Floquet
Higher-order
topological insulators

Yang Peng (Caltech/Northridge)
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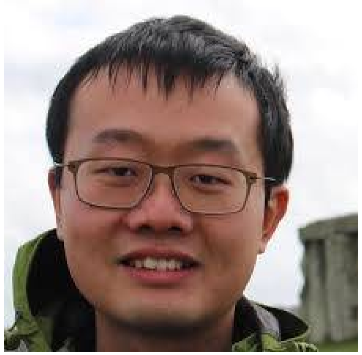
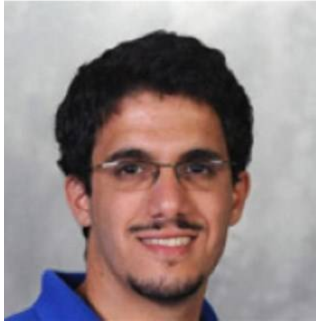
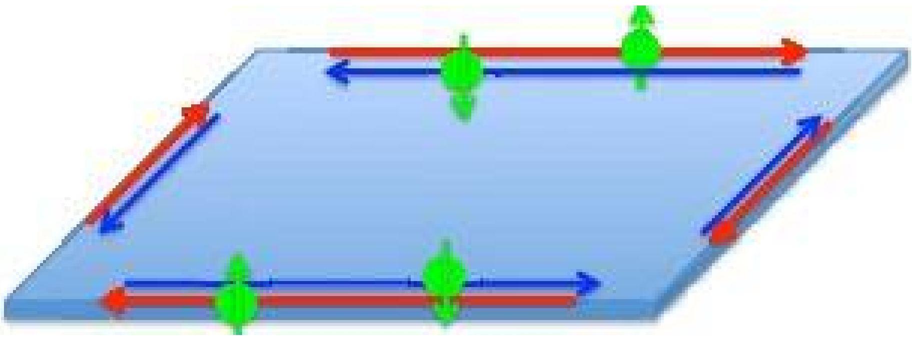
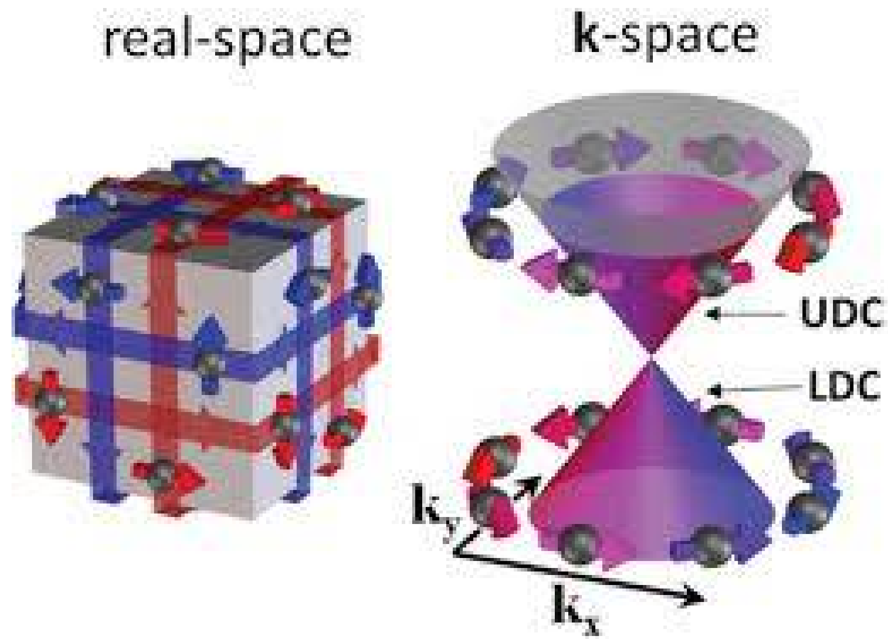




INSTITUTE FOR QUANTUM INFORMATION AND MATTER

Topological insulators

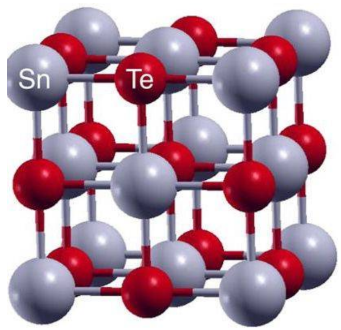
- *Ver. 1: Antiunitrary symmetries - 'Classical' TI's*



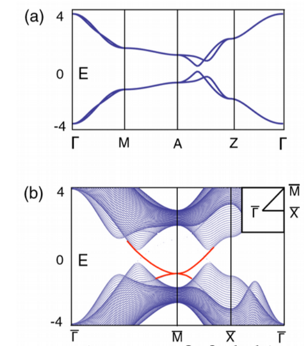


Topological insulators

- *Ver. 2: Additional unitary symmetries – Crystalline TI's*



C_4 ⇨

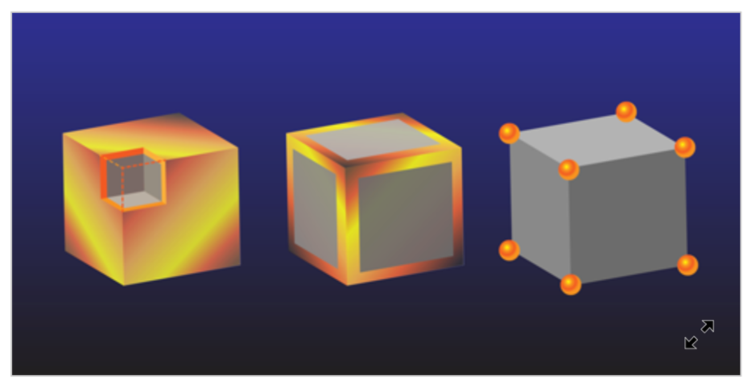


Fu (PRL,2011)

- *Ver. 3: Symmetries broken at surface – Higher-order TI's*

HOTI's

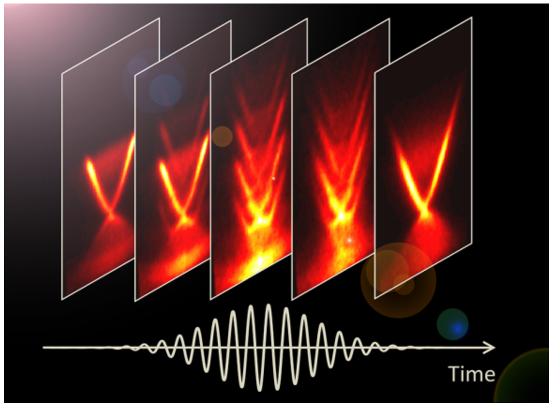
F. Schindler, A. M. Cook, M. G. Vergniory, Z. Wang, S. S. Parkin, B. A. Bernevig, and T. Neupert, arXiv:1708.03636.
W. A. Benalcazar, J. C. Y. Teo, and T. L. Hughes, Phys. Rev. B **89**, 224503 (2014).
W. A. Benalcazar, B. A. Bernevig, and T. L. Hughes, Science **357**, 61 (2017).
W. A. Benalcazar, B. A. Bernevig, and T. L. Hughes, Phys. Rev. B **96**, 245115 (2017).



