Algebraic and Geometric Methods
in Engineering and Physics

Homework 1

Due on September 20

1. Which of the following sets of matrices form groups under matrix multiplication? Provide either a short proof or a counter-example.

   (a) $M_{2\times 2}(\mathbb{Z})$;
   (b) $\{A \in M_{2\times 2}(\mathbb{Z}) \ : \ \det A \neq 0\}$;
   (c) $\{A \in M_{2\times 2}(\mathbb{Z}) \ : \ \det A = 1\}$.

2. Given a group $(G, \cdot)$, let us define the following relation: $x \sim y$ if and only if $y = gxg^{-1}$ for some $g \in G$.

   (a) Prove that $\sim$ is an equivalence relation.
   (b) Compute the equivalence classes in the following two cases:
      (i) $(G, \cdot) = (\mathbb{Z}, +)$;
      (ii) $(G, \cdot) = (S_3, \circ)$.