Algebraic and Geometric Methods in Engineering and Physics

Homework 5

Due on October 18

1. Let G be the symmetry group of a regular tetrahedron (that is, the group of isometries of Euclidean space that leave the tetrahedron invariant), and $H \subset G$ the rotation group of the tetrahedron (that is, the group of rotations of Euclidean space that leave the tetrahedron invariant).

   (a) Show that G is isomorphic to S_4.

   (b) Show that H is isomorphic to A_4.

   (**Hint:** Consider the action of G on the vertices of the tetrahedron, and recall that any isometry of the Euclidean space is the composition of a finite number of reflections).

2. Compute the sign of the permutation $\sigma \in S_{13}$ given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 3 & 10 & 2 & 9 & 4 & 7 & 12 & 5 & 13 & 1 & 11 & 6 & 8 \end{pmatrix}$$