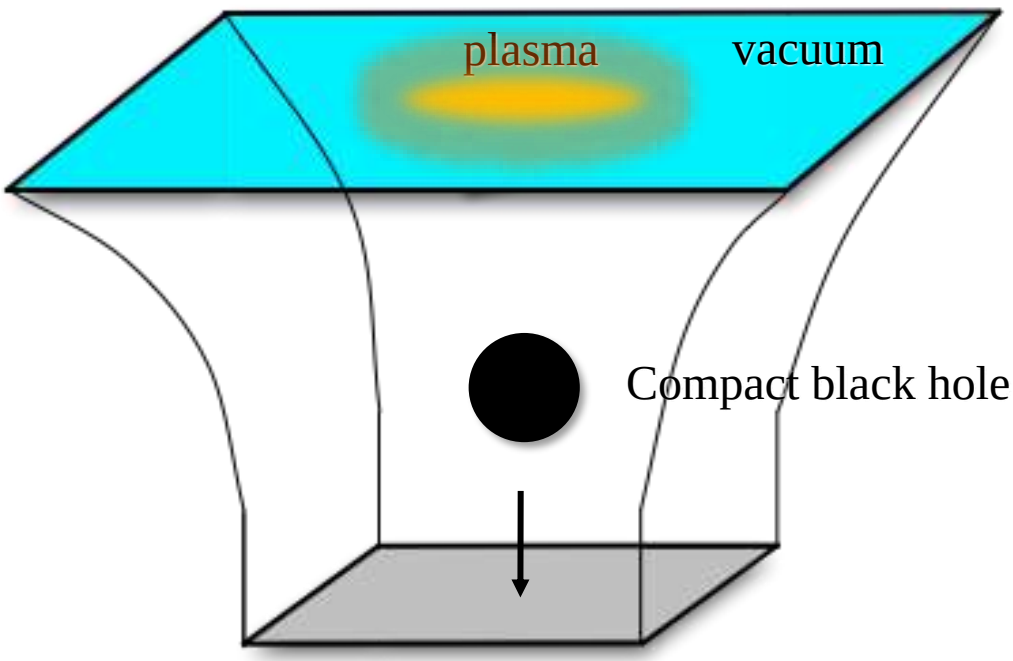
Black hole



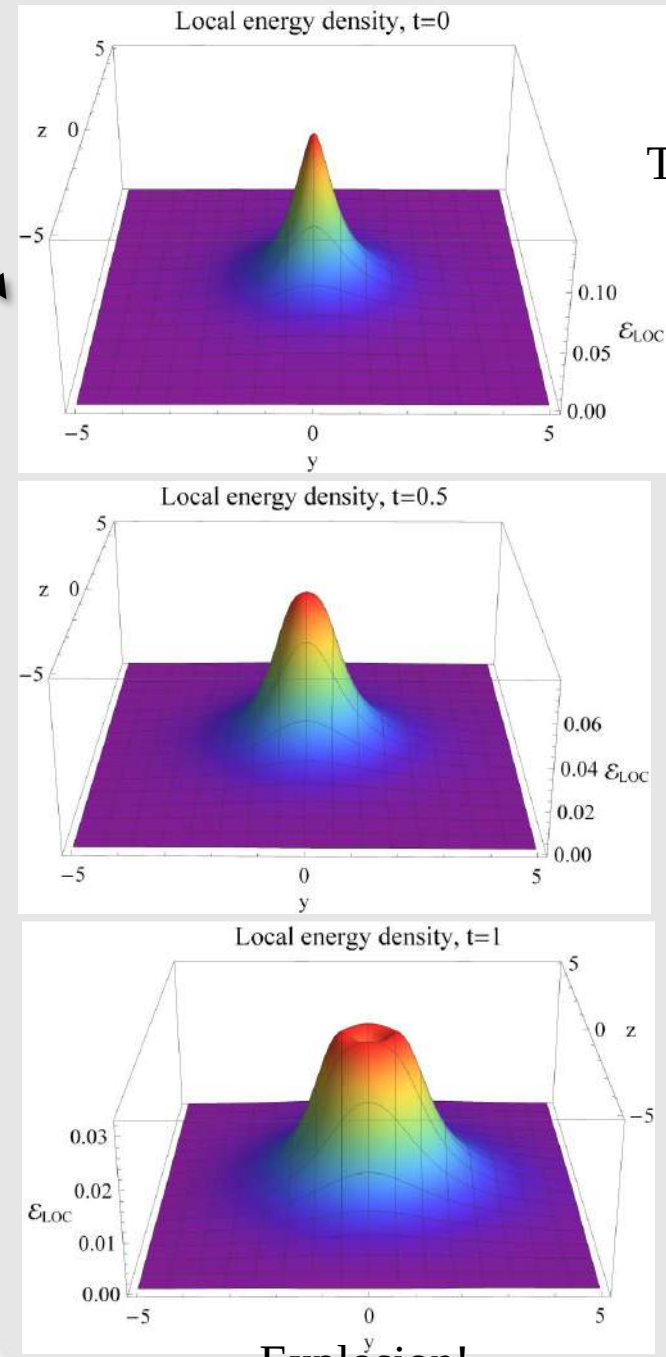
Local energy density



Black hole



Local energy density



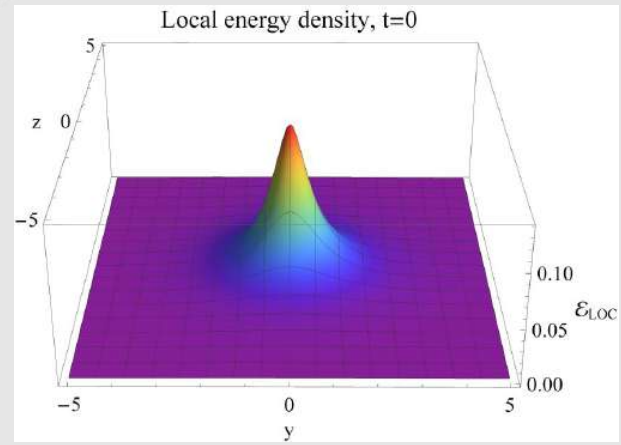
What are the differences between expanding in a plasma and in vacuum?

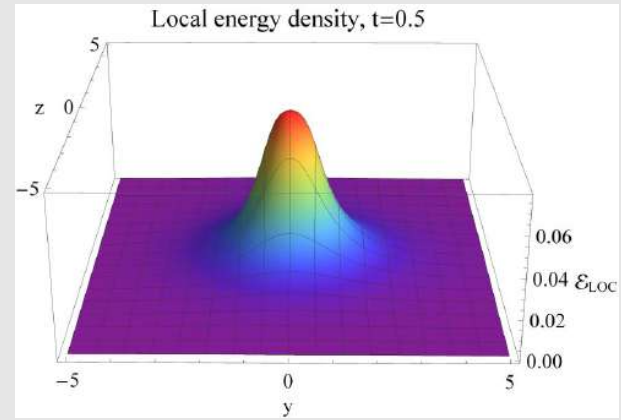
Hydrodynamics: constitutive relations

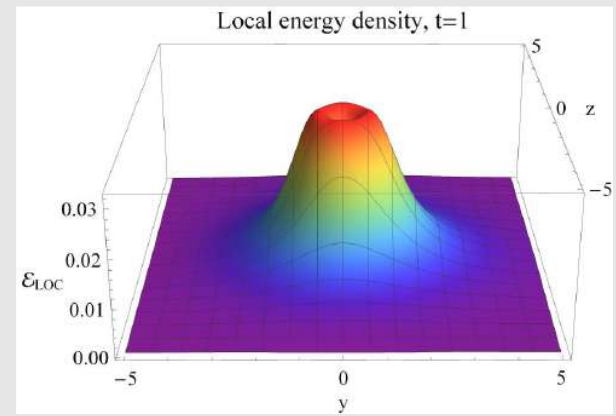
⟶ We find that it is described by hydro at the center, at all times.

- Symmetry argument? ⟶ Yes!

Rotational symmetry
Conformal symmetry
➡ (Ideal) hydro describes the center at all times







