Algebraic and Geometric Methods in Engineering and Physics

Homework 5

Due on October 16

1. Let G be the symmetry group of a regular tetrahedron (that is, the group of isometries of Euclidean space that leave the tetrahedron invariant), and $H \subset G$ the rotation group of the tetrahedron (that is, the group of rotations of Euclidean space that leave the tetrahedron invariant).

   (a) Show that G is isomorphic to S_4.

   (b) Show that H is isomorphic to A_4.

   (**Hint:** Consider the action of G on the vertices of the tetrahedron, and recall that any isometry of the Euclidean space is the composition of a finite number of reflections).

2. How many different colorings of a tetrahedron are there, if each face can be painted with any one of n possible colors?