Geometric Mechanics

Homework 7

Due on November 9

1. A **symmetry** of a rigid body is an isometry $S \in O(3)$ which preserves the mass distribution (i.e. $m(SA) = m(A)$ for any measurable set $A \subset \mathbb{R}^3$). Show that:
   (a) $SI = IS$, where I is the matrix representation of the inertia tensor;
   (b) if S is a reflection with respect to a plane then there exists a principal axis orthogonal to the plane;
   (c) if S is a nontrivial rotation about an axis then that axis is principal;
   (d) if moreover the rotation is not by π then all axes orthogonal to the rotation axis are principal.

2. Consider a rigid body with a fixed point in the absence of exterior forces (**Euler top**). Show that:
   (a) if $I_1 = I_2 = I_3$ then the rigid body rotates about a fixed axis with constant angular velocity;
   (b) if $I_1 = I_2 \neq I_3$ then there exist infinitely many principal inertia axes, but only the rotations about e_3 are stable;
   (c) if $I_1 > I_2 > I_3$ then for a given angular momentum the kinetic energy is minimized by rotations about e_1.
   (**Remark:** This is why most asteroids rotate about their minor axis: small perturbations due to other bodies in the Solar System conserve angular momentum but tend to dissipate energy).