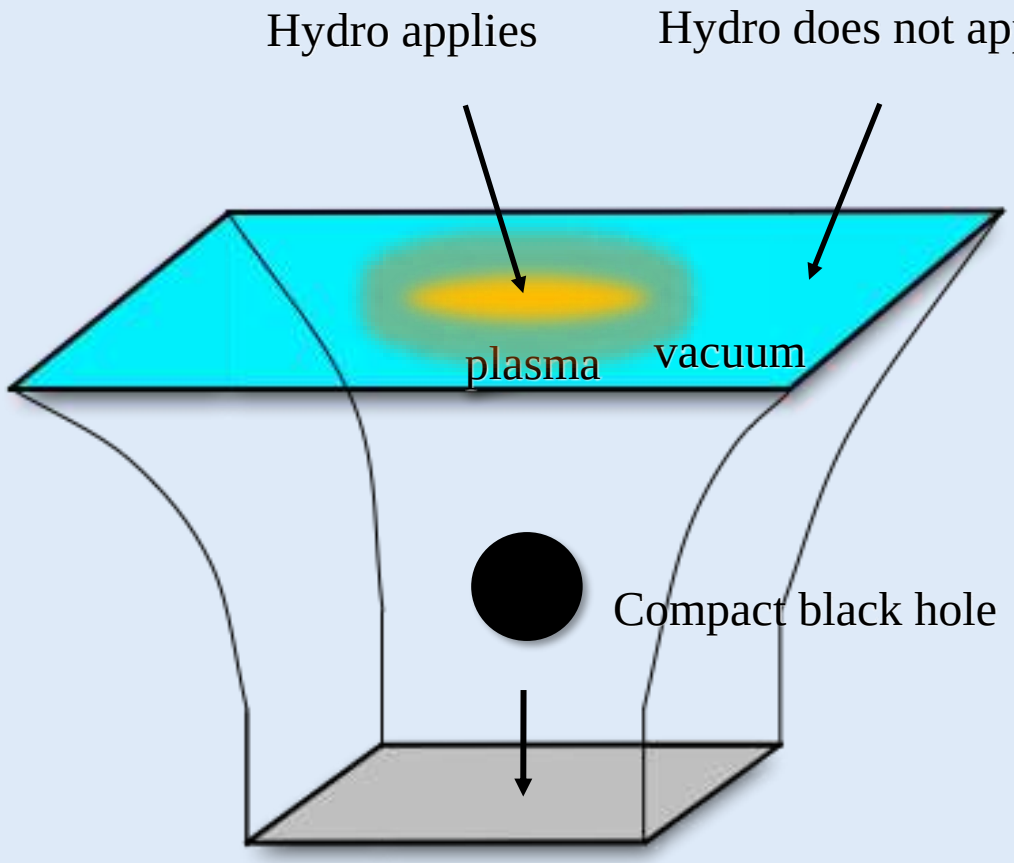
Time

Hydrodynamics: constitutive relations

→ We f
at all

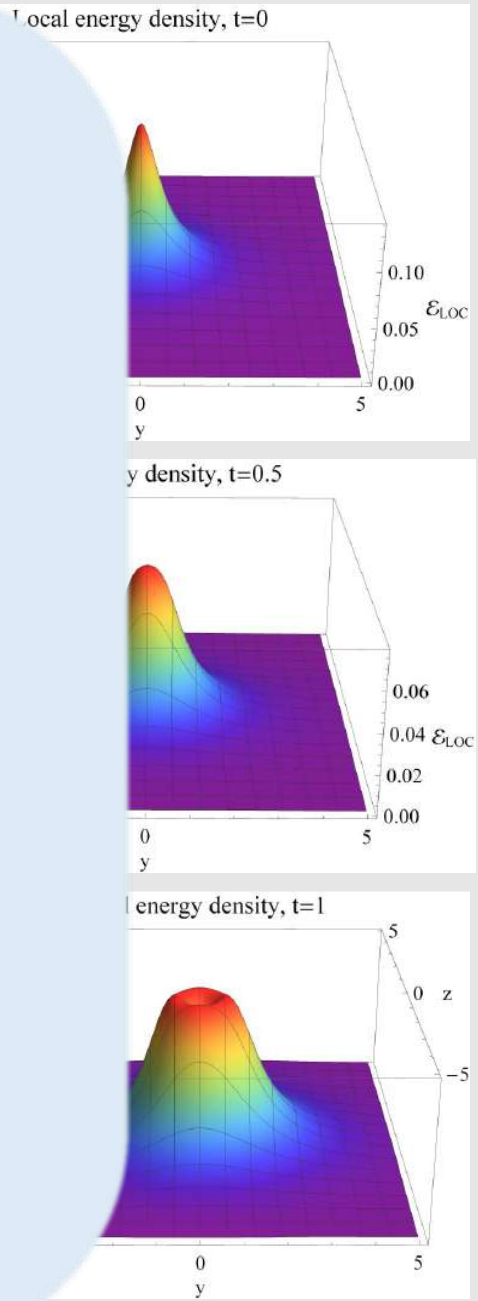
• Symm

Rotational
Conformal



Interesting questions:

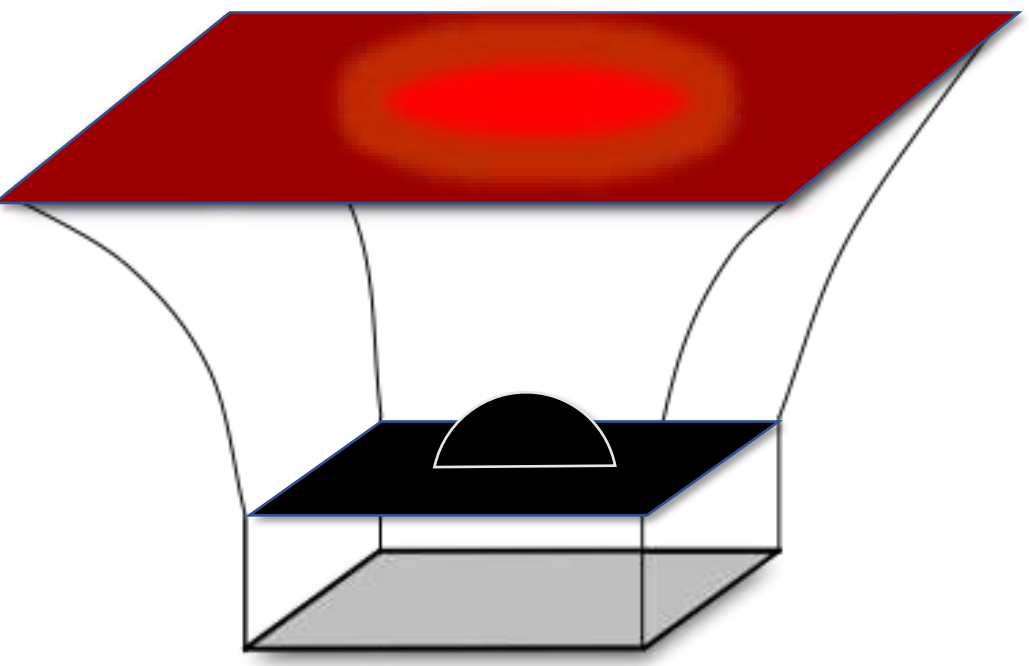
- Where and when hydro stops applying?
- Why specifically it stops applying?
- …



