

Algebraic and Geometric Methods in Engineering and Physics

Homework 2

Due on September 25

1. Mark as true or false, and explain why:

- (a) \mathbb{Z}_n is cyclic for any $n \in \mathbb{N}$.
- (b) If $p = |G|$ is prime then G is isomorphic to \mathbb{Z}_p .
- (c) If all nontrivial subgroups of G are abelian then G is abelian.
- (d) If H is a normal subgroup of G then G/H is abelian.

2. Consider the **unitary group**

$$U_n = \{A \in GL_n(\mathbb{C}) : A^*A = I\},$$

and the **special unitary group**

$$SU_n = \{A \in U_n : \det A = 1\}.$$

Prove that SU_n is a normal subgroup of U_n , and also that $U_n/SU_n \cong S^1$.