

Algebraic and Geometric Methods in Engineering and Physics

Homework 4

Due on October 9

1. Consider the action of $D_4 = \{e, r, r^2, r^3, s, sr, sr^2, sr^3\}$ on $R(4) = \{1, -1, i, -i\}$.

- (a) Determine whether this action is effective, transitive or free.
- (b) Find the isotropy group of i .

2. Consider the action $SO(3) \curvearrowright S^2$ defined by $\varphi_A(x) = Ax$, where S^2 is the set of unit vectors in \mathbb{R}^3 .

- (a) Show that

$$SO(3)_{(1,0,0)} = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} : \theta \in [0, 2\pi) \right\}.$$

- (b) Write the left coset $[A_\alpha]$ explicitly, where

$$A_\alpha = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- (c) Check explicitly that $T([A_\alpha])$ does not depend on the choice of representative of $[A_\alpha]$, where $T : SO(3)/SO(3)_{(1,0,0)} \rightarrow S^2$ is the natural equivalence between $SO(3) \curvearrowright S^2$ and the canonical action $SO(3) \overset{\varphi^{\text{can}}}{\curvearrowright} SO(3)/SO(3)_{(1,0,0)}$.