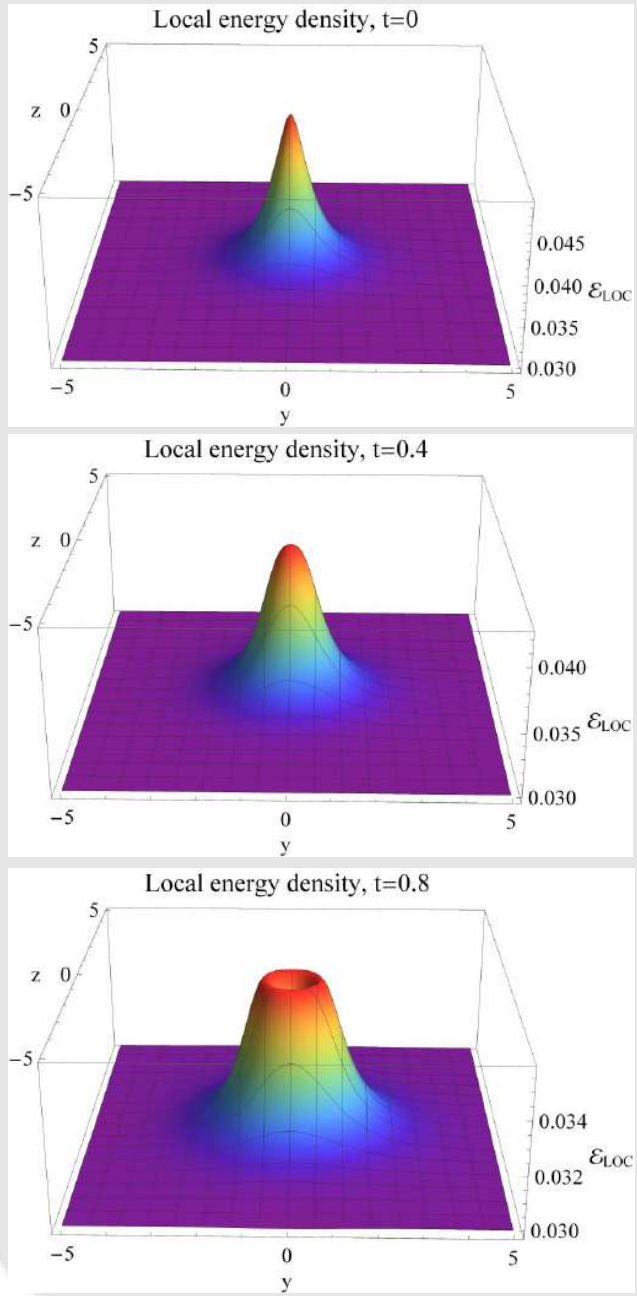
Time

2 – Dynamical evolutions of hydrodynamics

Causal hydrodynamics

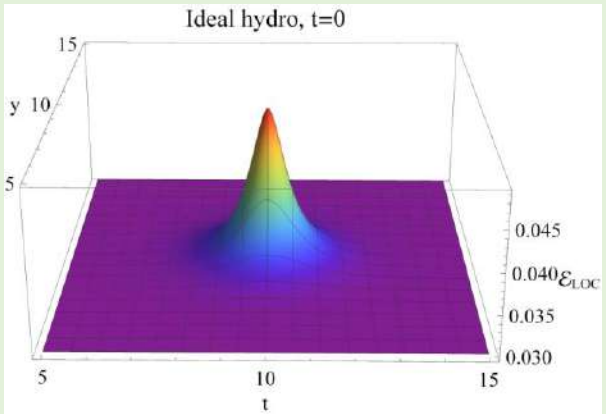


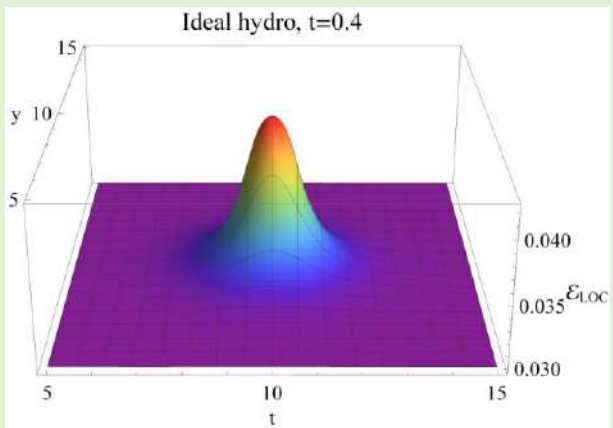
Microscopic evolution

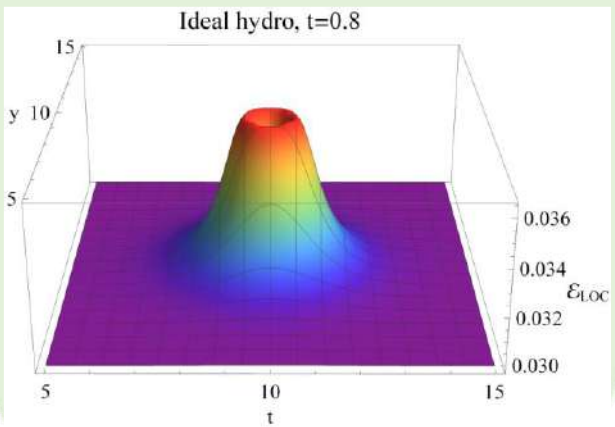


Causal hydrodynamics

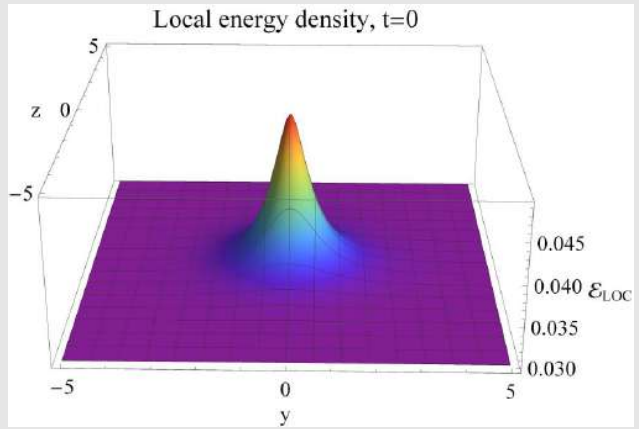
Ideal hydro evolution

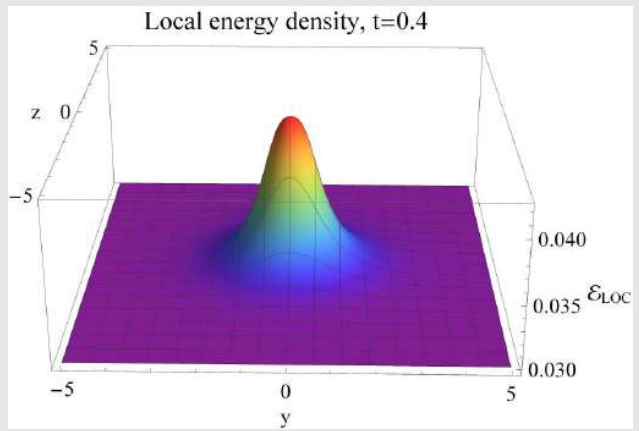


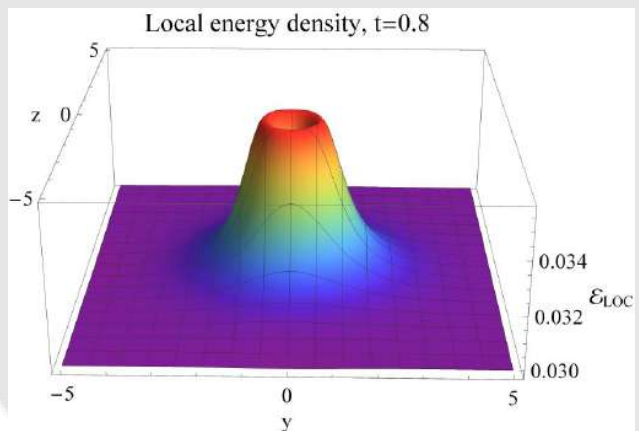




Microscopic evolution



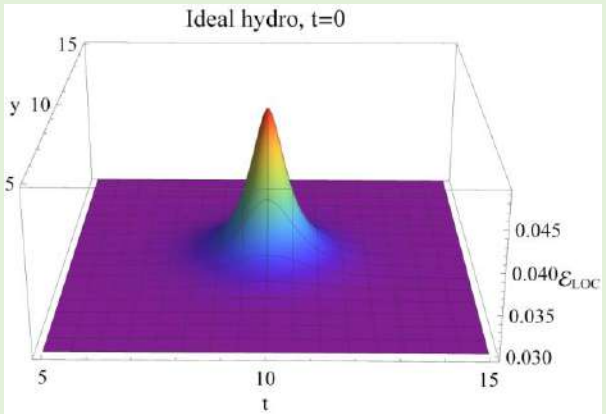


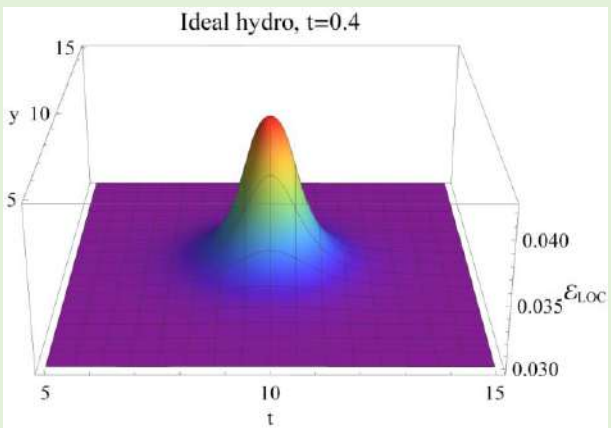


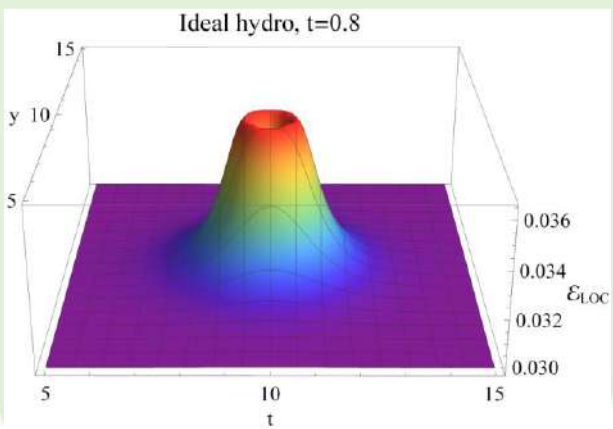
Time

Causal hydrodynamics

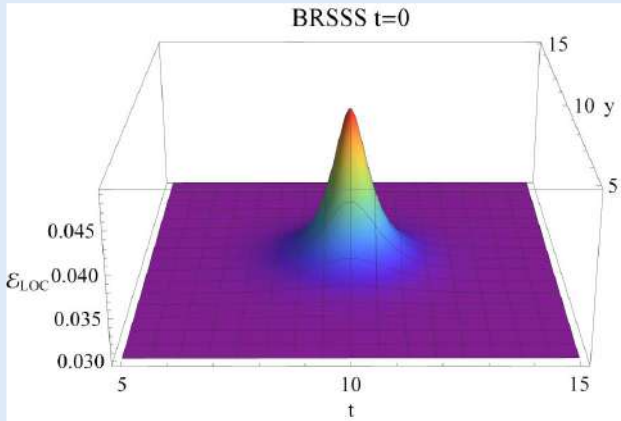
Ideal hydro evolution

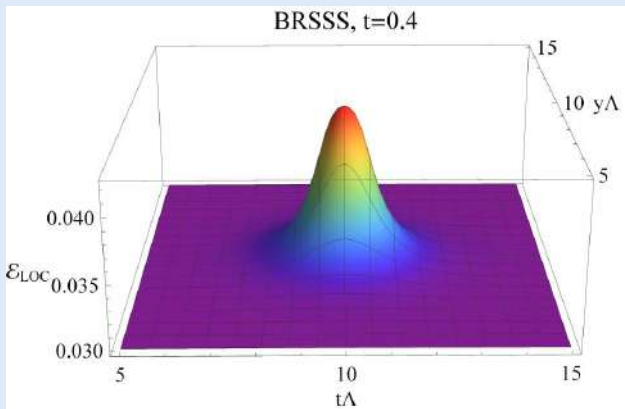


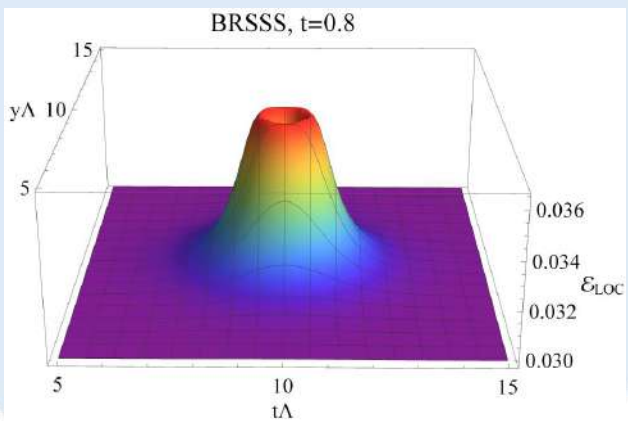




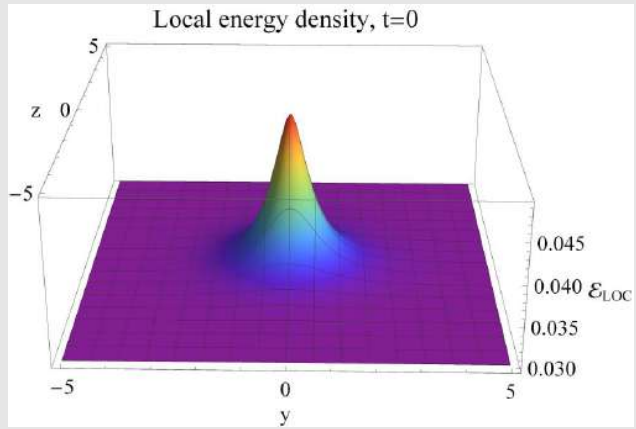
BRSSS (viscous) evolution

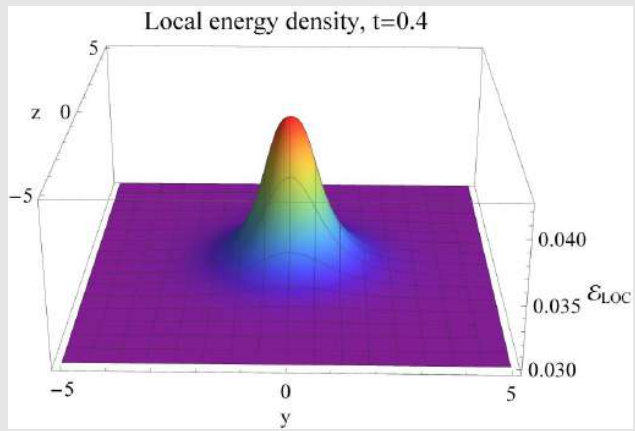


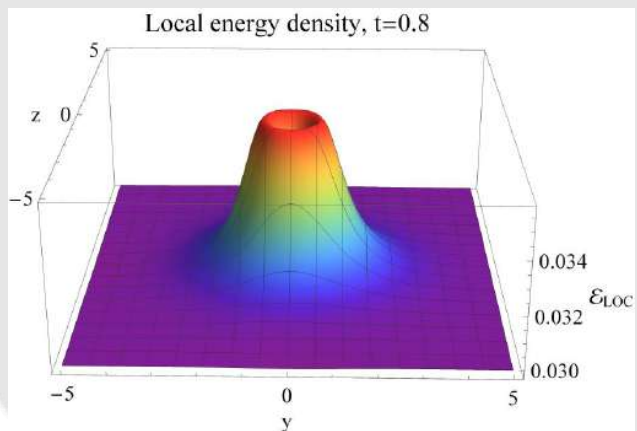




Microscopic evolution



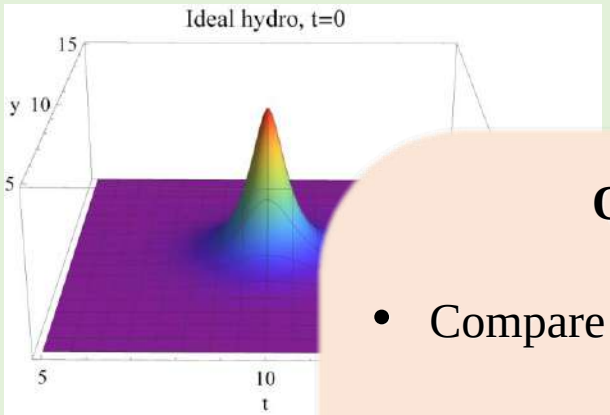


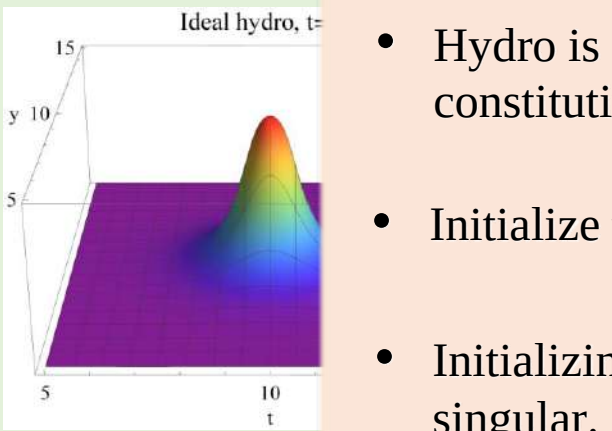


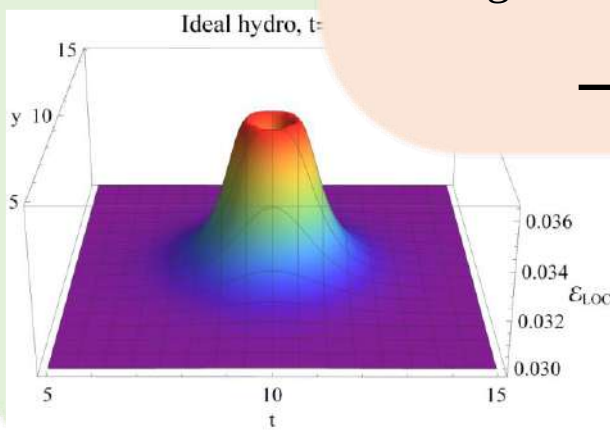
Time

Causal hydrodynamics

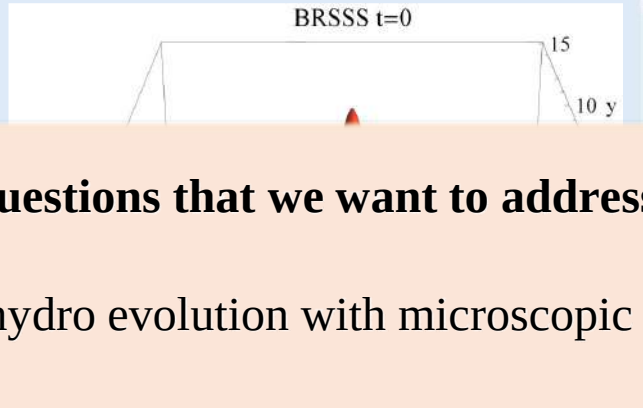
Ideal hydro evolution