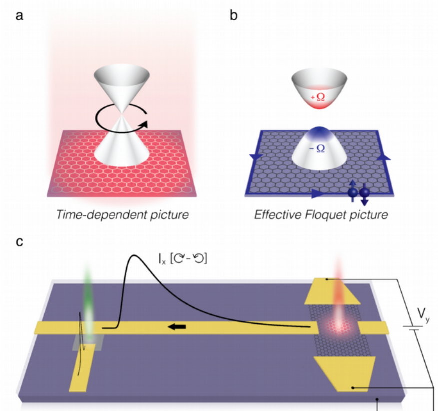
APS/Alan Stonebraker

Floquet Topological insulators

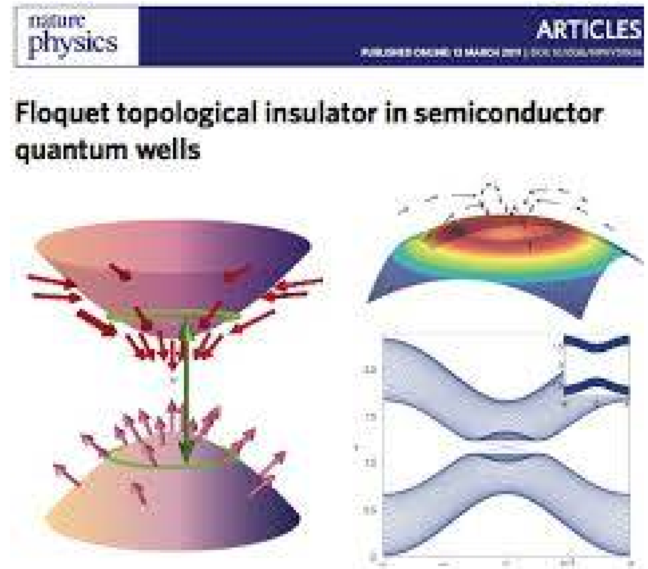
- *Ver. 1: 'Classical' TI's → Floquet TI's*



Gedik group



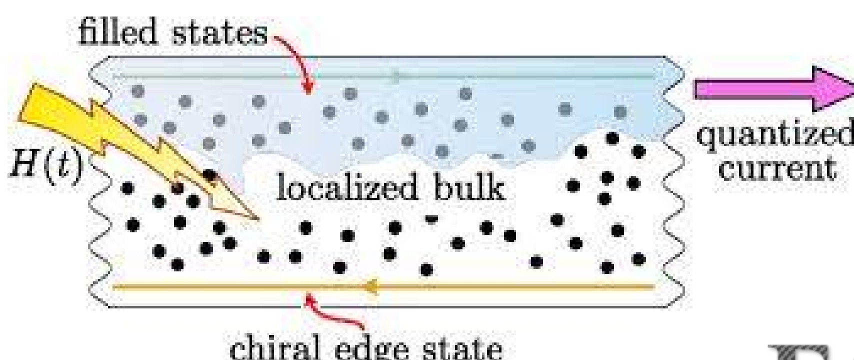
Cavalleri group


nature physics ARTICLES

Floquet topological insulator in semiconductor quantum wells

Lindner, Refael, Galitski (2011)
Aoki, Oka (2010)

- *Ver. 1.5: Disorder +chirality → Anomalous Anderson Floquet TI's (no static analog)*



AFAI

FAATI

Rudner, Lindner, Berg, Levin (2013)
Titum, Rudner, Lindner, Refael (2014)

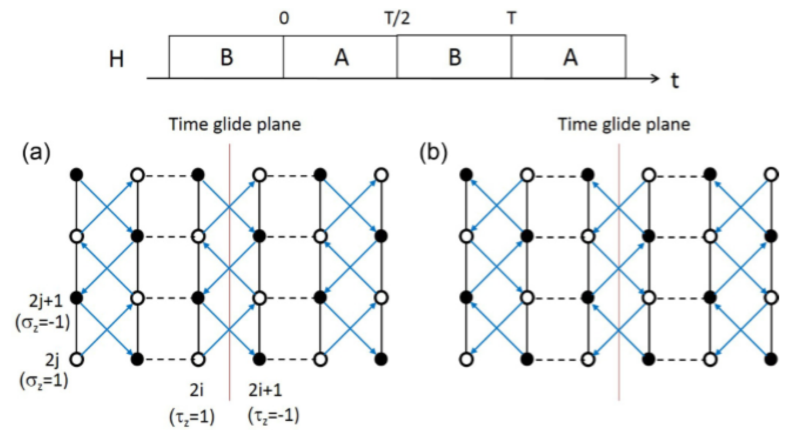
Floquet Topological insulators

- *Ver. 2: Unitary + time glide symmetry → Floquet crystalline TI*

PHYSICAL REVIEW B **95**, 195155 (2017)

Floquet topological phases protected by time glide symmetry

Takahiro Morimoto,[1] Hoi Chun Po,[1,2] and Ashvin Vishwanath[1,2]

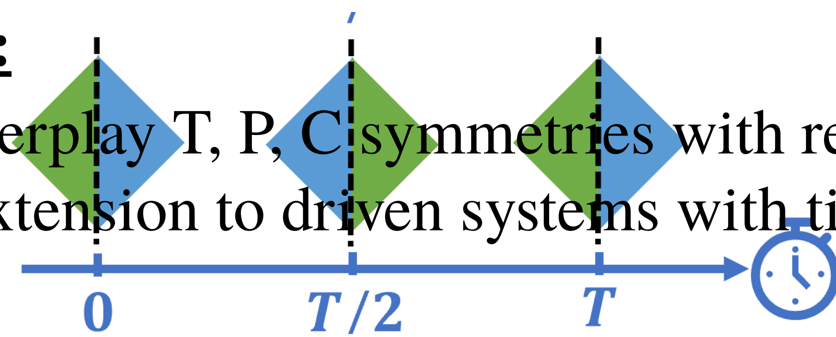


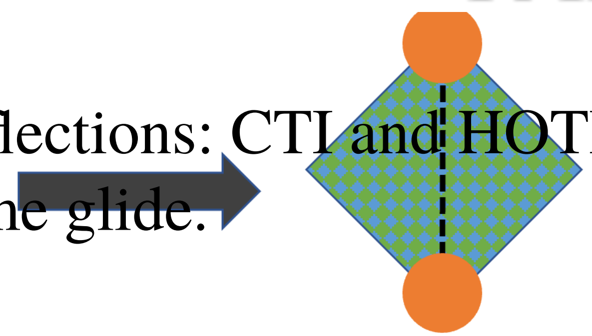
- *Ver. 3: Symmetries broken at surface + time glide – Higher-order Floquet TI's*

HOFTI
& SOFTI
FI-HOTI?

Plan:

- Inerplay T, P, C symmetries with reflections: CTI and HOTI.
- Extension to driven systems with time glide.





(Yang Peng, Refael, 2018)

From Crystalline to high-order TIs

- Start with chiral symmetry (class AIII):

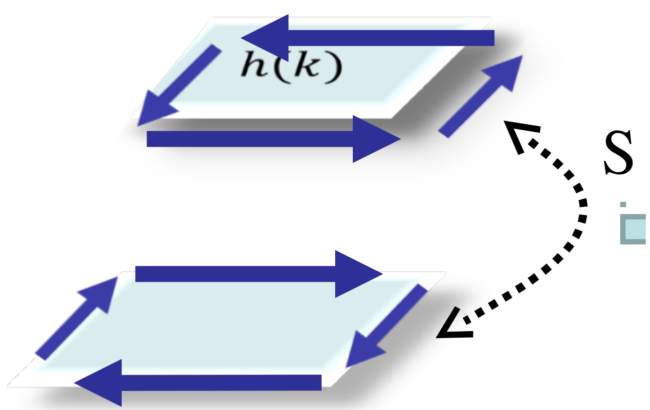
$$S\,H(k_x,k_y)\,S^{-1} = -H(k_x,k_y)$$

- Edge Hamiltonian:

$$H_{edge} = \sigma^z k_x \qquad and,\ e.g.,\ S = \sigma^x \quad \Rightarrow \quad \{H_{edge}, S\} = 0$$

- Mass term? Possible:

$$V_m = m\sigma^y,\ \{V_m, S\} = 0$$ Induces a gap...



