Algebraic and Geometric Methods in Engineering and Physics

Homework 7

Due on November 15

1. Consider the standard representation of S_3 on $\mathbb{C}^3$ by permutation matrices. Recall that $W = \mathrm{span}\{(1,1,1)\}$ is an invariant suspace for this representation.

   (a) What is the irreducible representation $\varphi^{(1)}$ of S_3 obtained by restricting the standard representation to W?

   (b) Obtain an orthonormal basis $\{u_1, u_2\}$ for $W^\perp$ by applying the Gram-Schmidt process to the basis $\{(1,-1,0);(1,0,-1)\}$.

   (c) Find the irreducible representation $\varphi^{(2)}$ of S_3 obtained by restricting the standard representation to $W^\perp$, written in the basis $\{u_1, u_2\}$, and check that it is unitary.

   (d) Check that Schur's orthogonality relations hold between the functions $\varphi^{(1)}_{11}$, $\varphi^{(2)}_{11}$ and $\varphi^{(2)}_{12}$.