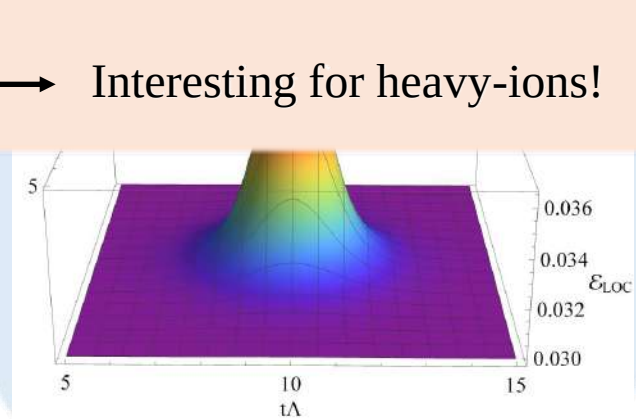




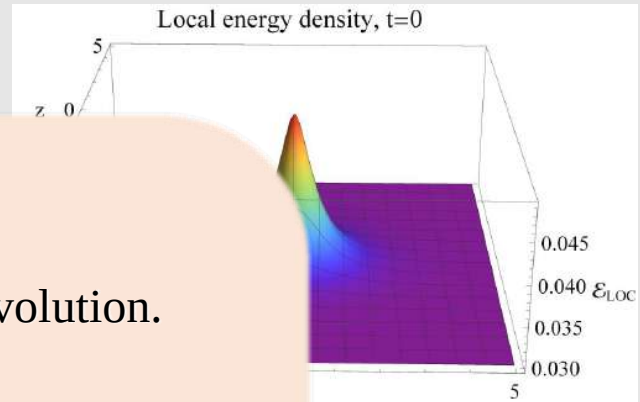


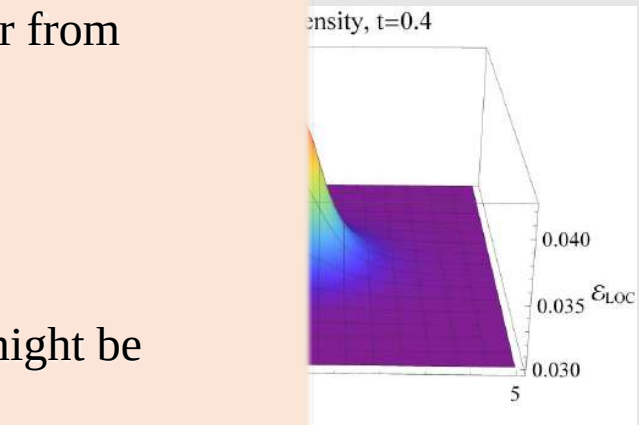
BRSSS (viscous) evolution

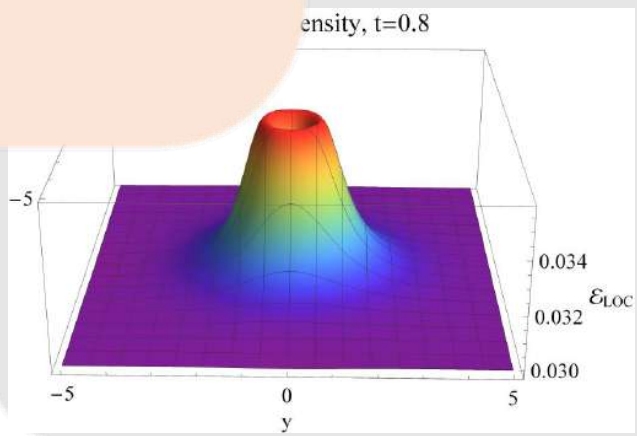




Microscopic evolution







Time

Questions that we want to address:

- Compare hydro evolution with microscopic evolution.
- Hydro is non-linear: time evolution may differ from constitutive relations.
- Initialize the data at different timesteps.
- Initializing the data with regions of vacuum might be singular.

⟶ Interesting for heavy-ions!

3 – Evolutions in BDNK

BDNK

- Recently, a new causal formulation of viscous hydrodynamics was proposed.

Bemfica, Disconzi, Noronha ’19
Kovtun ’19

- The idea is to change from the usual Landau frame.

⟶ This allows to obtain hyperbolicity without adding extra equations or extra variables.

- This formulation only includes first order terms, contrary to MIS.

BDNK

- Recently, a new causal formulation of viscous hydrodynamics was proposed.

Bemfica, Disconzi, Noronha ’19
Kovtun ’19

- The idea is to change from the usual Landau frame.

⟶ This allows to obtain hyperbolicity without adding extra equations or extra variables.

