Algebraic and Geometric Methods
in Engineering and Physics

Homework 10

Due on December 4

1. Consider the interval $M = (-1, 1)$ as a metric space with the usual distance function $d(x, y) = |x - y|$. Show that M is closed and bounded but it is not compact.

2. A topological space $(M, \mathcal{T})$ is said to be **path-connected** if for every $x, y \in M$ there exists a continuous function $c : [0, 1] \to M$ (a **path**) such that $c(0) = x$ and $c(1) = y$. Show that if $(M, \mathcal{T})$ is path-connected then it is connected.

3. Prove that the unit circle $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ is a 1-dimensional differentiable manifold.