

Algebraic and Geometric Methods in Engineering and Physics

Homework 2

Due on September 27

1. Show that:

(a) If G and H are groups then $G \times H$ with the binary operation

$$(g_1, h_1) \cdot (g_2, h_2) = (g_1 g_2, h_1 h_2)$$

is also a group.

(b) $\mathbb{Z}_2 \times \mathbb{Z}_2$ is not cyclic, and therefore is not isomorphic to \mathbb{Z}_4 .

(c) $\mathbb{Z}_2 \times \mathbb{Z}_2$ and \mathbb{Z}_4 are the only groups of order 4 (up to isomorphism).

(d) All groups of order ≤ 5 are abelian.

2. Consider the **unitary group**

$$U_n = \{A \in GL_n(\mathbb{C}) : A^* A = I\},$$

and the **special unitary group**

$$SU_n = \{A \in U_n : \det A = 1\}.$$

Prove that SU_n is a normal subgroup of U_n , and also that $U_n/SU_n \cong S^1$.