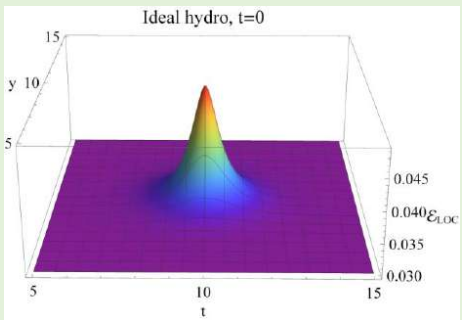
- This formulation only includes first order terms, contrary to MIS.

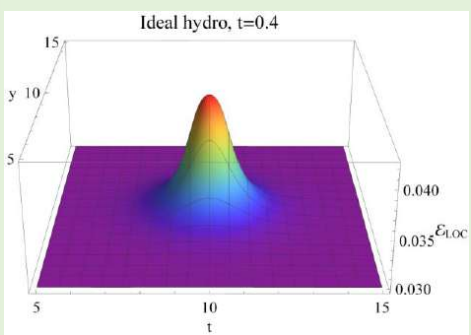
We perform, for the first time, real-time evolutions using this formulation.
(except for highly symmetric cases as Bjorken flow).

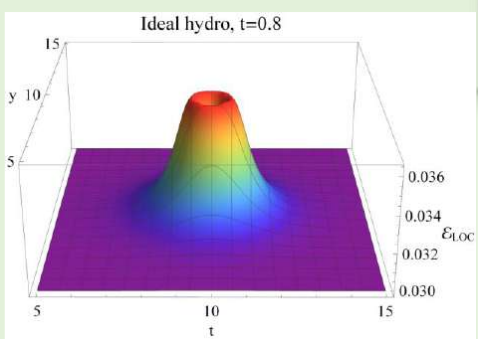
This is a first step towards its implementation in heavy-ions or other relevant scenarios.

