

Algebraic and Geometric Methods in Engineering and Physics

Homework 8

Due on November 20

1. Consider the stiffness matrix

$$K = \begin{pmatrix} 2k_1 + k_2 + k_3 & -k_1 & -k_2 & -k_1 \\ -k_1 & 2k_1 + k_2 + k_3 & -k_1 & -k_2 \\ -k_2 & -k_1 & 2k_1 + k_2 + k_3 & -k_1 \\ -k_1 & -k_2 & -k_1 & 2k_1 + k_2 + k_3 \end{pmatrix}$$

and the mass matrix $M = mI$, where where k_1, k_2, k_3 and m are positive numbers.

(a) Show that these matrices define intertwiners for the representation $D_4 \overset{\psi}{\curvearrowright} \mathbb{C}^4$ of the group $D_4 \equiv \{e, r, r^2, r^3, s, sr, sr^2, sr^3\}$ defined by

$$\psi_r = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \psi_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

(b) Recall that $\psi \sim \varphi^{(1)} \oplus \varphi^{(3)} \oplus \varphi^{(5)}$, where the irreducible representations $\varphi^{(1)}, \varphi^{(3)}$ and $\varphi^{(5)}$ are defined by $\varphi_r^{(1)} = \varphi_s^{(1)} = 1, \varphi_r^{(3)} = -1, \varphi_s^{(3)} = 1$ and

$$\varphi_r^{(5)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \varphi_s^{(5)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Find the orthogonal projectors P_1, P_3 and P_5 on the invariant subspaces corresponding to these irreducible representations.

(c) Obtain the vibration frequencies of the structure described by K and M , that is, the values of ω such that $\omega^2 Mu = Ku$ for some nonvanishing vector u . Is the structure stable?