ivial

Protection by reflection

- Start with chiral symmetry (class AIII):

$$S\,H(k_x,k_y)\,S^{-1} = -H(k_x,k_y)$$

$$H_{edge} = \sigma^z k_x \qquad S = \sigma^x$$

$$\{H_{edge}, S\} = 0$$

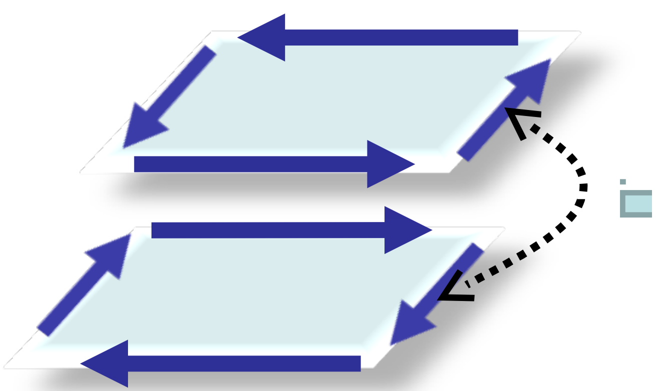
- Reflection:

$$U_R\,H(k_x,k_y)\,U_R^{+} = R\,H(-k_x,k_y)\,R^{+}$$

Must have: $\{R, H_{edge}\} = 0$, Assume: *[R,S]=0* ⇨ $R = \sigma^x$

- Mass term? Impossible!

$$\{V_m, S\} = 0,\ and\ [V_m, R] = 0 \qquad \text{But } R=S\,!$$



Protection by reflection

- Start with chiral symmetry (class AIII):

$$S\,H(k_x,k_y)\,S^{-1} = -H(k_x,k_y)$$

$$H_{edge} = \sigma^z k_x + V_m \qquad S = \sigma^x$$

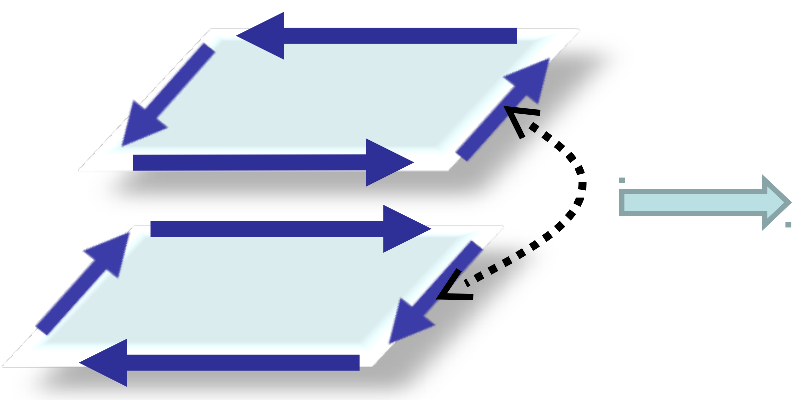
$$\{H_{edge}, S\} = 0$$

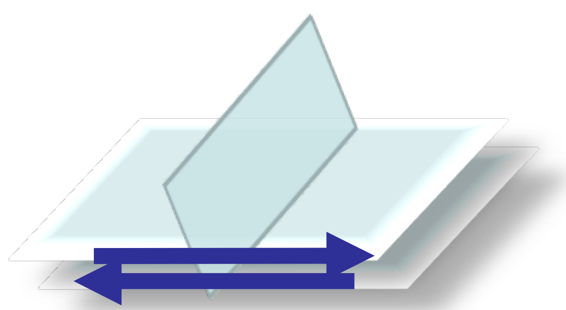
- Reflection:

$$U_R\,H(k_x,k_y)\,U_R^{+} = R\,H(-k_x,k_y)\,R^{+}$$

More generally, assume: *[R,S]=0* and diagonalize *RS*

$\{V_m, R\,S\} = 0$ ⇨ *Gaps (mixes) Opposite RS eigenvalue states*





Tr(*SR*)/2 *helical modes*

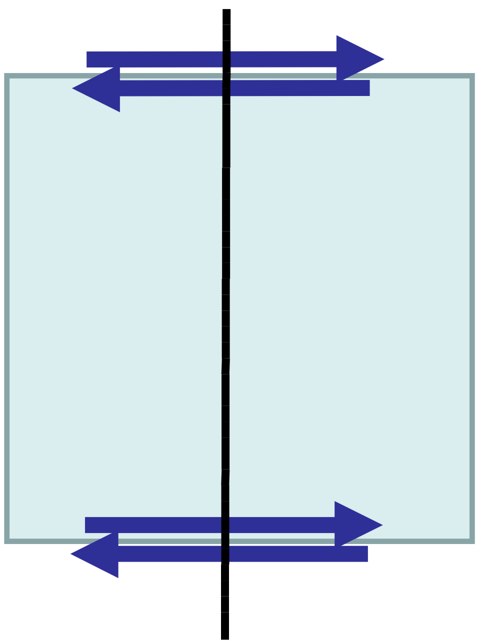
Higher-order TI: CTI's pushed to the corner

(Langbehn, Trifunovic, Peng, von Oppen, Brouwer, 2017)

$$H_{edge}(k_x) = k_x \Gamma_x$$

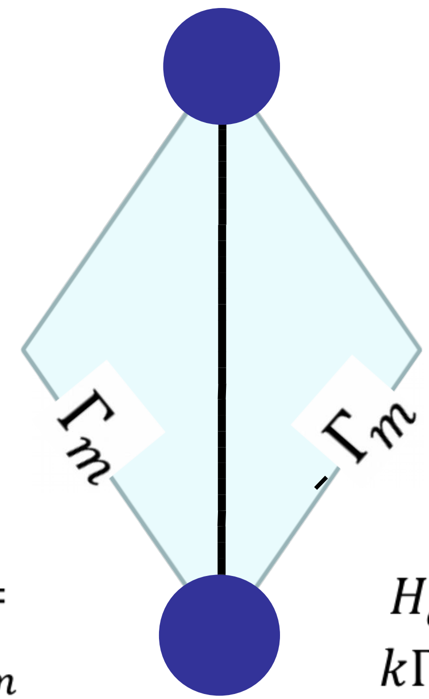
Γ_m respects only the chiral symmetry: $\{\hat{\mathcal{S}}, \Gamma_m\} = 0$

but *also*: $\{\hat{\mathcal{R}}, \Gamma_m\} = 0$



$$H_{edge}(k_x) = k_x \Gamma_x$$

$H^L_{edge} =$
$k\Gamma_x + \Gamma_m$



$H^R_{edge} =$
$k\Gamma_x - \Gamma_m$

Berlin style HOTI Classification

(Langbehn, Trifunovic, Peng, von Oppen, Brouwer, 2017)

$$\hat{R}\hat{T} = \eta_T \hat{T}\hat{R} \qquad \hat{R}\hat{P} = \eta_P \hat{P}\hat{R} \qquad \hat{C}\hat{R} = \eta_C \hat{R}\hat{C}$$

Cartan	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$	$d = 2$		$d = 3$	
A	0	0	0	0	$\cdots$	$\mathbb{Z}$	$\mathcal{R}$
AIII	0	0	1	$\mathbb{Z}$	$\mathcal{R}_+$	0	$\cdots$
AI	1	0	0	0	$\cdots$	0	$\cdots$
BDI	1	1	1	$\mathbb{Z}$	$\mathcal{R}_{++}$	0	$\cdots$
D	0	1	0	$\mathbb{Z}_2$	$\mathcal{R}_+$	$\mathbb{Z}$	$\mathcal{R}_+$
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathcal{R}_{++}, \mathcal{R}_{--}\ \mathcal{R}_{-+}$	$\mathbb{Z}_2$	$\mathcal{R}_{++}, \mathcal{R}_{-+}$
AII	−1	0	0	0	$\cdots$	$\mathbb{Z}_2$	$\mathcal{R}_+, \mathcal{R}_-$
CII	−1	−1	1	$\mathbb{Z}$	$\mathcal{R}_{++}, \mathcal{R}_{--}$	0	$\cdots$
C	0	−1	0	0	$\cdots$	$\mathbb{Z}$	$\mathcal{R}_+, \mathcal{R}_-$
CI	1	−1	1	0	$\cdots$	0	$\cdots$

Floquet HOTI'