BDNK

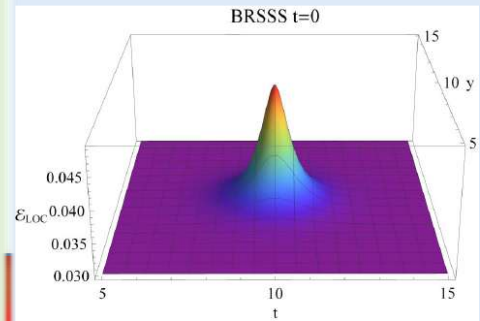
Ideal hydro evolution

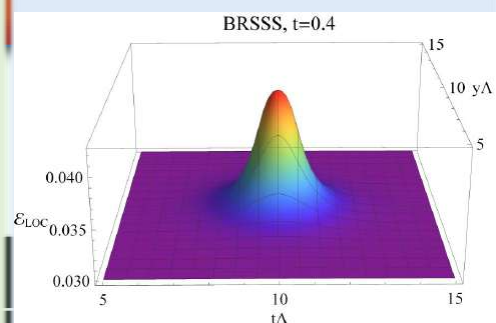


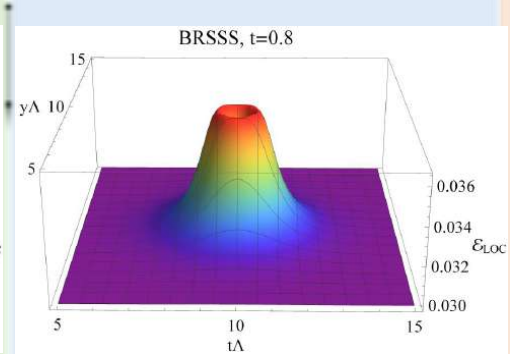




BRSSS (viscous) evolution

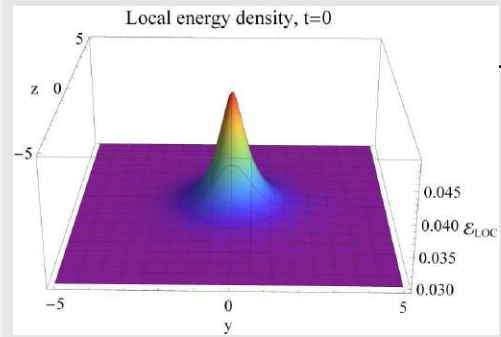


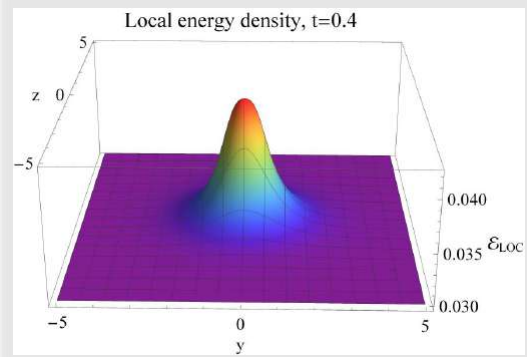


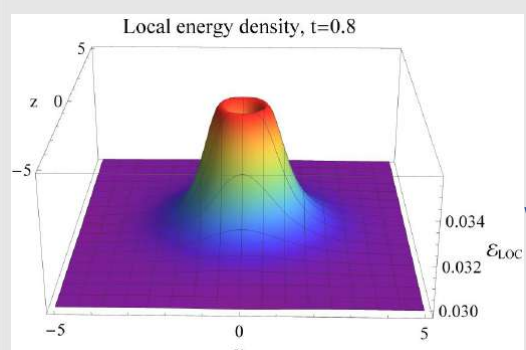


BDNK (viscous) evolution

Microscopic evolution



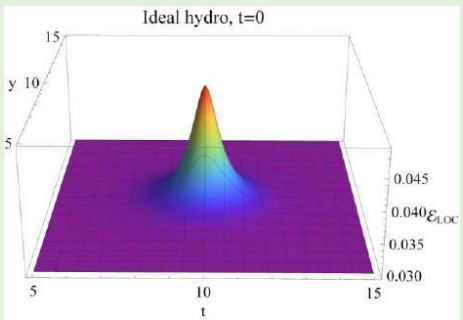


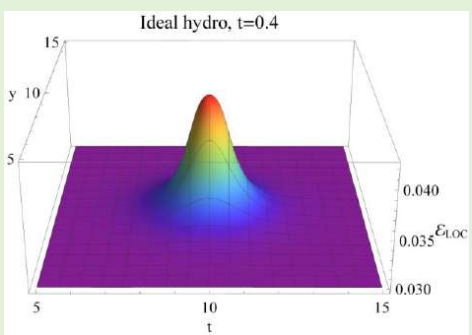


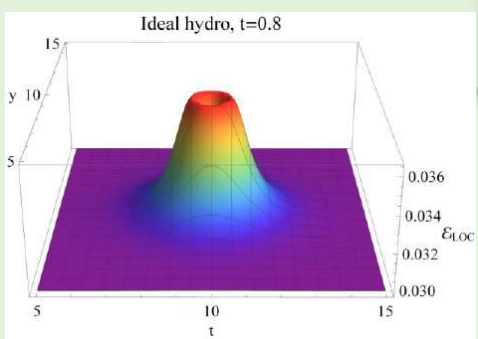
Time

BDNK

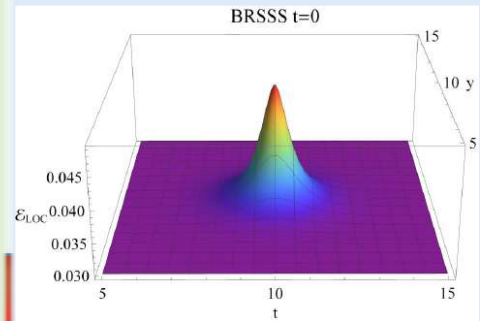
Ideal hydro evolution

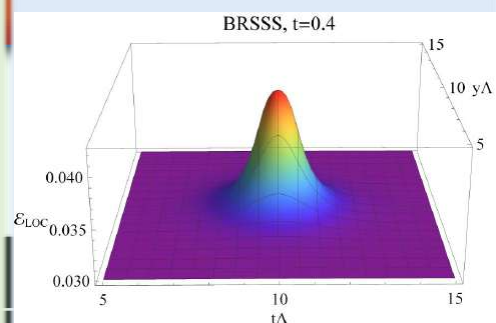


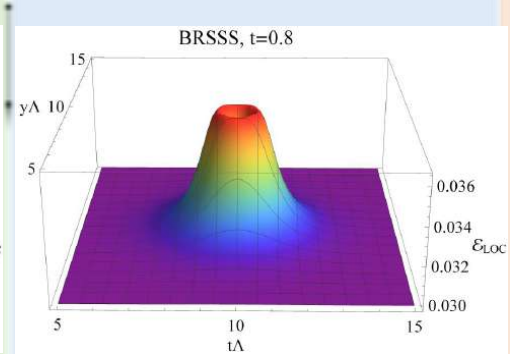




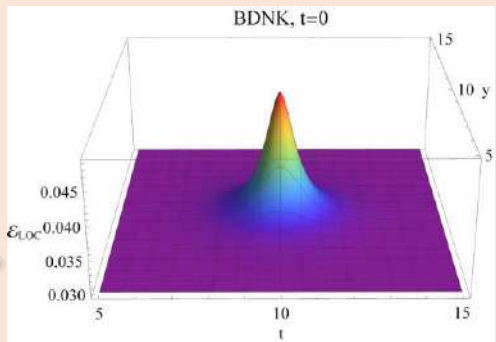
BRSSS (viscous) evolution

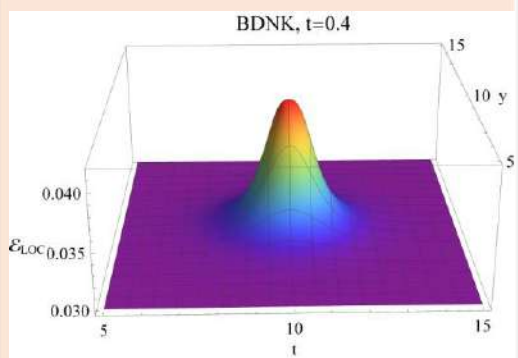


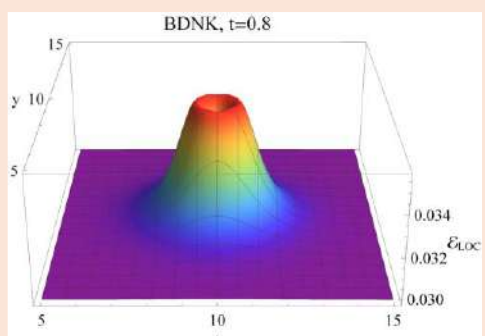




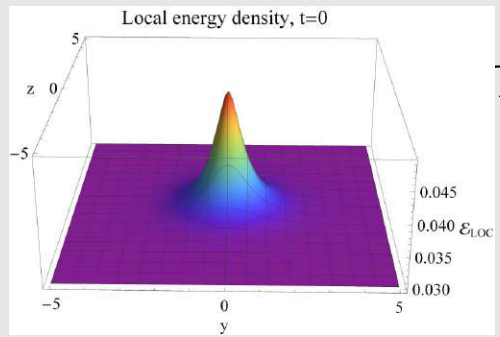
BDNK (viscous) evolution

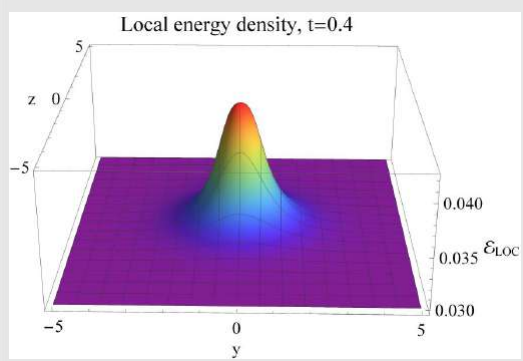


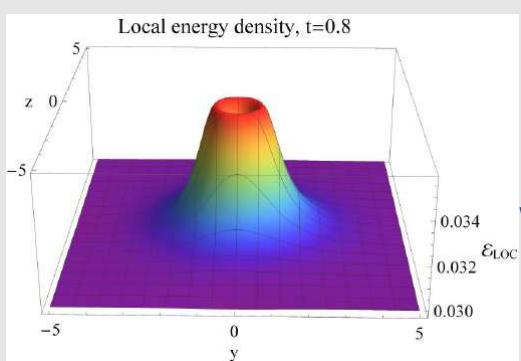




Microscopic evolution



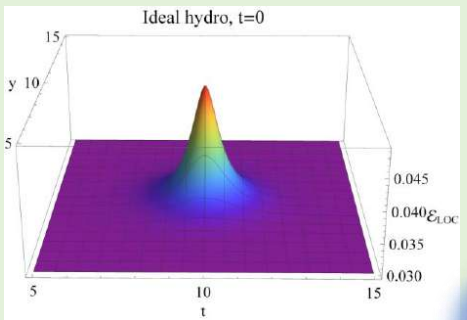


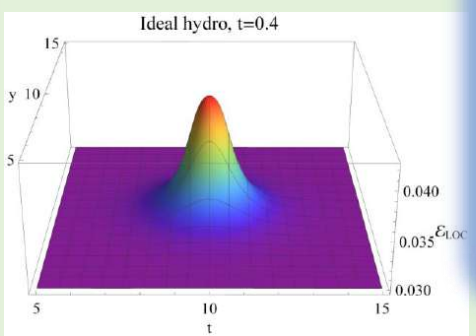


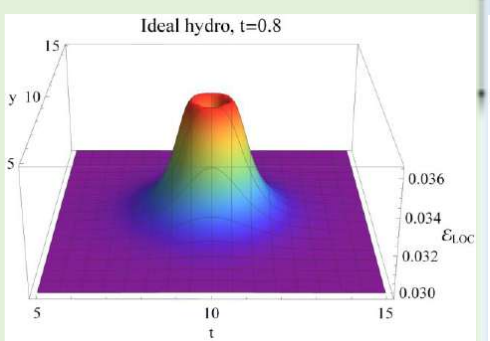
Time

BDNK

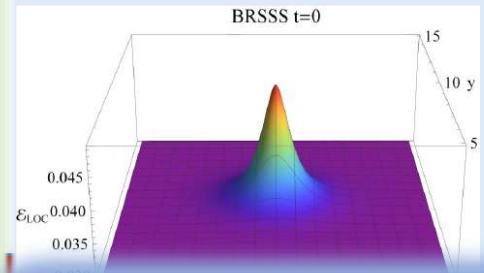
Ideal hydro evolution

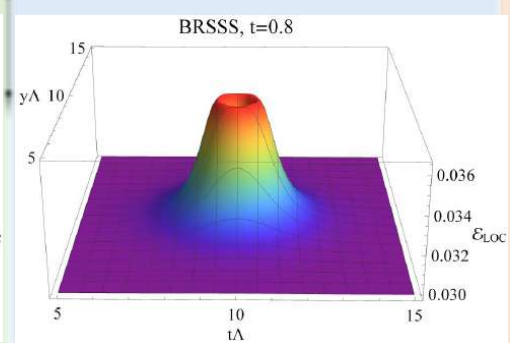




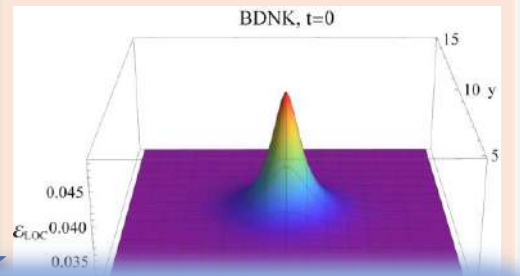


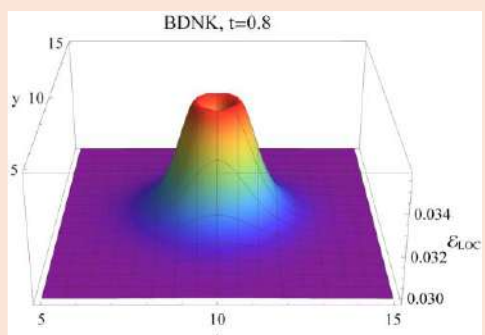
BRSSS (viscous) evolution



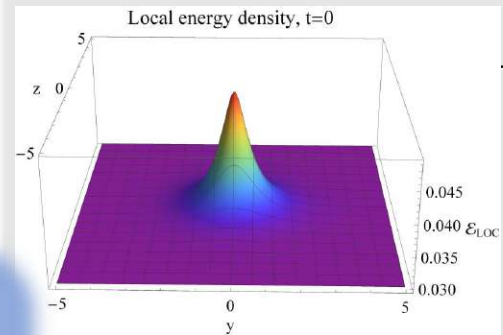


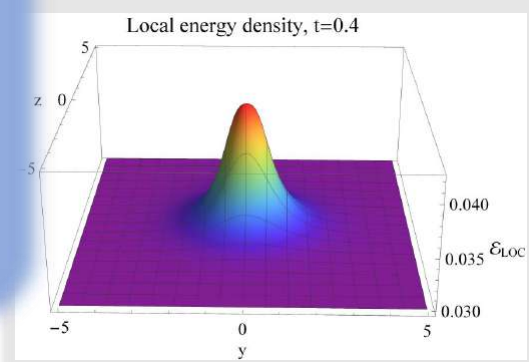
BDNK (viscous) evolution

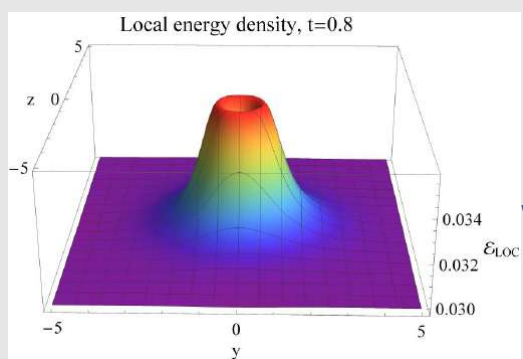




Microscopic evolution







Time

Interesting question:

- Causal theories of viscous hydro differ at UV: compare BRSSS with BDNK.

4 - Fluid/gravity: new examples?

Fluid/gravity

Relativistic hydrodynamic equations (D dim)

Non-linear map

Specific regime of Einstein equations with AdS asymptotics (D+1 dim)

Classical theory

FLUID/GRAVITY

Classical theory

- Found in the context of AdS/CFT, but independent statement.

Fluid/gravity

Relativistic hydrodynamic equations (D dim)

Non-linear map

Specific regime of Einstein equations with AdS asymptotics (D+1 dim)

Classical theory

FLUID/GRAVITY

Classical theory

- Found in the context of AdS/CFT, but independent statement.

- Natural question: Does the dual of a hydrodynamics solution has a regular horizon?

  ⟶ Under some assumptions, yes!

- Usual asumption: homogeneous black brane at asymptotically late times.

  ⟶ Our solution does not assume that final state.

Fluid/gravity

Relativistic hydrodynamic equations (D dim)

Non-linear map

Specific regime of Einstein equations with AdS asymptotics (D+1 dim)

Classical theory

FLUID/GRAVITY

Classical theory

- Found in the context of AdS/CFT, but independent statement.

- Natural question: Does the dual of a hydrodynamics solution has a regular horizon?

