Differential Geometry of Curves and Surfaces

Homework 4

Due on October 11

1. Consider the set
$$M = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = e^{2z}\}$$
and the map $\mathbf{g} : (0, 2\pi) \times \mathbb{R} \to \mathbb{R}^3$ given by
$$\mathbf{g}(u, v) = (e^v \cos u, e^v \sin u, v).$$
   (a) Prove that M is a diferentiable manifold, and determine its dimension.
   (b) Show that $\mathbf{g}$ is a parameterization of M.
   (c) Find $T^{\perp}_{(1,0,0)}M$.

2. Because $\binom{3}{1} = \binom{3}{2} = 3$, it is possible to identify $\mathbb{R}^3$ both with $\Lambda^1\left(\mathbb{R}^3\right)$ and with $\Lambda^2\left(\mathbb{R}^3\right)$: if $\mathbf{v} \in \mathbb{R}^3$, we define
$$\omega_{\mathbf{v}} = v^1 dx + v^2 dy + v^3 dz$$
and
$$\Omega_{\mathbf{v}} = v^1 dy \wedge dz + v^2 dz \wedge dx + v^3 dx \wedge dy.$$
Show that:
   (a) $\omega_{\mathbf{v}}(\mathbf{w}) = \mathbf{v} \cdot \mathbf{w}$;
   (b) $\Omega_{\mathbf{u}}(\mathbf{v}, \mathbf{w}) = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$;
   (c) $\omega_{\mathbf{v}} \wedge \omega_{\mathbf{w}} = \Omega_{\mathbf{v}\times\mathbf{w}}$;
   (d) $\omega_{\mathbf{v}} \wedge \Omega_{\mathbf{w}} = (\mathbf{v} \cdot \mathbf{w})\, dx \wedge dy \wedge dz$.