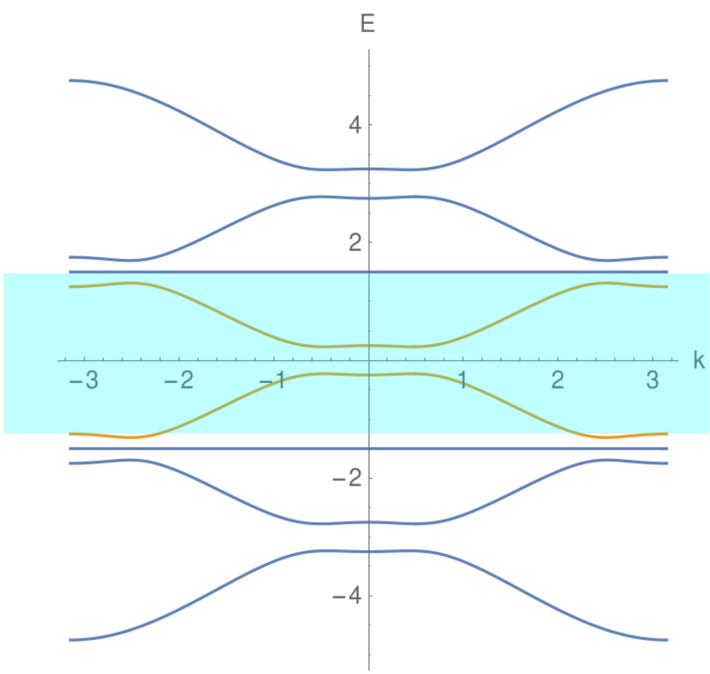
- Add periodic time dependence:

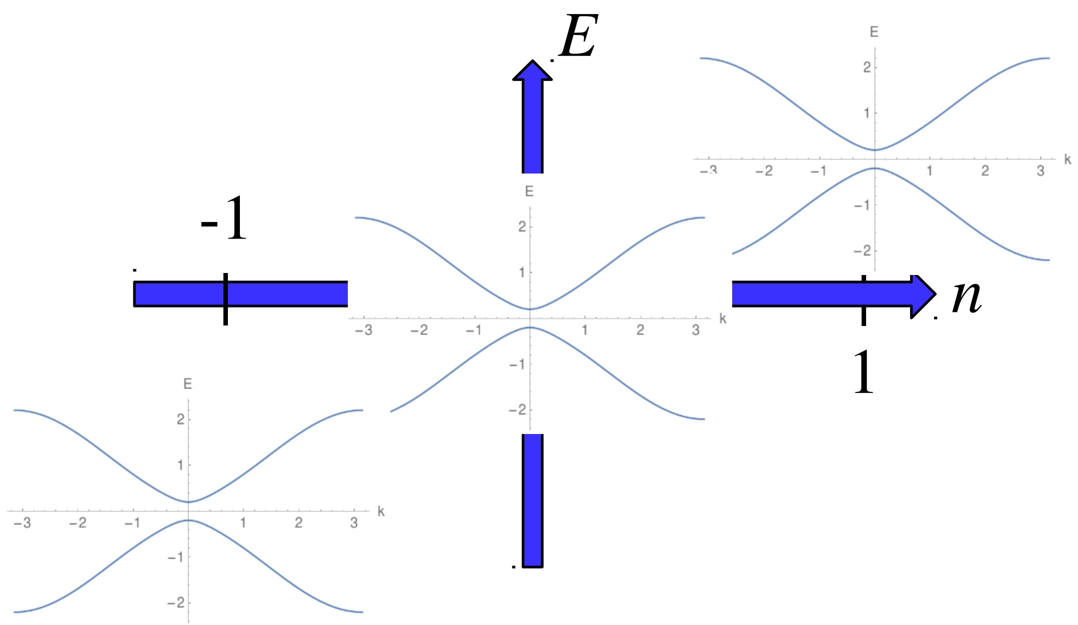
$$H = \hat{H}_0 + \hat{V}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

- Floquet states:

$$\psi_\varepsilon(t) = e^{-i\varepsilon t}\left[\psi_0 + \sum_{|n|>0}\psi_n e^{i\omega n t}\right]$$

Fast drive regime





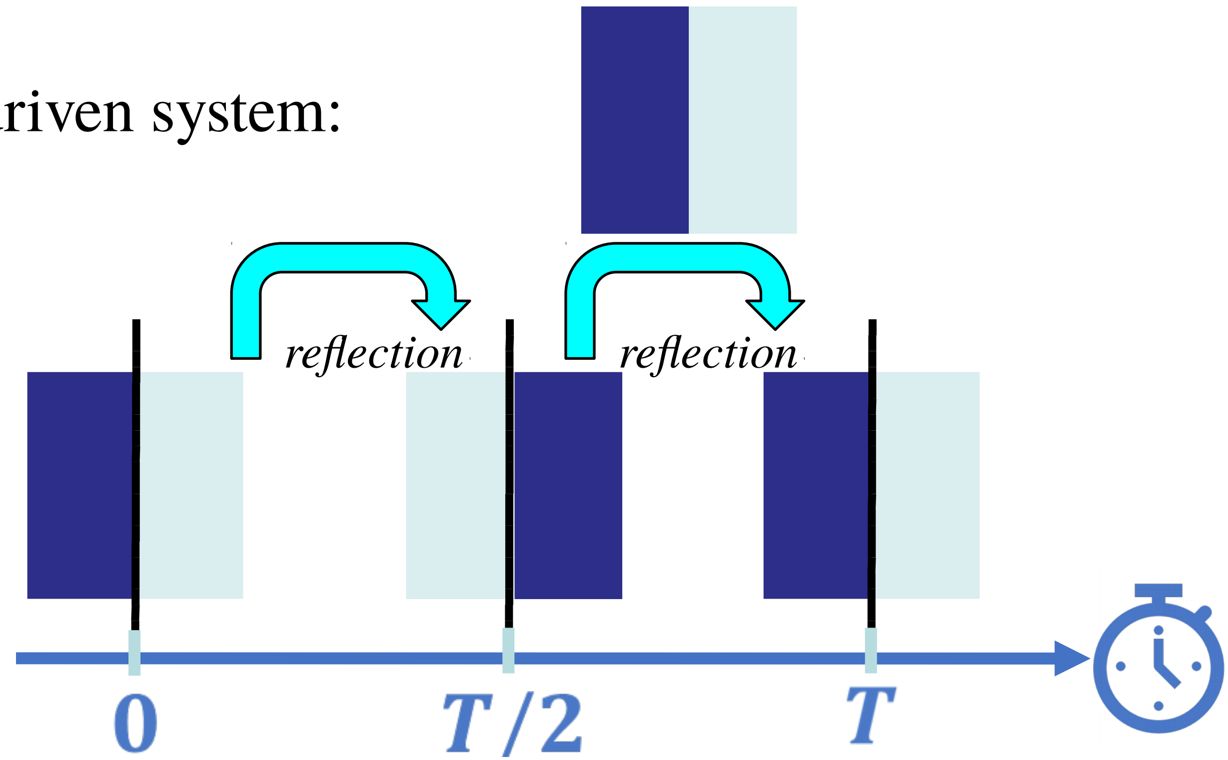
Time-glide Floquet Crytalline TI's

- Reflection-time-glide symmetries: (Morimoto, Po, Vishwanath, 2017)

$$U_{TG} H(k_x, k_y, t) U_{TG}^{+} = M H_{edge}(-k_x, k_y, t + T/2) M^{+}$$

Periodic driven system:

Time-glide



Time-glide Floquet Crytalline TI's

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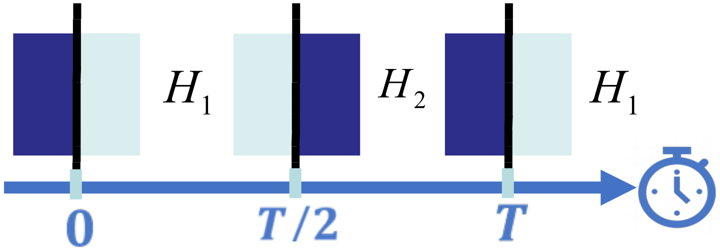
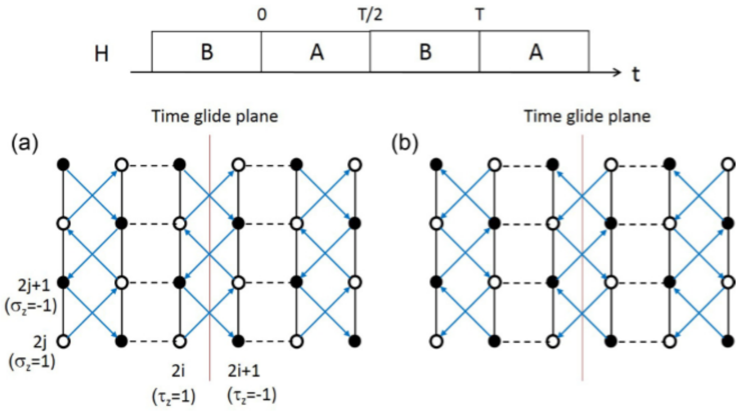
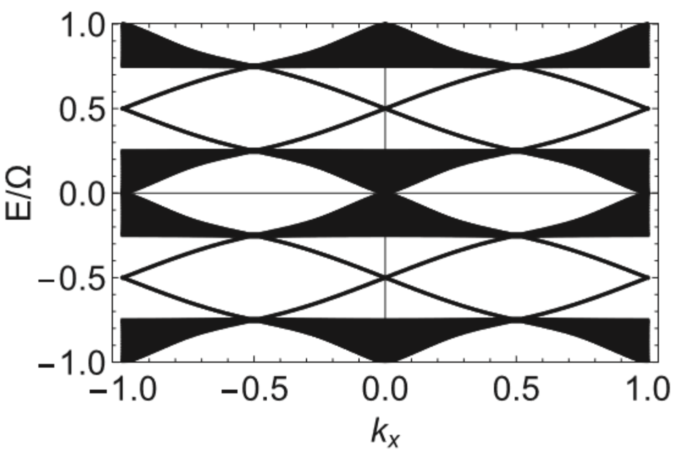
$$U_{TG} H(k_x, k_y, t) U_{TG}^{+} = M H_{edge}(-k_x, k_y, t + T/2) M^{+}$$

- Stroboscopic Models:







Odd resonance

- What about high-order Floquet TI's?

Higher-order Floquet Topological Insulators (Peng, Refael, 2018)

- Additional reflection and time glide symmetries

$$U_{TG}\,H(k_x,k_y,t)U_{TG}^{+} = M\,H_{edge}(-k_x,k_y,t+T/2)M^{+}$$

- Simple Floquet Hamiltonian:

$$H = \hat{H}_0 + \hat{V}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$H_F = \begin{pmatrix} \cdots & V & & & & \\ V^{+} & H_0-\omega & V & & & \\ & V^{+} & H_0 & V & & \\ & & V^{+} & H_0+\omega & V & \\ & & & V^{+} & H_0+2\omega & V \\ & & & & V^{+} & \cdots \end{pmatrix}$$

- Concentrate on odd resonances.

$$\psi(t) = e^{i\omega t/2}\left[\psi_0 + \sum_{|n|>0}\psi_n e^{i\omega n t}\right]$$

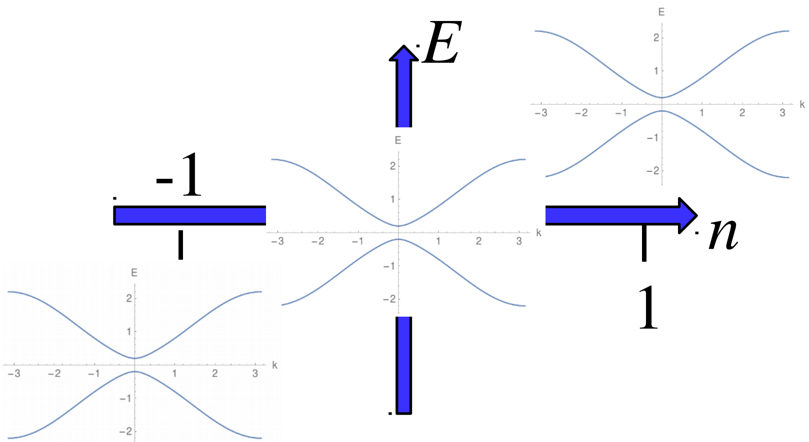
$$\psi_{(t)} = -\psi_{(t+T)}$$

Higher-order Floquet Topological Insulators (Peng, Refael, 2018)

- Additional reflection and time glide symmetries

$$U_{TG}\,H(k_x,k_y,t)\,U_{TG}^{+}=M\,H_{edge}(-k_x,k_y,t+T/2)M^{+}$$

- Simple Floquet Hamiltonian:



$$H_F=\begin{pmatrix} \ddots & V & & & & \\ V^{+} & H_0-\omega & V & & & \\ & V^{+} & H_0 & V & & \\ & & V^{+} & H_0+\omega & V & \\ & & & V^{+} & H_0+2\omega & V \\ & & & & V^{+} & \ddots \end{pmatrix}$$

- Concentrate on odd resonances.

$$\psi(t)=e^{i\omega t/2}\left(\psi_0+\sum_{|n|>0}\psi_n e^{i\omega nt}\right)=\psi_0 e^{i\omega t/2}+\psi_1 e^{-i\omega t/2}$$

Higher-order Floquet Topological Insulators (Peng, Refael, 2018)

- Additional reflection and time glide symmetries

$$U_{TG} H(k_x, k_y, t) U_{TG}^{+} = M H_{edge}(-k_x, k_y, t + T/2) M^{+}$$

- Effective Floquet Hamiltonian:

$$H_F^{eff} = \begin{pmatrix} H_0 & V \\ V^{+} & H_0 + \omega \end{pmatrix} = (H_0 + \omega/2) \cdot 1 + \hat{\rho}_z \omega/2 + \mathrm{Re} V \hat{\rho}_x + \mathrm{Im} V \hat{\rho}_y$$

$$H_F^{eff}(k_x, k_y) \to (\hat{\rho}_z \otimes M) H_F^{eff}(-k_x, k_y) (\hat{\rho}_z \otimes M)^{-1}$$

- Time glide:

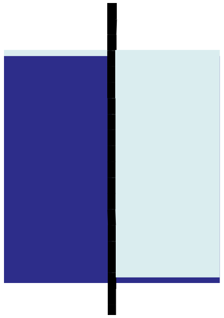
$$V_{(k_x, k_y)} \to -M V_{(-k_x, k_y)} M^{-1}$$

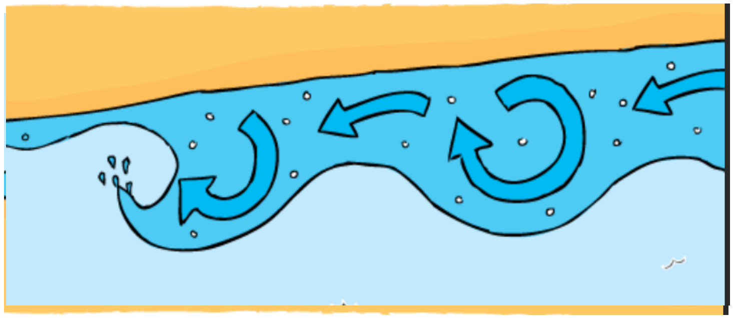
$$H_{0(k_x, k_y)} \to M H_{0(-k_x, k_y)} M^{-1}$$

$$\exp(i\omega t + i\omega T/2) = \exp(i\omega t + i\pi) = -\exp(i\omega t)$$

- New reflection operator:

$$R^{eff} = \begin{pmatrix} M & \\ & -M \end{pmatrix}$$





Higher-order Floquet Topological Insulators (Peng, Refael, 2018)