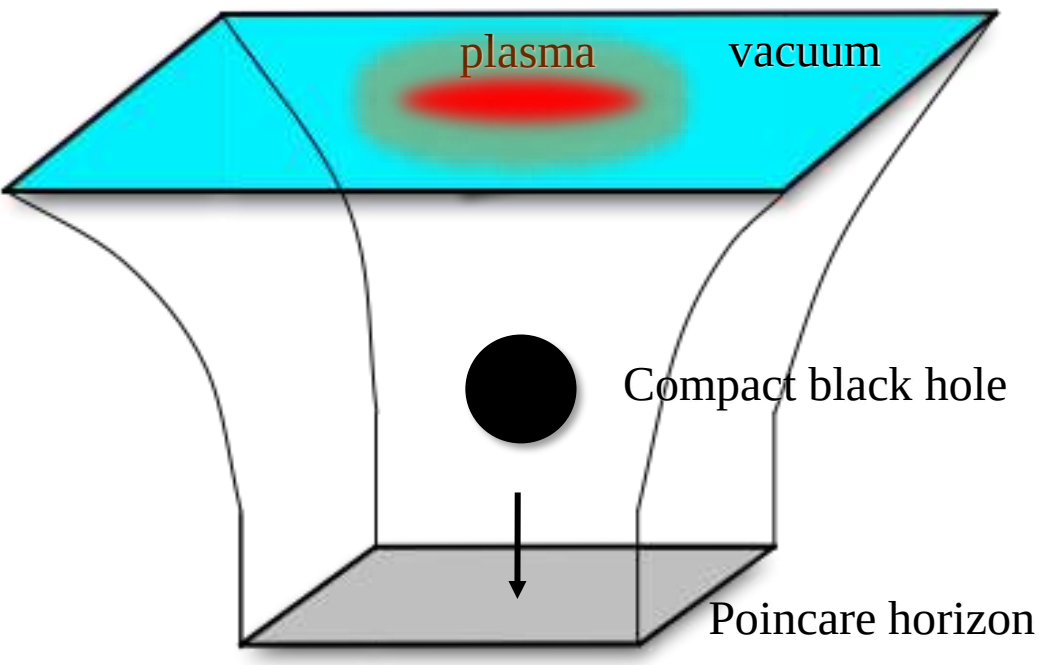
  ⟶ Under some assumptions, yes!

- Usual asumption: homogeneous black brane at asymptotically late times.

  ⟶ Our solution does not assume that final state.

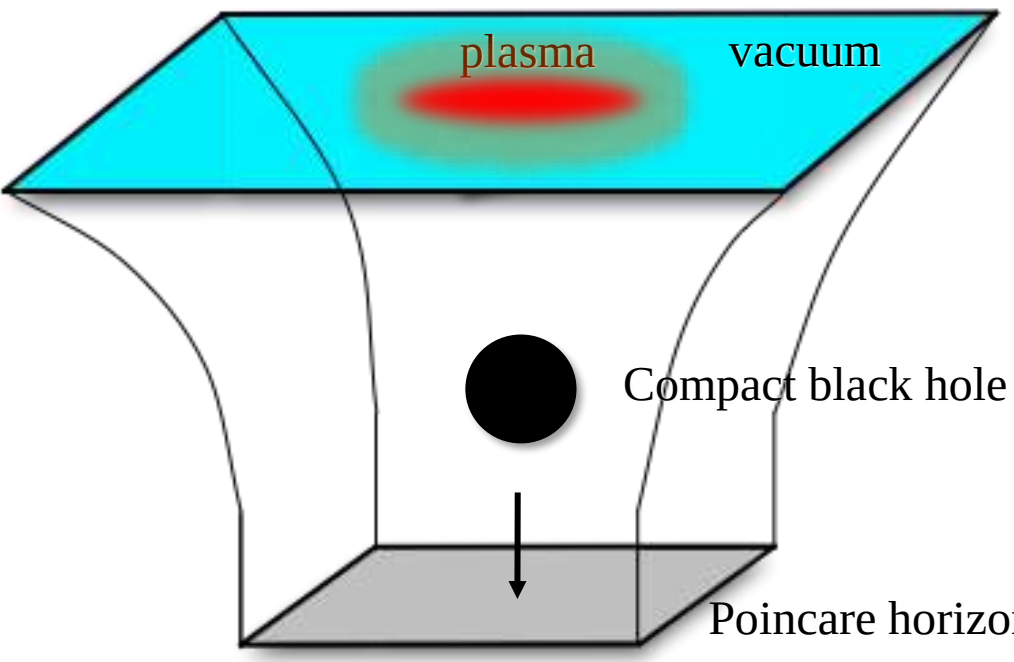
Does it correspond to a new example of fluid/gravity?

