

Algebraic and Geometric Methods in Engineering and Physics

Homework 1

Due on September 18

1. Which of the following sets of matrices are subgroups of the group $GL(n)$ of invertible $n \times n$ matrices?
 - (a) $SL(n) = \{A \in M_{n \times n}(\mathbb{R}) : \det A = 1\}$;
 - (b) $S(n) = \{A \in M_{n \times n}(\mathbb{R}) : A^t = A \text{ and } \det A \neq 0\}$;
 - (c) $O(n) = \{A \in M_{n \times n}(\mathbb{R}) : A^t A = I\}$.

2. Given a group (G, \cdot) , let us define the following relation: $x \sim y$ if and only if $y = gxg^{-1}$ for some $g \in G$.
 - (a) Prove that \sim is an equivalence relation.
 - (b) Compute the equivalence classes in the following two cases:
 - (i) $(G, \cdot) = (\mathbb{Z}, +)$;
 - (ii) $(G, \cdot) = (S_3, \circ)$.