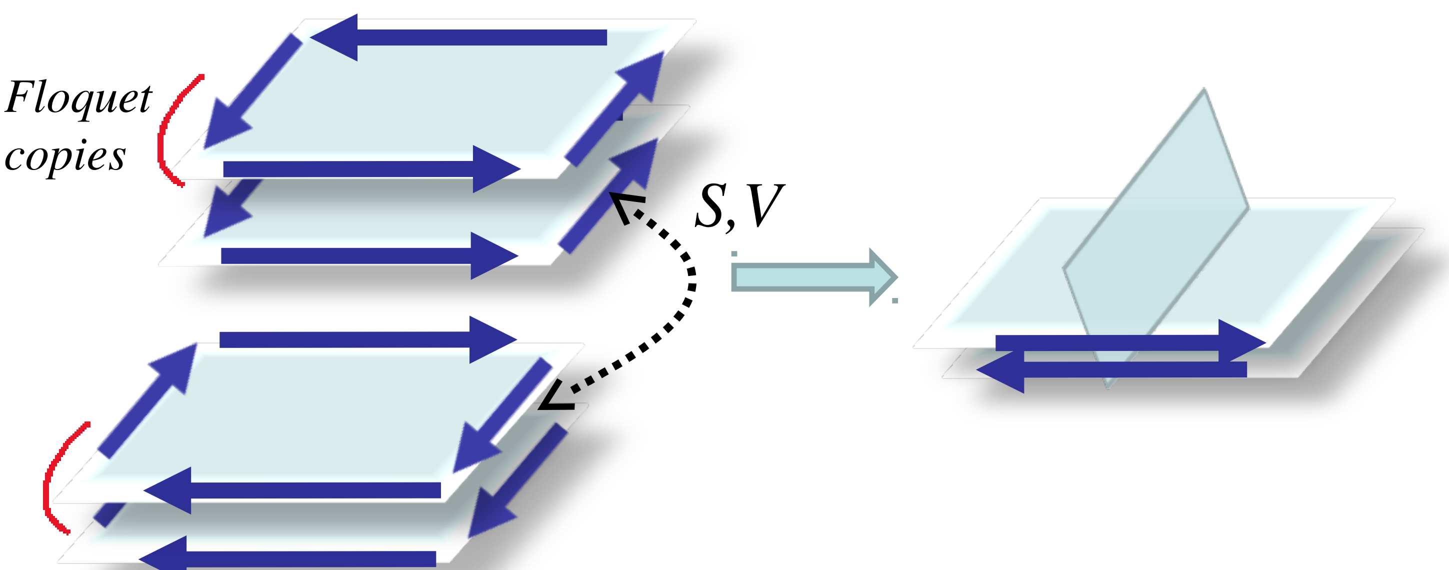
- Class AIII example:

$$=(H_0+\omega/2)\cdot 1+\hat{\rho}_z\,\omega/2+\mathrm{Re}\,V\hat{\rho}_x+\mathrm{Im}\,V\,\hat{\rho}_y$$

$$S^{eff}=\begin{pmatrix} & -iS \\ iS & \end{pmatrix}=S\cdot\hat{\rho}_y \qquad R^{eff}=\begin{pmatrix} M & \\ & -M \end{pmatrix}=M\cdot\hat{\rho}_z$$

$[t\rightarrow -t,\ \omega\rightarrow-\omega$ and $V\rightarrow V^{\dagger}]$

- Need: $[R,S]=0 \Rightarrow \{M,S\}=0$



Explicit AIII model

- Hamiltonian and symmetries:

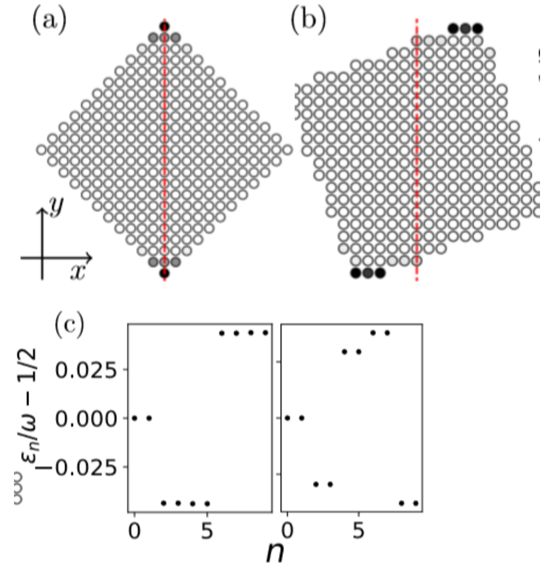
$$H_0 = (m - \cos k_x - \cos k_y)\tau^z + b\sigma^z$$

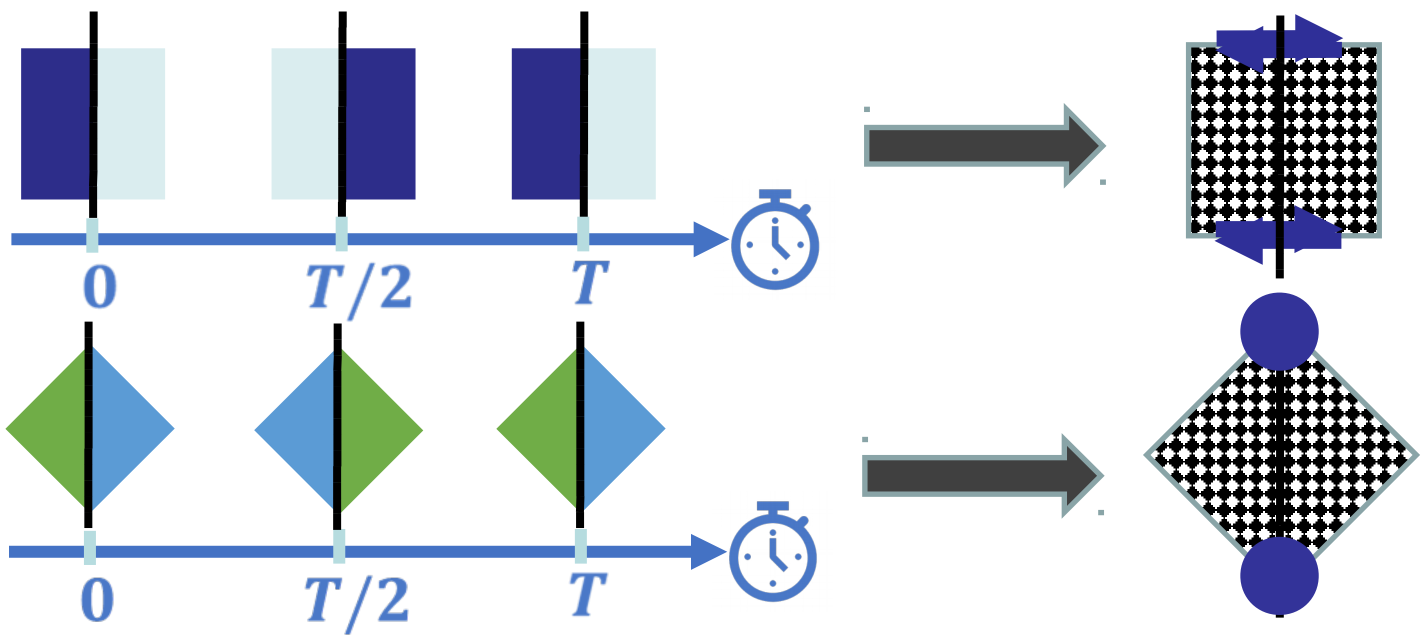
$$V(t) = \exp(i\omega t)\left(-i\sin k_x + \sigma^y \sin k_y\right)\tau^- + HC$$

$$S = \tau^x \sigma^x$$

$$M = \sigma^z$$

$\{S,M\}=0$ as needed





SOFT-SC

- Static Hamiltonian: (Class D/BDI)

$$H_0(\mathbf{k}) = (m_0 - 2t_0 \cos k_x - 2t_0 \cos k_y)\tau_z + \Delta\tau_x + b\sigma_x$$