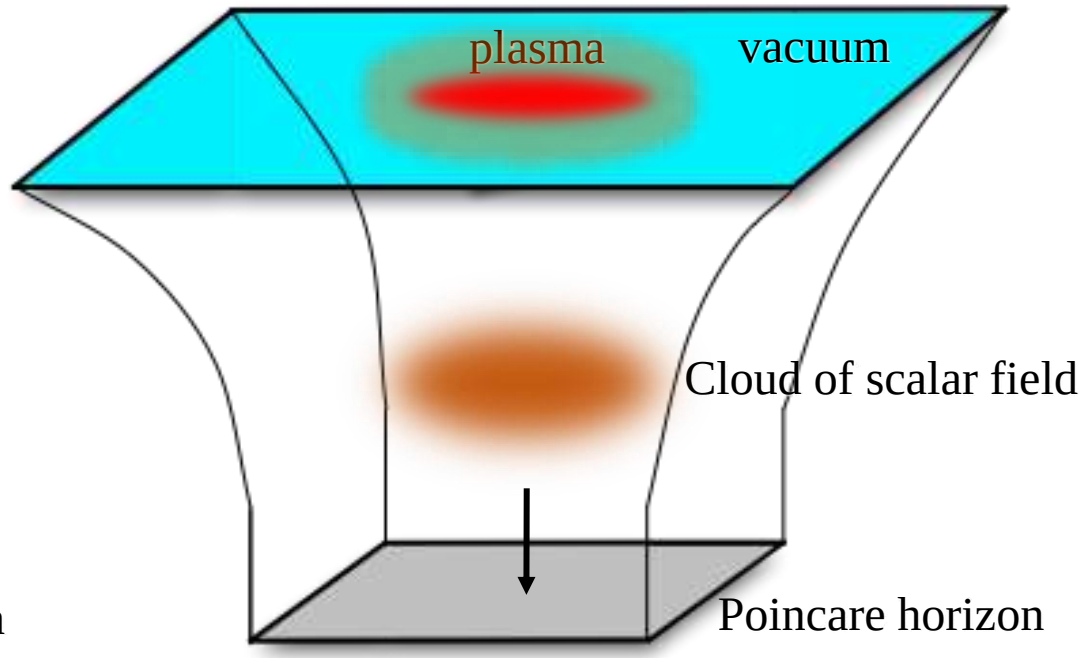
Fluid/gravity



Fluid/gravity

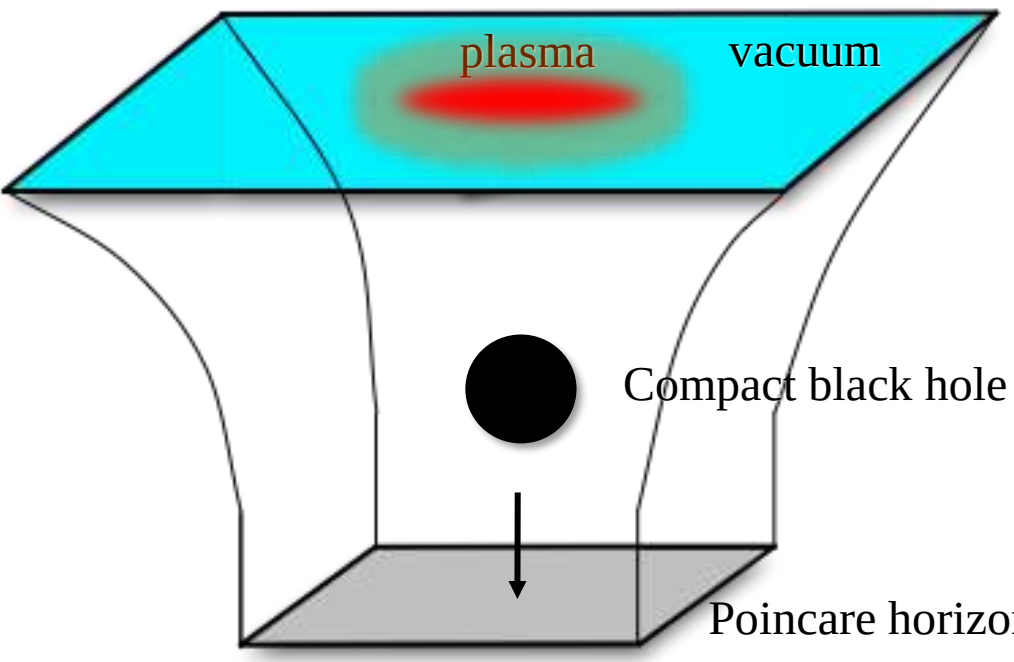
Another new solution

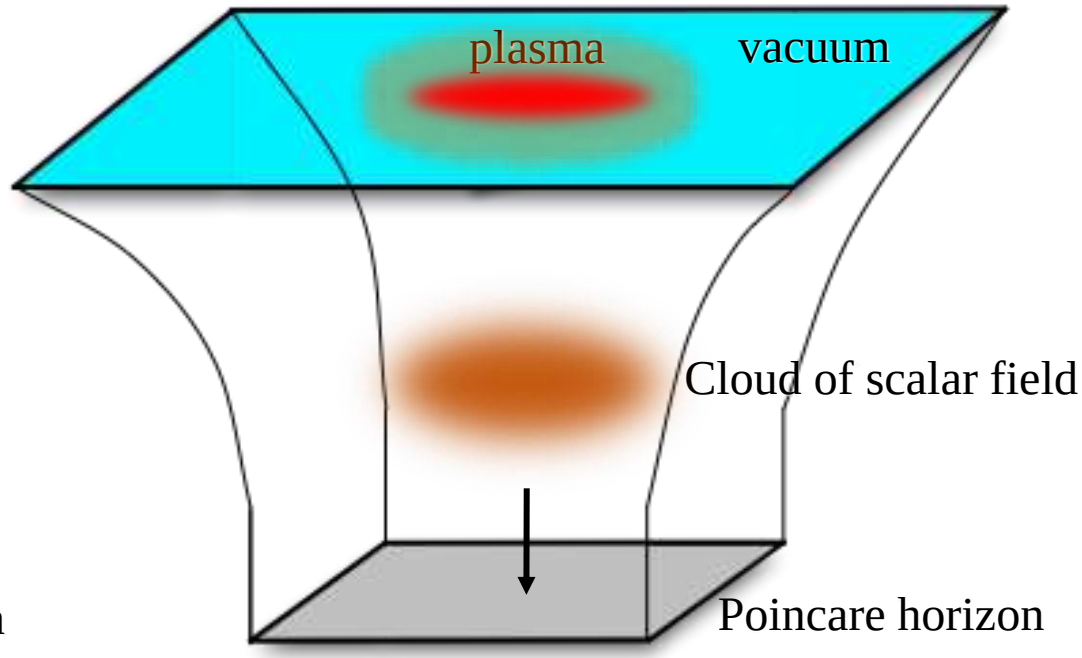




Fluid/gravity

Another new solution

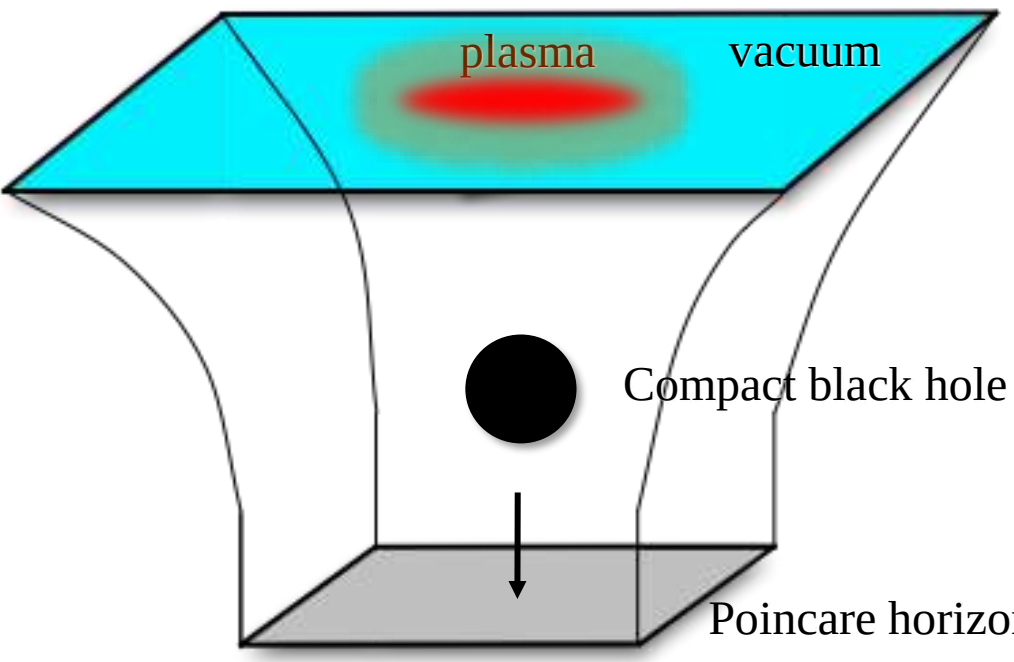


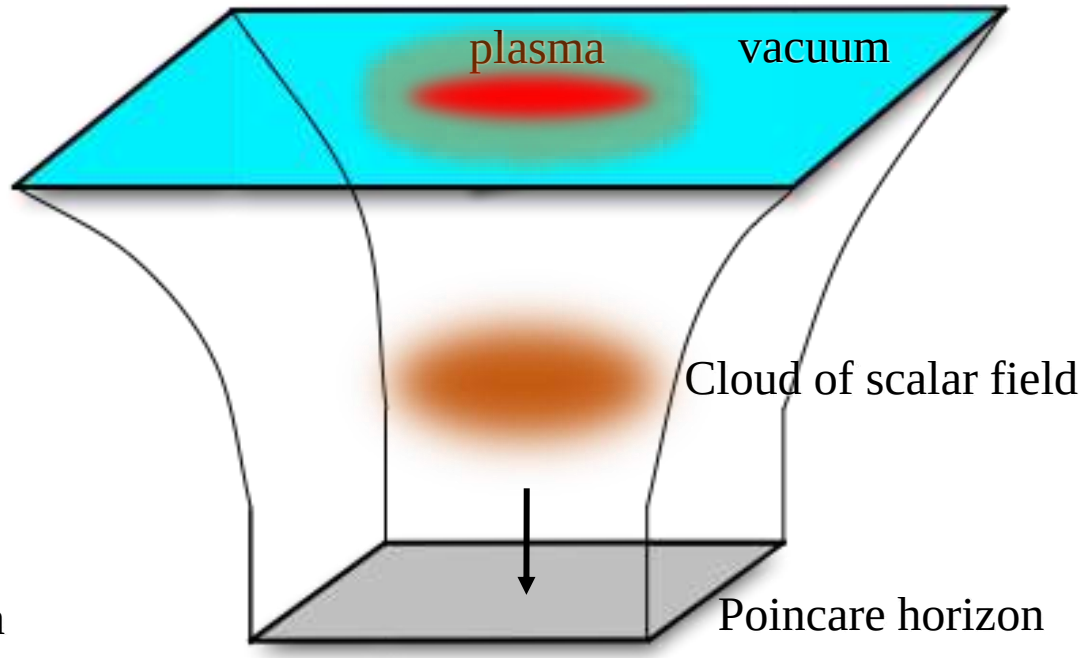


- No event horizon!
- Hydro applies at the center (same argument)

Fluid/gravity

Another new solution





- No event horizon!
- Hydro applies at the center (same argument)

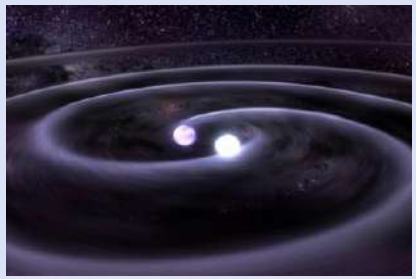
⟶ New examples of Fluid/Gravity?

Future directions

Future directions

Causal Hydrodynamics:

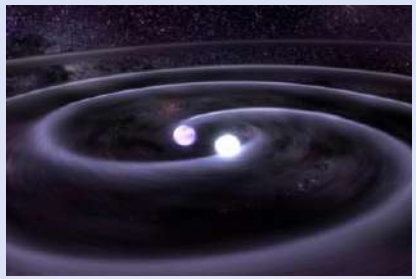
⟶ Magnetohydrodynamics (beyond weak coupling)

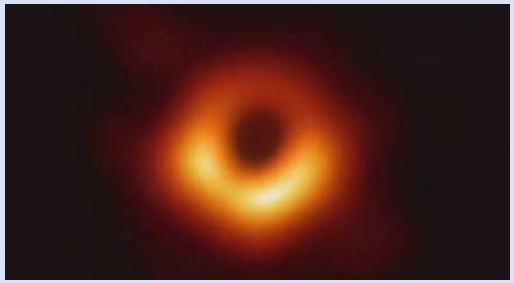


Future directions

Causal Hydrodynamics:

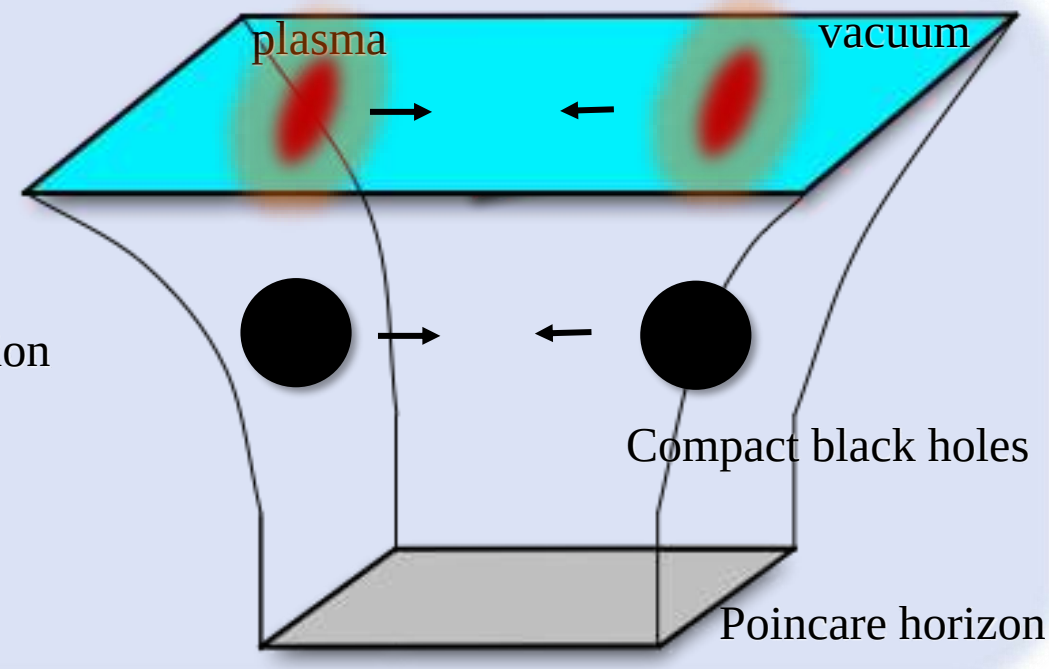
→ Magnetohydrodynamics (beyond weak coupling)





Holography

→ Holographic collisions: mimic heavy-ion collisions surrounded by vaccum



Thank you

Thank you

Hydrodynamics: constitutive relations

⟶ We find that it is described by hydro at the center, at all times.

- A bit counterintuitive: one may expect a significant deviation from hydro
- Symmetry argument? ⟶ Yes!

Rotational symmetry
Conformal symmetry
➡ (Ideal) hydro describes the center at all times

$$T^{\mu\nu}_{y=0,z=0} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$$

All higher order terms vanish!