$$M = \sigma_x$$

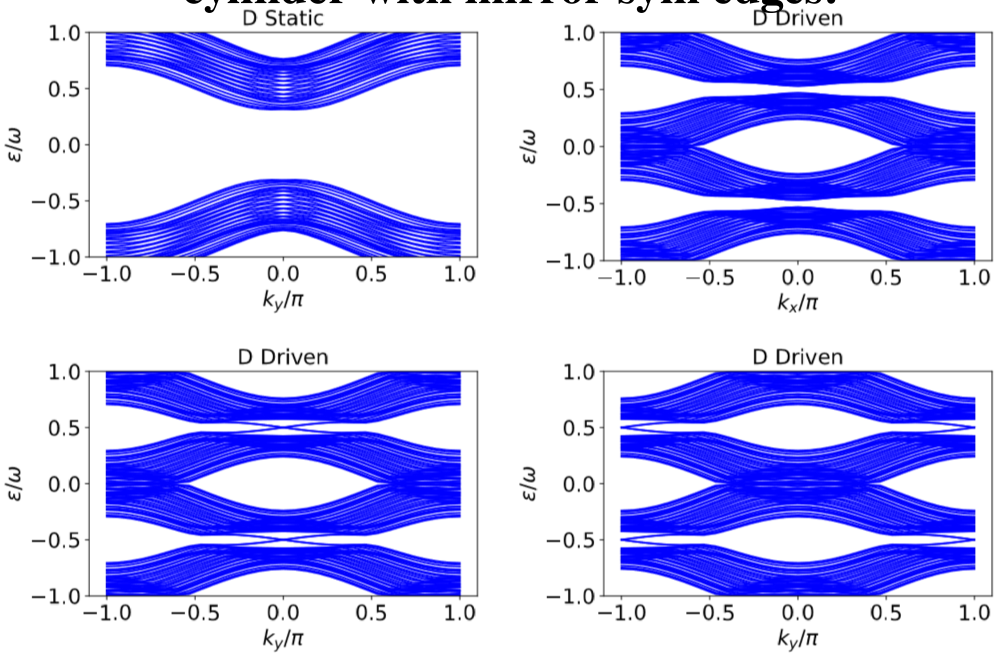
$$C = \tau_y\sigma_y$$

- Time-glide reflection (y) symmetric drive:
  Oscillating Rashba terms

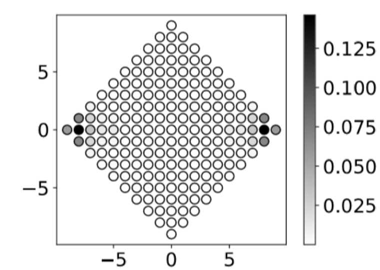
$$H(\mathbf{k}, t) = 2\alpha_0 \cos\omega t(\sin k_x\sigma_y - \sin k_y\sigma_x)\tau_z.$$

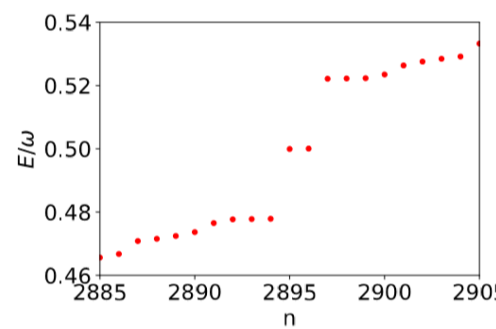
- Exact diagonalization:

cylinder with mirror sym edges:



non-mirror sym edges:





$$m_0 = \omega/2 + 1,\ \ \Delta = 0.9,\ \ \omega = 4.8,\ \ L = 15,\ \ b = 0.15$$

SOFT-SC – Realization with phonons

- Static Hamiltonian: (Class D/BDI)

$$H_0(\mathbf{k}) = (m_0 - 2t_0 \cos k_x - 2t_0 \cos k_y)\tau_z + \Delta\tau_x + b\sigma_x$$

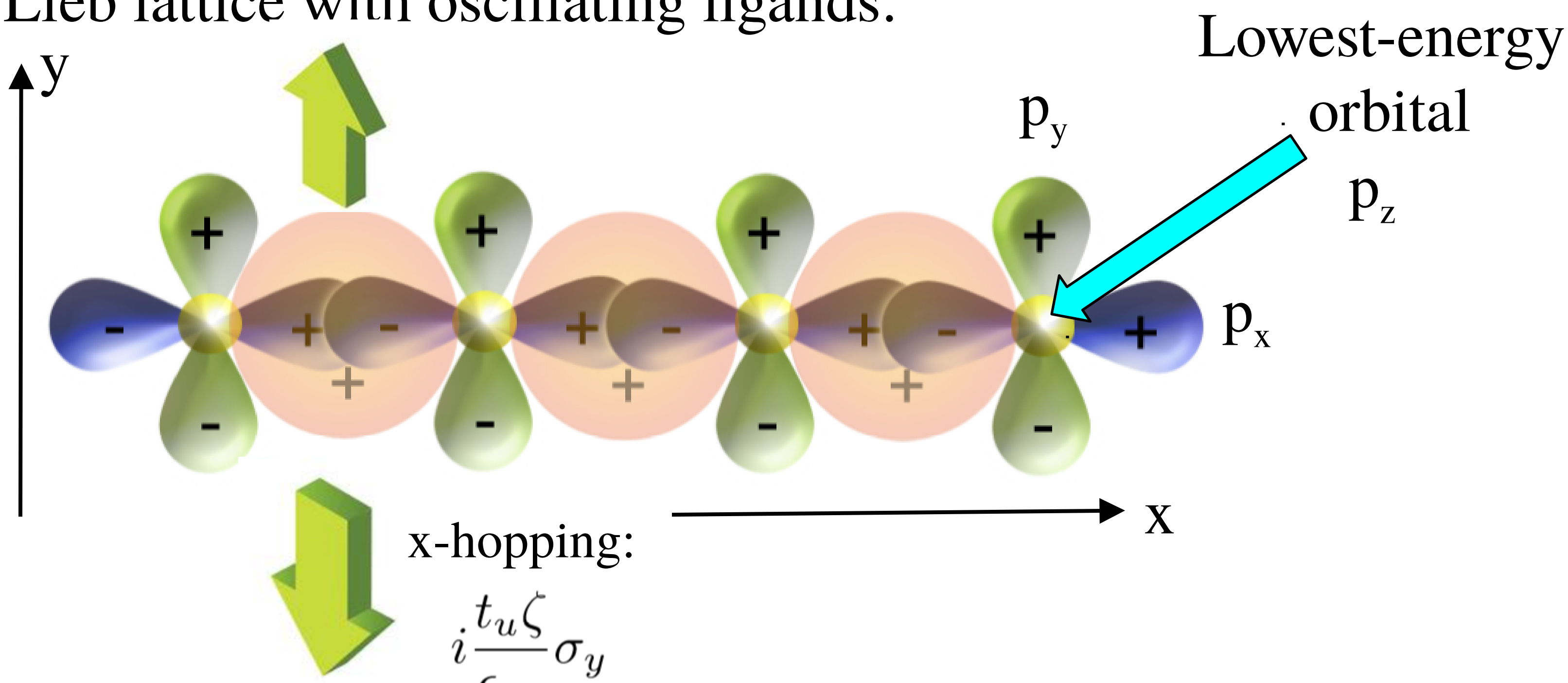
$$M = \sigma_x$$

$$C = \tau_y \sigma_y$$

- Time-glide reflection symmetric drive:
  Oscillating Rashba terms

$$H(\mathbf{k}, t) = 2\alpha_0 \cos \omega t (\sin k_x \sigma_y - \sin k_y \sigma_x)\tau_z.$$

- Lieb lattice with oscillating ligands:



x-hopping:

$$i\frac{t_u \zeta}{\epsilon_{zx}}\sigma_y$$

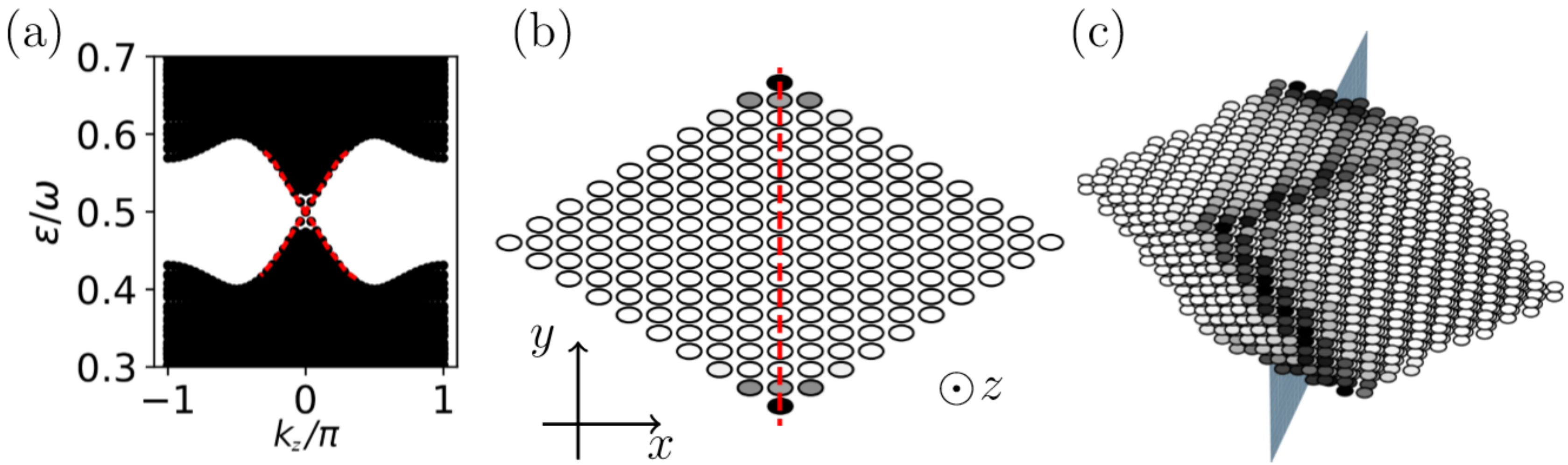
Explicit class A model in 3D

- Hamiltonian and symmetries:

$$H_0 = (m - \cos k_x - \cos k_y - \cos k_z)\tau^z + b\sigma^x$$

$$V(t) = \exp(i\omega t)\left(\sigma^x \sin k_x + \sigma^y \sin k_y + \sigma^z \sin k_z\right)\tau^- + HC$$

$$M = \sigma^x$$



From AIII to All

$$H_F = (H_0 + \omega/2)\cdot 1 + \hat{\rho}_z\,\omega/2 + \mathrm{Re}V\hat{\rho}_x + \mathrm{Im}V\,\hat{\rho}_y$$

T | **P** | **C** | **R**

T

$t \to -t, \mathrm{i} \to -\mathrm{i}$

$T_{eff} = T \cdot \rho_0$

P

$\mathrm{i} \to -\mathrm{i}$

$C_{eff} = C \cdot \rho_x$

C

$S_{eff} = T_{eff} \cdot C_{eff}$

$= T \cdot C \cdot \rho_x$

R

$R_{eff} = M \cdot \hat{\rho}_z$

$$\hat{R}\hat{T} = \eta_T \hat{T}\hat{R} \qquad \hat{R}\hat{P} = \eta_P \hat{P}\hat{R} \qquad \hat{C}\hat{R} = \eta_C \hat{R}\hat{C}$$

HOTI

η_T, η_P, η_C

Floquet HOTI

η_T, -η_P, $-\eta_C$

From AIII to All

HOTI

η_T, η_P, η_C

Cartan	$\mathcal{T}$	$\mathcal{P}$	$\mathcal{C}$		$d=2$		$d=3$
A	0	0	0	0	$\cdots$	$\mathbb{Z}$	$\mathcal{R}$
AIII	0	0	1	$\mathbb{Z}$	$\mathcal{R}_+$	0	$\cdots$
AI	1	0	0	0	$\cdots$	0	$\cdots$
BDI	1	1	1	$\mathbb{Z}$	$\mathcal{R}_{++}$	0	$\cdots$
D	0	1	0	$\mathbb{Z}_2$	$\mathcal{R}_+$	$\mathbb{Z}$	$\mathcal{R}_+$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathcal{R}_{++}$, $\mathcal{R}_{--}$ $\mathcal{R}_{-+}$	$\mathbb{Z}_2$	$\mathcal{R}_{++}$, $\mathcal{R}_{-+}$
AII	-1	0	0	0	$\cdots$	$\mathbb{Z}_2$	$\mathcal{R}_+$, $\mathcal{R}_-$
CII	-1	-1	1	$\mathbb{Z}$	$\mathcal{R}_{++}$, $\mathcal{R}_{--}$	0	$\cdots$
C	0	-1	0	0	$\cdots$	$\mathbb{Z}$	$\mathcal{R}_+$, $\mathcal{R}_-$
CI	1	-1	1	0	$\cdots$	0	$\cdots$

Floquet HOTI

η_T, -η_P, $-\eta_C$

Class	$\mathcal{T}$	$\mathcal{C}$	$\mathcal{S}$	$d=2$		$d=3$	
A	0	0	0	$\ldots$	0	$\mathcal{M}$	$\mathbb{Z}$
AIII	0	0	1	$\mathcal{M}_-$	$\mathbb{Z}$	$\ldots$	0
AI	1	0	0	$\ldots$	0	$\ldots$	0
BDI	1	1	1	$\mathcal{M}_{+-}$	$\mathbb{Z}$	$\ldots$	0
D	0	1	0	$\mathcal{M}_-$	$\mathbb{Z}_2$	$\mathcal{M}_-$	$\mathbb{Z}$
DIII	-1	1	1	$\mathcal{M}_{+-}$,$\mathcal{M}_{-+}$, $\mathcal{M}_{--}$	$\mathbb{Z}_2$	$\mathcal{M}_{+-}$,$\mathcal{M}_{--}$	$\mathbb{Z}_2$
AII	-1	0	0	$\ldots$	0	$\mathcal{M}_+$, $\mathcal{M}_-$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathcal{M}_{+-}$,$\mathcal{M}_{-+}$	$2\mathbb{Z}$	$\ldots$	0
C	0	-1	0	$\ldots$	0	$\mathcal{M}_-$,$\mathcal{M}_+$	$2\mathbb{Z}$
CI	1	-1	1	$\ldots$	0	$\ldots$	0

Summary and conclusions

- Periodic drives induce additional toplogical phases.

- Driven trivial systems give rise to new kinds of high-order TI's

- Pathways to realizing Floquet HOTI's
  using phonons and SO coupling.

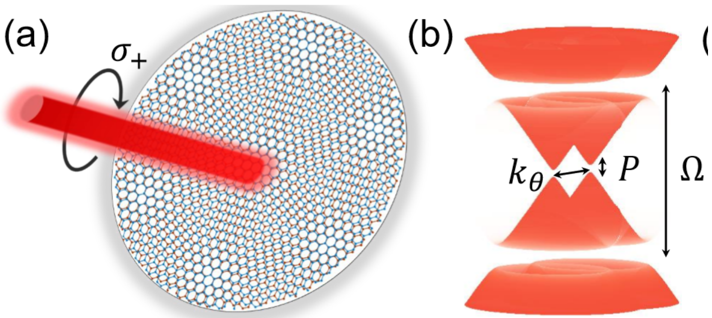
If there's time..

Dynamically induced band-flattening

Or Katz (Technion)
Netanel Lindner (Technion)
Gil Refael

Applications to Twisted Bilayer Graphene?

- "Flattening TBG with light": (Katz, Refael, Lindner, 2019)



(No interlayer coupling)

- Small fields needed!
  ~5MV/cm